

**CS 325**  
**Homework 6**

**Problem 1 (6 points)**

Shortest paths can be cast as an LP using distances  $d_v$  from the source  $s$  to a particular vertex  $v$  as variables.

- We can compute the shortest path from  $s$  to  $t$  in a weighted directed graph by solving.

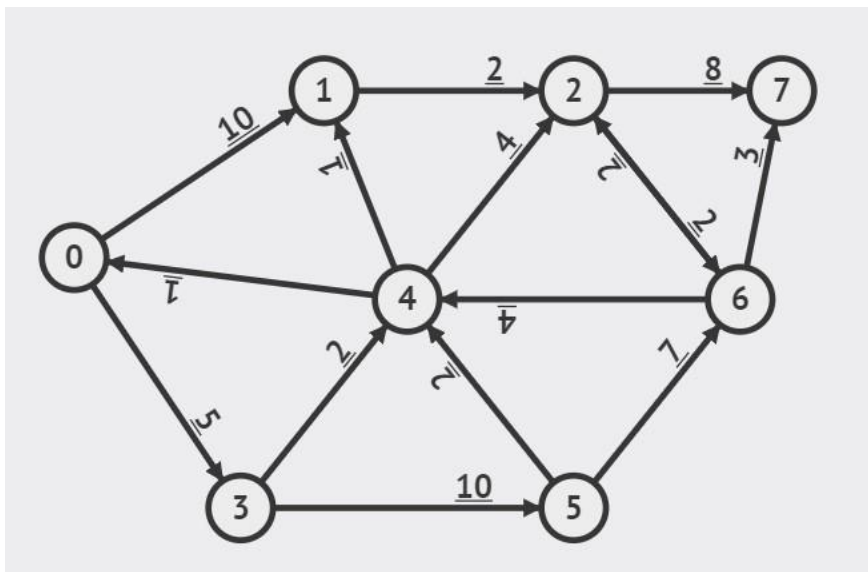
$$\begin{array}{ll}\max & d_t \\ \text{subject to} & \\ & d_s = 0 \\ & d_v - d_u \leq w(u,v) \text{ for all } (u,v) \in E\end{array}$$

- We can compute the single-source by changing the objective function to

$$\max \sum_{v \in V} d_v$$

Use linear programming to answer the questions below. State the objective function and constraints for each problem and include a copy of the LP code and output.

- Find the distance of the shortest path from vertex 0 to vertex 7 in the graph below.
- Find the distances of the shortest paths from vertex 0 to all other vertices.



**Problem 2 (6 points)**

Acme Industries produces four types of men's ties using three types of material. Your job is to determine how many of each type of tie to make each month. The goal is to maximize profit, profit per tie = selling price - labor cost – material cost. Labor cost is \$0.75 per tie for all four types of ties. The material requirements and costs are given below.

Material	Cost per yard	Yards available per month
Silk	\$20	1,000
Polyester	\$6	2,000
Cotton	\$9	1,250

Product Information	Type of Tie			
	Silk = s	Poly = p	Blend1 = b	Blend2 = c
Selling Price per tie	\$6.70	\$3.55	\$4.31	\$4.81
Monthly Minimum units	6,000	10,000	13,000	6,000
Monthly Maximum units	7,000	14,000	16,000	8,500

Material Information in yards	Type of Tie			
	Silk	Polyester	Blend 1 (50/50)	Blend 2 (30/70)
Silk	0.125	0	0	0
Polyester	0	0.08	0.05	0.03
Cotton	0	0	0.05	0.07

Formulate the problem as a linear program with an objective function and all constraints. Determine the optimal solution for the linear program using any software you want. Include a copy of the code and output. What are the optimal numbers of ties of each type to maximize profit?

### Problem 3 (12 points)

Veronica the owner of Very Veggie Vegeria is creating a new healthy salad that is low in calories but meets certain nutritional requirements. A salad is any combination of the following ingredients: Tomato, Lettuce, Spinach, Carrot, Smoked Tofu, Sunflower Seeds, Chickpeas, Oil.

Each salad must contain:

- At least 15 grams of protein
- At least 2 and at most 8 grams of fat
- At least 4 grams of carbohydrates
- At most 200 milligrams of sodium
- At least 40% leafy greens (lettuce and spinach) by mass.

The nutritional contents of these ingredients (per 100 grams) and cost are

<b>Ingredient</b>	<b>Energy (cal)</b>	<b>Protein (grams)</b>	<b>Fat (grams)</b>	<b>Carbohydrate (grams)</b>	<b>Sodium (mg)</b>	<b>Cost (100g)</b>
<b>Tomato</b>	21	0.85	0.33	4.64	9.00	\$1.00
<b>Lettuce</b>	16	1.62	0.20	2.37	28.00	\$0.75
<b>Spinach</b>	40	2.86	0.39	3.63	65.00	\$0.50
<b>Carrot</b>	41	0.93	0.24	9.58	69.00	\$0.50
<b>Sunflower Seeds</b>	585	23.4	48.7	15.00	3.80	\$0.45
<b>Smoked Tofu</b>	120	16.00	5.00	3.00	120.00	\$2.15
<b>Chickpeas</b>	164	9.00	2.6	27.0	78.00	\$0.95
<b>Oil</b>	884	0	100.00	0	0	\$2.00

**Part A:** Determine the combination of ingredients that minimizes calories but meets all nutritional requirements.

- i. Formulate the problem as a linear program with an objective function and all constraints.
- ii. Determine the optimal solution for the linear program using any software you want.  
Include a copy of the code/file in the report.
- iii. What is the cost of the low calorie salad?

