MATH 511 HOMEWORK (ON GRADESCOPE)

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Homework # 5

Due on July 14, 2024, 11:59 PM

Section 5.1: P241 #5, P242 #14,

Section 5.2: P250 #3,

Section 5.3: P264 #11,

Section 5.4: P277 #15, P278 #21

Section 5.5: P291 #31,

Section 5.6: P303 #22

5.1.p241.#05 [chapter].[section].[page].[problem

5. Find the eigenvalues and eigenvectors of

$$A = \begin{bmatrix} 3 & 4 & 2 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} 0 & 0 & 2 \\ 0 & 2 & 0 \\ 2 & 0 & 0 \end{bmatrix}.$$

Check that $\lambda_1+\lambda_2+\lambda_3$ equals the trace and $\lambda_1\lambda_2\lambda_3$ equals the determinant.

A)
$$\begin{bmatrix} 3 & 4 & 2 \\ A & = \emptyset & 1 & 2 \\ \emptyset & \emptyset & \emptyset \end{bmatrix} \xrightarrow{3-\lambda} 4 \xrightarrow{2} \xrightarrow{\lambda} (3-\lambda)(1-\lambda)(\emptyset-\lambda) = \emptyset$$

$$A = \begin{bmatrix} 3 & 4 & 2 \\ 0 & 1 & 2 \\ 0 & \emptyset & 0 \end{bmatrix} \xrightarrow{2} 0 \xrightarrow{1-\lambda} 2 \xrightarrow{2} 1 \xrightarrow{\lambda=3} 1 \xrightarrow{\lambda=1} 1 \xrightarrow{\lambda=0} 1 \xrightarrow$$

$$\emptyset \ \emptyset \ \neg 3$$

$$\neg 3x_3 = \emptyset$$

$$x_2 = \emptyset$$

$$x_2 = \emptyset$$

$$2-\lambda = 4(2-\lambda)$$

$$\lambda = 2 \quad \lambda = 2 \Rightarrow \lambda$$

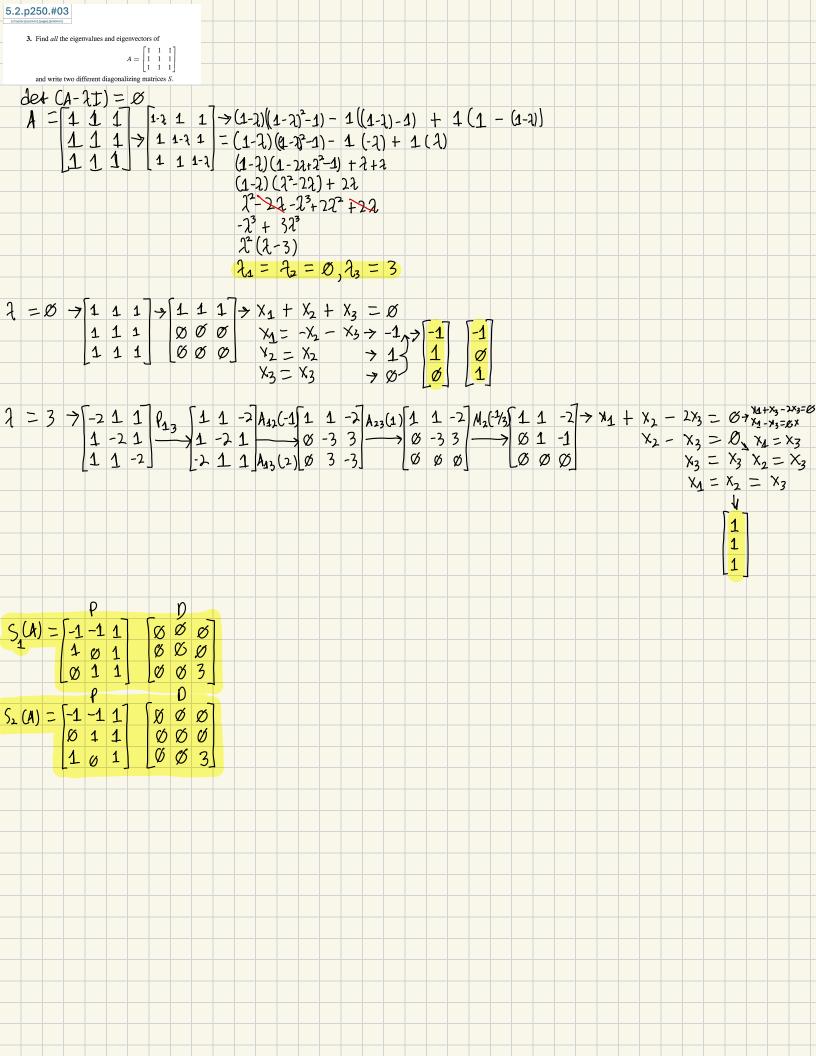
$$(\lambda) - 4(2-2)$$
 = $(\lambda - 2)$ =

A11 + A22 + A33 -

TRACE (A) = $\lambda_1 + \lambda_2 + \lambda_3$ = $\omega + 1 + 3$

Y1 = 24

X1 - -2x2



11. (a) From the fact that column 1 + column 2 = 2(column 3), so the columns are linearly dependent, find one eigenvalue and one eigenvector of A:

$$A = \begin{bmatrix} .2 & .4 & .3 \\ .4 & .2 & .3 \\ .4 & .4 & .4 \end{bmatrix}.$$

