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7. Find a vector x orthogonal to the row space of A, and a vector y orthogonal to the column space, and a vector z orthogonal to the nullspace:

$$A = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 4 & 3 \\ 3 & 6 & 4 \end{bmatrix}.$$

$$A = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 4 & 3 \\ 3 & 6 & 4 \end{bmatrix} \xrightarrow{A_{12}(-2)} \begin{bmatrix} 1 & 2 & 1 \\ 0 & 0 & 1 \\ 3 & 6 & 4 \end{bmatrix} \xrightarrow{A_{13}(-3)} \begin{bmatrix} 1 & 2 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix} \xrightarrow{A_{23}(-1)} \begin{bmatrix} \boxed{1} & 2 & 1 \\ 0 & 0 & \boxed{1} \\ 0 & 0 & 0 \end{bmatrix}$$

$x_1 + x_3 =$ PIVOTS
$x_2 =$ FREE VARIABLE

$x_1 + 2x_2 + x_3 = 0$
$x_1 = -2x_2 - x_3$
$x_2 = 1 \rightarrow x_1 = -2(1) - 0$
$x_3 = 0 \quad x_1 = -2$

$x_3 = 1 \quad x_1 = -2(0) - 1$
$x_2 = 0 \quad x_1 = -1$

$x_2 = 1 \quad x_2 = 0$
$x_3 = 0 \quad x_3 = 1$

$$\begin{bmatrix} -2x_2 - x_3 \\ x_2 \\ x_3 \end{bmatrix} \rightarrow x_2 \begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix} + x_3 \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix} = ?$$

$$A^T = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 1 & 3 & 4 \end{bmatrix} \xrightarrow[A_{13}(-1)]{A_{12}(-2)} \begin{bmatrix} 1 & 2 & 3 \\ 0 & 0 & 0 \\ 0 & 1 & 1 \end{bmatrix} \xrightarrow{P_{23}} \begin{bmatrix} \boxed{1} & 2 & 3 \\ 0 & \boxed{1} & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

$x_1 + 2x_2 + 3x_3 = 0 \longrightarrow x_1 = -2x_2 - 3x_3$
$x_2 + x_3 = 0 \qquad = -2(-x_3) - 3x_3$
$x_2 = -x_3 \qquad x_1 = 2x_3 - 3x_3$
$x_3 = x_3 \qquad x_1 = -x_3$

$x_3 = 1 \quad x_3 = 0$

$$\begin{bmatrix} -x_3 \\ -x_3 \\ x_3 \end{bmatrix} \rightarrow \begin{bmatrix} -1 \\ -1 \\ 0 \end{bmatrix} \quad \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

X ORTHOGONAL TO THE
ROW SPACE OF A = NULL SPACE $\Rightarrow x = \begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix}$

Y ORTHOGONAL TO THE NULLSPACE
FIND LEFT NULL SPACE
$A^T y = 0$
$y = \begin{bmatrix} -1 \\ -1 \\ 0 \end{bmatrix}$

Z ORTHOGONAL TO THE NULLSPACE
FIND ROW SPACE OF A
$z = \begin{bmatrix} 1 & 2 & 1 \end{bmatrix}$
$+$
$\begin{bmatrix} 3 & 6 & 4 \end{bmatrix}$

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14. What matrix P projects every point in $\mathbf{R}^3$ onto the line of intersection of the planes $x+y+t=0$ and $x-t=0$?

$$P = \frac{aa^T}{a^Ta} = \frac{\begin{bmatrix} c \\ s \end{bmatrix}\begin{bmatrix} c & s \end{bmatrix}}{\begin{bmatrix} c & s \end{bmatrix}\begin{bmatrix} c \\ s \end{bmatrix}} = \begin{bmatrix} c^2 & cs \\ cs & s^2 \end{bmatrix}$$

$$x + y + t = 0$$
$$x - t = 0 \Rightarrow \boxed{x = t}$$
$$t + y + t = 0$$
$$y + 2t = 0$$
$$\boxed{y = -2t}$$

$$\begin{bmatrix} t \\ -2t \\ t \end{bmatrix} \Rightarrow \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix} = a$$

$$a = \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix} \quad a^T = \begin{bmatrix} 1 & -2 & 1 \end{bmatrix}$$

$$P = \frac{\begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}\begin{bmatrix} 1 & -2 & 1 \end{bmatrix}}{\begin{bmatrix} 1 & -2 & 1 \end{bmatrix}\begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}} = \frac{\begin{bmatrix} 1 & -2 & 1 \\ -2 & 4 & -2 \\ 1 & -2 & 1 \end{bmatrix}}{1+4+1} = \frac{1}{6}\begin{bmatrix} 1 & -2 & 1 \\ -2 & 4 & -2 \\ 1 & -2 & 1 \end{bmatrix} = \begin{bmatrix} 1/6 & -1/3 & 1/6 \\ -1/3 & 2/3 & -1/3 \\ 1/6 & -1/3 & 1/6 \end{bmatrix}$$

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3. Solve $Ax = b$ by least squares, and find $p = A\hat{x}$ if

$$A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 1 \end{bmatrix}, \qquad b = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}.$$

Verify that the error $b - p$ is perpendicular to the columns of A.

$$A^T A\hat{x} = A^T b$$

$$A = \begin{bmatrix} 1 & \emptyset \\ \emptyset & 1 \\ 1 & 1 \end{bmatrix} \quad A^T = \begin{bmatrix} 1 & \emptyset & 1 \\ \emptyset & 1 & 1 \end{bmatrix}$$

$$A^T A = \begin{bmatrix} 1 & \emptyset & 1 \\ \emptyset & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & \emptyset \\ \emptyset & 1 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$$

$$A^T b = \begin{bmatrix} 1 & \emptyset & 1 \\ \emptyset & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ \emptyset \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$A^T A x = A^T b$$

$$\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$\left[\begin{array}{cc|c} 2 & 1 & 1 \\ 1 & 2 & 1 \end{array}\right] \xrightarrow{P_{12}} \left[\begin{array}{cc|c} 1 & 2 & 1 \\ 2 & 1 & 1 \end{array}\right] \xrightarrow{A_{12}(-2)} \left[\begin{array}{cc|c} 1 & 2 & 1 \\ \emptyset & -3 & -1 \end{array}\right] \xrightarrow{A_{12}(-1/3)} \left[\begin{array}{cc|c} 1 & 2 & 1 \\ \emptyset & 1 & 1/3 \end{array}\right]$$

$$\boxed{x_2 = 1/3}$$

$$x_1 + 2x_2 = 1$$
$$x_1 + 2(1/3) = 1$$
$$x_1 + 2/3 = 1$$
$$\boxed{x_1 = 1/3}$$

$$\hat{x} = \begin{bmatrix} 1/3 \\ 1/3 \end{bmatrix}$$

$$p = A\hat{x}$$

$$A = \begin{bmatrix} 1 & \emptyset \\ \emptyset & 1 \\ 1 & 1 \end{bmatrix} \quad \hat{x} = \begin{bmatrix} 1/3 \\ 1/3 \end{bmatrix}$$

$$p = \begin{bmatrix} 1 & \emptyset \\ \emptyset & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 1/3 \\ 1/3 \end{bmatrix} \quad p = \begin{bmatrix} 1/3 \\ 1/3 \\ 2/3 \end{bmatrix}$$

$3\times 2 \quad 2\times 1$

$$b - p = \begin{bmatrix} 1 \\ 1 \\ \emptyset \end{bmatrix} - \begin{bmatrix} 1/3 \\ 1/3 \\ 2/3 \end{bmatrix} = \begin{bmatrix} 2/3 \\ 2/3 \\ -2/3 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & \emptyset \\ \emptyset & 1 \\ 1 & 1 \end{bmatrix} \quad COL(A) = SPAN\left\{ \overset{A_1}{\begin{bmatrix} 1 \\ \emptyset \\ 1 \end{bmatrix}}, \overset{A_2}{\begin{bmatrix} \emptyset \\ 1 \\ 1 \end{bmatrix}} \right\}$$

$$(b-p) \cdot A_1 = (2/3 + \emptyset - 2/3) = \emptyset$$
$$(b-p) \cdot A_2 = (\emptyset + 2/3 - 2/3) = \emptyset$$

6. Find the projection of b onto the column space of A:

$$A = \begin{bmatrix} 1 & 1 \\ 1 & -1 \\ -2 & 4 \end{bmatrix}, \quad b = \begin{bmatrix} 1 \\ 2 \\ 7 \end{bmatrix}.$$

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Split b into $p + q$, with p in the column space and q perpendicular to that space. Which of the four subspaces contains q?

$$A^T A \hat{x} = A^T b$$

$$A = \begin{bmatrix} 1 & 1 \\ 1 & -1 \\ -2 & 4 \end{bmatrix} \quad A^T = \begin{bmatrix} 1 & 1 & -2 \\ 1 & -1 & 4 \end{bmatrix}$$

$$A^T A = \underbrace{\begin{bmatrix} 1 & 1 & -2 \\ 1 & -1 & 4 \end{bmatrix}}_{2\times 3} \underbrace{\begin{bmatrix} 1 & 1 \\ 1 & -1 \\ -2 & 4 \end{bmatrix}}_{3\times 2} = \begin{bmatrix} 6 & -8 \\ -8 & 18 \end{bmatrix} = A^T A$$

$$A^T b = \underbrace{\begin{bmatrix} 1 & 1 & -2 \\ 1 & -1 & 4 \end{bmatrix}}_{2\times 3} \underbrace{\begin{bmatrix} 1 \\ 2 \\ 7 \end{bmatrix}}_{3\times 1} = \begin{bmatrix} 1 + 2 - 14 \\ 1 - 2 + 28 \end{bmatrix} = \begin{bmatrix} -11 \\ 27 \end{bmatrix}$$

$$A^T A \hat{x} = A^T b$$

$$\begin{bmatrix} 6 & -8 \\ -8 & 18 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} -11 \\ 27 \end{bmatrix} \Rightarrow \left[\begin{array}{cc|c} 6 & -8 & -11 \\ -8 & 18 & 27 \end{array}\right] \xrightarrow{A_{12}(8/6)} \left[\begin{array}{cc|c} 6 & -8 & -11 \\ 0 & 22/3 & 37/3 \end{array}\right] \xrightarrow[M_2(3/22)]{M_1(1/6)} \left[\begin{array}{cc|c} 1 & -4/3 & -11/6 \\ 0 & 1 & 37/22 \end{array}\right] \xrightarrow{A_{21}(4/3)} \left[\begin{array}{cc|c} 1 & 0 & 9/22 \\ 0 & 1 & 37/22 \end{array}\right]$$

$x_1 = 9/22$ $\quad \hat{x} = \begin{bmatrix} 9/22 \\ 37/22 \end{bmatrix}$

$x_2 = 37/22$

$$p = A\hat{x}$$

$$A = \begin{bmatrix} 1 & 1 \\ 1 & -1 \\ -2 & 4 \end{bmatrix} \quad \hat{x} = \begin{bmatrix} 9/22 \\ 37/22 \end{bmatrix}$$

$$p = \underbrace{\begin{bmatrix} 1 & 1 \\ 1 & -1 \\ -2 & 4 \end{bmatrix}}_{3\times 2} \underbrace{\begin{bmatrix} 9/22 \\ 37/22 \end{bmatrix}}_{2\times 1} = \begin{bmatrix} 9/22 + 37/22 \\ 9/22 - 37/22 \\ -18/22 + 148/22 \end{bmatrix} = \begin{bmatrix} 23/11 \\ -14/11 \\ 65/11 \end{bmatrix} = p$$

$$\text{ERROR} = b - p = \begin{bmatrix} 1 \\ 2 \\ 7 \end{bmatrix} - \begin{bmatrix} 23/11 \\ -14/11 \\ 65/11 \end{bmatrix} = \begin{bmatrix} -12/11 \\ 25/11 \\ -54/11 \end{bmatrix}$$

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12. If **V** is the subspace spanned by (1, 1, 0, 1) and (0, 0, 1, 0), find

(a) a basis for the orthogonal complement $\mathbf{V}^\perp$.
(b) the projection matrix P onto **V**.
(c) the vector in **V** closest to the vector $b = (0, 1, 0, -1)$ in $\mathbf{V}^\perp$.

A) $A = \begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$

$A x = 0$

$$\begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \Rightarrow \begin{aligned} x_1 + x_2 + + x_4 &= 0 \\ x_3 &= 0 \end{aligned}$$

$$\begin{aligned} x_1 &= -x_2 - x_4 \\ x_3 &= 0 \end{aligned} \Rightarrow \begin{bmatrix} -x_2 - x_4 \\ x_2 \\ 0 \\ x_4 \end{bmatrix}$$

$x_2 = 1, 0 \quad x_4 = 1, 0$

$$\begin{bmatrix} -x_2 - x_4 \\ x_2 \\ 0 \\ x_4 \end{bmatrix} \Rightarrow x_2 \begin{bmatrix} -1 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \; x_4 \begin{bmatrix} -1 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

$$V^\perp = \left\{ \begin{bmatrix} -1 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ 0 \\ 0 \\ 1 \end{bmatrix} \right\}$$

B) $P = \dfrac{V V^T}{V^T V} \quad V = \begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \quad V^T = \begin{bmatrix} 1 & 0 \\ 1 & 0 \\ 0 & 1 \\ 1 & 0 \end{bmatrix}$

$$P = \frac{\underbrace{\begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}}_{2\times4} \underbrace{\begin{bmatrix} 1 & 0 \\ 1 & 0 \\ 0 & 1 \\ 1 & 0 \end{bmatrix}}_{4\times2}}{\underbrace{\begin{bmatrix} 1 & 0 \\ 1 & 0 \\ 0 & 1 \\ 1 & 0 \end{bmatrix}}_{4\times2} \underbrace{\begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}}_{2\times4}} = \frac{\begin{bmatrix} 3 & 0 \\ 0 & 1 \end{bmatrix}}{\begin{bmatrix} 1 & 1 & 0 & 1 \\ 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 \end{bmatrix}}$$

$P = v^T (v\, v^T)^{-1} v$

$$(v\, v^T)^{-1} = \left[\begin{array}{cc|cc} 3 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{array}\right] \xrightarrow{M_1(1/3)} \left[\begin{array}{cc|cc} 1 & 0 & 1/3 & 0 \\ 0 & 1 & 0 & 1 \end{array}\right] \quad (v\, v^T)^{-1}$$

$$= \underbrace{\begin{bmatrix} 1 & 0 \\ 1 & 0 \\ 0 & 1 \\ 1 & 0 \end{bmatrix}}_{4\times2} \underbrace{\begin{bmatrix} 1/3 & 0 \\ 0 & 1 \end{bmatrix}}_{2\times2} \underbrace{\begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}}_{2\times4} = \underbrace{\begin{bmatrix} 1/3 & 0 \\ 1/3 & 0 \\ 0 & 1 \\ 1/3 & 0 \end{bmatrix}}_{4\times2} \underbrace{\begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}}_{2\times4} = \begin{bmatrix} 1/3 & 1/3 & 0 & 1/3 \\ 1/3 & 1/3 & 0 & 1/3 \\ 0 & 0 & 1 & 0 \\ 1/3 & 1/3 & 0 & 1/3 \end{bmatrix}$$

C) $p = P b$

$$= \underbrace{\begin{bmatrix} 1/3 & 1/3 & 0 & 1/3 \\ 1/3 & 1/3 & 0 & 1/3 \\ 0 & 0 & 1 & 0 \\ 1/3 & 1/3 & 0 & 1/3 \end{bmatrix}}_{4\times4} \underbrace{\begin{bmatrix} 0 \\ 1 \\ 0 \\ -1 \end{bmatrix}}_{4\times1} = \begin{bmatrix} 0 + 1/3 + 0 - 1/3 \\ 0 + 1/3 + 0 - 1/3 \\ 0 + 0 + 0 + 0 \\ 0 + 1/3 + 0 - 1/3 \end{bmatrix}$$

$$p = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

15. Find an orthonormal set q_1, q_2, q_3 for which q_1, q_2 span the column space of

$$A = \begin{bmatrix} 1 & 1 \\ 2 & -1 \\ -2 & 4 \end{bmatrix}.$$

Which fundamental subspace contains q_3? What is the least-squares solution of $Ax = b$ if $b = \begin{bmatrix} 1 & 2 & 7 \end{bmatrix}^T$?

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$$A = \begin{bmatrix} 1 & 1 \\ 2 & -1 \\ -2 & 4 \end{bmatrix} \quad A_1 = \begin{bmatrix} 1 \\ 2 \\ -2 \end{bmatrix} \quad A_2 = \begin{bmatrix} 1 \\ -1 \\ 4 \end{bmatrix}$$

$$q_1 = \frac{a_1}{\|a_1\|} = \frac{1}{\sqrt{1^2 + 2^2 + -2^2}} \begin{bmatrix} 1 \\ 2 \\ -2 \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 \\ 2 \\ -2 \end{bmatrix} = \begin{bmatrix} 1/3 \\ 2/3 \\ -2/3 \end{bmatrix} = q_1$$

$$\underbrace{P_{q_1} a_2} = (a_2 \cdot q_1) q_1 = \left(\begin{bmatrix} 1 \\ -1 \\ 4 \end{bmatrix} \cdot \begin{bmatrix} 1/3 \\ 2/3 \\ -2/3 \end{bmatrix} \right) \begin{bmatrix} 1/3 \\ 2/3 \\ -2/3 \end{bmatrix} = \left(1/3 + -2/3 + -8/3\right) \begin{bmatrix} 1/3 \\ 2/3 \\ -2/3 \end{bmatrix} = -3 \begin{bmatrix} 1/3 \\ 2/3 \\ -2/3 \end{bmatrix} = \begin{bmatrix} -1 \\ -2 \\ 2 \end{bmatrix}$$

$$v_2 = a_2 - P_{q_1} a_2 = \begin{bmatrix} 1 \\ -1 \\ 4 \end{bmatrix} - \begin{bmatrix} -1 \\ -2 \\ 2 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ 2 \end{bmatrix}$$

$$q_2 = \frac{v_2}{\|v_2\|} = \frac{1}{\sqrt{2^2 + 1^2 + 2^2}} \begin{bmatrix} 2 \\ 1 \\ 2 \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 2 \\ 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 2/3 \\ 1/3 \\ 2/3 \end{bmatrix} = q_2$$

NULL $(A^T) \perp$ COL SPACE

$$A = \begin{bmatrix} 1 & -1 \\ 2 & -1 \\ -2 & 4 \end{bmatrix} \quad A^T = \begin{bmatrix} 1 & 2 & -2 \\ -1 & -1 & 4 \end{bmatrix} \xrightarrow{A_{12}(-1)} \begin{bmatrix} 1 & 2 & -2 \\ 0 & -3 & 6 \end{bmatrix} \xrightarrow{M_2(-1/3)} \begin{array}{c} \begin{bmatrix} 1 & 2 & -2 \\ 0 & 1 & -2 \end{bmatrix} \\ x_1 \quad x_2 \quad x_3 \end{array}$$

$$x_1 + 2x_2 - 2x_3 = 0 \qquad x_1 = -2x_2 + 2x_3$$
$$x_2 - 2x_3 = 0 \qquad x_1 = -2(2x_3) + 2x_3$$
$$x_2 = 2x_3 \qquad x_1 = -4x_3 + 2x_3$$
$$x_1 = -2x_3$$

$x_3 = 1$

$$\begin{bmatrix} -2x_3 \\ 2x_3 \\ x_3 \end{bmatrix} \rightarrow \begin{bmatrix} -2 \\ 2 \\ 1 \end{bmatrix} = v_3$$

$$q_3 = \frac{v_3}{\|v_3\|} = \frac{1}{\sqrt{-2^2 + 2^2 + 1^2}} \begin{bmatrix} -2 \\ 2 \\ 1 \end{bmatrix} = \frac{1}{3}\begin{bmatrix} -2 \\ 2 \\ 1 \end{bmatrix} = \begin{bmatrix} -2/3 \\ 2/3 \\ 1/3 \end{bmatrix} = q_3$$

$$v_3 = \begin{bmatrix} -2 \\ 2 \\ 1 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & -1 \\ 2 & -1 \\ -2 & 4 \end{bmatrix} \quad A^T = \begin{bmatrix} 1 & 2 & -2 \\ -1 & -1 & 4 \end{bmatrix}$$

$$A^T A \hat{x} = A^T b$$

$$\underbrace{\begin{bmatrix} 1 & 2 & -2 \\ 1 & -1 & 4 \end{bmatrix}}_{2\times 3} \underbrace{\begin{bmatrix} 1 & 1 \\ 2 & -1 \\ -2 & 4 \end{bmatrix}}_{3\times 2} \hat{x} = \underbrace{\begin{bmatrix} 1 & 2 & -2 \\ 1 & -1 & 4 \end{bmatrix}}_{2\times 3} \underbrace{\begin{bmatrix} 1 \\ 2 \\ 7 \end{bmatrix}}_{3\times 1}$$

$$\begin{bmatrix} 9 & -9 \\ -9 & 18 \end{bmatrix} \hat{x} = \begin{bmatrix} -9 \\ 27 \end{bmatrix} \Rightarrow \left[\begin{array}{cc|c} 9 & -9 & -9 \\ -9 & 18 & 27 \end{array}\right] \xrightarrow[A_{12}(9)\ (2)]{M_1(1/9)\ (1)} \left[\begin{array}{cc|c} 1 & -1 & -1 \\ 0 & 9 & 18 \end{array}\right] \xrightarrow[A_{21}(1)\ (2)]{M_2(1/9)\ (1)} \left[\begin{array}{cc|c} 1 & 0 & 1 \\ 0 & 1 & 2 \end{array}\right]$$

$$\hat{x} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

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11. Compute $y = F_4c$ by the three steps of the Fast Fourier Transform if $c = (1, 0, 1, 0)$.

$c = (1, 0, 1, 0)$

$$\begin{bmatrix} c_0 \\ c_1 \\ c_2 \\ c_3 \end{bmatrix} \rightarrow \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix} \Rightarrow \overset{C_{bit}\text{-REVERSED}}{\begin{bmatrix} c_0 \\ c_2 \\ c_1 \\ c_3 \end{bmatrix}} \rightarrow \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix}$$

$C_e = (1, 1)$

$C_o = (0, 0)$

$$\begin{array}{c} y' \\ \\ y'' \\ \end{array} \begin{bmatrix} 1+1 \\ 1-1 \\ 0+0 \\ 0-0 \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \\ 0 \\ 0 \end{bmatrix} \Rightarrow Y \Rightarrow \begin{bmatrix} C_e(0) + W_4^0 C_o(0) \\ C_e(1) + W_4^1 C_o(1) \\ C_e(0) - W_4^2 C_o(0) \\ C_e(1) - W_4^3 C_o(1) \end{bmatrix} = \begin{bmatrix} 2 + 1 \cdot 0 \\ 0 + (-i) \cdot 0 \\ 2 - 1 \cdot 0 \\ 0 - (-i) \cdot 0 \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \\ 2 \\ 0 \end{bmatrix} = Y$$

$Y_x = C_{ex} + W_4^k C_{ox}$

$W_4^0 = 1$

$W_4^1 = -i$

$W_4^2 = -1$

$W_4^3 = i$