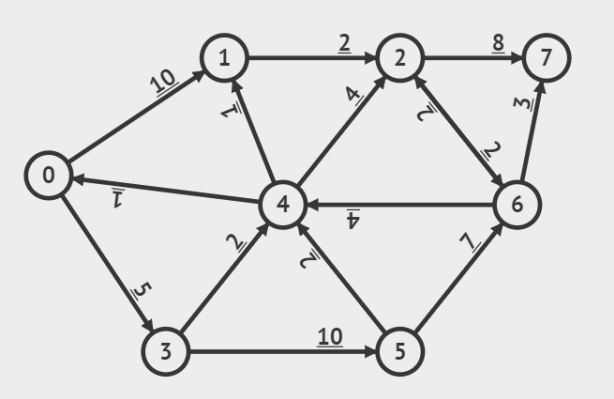
CS 325 HW 6 – Solutions

1. (6 points)



Use linear programming to answer the questions below. State the objective function and constraints for each problem and include a copy of the LP code and output.

a) Find the distance of the shortest path from vertex 0 to vertex 7 in the graph below.

```
max d7
ST
        d0 = 0
        d1 - d0 <= 10
        d3 - d0 <= 5
        d2 - d1 <= 2
        d7 - 2 <= 8
        d7 - d6 <= 3
        d6 - d2 <= 2
        d2 - d6 <= 2
        d4 - d6 <= 4
        d2 - d4 <= 4
        d0 - d4 <= 1
        d5 - d3 <= 10
        d6 - d5 <= 7
        d4 - d5 <= 2
        d4 - d3 <= 2
        d1 - d4 <= 1
```

```
LP OPTIMUM FOUND AT STEP      8

        OBJECTIVE FUNCTION VALUE

        1)      15.00000

 VARIABLE        VALUE          REDUCED COST
       D7       15.000000          0.000000
       D0        0.000000          0.000000
       D1        8.000000          0.000000
       D2       10.000000          0.000000
       D6       12.000000          0.000000
       D4        7.000000          0.000000
       D3        5.000000          0.000000
       D5        5.000000          0.000000

      ROW    SLACK OR SURPLUS     DUAL PRICES
        2)         0.000000          1.000000
        3)         2.000000          0.000000
        4)         0.000000          1.000000
        5)         3.000000          0.000000
        6)         0.000000          1.000000
        7)         0.000000          1.000000
        8)         4.000000          0.000000
        9)         9.000000          0.000000
       10)         1.000000          0.000000
       11)         8.000000          0.000000
       12)         0.000000          1.000000
       13)         0.000000          1.000000
       14)        10.000000          0.000000
       15)         0.000000          0.000000
       16)         0.000000          0.000000
       17)         0.000000          1.000000
```

The distance of the shortest path from 0 to 7 is 15.

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b) Find the distances of the shortest paths from vertex 0 to all other vertices.

```
max d1 + d2 + d3 + d4 + d5 + d6 + d7
ST
        d0 = 0
        d1 - d0 <= 10
        d3 - d0 <= 5
        d2 - d1 <= 2
        d7 - 2 <= 8
        d7 - d6 <= 3
        d6 - d2 <= 2
        d2 - d6 <= 2
        d4 - d6 <= 4
        d2 - d4 <= 4
        d0 - d4 <= 1
        d5 - d3 <= 10
        d6 - d5 <= 7
        d4 - d5 <= 2
        d4 - d3 <= 2
        d1 - d4 <= 1
```

```
        1)        72.00000

  VARIABLE        VALUE          REDUCED COST
        D1         8.000000          0.000000
        D2        10.000000          0.000000
        D3         5.000000          0.000000
        D4         7.000000          0.000000
        D5        15.000000          0.000000
        D6        12.000000          0.000000
        D7        15.000000          0.000000
        D0         0.000000          0.000000
```

V0 to	V1	V2	V3	V4	V5	V6	V7
min dist	8	10	5	7	15	12	15

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2. (6 points)

Formulate the problem as a linear program with an objective function and all constraints. (2 points)

Max 3.45s + 2.32p + 2.81b + 3.25c

ST 0.125s <= 1000 : silk

0.08p + 0.05b + 0.03c <= 2000 : poly

0.05b + 0.07c <= 1250 :cotton

S >= 6000 ; S <= 7000

P >= 10,000 ; p <= 14,000

B >= 13,000; b <= 16000

C >= 6000; c <= 8500

Determine the optimal solution for the linear program using any software you want. Include a copy of the code and output. (2 points)

```
Max 3.45s + 2.32p + 2.81b + 3.25c
ST      0.125s <= 1000
        0.08p + 0.05b + 0.03c <= 2000
        0.05b + 0.07c <= 1250
s >= 6000
s <= 7000
p >= 10000
p <= 14000
b >= 13000
b <= 16000
c >= 6000
c <= 8500
```

```
LP OPTIMUM FOUND AT STEP      4

       OBJECTIVE FUNCTION VALUE

       1)      120196.0

  VARIABLE        VALUE          REDUCED COST
         S      7000.000000          0.000000
         P     13625.000000          0.000000
         B     13100.000000          0.000000
         C      8500.000000          0.000000

       ROW   SLACK OR SURPLUS     DUAL PRICES
         2)       125.000000          0.000000
         3)         0.000000         29.000000
         4)         0.000000         27.200001
         5)      1000.000000          0.000000
         6)         0.000000          3.450000
         7)      3625.000000          0.000000
         8)       375.000000          0.000000
         9)       100.000000          0.000000
        10)      2900.000000          0.000000
        11)      2500.000000          0.000000
        12)         0.000000          0.476000

NO. ITERATIONS=       4
```

