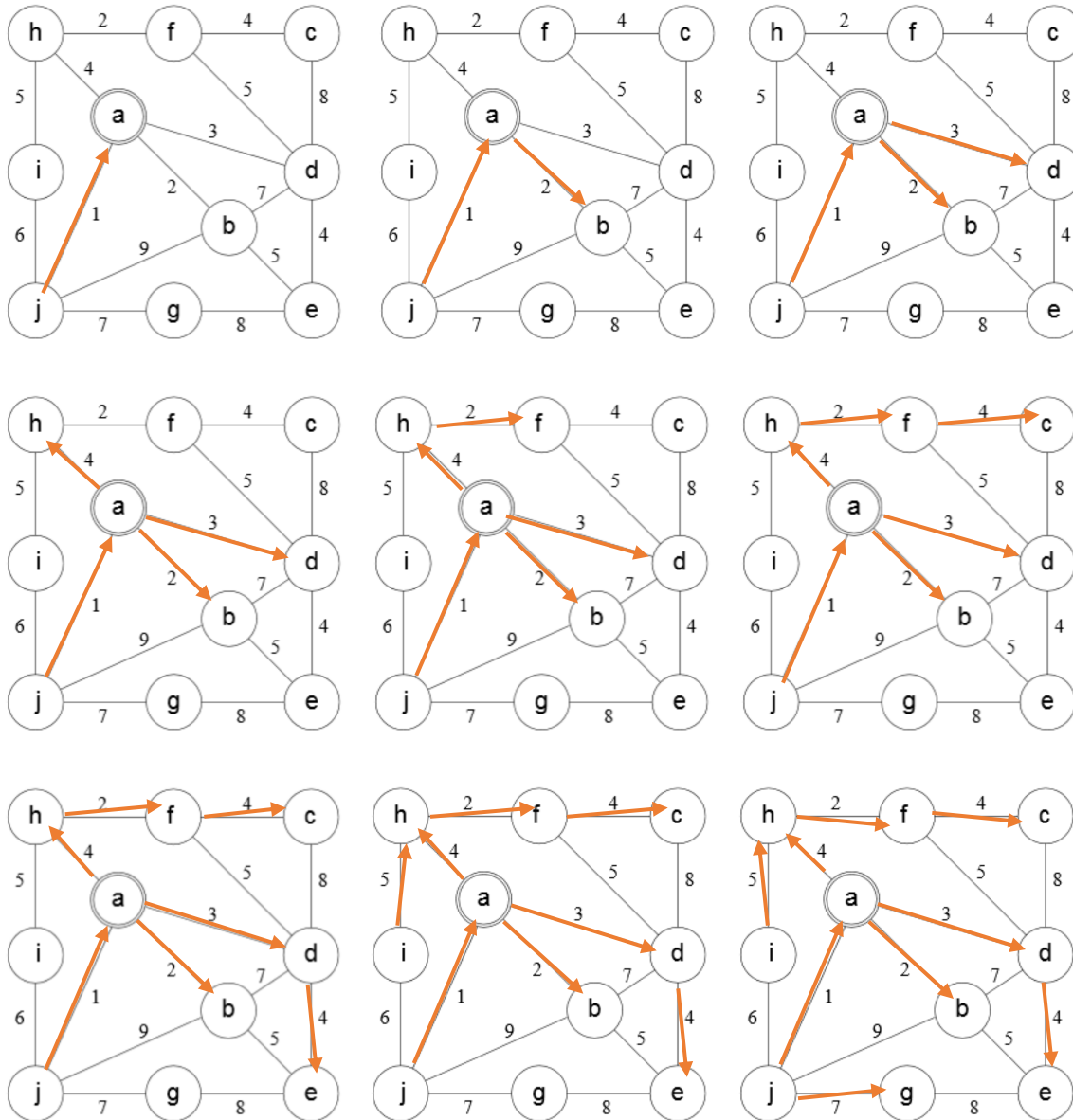


Casey Levy – CS 325 – HW 5

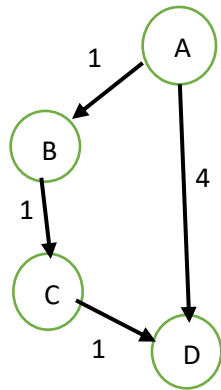
Problem 1



Order would be **j – a – b – d – h – f – c – e – h – g**

Total Weight: **32**

Problem 2



- a) MST before all edges are increased by 1 is **A – B – C – D**. The MST holds if we use Kruskal's since the lowest value edge is always added, no matter its connection. This proves the MST remains unchanged if each edge is +1.
- b) The shortest path would change if all edges are increased by one. If this happens, all connections between A, B, C, and D would be 2 instead of 1, making the direct path from A to D the shortest path since $5 < 6$.

Problem 3

- a) We could use a modified breadth-first search algorithm and modify it to skip over any edges that are less than W.

