CS 325

Homework 7

Problem 1 (6 points)

Let X and Y be two decision problems. Suppose we know that X reduces to Y in polynomial time. Which of the following can we infer? Give a YES/NO answer.

a. If Y is NP-complete then so is X.

b. If X is NP-complete then so is Y.

c. If Y is NP-complete and X is in NP then X is NP-complete.

d. If X is NP-complete and Y is in NP then Y is NP-complete.

e. If X is in P, then Y is in P.

f. If Y is in P, then X is in P.

Problem 2 (4 points)

Consider the problem COMPOSITE: given an integer y, does y have any factors other than one and itself? For this exercise, you may assume that COMPOSITE is in NP, and you will be comparing it to the well-known NP-complete problem SUBSET-SUM: given a set S of n integers and an integer target t, is there a subset of S whose sum is exactly t? Clearly explain whether or not each of the following statements follows from that fact that COMPOSITE is in NP and SUBSET-SUM is NP-complete:

a. SUBSET-SUM $\leq_p$ COMPOSITE.

b. If there is an $O(n^3)$ algorithm for SUBSET-SUM, then there is a polynomial time algorithm for COMPOSITE.

c. If there is a polynomial algorithm for COMPOSITE, then P = NP.

d. If P $\neq$ NP, then no problem in NP can be solved in polynomial time.

Problem 3 (8 points)

A Hamiltonian path in a graph is a simple path that visits every vertex exactly once. Prove that HAM-PATH = { (G, u, v): there is a Hamiltonian path from u to v in G } is NP-complete. You may use the fact that HAM-CYCLE is NP-complete.

Problem 4 (12 points)

K-COLOR. Given a graph $G = (V,E)$, a k-coloring is a function $c: V \to \{1, 2, \ldots, k\}$ such that $c(u) \neq c(v)$ for every edge $(u,v) \in E$. In other words the number 1, 2, ..., k represent the k colors and adjacent vertices must have different colors. The decision problem K-COLOR asks if a graph can be colored with at most K colors.

a. The 2-COLOR decision problem is in P. Describe an efficient algorithm to determine if a graph has a 2-coloring. What is the running time of your algorithm?

b. It is known that the 3-COLOR decision problem is NP-complete by using a reduction from SAT. Use the fact that 3-COLOR is NP-complete to prove that 4-COLOR is NP-complete.