Maximum profit is $120,196 from producing 7000 silk ties, 13625 polyester ties, 13,100 blend1 and 8,500 blend 2. (2 points)

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3. (12 points) **Part A**: Determine the combination of ingredients that minimizes calories but meets all nutritional requirements.

i. Formulate the problem as a linear program with an objective function and all constraints.

C:\Windows\system32\prob2a.ltx

```
MIN 21 F1 + 16 F2 + 40 F3 + 41 F4 + 585 F5 + 120 F6 + 164 F7 + 884 F8
ST
     .85 F1 + 1.62 F2 + 2.86 F3 + .93 F4 + 23.4 F5 + 16 F6 + 9 F7 > 15
     .33 F1 + .2F2 + .39 F3 + .24 F4 + 48.7 F5 + 5 F6 + 2.6 F7 + 100 F8 > 2
     .33 F1 + .2F2 + .39 F3 + .24 F4 + 48.7 F5 + 5 F6 + 2.6 F7 + 100 F8 < 8
     4.64 F1 + 2.37 F2 + 3.63 F3 + 9.58 F4 + 15 F5 + 3  6 + 27 F7 > 4
     9 F1 + 28 F2 + 65 F3 + 69 F4 + 3.8 F5 + 120 F6 + 7  F7 < 200
     F2 + F3 - .4 F1 - .4 F2 - .4 F3 - .4 F4 -    .4 F5    .4 F6 - .4 F7 - .4 F8  > 0
     F1 > 0
     F2 > 0
     F3 > 0
     F4 > 0
     F5 > 0
     F6 > 0
     F7 > 0
     F8 > 0
END
```

ii. Determine the optimal solution for the linear program using any software you want. Include a copy of the code. **114.75 calories**

Lettuce	.585480	58.548 g
Smoked Tofu	.878220	87.822 g

Reports Window

```
 LP OPTIMUM FOUND AT STEP      12

        OBJECTIVE FUNCTION VALUE

        1)       114.7541

  VARIABLE        VALUE          REDUCED COST
        F1         0.000000         16.901640
        F2         0.585480          0.000000
        F3         0.000000         14.513662
        F4         0.000000         36.289616
        F5         0.000000        408.387970
        F6         0.878220          0.000000
        F7         0.000000         97.551910
        F8         0.000000        886.404358
```

iii. What is the cost of the low calorie salad? **The total cost is $2.33.**

	Servings	Cost	Cost/Item
Lettuce	0.58548	0.75	0.43911
Smoked Tofu	0.87822	2.15	1.888173
Total			**$2.33**

Part B: Determine the combination of ingredients that minimizes cost.

i. Formulate the problem as a linear program with an objective function and all constraints.

```
C:\Windows\system32\prob2a.ltx
MIN   F1 + .75 F2 + .5 F3 + .5 F4 + .45 F5 + 2.15 F6 + .95 F7 + 2 F8
ST
     .85 F1 + 1.62 F2 + 2.86 F3 + .93 F4 + 23.4 F5 + 16 F6 + 9 F7 > 15
     .33 F1 + .2F2 + .39 F3 + .24 F4 + 48.7 F5 + 5 F6 + 2.6 F7 + 100 F8 > 2
     .33 F1 + .2F2 + .39 F3 + .24 F4 + 48.7 F5 + 5 F6 + 2.6 F7 + 100 F8 < 8
     4.64 F1 + 2.37 F2 + 3.63 F3 + 9.58 F4 + 15 F5 + 3 F6 + 27 F7 > 4
     9 F1 + 28 F2 + 65 F3 + 69 F4 + 3.8 F5 + 120 F6 + 78 F7 < 200
     F2 + F3 - .4 F1 - .4 F2 - .4 F3 - .4 F4 - .4 F5 - .4 F6 - .4 F7 - .4 F8 >0
     F1 > 0
     F2 > 0
     F3 > 0
     F4 > 0
     F5 > 0
     F6 > 0
     F7 > 0
     F8 > 0
END
```

