167181

MUGENI AMY ZAWADI

STRATHMORE UNIVERSITY

PROBABILITY AND STATISTICS ASSIGNMENT 1

Probability and Statistics

Question 1.

a) $P(H) = 3P(T)$
$= P(T) = 1/4 \quad , P(H) = 3/4$

PMF of total number of heads
when P(TTT)
$X = 0, 1, 2, 3$
$P(X=0) = P(T\,T\,T)$
$\frac{1}{4} \times \frac{1}{4} \times \frac{1}{4} = \frac{1}{64}$
$P(X=1)$
$\frac{3}{4} \times \frac{1}{4} \times \frac{1}{4} \times = \frac{3}{64} \times 3 = \frac{9}{64}$
$P(X=2)$
$\frac{3}{4} \times \frac{3}{4} \times \frac{1}{4} = \frac{9}{64} \times 3 = \frac{27}{64}$
$P(X=3)$
$\frac{3}{4} \times \frac{3}{4} \times \frac{3}{4} = \frac{27}{64}$

PMF of At most 2 heads
$P(X \leq 2) = 1 - P(X=3)$
$\frac{3}{4} \times \frac{3}{4} \times \frac{3}{4} = \frac{27}{64}$
$\frac{64}{64} - \frac{27}{64} = \underline{\underline{\frac{37}{64}}}$

b) When $x = 0$, $Y = 4 \times (-5) = -20$
1 = 1 head, 3 tails $= 2 \times 1 + (-5) \times 3 = -13$
2 = 2 heads, 2 tails $= 2 \times 2 + (-5) \times 2 = -6$
3 = 3 heads, 1 tail $= 2 \times 3 + (-5) \times 1 = 1$
4 = 4 heads, 0 tails $= 4 \times 2 = 8$

By Binomial distribution:
$x=0, \; 1 \times 1 \times \left(\frac{1}{4}\right)^4 = \frac{1}{256}$
$x=1 \quad 4 \times \frac{3}{4} \times \left(\frac{1}{4}\right)^3 = \frac{12}{256}$
$x=2 \quad 6 \times \frac{9}{16} \times \left(\frac{1}{4}\right)^2 = \frac{54}{256}$
$x=3 \quad 4 \times \frac{27}{64} \times \left(\frac{1}{4}\right)^1 = \frac{108}{256}$
$x=4 = 1 \times \frac{81}{256} \times \left(\frac{1}{4}\right)^0 = \frac{81}{256}$

Assuming Y = score
Hence
$P(Y=-20) = 1/256$
$P(Y=-13) = 12/256$
$P(Y=-6) = 54/256$
$P(Y=1) = 108/256$
$P(Y=8) = 81/256$

Question 2

a) $f(t) = c(81-t^2)$, $0 < t < 9$

$= \int_0^9 c(81-t^2)$

$c\left(81t - \frac{t^3}{3} + c\right)$ when $t = 9, 0$

$= 81(9) - \frac{9(3)}{3} - 81(0) - \frac{0(3)}{3} = \underline{486}$

$486c = 1$

$c = \frac{1}{486}$

b) CDF

$f(x) = \begin{cases} \frac{1}{486}(81-t^2) & 0 < t < 9. \\ 0, \text{ otherwise} \end{cases}$

$\frac{1}{486}(81-t^2)$

$CDF = \int \frac{1}{486}(81-t^2)$

$= \frac{1}{486}\left[81x - \frac{x^3}{3}\right]$

$\begin{cases} \dfrac{81x - x^3/3}{486}, & 0 < t < 9. \\ 0, \text{ otherwise} \end{cases}$

Find $P(T > 3) = 1 - F(3)$

When $x = 3$

$\dfrac{81(3) - 3^3/3}{486} = 39/81$

$81/81 - 39/81 = 42/81 \quad \frac{14}{27}$

c) Find $P(T \geq 7 | T \geq 3)$

for $P(T \geq 7)$

$= 1 - F(7)$

When $x = 7$

$f(7) = \dfrac{81(7) - 7^3/3}{486} = 0.93105$

$1 - 0.91305$

$= 0.0685$

And $P(T \geq 3)$

$= 14/27$

With conditional probabilities.

$\dfrac{P(T \geq 7)}{P(T \geq 3)} \quad \dfrac{0.0685}{14/27}$

$= 0.1321$

d) The $P(T > 3) = 14/27$.

