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PRACTICE FINAL EXAM MA511

EACH QUESTION IS WORTH 10 POINTS.
CIRCLE EXACTLY ONE ANSWER CHOICE FOR EACH PROBLEM.

- 1. Which of the following sets of vectors are linearly independent?
  i) $(0, 0, \pi), (0, -1, 1), (0, 2, 3)$
  ii) $(1, 2, 3), (4, 5, 6), (7, 8, 9)$
  iii) $(1, 0, 0), (0, 0, 0), (0, 0, 1)$.

- A. i)
- B. ii)
- C. iii)
- D. i) and iii)
- E. None of these

- 2. Find the inverse of the matrix $L = \begin{pmatrix} 2 & -1 \\ 8 & -5 \end{pmatrix}$.

A. $\begin{pmatrix} 5 & -1 \\ 4 & -1 \end{pmatrix}$ B. $\begin{pmatrix} \frac{5}{2} & -\frac{1}{2} \\ 4 & 1 \end{pmatrix}$ C. $\begin{pmatrix} \frac{5}{2} & 1 \\ 4 & 2 \end{pmatrix}$ D. $\begin{pmatrix} \frac{5}{2} & -\frac{1}{2} \\ 4 & -1 \end{pmatrix}$ E. L has no inverse

- 3 . Suppose that the system $Bx = b$, where B is a $n \times n$ matrix and $b \in \mathbb{R}^n$, has no solution. Which of the following statements are true?
  i) B does not have 0 as an eigenvalue.
  ii) The system $Bx = 0$ has infinitely many solutions.
  iii) The rank of B is less than n.

  - A. iii) only
  - B. i) and ii)
  - C. i) and iii)
  - D. ii) and iii)
  - E. All of these

- 4. Find the rank of the matrix $C = \begin{pmatrix} 0 & 1 & 2 & 3 \\ 0 & -2 & -4 & -6 \\ 0 & \sqrt{2} & 2\sqrt{2} & 3\sqrt{2} \end{pmatrix}$.

  - A. 0
  - B. 1
  - C. 2
  - D. 3
  - E. 4

- 5. Let $a \in \mathbb{R}^7$ be a unit vector and let $P = aa^T$ be the projection matrix onto the line

$$l = \{sa, \quad s \in \mathbb{R}\}.$$

Then, the eigenvalues of P are

- A. 1 and 7
- B. 0 and 7
- C. 0 and 1
- D. 0, 1, and 7
- E. 0, 1, and $\sqrt{7}$

- 6. Let D be a 5×5 matrix such that $D^2 = 0$. Then, e^{-Dt} is equal to

- A. $e^{\mathbb{I}t}$
- B. $e^{-\mathbb{I}t}$
- C. $e^{\mathbb{I}+Dt}$
- D. $\mathbb{I} - Dt$
- E. $\mathbb{I} + Dt$

- 7. The system $\dfrac{d}{dt}\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} 3 & 4 \\ 3 & 2 \end{pmatrix}\begin{pmatrix} y_1 \\ y_2 \end{pmatrix}$

- A. does not have any nonzero solution
- B. has only periodic solutions
- C. has only unbounded solutions
- D. has only solutions that decay to 0 as $t \to \infty$
- E. has some unbounded solutions

- 8. $\lambda = 0$ is an eigenvalue of $E = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix}$. Find a basis for the eigenspace S_0.

A. $\begin{pmatrix} 1 \\ 4 \\ 7 \end{pmatrix}, \begin{pmatrix} 2 \\ 5 \\ 8 \end{pmatrix}$ B. $\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}, \begin{pmatrix} 4 \\ 5 \\ 6 \end{pmatrix}$ C. $\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$ D. $\begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}$ E. None of these

- 9. Let V and W be subspaces of $\mathcal{C}[0,1]$, the space of continuous functions on the interval $[0,1]$. If $\dim V = 13$, $\dim W = 12$, and $\dim V \cap W = 7$, then $\dim(V + W)$
  - A. $=18$
  - B. $=1$
  - C. $=25$
  - D. $=20$
  - E. cannot be determined

- 10. Let F be a singular $n \times n$ matrix. Which of the following is <u>not</u> necessarily true for F?
  - A. $\det e^F \neq 0$
  - B. $\det(F + \mathbb{I}) = 1 + \det F$
  - C. F is unitarily similar to an upper triangular matrix
  - D. $\det(Fe^F) = \frac{\det F}{\det(e^{-F})}$
  - E. $\text{rank}\,(FF^T) < n$

- 11. Let H be a $n \times n$ (real) symmetric matrix. Which of the following statements is false?