- (b) Find the other eigenvalues of A (it is Markov).
- (c) If $u_0 = (0, 10, 0)$, find the limit of $A^k u_0$ as $k \to \infty$.
- a) DUE TO LINEAR DEPENDENCE, det(A) = 0

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W DUE TO MARKOV, 2 = 1

TRACE =
$$\lambda_1 + \lambda_2 + \lambda_3 = 9_{11} + a_{22} + a_{33}$$

 $\emptyset + 1 + \lambda_3 = .2 + .2 + .4$
 $1 + \lambda_3 = .9$
 $\lambda_3 = -.2$
 $\lambda_1 = \emptyset, \lambda_2 = 1, \lambda_3 = -.2$

$$U_{\emptyset} = (\emptyset, 1\emptyset, \emptyset)$$
 $\begin{bmatrix} .3 \\ .3 \end{bmatrix} = \begin{bmatrix} 3\emptyset \\ 3\emptyset \end{bmatrix} = LIMIT OFA^{k} U_{\emptyset} as k \rightarrow \infty$

5.4.p277.#15 15. Solve the second-order equation $\frac{d^2u}{dt^2} = \begin{bmatrix} -5 & -1 \\ -1 & -5 \end{bmatrix} u \quad \text{with} \quad u(0) = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \quad \text{and} \quad u'(0) = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ $A = \begin{bmatrix} -5 & -1 \\ -1 & -5 \end{bmatrix} \xrightarrow{5 \cdot 2} -1 \xrightarrow{-9} (-5 \cdot 2)(-5 \cdot 2) - (-1)(-1)$ $-1 & -5 & -1 & -5 \cdot 2 \\ -1 & -5 \cdot 2 & (2+5)(2+5) - 1 \\ 2^{2} + 1 \emptyset 2 + 25 - 1 \\ 2^{2} + 1 \emptyset 2 + 24 \\ 2^{4} & 2^{4}$ $\lambda_{1} = -4 \rightarrow \begin{bmatrix} -1 & -1 \\ -1 & -1 \end{bmatrix} \rightarrow \begin{bmatrix} -1 & -1 \\ \emptyset & \emptyset \end{bmatrix} \rightarrow -x_{1} - x_{2} = \emptyset \rightarrow -x_{1} = x_{2} \rightarrow \begin{bmatrix} -1 \\ 1 \end{bmatrix}$ $\begin{cases} 2 = -6 \rightarrow \begin{bmatrix} 1 & -1 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & -1 \end{bmatrix} \rightarrow \begin{cases} 1 & -1 \end{cases} \rightarrow \begin{cases} 1 &$ **21.** Find λ 's and x's so that $u = e^{\lambda t}x$ solves

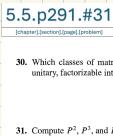
$$\frac{du}{dt} = \begin{bmatrix} 4 & 3 \\ 0 & 1 \end{bmatrix} u.$$

What combination $u = c_1 e^{\lambda_1 t} x_1 + c_2 e^{\lambda_2 t} x_2$ starts from u(0) = (5, -2)?

$$\begin{array}{cccc}
A & = \begin{bmatrix} 4 & 3 \\ \emptyset & 1 \end{bmatrix} \Rightarrow \begin{bmatrix} 4-\lambda & 3 \\ \emptyset & 1-\lambda \end{bmatrix} \Rightarrow \begin{pmatrix} 4-\lambda \end{pmatrix} \begin{pmatrix} 1-\lambda \\ 1-\lambda \end{bmatrix}$$

$$\lambda_{1} = 4 \rightarrow \begin{bmatrix} \emptyset & 3 \end{bmatrix} \rightarrow \begin{bmatrix} \emptyset & 3 \end{bmatrix} \rightarrow \chi_{1} = \chi_{1} \rightarrow \begin{bmatrix} 1 \end{bmatrix} \rightarrow U_{1} = e^{4t} \begin{bmatrix} 1 \\ \emptyset \end{bmatrix}$$

$$u(t) = 3e^{4t} \begin{bmatrix} 1 \\ \emptyset \end{bmatrix} + 2e^{t} \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$



30. Which classes of matrices does P belong to: orthogonal, invertible, Hermitian, unitary, factorizable into LU, factorizable into QR?

$$P = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}.$$

31. Compute P^2 , P^3 , and P^{100} in Problem 30. What are the eigenvalues of P?

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$$P^{3} = P^{2} \cdot P = \begin{bmatrix} \varnothing & \varnothing & 1 \\ 1 & \varnothing & \varnothing \\ 0 & 1 & \varnothing \end{bmatrix} \begin{bmatrix} \varnothing & 1 & \varnothing \\ 0 & \varnothing & 1 \\ 0 & 0 & \varnothing \end{bmatrix} = \begin{bmatrix} 1 & \varnothing & \varnothing \\ 0 & 1 & \varnothing \\ 0 & \varnothing & 1 \end{bmatrix} = \begin{bmatrix} 1 & \varnothing & \varnothing \\ 0 & 0 & 1 \end{bmatrix}$$