Casey Levy - CS 325 - HW 1

Problem 1

- $\frac{2}{N} < 37 < \sqrt{N} < N < N \log(\log N) < N \log N \le N \log(N^2) < N \log^2 N < N^{1.5} < N^2 < N^2 \log N < N^3 < N^{N/2} < 2^N$

- **N log N** and **N log(N²)** are the only functions that grow at the same rate
  - $N \log(N^2) = 2N \log N = O(N \log N)$

Problem 2

- claim that $N \log N < N^{(1+\frac{\mathcal{E}}{\sqrt{\log N}})}$

$N^{(1+\frac{\mathcal{E}}{\sqrt{\log N}})} = N * N^{\frac{\mathcal{E}}{\sqrt{\log N}}}$

$N * N^{\frac{\mathcal{E}}{\sqrt{\log N}}} < N \log N$

$N^{\frac{\mathcal{E}}{\sqrt{\log N}}} < \log N$

$\frac{\mathcal{E}}{\sqrt{\log N}} \log N < \log \log N$

Simplified gives us $\frac{\mathcal{E}}{\sqrt{\log N}}$ which then shows us that $\boldsymbol{\frac{\mathcal{E}}{\sqrt{\log N}} < \log \log N}$

Problem 3

a)
```
sum = 0;                  O(1)
   for( i = 0; i < n; ++i )   O(n)
      ++sum;                  O(1)
```

O(1) + O(n) + O(1)
Fragment runs in O(n)

b)
```
sum = 0;                      O(1)
   for(i = 0; i < n; ++i)        O(n)
      for(j = 0; j < n; ++j)     O(n)
         ++sum;                  O(1)
```

O(1) + O(n * n) + O(1)
Fragment runs in O(n²)

c)	`sum = 0;`	**$O(1)$**
	`for(i = 0; i < n; ++i)`	**$O(n)$**
	`    for(j = 0; j < n*n; ++j)`	**$O(n^2)$**
	`        ++sum;`	**$O(1)$**

$O(1) + O(n * n^2) + O(1)$
Fragment runs in $O(n^3)$

d)	`sum = 0;`	**$O(1)$**
	`    for( i = 0; i < n; ++i )`	**$O(n)$**
	`        for(j = 0; j < i; ++j )`	**$O(n)$**
	`            ++sum;`	**$O(1)$**

$O(1) + O(n * n) + O(1)$
Fragment runs in $O(n^2)$

e)	`sum = 0;`	**$O(1)$**
	`for( i = 0; i < n; ++i )`	**$O(n)$**
	`    for( j = 0; j < i * i; ++j )`	**$O(n^2)$**
	`        for( k = 0; k < j; ++k )`	**$O(n^2)$**
	`            ++sum;`	**$O(1)$**

$O(1) + O(n * n^2 * n^2) + O(1)$
Fragment runs in $O(n^5)$

f)	`sum = 0;`	**$O(1)$**
	`for(i = 1; i < n; ++i)`	**$O(n)$**
	`    for(j = 1; j < i*i; ++j)`	**$O(n^2)$**
	`        if(j % i == 0)`	**$O(1)$**
	`            for(k = 0; k < j; ++k)`	**$O(n^2)$**
	`                ++sum;`	**$O(1)$**

$O(1) + O(n * n^2) + O(1) + O(n^2) + O(1)$
Fragment runs in $O(n^4)$

Problem 5

b and c)

n	t (sec)
5,000	2.134
6,000	2.96
7,000	3.863
8,000	5.052
9,000	6.315
10,000	7.908
11,000	9.392