ii. Determine the optimal solution. **$1.55**

Food	Servings	Grams
Spinach	.832298	83.2298
Sunflower Seeds	.096083	9.6083
Chick Peas	1.152364	115.2364

```
Reports Window

LP OPTIMUM FOUND AT STEP      10

        OBJECTIVE FUNCTION VALUE

        1)       1.554133

 VARIABLE        VALUE          REDUCED COST
       F1         0.000000          1.002081
       F2         0.000000          0.402912
       F3         0.832298          0.000000
       F4         0.000000          0.486914
       F5         0.096083          0.000000
       F6         0.000000          0.405609
       F7         1.152364          0.000000
       F8         0.000000          7.281258

      ROW    SLACK OR SURPLUS      DUAL PRICES
        2)         0.000000          -0.131261
        3)         6.000000           0.000000
        4)         0.000000           0.051847
        5)        31.576324           0.000000
        6)        55.651089           0.000000
        7)         0.000000          -0.241358
        8)         0.000000           0.000000
        9)         0.000000           0.000000
       10)         0.832298           0.000000
       11)         0.000000           0.000000
       12)         0.096083           0.000000
       13)         0.000000           0.000000
       14)         1.152364           0.000000
       15)         0.000000           0.000000

NO. ITERATIONS=      10
```

iii. How many calories are in the low cost salad? **278.5**

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4. (6 points)

min 10pw11 + 15pw12 + 11pw21 + 8pw22 + 13pw31 + 8pw32 + 9pw33 +
14pw42 + 8pw43 + 5wr11 + 6wr12 + 7wr13 + 10wr14 + 12wr23 + 8wr24 +
10wr25 + 14wr26 + 14wr34 + 12wr35 + 12wr36 + 6wr37

The number of refrigerators shipped on a route cannot be negative, so all variables must be greater than or equal to 0, resulting in all of the non-negativity constraints.

The amount of refrigerators shipped out of a plant must be less than or equal to that plants initial supply. This leads to the following constraints:

pw11 + pw12 <= 150
pw21 + pw22 <= 450
pw31 + pw32 + pw33 <= 250
pw42 + pw43 <= 150

The amount of refrigerators shipped into a retailer must be as least as great as that retailers demand. This results in the following constraints:

wr11 >= 100
wr12 >= 150
wr13 + wr23 >= 100
wr14 + wr24 + wr34 >= 200
wr25 + wr35 >= 200
wr26 + wr36 >= 150
wr37 >= 100

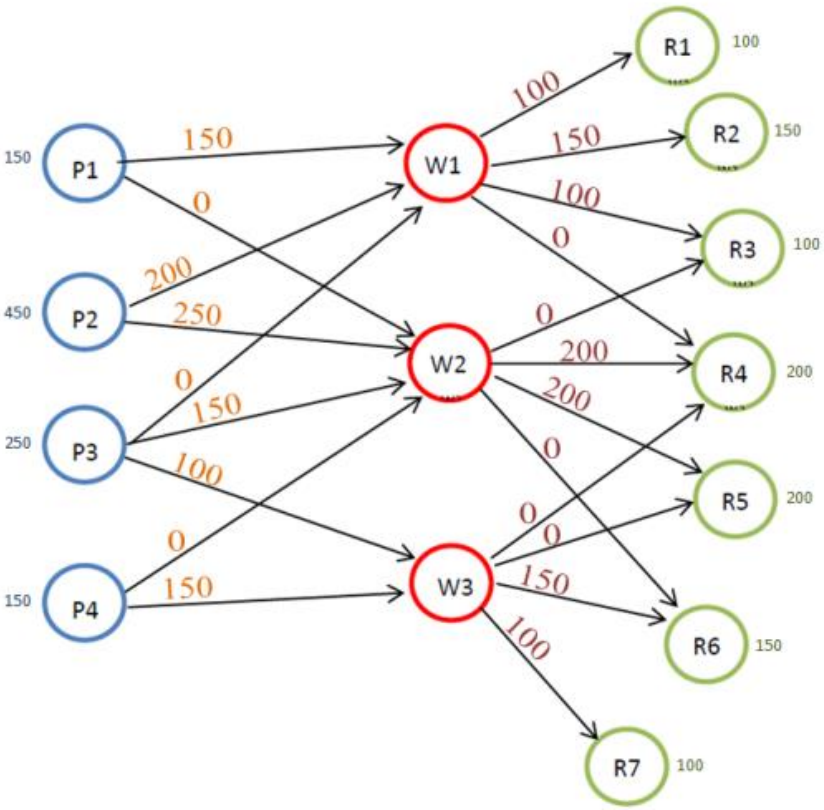
Each warehouse must have less or the same amount of refrigerators being shipped out than being shipped in. This leads to the following constraints:

pw11 + pw21 + pw31 - wr11 - wr12 - wr13 - wr14 >= 0
pw12 + pw22 + pw32 + pw42 - wr23 - wr24 - wr25 - wr26 >= 0
pw33 + pw43 - wr34 - wr35 - wr36 - wr37 >= 0

Include a copy of the code and output.

What are the optimal shipping routes and minimum cost. **Minimal value is $17,100**

The amount shipped is shown on the graph below


150
P1
450
P2
250
P3
150
P4
W1
W2
W3
R1
100
R2
150
R3
100
R4
200
R5
200
R6
150
R7
100
150
0
200
250
0
150
100
0
150
100
150
0
0
0
200
200
0
0
0
150
100