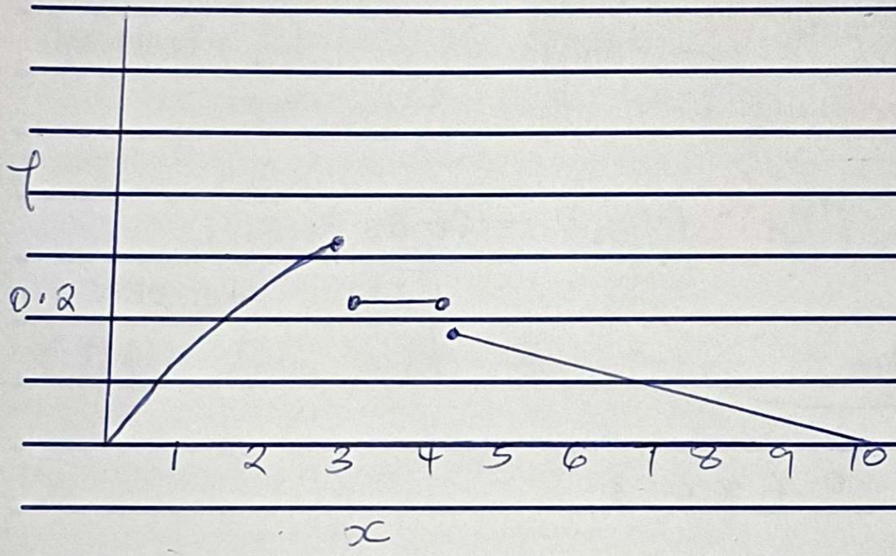
PMF of a Binomial distribution

$P(X = k) = \binom{n}{k} p^k (1-p)^{n-k}$

for Exactly 2 members
$k = 2$

$3C_2 (14/27)^2 (13/27)^1 = \frac{2548}{6561} = 0.388$

Question 3 Graph



0 1 2 3 4 5 6 7 8 9 10

x

b) Find CDF

$$\int_0^x x^3/45 = 1/45 \int_0^x x^2 = \left(\left[\frac{x^3}{3}\right]_0^x\right) 1/45 = \frac{x^3}{135}$$

$$\int_3^x 0.2 + \left(\frac{3^3}{135}\right) = \left[0.2x\right]_3^x \; (0.2x - 0.6) + 0.2 = 0.2x - 0.4$$

$$\int_4^x \frac{10-x}{30} + (0.2(4) - 0.4) = \frac{1}{30}\int_4^x 10 - x = \left[10x - \frac{x^2}{2}\right]_4^x \; \left(10x - \frac{x^2}{2}\right) - \left(10(4) - \left(\frac{4^2}{2}\right)\right)$$

$$\left(10x - \frac{x^2}{2}\right) - 32 + 0.4 = \left(10x - \frac{x^2}{2} - 32\right) \frac{1}{30} + 0.4$$

Hence CDF =

$$\begin{cases} \frac{x^3}{135}, & 0 < x < 3 \\ 0.2x - 0.4, & 3 \le x \le 4 \\ \frac{1}{30}\left(10x - \left(\frac{x^2}{2}\right) - 32\right) + 0.4, & 4 \le x \le 10 \\ 1, & x \ge 10 \\ 0, & \text{otherwise} \end{cases}$$

P(x<8) $P(X < 8)$

$$\frac{1}{30}\left(10(8) - \left(\frac{8^2}{2}\right) - 32\right) + 0.4$$

$$0.4 + \frac{16}{30}$$

$$= \frac{14}{15}$$

Question 4

$0.0001 = 1$ minute

$x \quad 20$ min

$= 1/500$

$\lambda = 1/500$

a) let x = no. of errors

Find $P(X=0)$

$$\frac{1/500^{0} e^{-1/500}}{0!}$$

$= 0.998$

b) $1 - P(X=0) = 0.9995$

$P(X=0) = 0.0005 = e^{-\lambda t}$

$-\lambda t = \ln(0.0005)$

but $\lambda = 0.0001$ $\quad \frac{\ln(0.005)}{0.0001}$ $\quad t = 76{,}009$ minutes

Question 5

a) $X \sim u(0.5, 4.0)$ $\quad fx \begin{cases} \frac{1}{3.5} & 0.5 < x < 4.0 \end{cases}$

$P(X>2) = 1 - P(\geq 2) = 1 - \left(\frac{2-0.5}{3.5}\right)$

$= 4/7$

b) $P(X>2 \mid X>1.5) = \frac{P(X>2)}{P(X>1.5)}$

$P(X>1.5) = 1 - P(X \leq 1.5) = 1 - f(1.5)$

$f(1.5) = \frac{1.5 - 0.5}{3.5} = 2/7$

$1 - 2/7 = 5/7$ $\quad = \frac{4/7}{5/7} = \underline{4/5}$

Question 6

$u = 10$ min $\quad \lambda = \frac{1}{u}$

$10 \text{ min} = \frac{1}{\lambda}$ $\quad \lambda = 1/10$

a) $f(x) = \lambda e^{-\lambda(x)}$

when $P(X < 15)$

$$\int_{15}^{\infty} 1/10\, e^{-1/10 t} = \left[e^{-1/10 t}\right]_{15}^{\infty}$$

$= 0.2231$

b) $P(T>15 \mid T>10) = P(T>5)$

$P(T>5) = 1 - F(5)$

$F(5) = 1 - e^{-1/10(5)}$

$= 0.6065$

Question 7

$n = 200$ and $P = 0.4$

$u = np$

$= 200 \times 0.4$

Mean $= \underline{80}$

Variance $(\sigma^2) = np(1-p)$

$= 200 \times 0.4 \times 0.6$

$= 48$

$S.D = \sqrt{\text{variance}}$

$\sqrt{48}$

$= 6.93$

for $P(X \geq 75) = P\left(K \geq \left(\frac{75-80}{6.93}\right)\right)$

$= P(K \geq -0.72$

$P(K \geq -0.72) = 1 - P(K < -0.72)$

$1 - 0.2358 = 0.7642$

$= \underline{0.7642}$

b) $P(x \leq 70)$

$= P\left(K \leq \left(\frac{70-80}{6.93}\right)\right)$

$P(K \leq -1.44)$

$= \underline{0.0749}$

c) $P(70 \leq x \leq 75)$

$= P(-1.44 \leq K -0.72)$

but

$P(-1.44 \leq K -0.72) = P(K \leq -0.72) - P(K \leq -1.44)$

$\left(\begin{matrix} P(K \leq -0.72) = 0.2358 \\ P(K \leq -1.44) = 0.0749 \end{matrix}\right) -$

$= \underline{0.1609}$