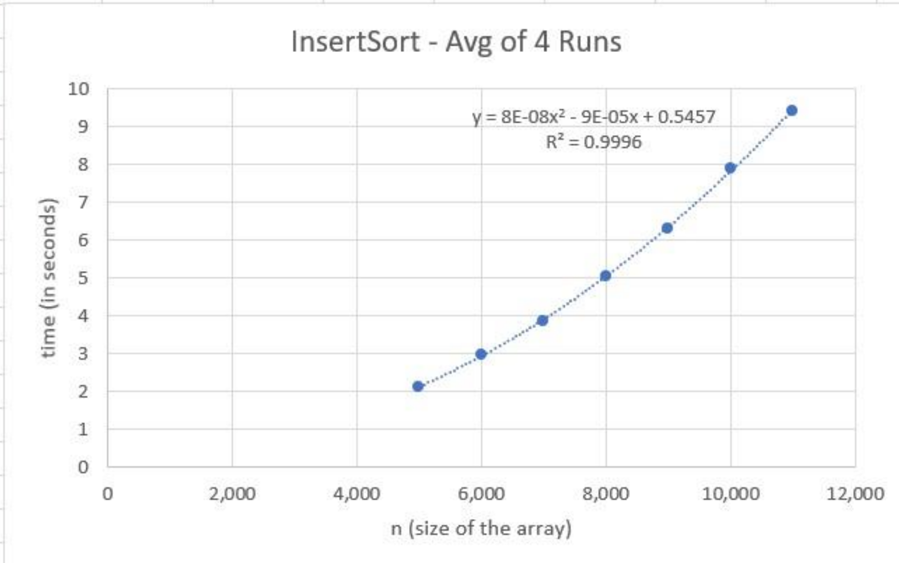
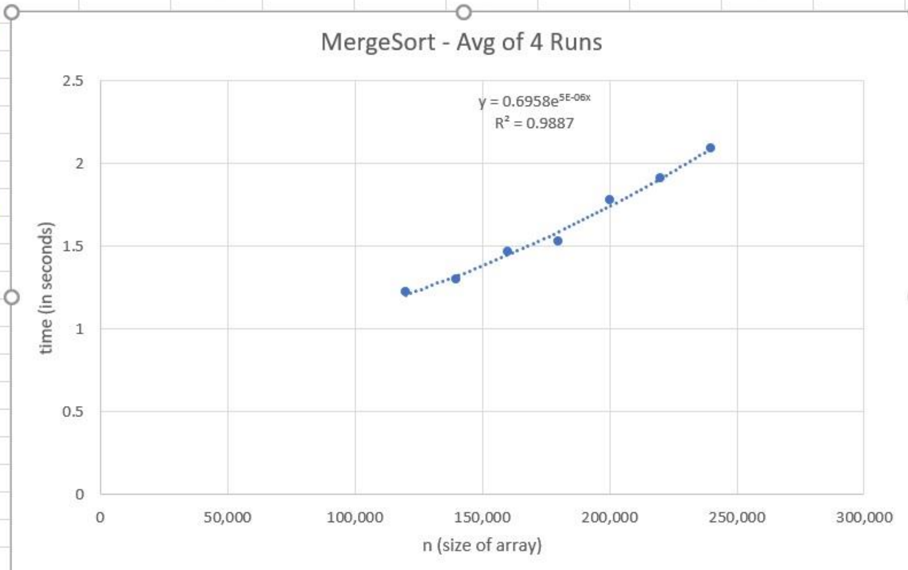


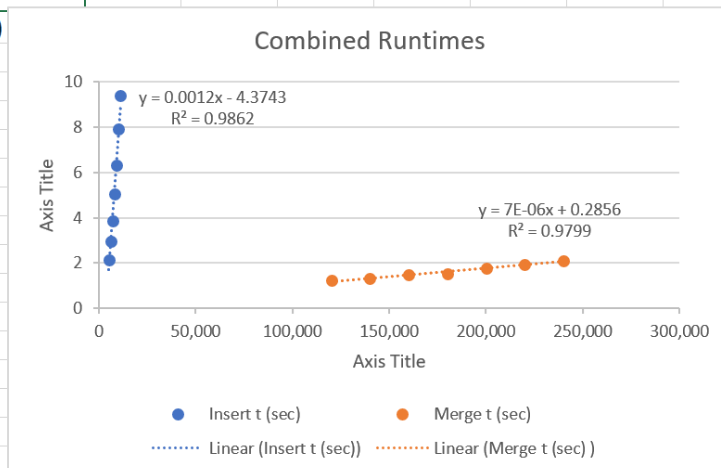
Chart 1

n	t (sec)
120,000	1.226
140,000	1.304
160,000	1.465
180,000	1.527
200,000	1.783
220,000	1.914
240,000	2.093



d)

Insert		Merge	
n	sert t (se	n	erge t (sec)
5,000	2.134	120,000	1.226
6,000	2.96	140,000	1.304
7,000	3.863	160,000	1.465
8,000	5.052	180,000	1.527
9,000	6.315	200,000	1.783
10,000	7.908	220,000	1.914
11,000	9.392	240,000	2.093



e)

Based on the theoretical runtime of **Merge Sort**, O (n log(n)), my actual runtimes seem to somewhat follow the trend line of such a function, trending linearly.

Based on the theoretical runtime of **Insert Sort**, $O(n^2)$, my actual runtimes seem to grow a bit faster than the theoretical time. Based on the graph above, my runtimes seem to follow more of a trend line of $O(2^n)$.