**Part B:** Veronica realizes that it is also important to minimize the cost associated with the new salad. Unfortunately some of the ingredients can be expensive. Determine the combination of ingredients that minimizes cost.

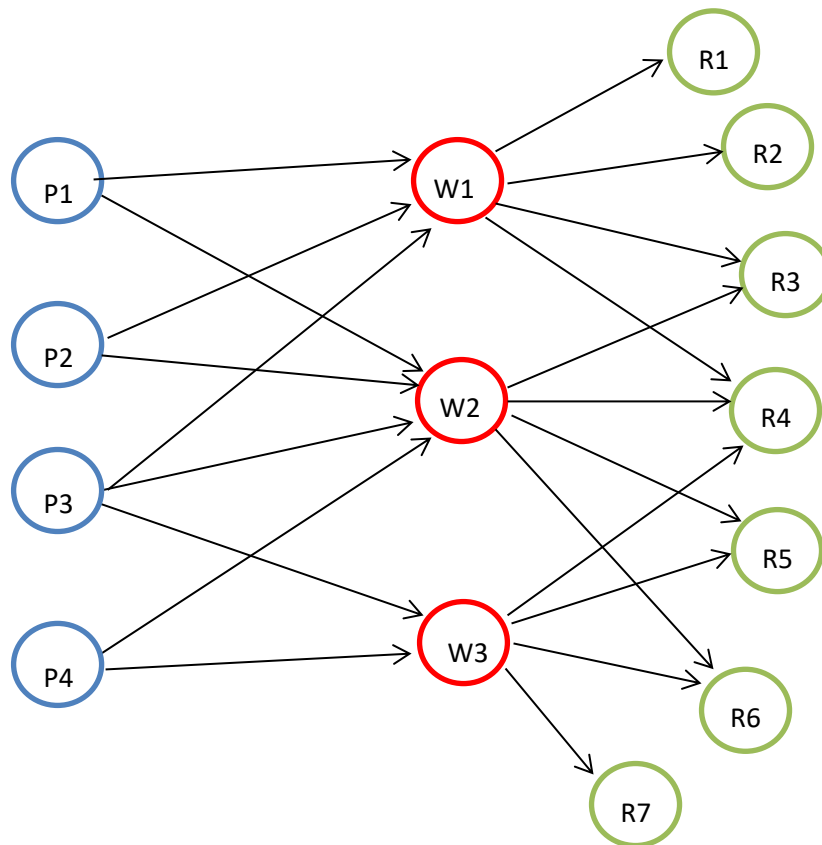
- i. Formulate the problem as a linear program with an objective function and all constraints.
- ii. Determine the optimal solution for the linear program using any software you want.  
Include a copy of the code/file in the report.
- iii. How many calories are in the low cost salad?

#### **Problem 4 (6 points)**

This is an extension of the transportation model (lecture notes: slides 52-55). There are now intermediate transshipment points added between the sources (plants) and destinations (retailers). Items being shipped from a Plant ( $p_i$ ) must be shipped to a Warehouse ( $w_j$ ) before being shipped to the Retailer ( $r_k$ ). Each Plant will have an associated supply ( $s_i$ ) and each Retailer will have a demand ( $d_k$ ). The number of plants is  $n$ , number of warehouses is  $q$  and the number of retailers is  $m$ . The edges  $(i,j)$  from plant ( $p_i$ ) to warehouse ( $w_j$ ) have costs associated denoted  $cp(i,j)$ . The edges  $(j,k)$  from a warehouse ( $w_j$ ) to a retailer ( $r_k$ ) have costs associated denoted  $cw(j,k)$ .

The graph below shows the transshipment map for a manufacturer of refrigerators.

Refrigerators are produced at four plants and then shipped to a warehouse (weekly) before going to the retailer.



Below are the costs of shipping from a plant to a warehouse and then a warehouse to a retailer.  
 If it is impossible to ship between the two locations an X is placed in the table.

cost	W1	W2	W3
P1	\$10	\$15	X
P2	\$11	\$8	X
P3	\$13	\$8	\$9
P4	X	\$14	\$8

cost	R1	R2	R3	R4	R5	R6	R7
W1	\$5	\$6	\$7	\$10	X	X	X
W2	X	X	\$12	\$8	\$10	\$14	X
W3	X	X	X	\$14	\$12	\$12	\$6

The tables below give the capacity of each plant (supply) and the demand for each retailer (per week).

	P1	P2	P3	P4
Supply	150	450	250	150

	R1	R2	R3	R4	R5	R6	R7
Demand	100	150	100	200	200	150	100

Determine the number of refrigerators to be shipped from the plants to the warehouses and then from the warehouses to retailers to minimize the cost. Formulate the problem as a linear program with an objective function and all constraints. Determine the optimal solution for the linear program using any software you want. What are the optimal shipping routes and minimum cost?

**Note:**

**You may solve the problems using your choice of software, State which software package(s)/language(s) you used, and provide the code or spreadsheet.**

**Upload one PDF file (only PDF) to Canvas. Please include the code in your pdf. If you use Excel, then submit your Excel file(s), too.**

**There is no submission to TEACH this week.**