MA511 Final Exam Review (Session 33 - 34)

July 26, 2024

1. Compute the determinant of A, where $\mathrm{A} = \begin{bmatrix} a & b & c & d \\ e & f & g & h \\ i & j & k & l \\ m & n & o & p \end{bmatrix} \begin{bmatrix} 2 & 5 & 4 & 2 \\ 0 & 0 & 0 & 2 \\ 0 & 3 & 0 & 4 \\ 1 & 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} a & b & c & d \\ e & f & g & h \\ i & j & k & l \\ m & n & o & p \end{bmatrix}^{-1}$

2. Let $\mathrm{A} = \begin{bmatrix} 0 & 0 & 2 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$. If $\mathrm{B} = [b_{ij}]$ is A^{-1}, what is b_{21}

3. Simplify $det(BAB^{-1}) + det(2A) - det(A^2) + det(A^T)$

4. Find the determinant of $\mathrm{A} = \begin{bmatrix} 0 & 3 & 4 & 0 & 0 \\ 0 & 3 & 2 & 0 & 5 \\ 1 & 2 & 0 & 2 & 3 \\ 0 & 0 & -3 & 0 & 0 \\ 3 & 2 & 0 & 2 & 4 \end{bmatrix}$

5. Given $\det \begin{bmatrix} 2c_1 & -2c_2 & 2c_3 \\ a_1 + 6c_1 & -a_2 - 6c_2 & a_3 + 6c_3 \\ -b1 & b_2 & b_3 \end{bmatrix} = -16$, find $\det \begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix}$

6. Let A be a $n \times n$ singular real matrix. Which of the following statements are always TRUE?

   (a) $\det(\mathrm{A}) = 0$

   (b) A is row equivalent to the identity matrix

   (c) $Ax = 0$ must have non-trivial solutions

   (d) $Ax = b$ has a unique solution for every $b \in \Re^n$

7. Suppose $\begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}$ is an eigenvector of $\begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 0 \\ a & 5 & 1 \end{bmatrix}$. What is a?

8. Which of the following value is a multiple eigenvalue of A, where $\mathrm{A} = \begin{bmatrix} 2 & 0 & 0 & 0 \\ 1 & -1 & 0 & 0 \\ 3 & 5 & -2 & -2 \\ -3 & 3 & 3 & 5 \end{bmatrix}$

   (a) -1

   (b) -2

   (c) 1

   (d) 2

(e) 4

9. Given $A = \begin{bmatrix} 1 & 1 \\ 0 & 2 \end{bmatrix}$, what is A^6?

10. Which of the following matrices are diagonalizable?

(a) $\begin{bmatrix} 1 & 4 \\ 1 & -1 \end{bmatrix}$

(b) $\begin{bmatrix} 1 & -2 \\ -2 & 1 \end{bmatrix}$

(c) $\begin{bmatrix} 1 & 1 & -2 \\ 0 & 0 & 4 \\ 0 & 0 & 6 \end{bmatrix}$

(d) $\begin{bmatrix} 1 & 2 & 3 \\ 2 & -1 & 2 \\ 3 & 2 & 2 \end{bmatrix}$

(e) $\begin{bmatrix} 10 & 0 & -2 \\ 0 & -6 & 1 \\ -2 & 1 & 0 \end{bmatrix}$

(f) $\begin{bmatrix} 1 & 1 & 1 \\ 0 & 2 & 2 \\ 0 & 2 & 5 \end{bmatrix}$

(g) $\begin{bmatrix} 3 & -1 & 4 \\ 0 & 5 & 2 \\ 0 & 0 & -1 \end{bmatrix}$

(h) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 3 & 4 \\ 0 & 0 & 3 \end{bmatrix}$

11. Find the particular solution to the differential equation $\begin{bmatrix} x'(t) \\ y'(t) \end{bmatrix} = \begin{bmatrix} 2 & 3 \\ 3 & 2 \end{bmatrix} \begin{bmatrix} x(t) \\ y(t) \end{bmatrix}$ with initial conditions $\begin{bmatrix} x(0) \\ y(0) \end{bmatrix} = \begin{bmatrix} 2 \\ 4 \end{bmatrix}$.

12. Let $A = \begin{bmatrix} 5 & 1 \\ 0 & 5 \end{bmatrix}$, which of the following statement is TRUE?

(a) A is both invertible and diagonalizable.

(b) A is invertible but not diagonalizable.

(c) A is diagonalizable but not invertible.

(d) A is neither invertible nor diagonalizable.

(e) None of the above.

13. if A has eigenvalues of 0,1,2, what are the eigenvalues of A(A-I)(A-2I)?

14. Find U and T for the matrix $A = \begin{bmatrix} 5 & -3 \\ 4 & -2 \end{bmatrix}$

15. What are the eigenvalues λ and frequencies ω and the general solution to $\frac{d^2u}{dt^2} = \begin{bmatrix} -5 & 4 \\ 4 & -5 \end{bmatrix} u$

16. Which 3×3 symmetric matrices A produce the following functions $f = x^T Ax$? which one is positive definite?

    - $f = 2(x_1^2 + x_2^2 + x_3^2 - x_1x_2 - x_2x_3)$
    - $f = 2(x_1^2 + x_2^2 + x_3^2 - x_1x_2 - x_1x_3 - x_2x_3)$

17. If A and B are positive definite, then A+B is positive definite. TRUE or FALSE?

18. Reduce to sum of squares, $3u^2 - 2\sqrt{2}uv + 2v^2 = 1$

19. Find 3 by 3 matrix A and its pivots, rank, eigenvalue, and determinant, where $\begin{bmatrix} x_1 & x_2 & x_3 \end{bmatrix} A \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 4(x_1 - x_2 + 2x_3)^2$