  - A. H is unitarily similar to a diagonal matrix
  - B. H has an orthonormal set of n eigenvectors
  - C. $\|Hx\| = \|x\|$ for all $x \in \mathbb{R}^n$
  - D. $\mathcal{N}(H)$ is the orthogonal complement of $\mathcal{C}(H)$
  - E. H has only real eigenvalues

- 12. A Jordan canonical form for $K = \begin{pmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ is

  A. $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ B. $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix}$ C. $\begin{pmatrix} 3 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix}$ D. $\begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix}$ E. $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 3 \end{pmatrix}$

- 13. Let $r \in \mathbb{R}$ satisfy $0 < r < 1$. Then, the norm of the matrix $\begin{pmatrix} r & 1 \\ -1 & r \end{pmatrix}$ is
  - A. 1
  - B. $\frac{1+r}{1-r}$
  - C. $\sqrt{1+r^2}$
  - D. r
  - E. $1+r$

- 14. The matrix $\begin{pmatrix} 2 & -1 & 0 \\ -1 & 2 & 0 \\ 0 & 0 & -1 \end{pmatrix}$ is

  A. indefinite B. positive semidefinite C. positive definite D. negative definite
  E. negative semidefinite

- 15. Determine which of the following functions $T : \mathcal{P}_4 \longrightarrow \mathcal{P}_2$ are linear transformations:
  a) $T(p) = \dfrac{d^2p}{dt^2}$
  b) $T(p) = t\,\dfrac{d^3p}{dt^3}$
  c) $T(p) = t + \dfrac{d^3p}{dt^3}$

- A. a) only
- B. a) and b)
- C. a) and c)
- D. All of these
- E. None of these

- 16. Determine which of the following subsets W of $V = \mathcal{C}[-\pi, \pi]$ are subspaces (V is the vector space of continuous functions defined on the interval $[-\pi, \pi]$):
  a) $W = \{f \in V : f(0) = 0\}$
  b) $W = \{f \in V : f(0) = f(1)\}$
  c) $W = \{f \in V : \int_0^1 f(t)\,dt = 0\}$

- A. a) only
- B. a) and b)
- C. a) and c)
- D. All of these
- E. None of these

- 17. Let $V = \mathcal{P}_5$ and $W = \{p \in V : p(-1) = 0 \text{ and } p'(2) = 0\}$. Then, the dimension of W is

  - A. 2
  - B. 3
  - C. 4
  - D. 5
  - E. 6

- 18. Consider the vector space $V = \mathcal{C}[0,1]$ with the inner product $(v,w) = \int_0^1 v(t)\,w(t)\,dt$. If you apply Gram-Schmidt orthonormalization to $v_1(t) = 1$ and $v_2(t) = t^2$, then v_2 is replaced by what function?

  - A. t^2
  - B. $\dfrac{3}{2}\sqrt{5}\left(t^2 - \dfrac{1}{3}\right)$
  - C. $t^2 - \frac{1}{3}$
  - D. $t^2 - \frac{2}{3}$
  - E. $\sqrt{5}\left(t^2 - \dfrac{2}{3}\right)$

- 19. The singular values of $\begin{pmatrix} 0 & 1 & 1 \\ -1 & 0 & -2 \\ 0 & 0 & 0 \end{pmatrix}$ are

  - A. $0, i$, and $-i$
  - B. $0, -1$, and -1
  - C. 1 and 6
  - D. 1 and $\sqrt{6}$
  - E. 1 and 1

- 20. Find the inverse of the matrix A,

$$A = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 1 & 1 \\ 1 & e^{2\pi i/3} & e^{4\pi i/3} \\ 1 & e^{4\pi i/3} & e^{2\pi i/3} \end{pmatrix}.$$

  - A. A is singular
  - B. $AA^H - A^H A$
  - C. A
  - D. A^H
  - E. None of these