

Greedy Practice - Sample Solutions

Problem 1: The Waiter's Dilemma

Alice and Bob are the only two waiters in the Acme Restaurant. Today, the restaurant received N orders. The amount of the tip for each order may differ when handled by different waiters, if Alice takes the i th order, she would be tipped A_i dollars and if Bob takes this order, the tip would be B_i dollars.

In order to maximize the total tips for the night they decided to distribute the orders among themselves. Each order will be handled by only one person and due to time constraints Alice cannot take more than X orders and Bob cannot take more than Y orders. It is guaranteed that $X + Y$ is greater than or equal to N , which means that all the orders can be handled by either Alice or Bob. Find out the maximum possible amount of total tip money after processing all the orders.

Example 1: Input $N = 5, X = 3, Y = 3$

A_i	1	2	3	4	5
B_i	5	4	3	2	1

Output: 21

Example 2: Input $N = 8, X = 4, Y = 4$

A_i	1	4	3	2	7	5	9	6
B_i	1	2	3	6	5	4	9	8

Output: 43

a) Give a verbal description and detailed pseudocode for an algorithm to find the maximum total tips.

We will use greedy approach to get the maximum tip earned by the two waiters.

Following steps will help in solving this problem:

1. Sort the absolute difference of the value of A_i and B_i ($|A_i - B_i|$) in decreasing order.
2. Compare the A_i and B_i and add the greater value to the sum.
3. If $A_i \geq B_i$ Increment the total orders taken by Alice otherwise increment the number of orders taken by Bob.

3a) If the number of orders given to Alice is X then give all remaining order to Bob and go to step 5

3b) If the number of orders given to Bob is Y then give all remaining orders to Alice and go to step 5

4. If i equals to N goto step 5 else goto step 2.
5. Return sum.

b) What is the running time of your algorithm?

Time to sort $\Theta(n \lg n)$

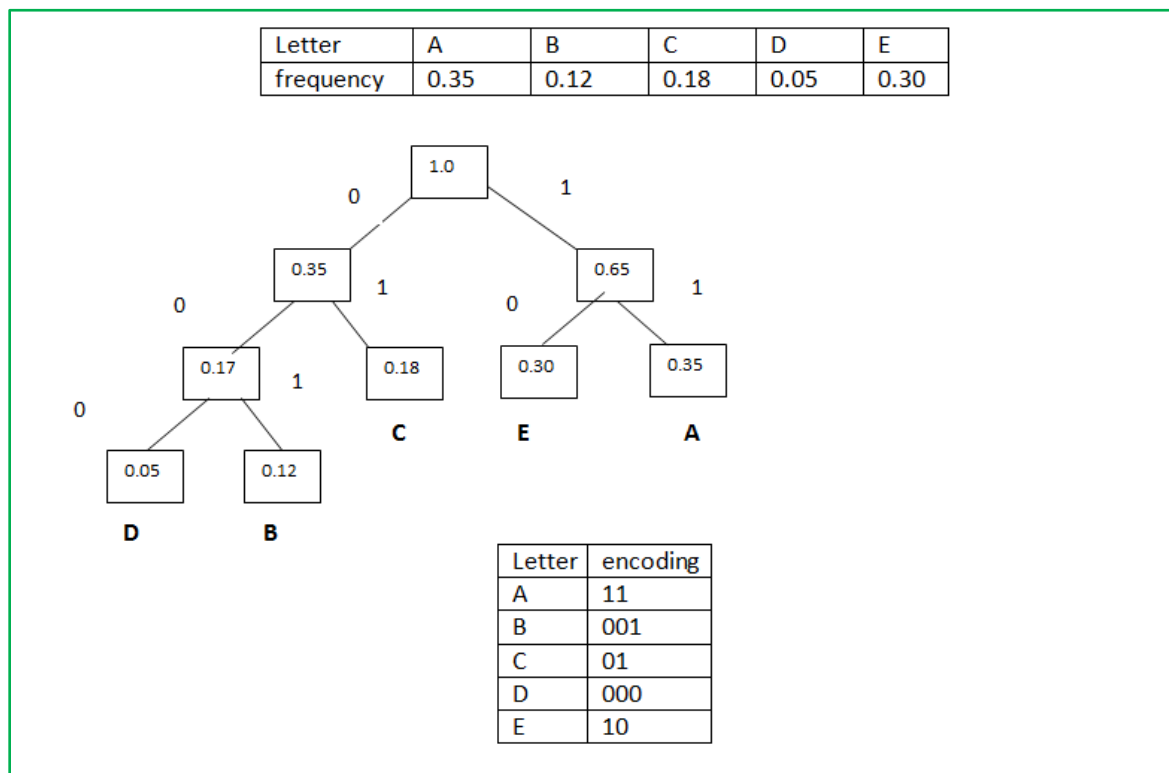
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Problem 2: Suppose we have an alphabet with only five letters A, B, C, D, E which occur with the following frequencies.

Letter	A	B	C	D	E
frequency	0.35	0.12	0.18	0.05	0.30

Use Huffman coding to find the optimal prefix-free variable-length binary encoding of the alphabet.

(a) Draw a binary tree that represents the optimal encoding.



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Problem 3: Work Schedules

You are the manager in a firm where the length of working time and the start time of work is different for different employees. For example, person X works every day from 8 to 11 AM, person Y from 9 AM to 1 PM, person Z from 2 to 10 PM etc. Your task is to create a workforce consisting of maximum number of people possible with non-overlapping work hours (if a person leaves at 7 AM and another starts at 7 AM, that is not an overlap).

You use the following greedy strategy:

- a) Choose the person (say X) with least number of working hour overlaps with other workers.
- b) Eliminate all people having working hour overlaps with X.
- c) Choose the next person with the least number of overlaps with the remaining people, and so on.

Does this strategy yield a workforce with the maximum number of people? If yes, prove it. Otherwise, just give a counterexample.

Solution:

No, the given greedy algorithm is not always optimal.

Here is a counter example: P1: 9 – 11, P2: 10 – 12, P3: 10 – 12, P4: 10 – 12, P5: 11 – 13, P6: 12.5 – 13.5, P7: 13 – 15, P8: 14 – 16, P9: 14 – 16, P10: 14 – 16, P11: 15 – 17.

A greedy algorithm will select P1, P6 and P8. While the optimal solution selects P1, P5, P7 and P11.

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Problem 3: