MATH 511 HOMEWORK (ON GRADESCOPE)

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Homework # 6

Due on July 21, 2024, 11:59 PM

Section 6.1: P316 #7, P317 #18
Section 6.2: P328 #24, P329 #32,
Section 6.3: P338 #15, #18

6.1.p316.#07

[chapter].[section].[page].[problem]

7. (a) What 3 by 3 symmetric matrices A_1 and A_2 correspond to f_1 and f_2?

$$f_1 = x_1^2 + x_2^2 + x_3^2 - 2x_1x_2 - 2x_1x_3 + 2x_2x_3$$

$$f_2 = x_1^2 + 2x_2^2 + 11x_3^2 - 2x_1x_2 - 2x_1x_3 - 4x_2x_3.$$

(b) Show that f_1 is a *single* perfect square and not positive definite. Where is f_1 equal to 0?

(c) Factor A_2 into LL^{T}. Write $f_2 = x^{\mathrm{T}} A_2 x$ as a sum of three squares.

6.1.p317.#18

[chapter].[section].[page].[problem]

Problems 14–18 are about tests for positive definiteness.

18. Test to see if $A^{\mathrm{T}}A$ is positive definite in each case:

$$A = \begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix}, \quad A = \begin{bmatrix} 1 & 1 \\ 1 & 2 \\ 2 & 1 \end{bmatrix}, \quad \text{and} \quad A = \begin{bmatrix} 1 & 1 & 2 \\ 1 & 2 & 1 \end{bmatrix}.$$

6.2.p328.#24

[chapter].[section].[page].[problem]

24. For which s and t do A and B have all $\lambda > 0$ (and are therefore positive definite)?

$$A = \begin{bmatrix} s & -4 & -4 \\ -4 & s & -4 \\ -4 & -4 & s \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} t & 3 & 0 \\ 3 & t & 4 \\ 0 & 4 & t \end{bmatrix}.$$

6.2.p329.#32

[chapter].[section].[page].[problem]

32. Apply any three tests to each of the matrices

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} 2 & 1 & 2 \\ 1 & 1 & 1 \\ 2 & 1 & 2 \end{bmatrix},$$

to decide whether they are positive definite, positive semidefinite, or indefinite.

6.3.p338.#15

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15. Find the SVD and the pseudoinverse $V\Sigma^+U^{\mathrm{T}}$ of

$$A = \begin{bmatrix} 1 & 1 & 1 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}, \quad \text{and} \quad C = \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}.$$

6.3.p338.#18

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18. What is the minimum-length least-squares solution $x^+ = A^+ b$ to the following?

$$Ax = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} C \\ D \\ E \end{bmatrix} = \begin{bmatrix} 0 \\ 2 \\ 2 \end{bmatrix}.$$

You can compute A^+, or find the general solution to $A^{\mathrm{T}} A \widehat{x} = A^{\mathrm{T}} b$ and choose the solution that is in the row space of A. This problem fits the best plane $C + Dt + Ez$ to $b = 0$ *and also* $b = 2$ at $t = z = 0$ (and $b = 2$ at $t = z = 1$).