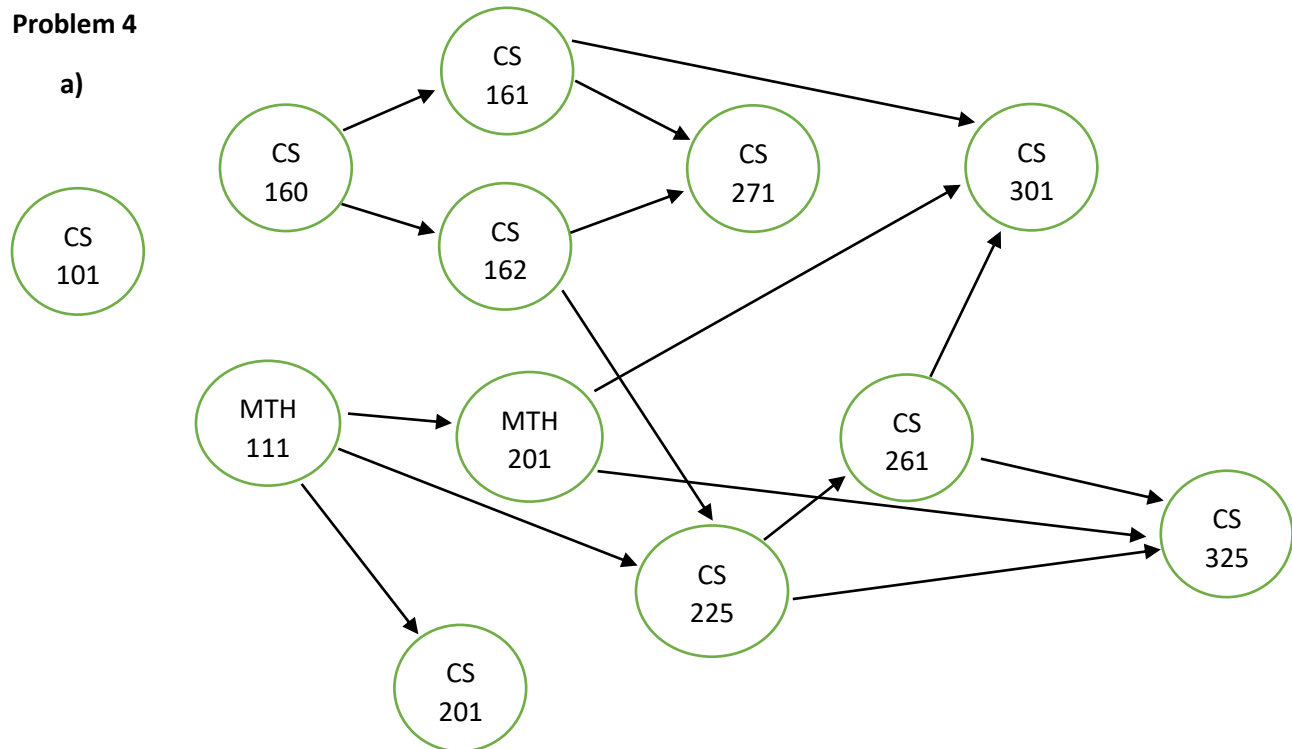
We could also use a modified Prim's Algorithm as well in a similar fashion. We can skip the min value extracted from Q if the value is $< W$.

Ex: If W is 7 and GetMin(Q) gives us 6, then nothing more is done but get the next min value in Q. Last step, we count the total number of E if $E = n - 1$ where n is the number of vertices. If it isn't, then no path is found.

- b) Runtime for BFS: **$O(V + E)$**
Runtime if using a binary heap: **$O(E \log V)$**

Problem 4

a)



b) Topological Sort: **CS 101, CS 160, CS 162, CS 271, MTH 111, CS 225, CS 201, MTH 201, CS 261, CS 301, CS 325**

c) Taking multiple courses, no pre-req conflict:

- **CS 101, MTH 111, CS 160**
- **MTH 201, CS 161, CS 162, CS 201**
- **CS 225, CS 271**
- **CS 261**
- **CS 301, CS 325**

d) Longest path: 4, 160 – 162 – 225 – 261 – 301.
This tells us that CS 301 has the most pre-requisites

Problem 5

a) A modified breadth-first search algorithm should be able to solve this problem. Below is the pseudocode taken from our textbook. “Babyfaces” would be assigned at lines 15/16 if **v.d** is even. If it is odd, then “Heels” would be assigned instead. If both *u* and *v* are on the same team, then “impossible” would be returned.

BFS(G, s)

```
1  for each vertex  $u \in G.V - \{s\}$ 
2       $u.color = \text{WHITE}$ 
3       $u.d = \infty$ 
4       $u.\pi = \text{NIL}$ 
5   $s.color = \text{GRAY}$ 
6   $s.d = 0$ 
7   $s.\pi = \text{NIL}$ 
8   $Q = \emptyset$ 
9  ENQUEUE( $Q, s$ )
10 while  $Q \neq \emptyset$ 
11      $u = \text{DEQUEUE}(Q)$ 
12     for each  $v \in G.Adj[u]$ 
13         if  $v.color == \text{WHITE}$ 
14              $v.color = \text{GRAY}$ 
15              $v.d = u.d + 1$ 
16              $v.\pi = u$ 
17             ENQUEUE( $Q, v$ )
18      $u.color = \text{BLACK}$ 
```

b) Runtime: **$O(V + E)$**