

Casey Levy

CS 225; Discrete Structures in CS

### Homework 7, Part 1

#### Set 9.2

##### #11c.

The first digit and last digit of a bit string can be chosen in only 1 way each, 2 ways total. So  $2^{8-2} = 2^6 = 64$

##### #14c.

Since there's only 1 way for a license plate to start with 1 certain letter, we have  $1 \times 1 \times 1 \times 1 \times 10 \times 10 \times 10 = 10^3 = 1000$

##### #14e.

There's only 1 way for a license plate to begin with A and then B, respectively, and then 24 ways for the 3<sup>rd</sup> letter and 23 ways for the 4<sup>th</sup> for the letters to be distinct. Also for the digits to be distinct, there's 10 ways for the 1<sup>st</sup>, 9 for the 2<sup>nd</sup> and 8 for the 3<sup>rd</sup>. That gives us  $1 \times 1 \times 24 \times 23 \times 10 \times 9 \times 8 = 397,440$

##### #17a.

To find how many integers between 1000 and 9999, we have only 9 ways for the first digit (since it can't be zero) and then 10 for the next 3 digits which gives us  $9 \times 10 \times 10 \times 10 = 9000$ .

##### b.

Similar to the above problem, we have 9000 possible integers but there are only 5 ways for any of them to be odd, which means only 5 ways for their last digit to be odd, instead of 10. That gives us  $9 \times 10 \times 10 \times 5 = 4500$ .

##### c.

For each digit to be distinct, we have 9 ways for the 1<sup>st</sup>, 9 for the 2<sup>nd</sup> (since it must be different from the 1<sup>st</sup> but can be zero), 8 for the 3<sup>rd</sup> and 7 for the 4<sup>th</sup> which gives us  $9 \times 9 \times 8 \times 7 = 4536$ .

##### d.

Since we need odd integers, the last number only has 5 ways, the 1<sup>st</sup> has 8 (can't be zero or the same as the last), the 2<sup>nd</sup> has 8 (different from first and last) and the 3<sup>rd</sup> has 7 which gives us  $5 \times 8 \times 8 \times 7 = 2240$ .

##### e.

We know we have 4536 possible distinct digit integers, therefore  $\frac{4536}{9000} = .504 = 50.4\%$

For distinct digits and is odd, we get  $\frac{2240}{9000} = .248 = 24.8\%$

### **Set 9.3**

#### **#5a.**

We first find that there are 90,000 integers between 10,000 and 99,999. Then we know there are only 2 ways an integer is divisible by 5, to have a 5 or 0 at the end. Using the addition rule we get  $9000 + 9000 = 18,000$ .

#### **#24a.**

We know there are 1000 integers from 1 to 1000.

A = integers from 1 through 1000 that are multiples of 2 = 500

B = integers from 1 through 1000 that are multiples of 9 = 111

There are 55 integers that are multiples of 18 ( $2 \times 9$ )

$$|A \cup B| = |A| + |B| - |A \cap B| = 500 + 111 - 55 = 556$$

#### **c.**

If we know there are 556 integers that are multiples of 2 or 9, then integers that are not a multiple of 2 or 9 gives us  $1000 - 556 = 444$ .

#### **33e.**

A = Students who checked #2 and #3 = 3

B = Students who checked #1, #2 and #3 = 2

$$|A \cup B| = |A| + |B| - |A \cap B| = 3 + 2 - 2 = 3$$

#### **f.**

A = Students who checked only #2 = 26

B = Students who checked any other statement except #2 = 74

$$|A \cup B| = |A| + |B| - |A \cap B| = 26 + 74 - 74 = 26.$$