MATH 511 HOMEWORK (ON GRADESCOPE)

SIAMAK YASSEMI

Homework # 4

Due on July 07, 2024, 11:59 PM

Section 4.2: P208 #12, #16, P210 #35,
Section 4.3: P216 #7,
Section 4.4: P226 #5, P227 #13, P228 #31,

4.2.p208.#12

[chapter].[section].[page].[problem]

12. Use row operations to verify that the 3 by 3 "Vandermonde determinant" is

$$\det \begin{bmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{bmatrix} = (b-a)(c-a)(c-b).$$

4.2.p208.#16

[chapter].[section].[page].[problem]

16. Find these 4 by 4 determinants by Gaussian elimination:

$$\det\begin{bmatrix} 11 & 12 & 13 & 14 \\ 21 & 22 & 23 & 24 \\ 31 & 32 & 33 & 34 \\ 41 & 42 & 43 & 44 \end{bmatrix} \quad \text{and} \quad \det\begin{bmatrix} 1 & t & t^2 & t^3 \\ t & 1 & t & t^2 \\ t^2 & t & 1 & t \\ t^3 & t^2 & t & 1 \end{bmatrix}.$$

4.2.p210.#35

[chapter].[section].[page].[problem]

35. Suppose the 4 by 4 matrix M has four equal rows all containing a, b, c, d. We know that $\det(M) = 0$. The problem is to find $\det(I + M)$ by any method:

$$\det(I+M) = \begin{vmatrix} 1+a & b & c & d \\ a & 1+b & c & d \\ a & b & 1+c & d \\ a & b & c & 1+d \end{vmatrix}.$$

Partial credit if you find this determinant when $a = b = c = d = 1$. Sudden death if you say that $\det(I + M) = \det I + \det M$.

4.3.p216.#07

[chapter].[section].[page].[problem]

7. (a) Evaluate this determinant by cofactors of row 1:

$$\begin{vmatrix} 4 & 4 & 4 & 4 \\ 1 & 2 & 0 & 1 \\ 2 & 0 & 1 & 2 \\ 1 & 1 & 0 & 2 \end{vmatrix}.$$

(b) Check by subtracting column 1 from the other columns and recomputing.

4.4.p226.#05

[chapter].[section].[page].[problem]

5. (a) Draw the triangle with vertices $A = (2, 2)$, $B = (-1, 3)$, and $C = (0, 0)$. By regarding it as half of a parallelogram, explain why its area equals

$$\text{area}\,(ABC) = \frac{1}{2}\det\begin{bmatrix} 2 & 2 \\ -1 & 3 \end{bmatrix}.$$

(b) Move the third vertex to $C = (1, -4)$ and justify the formula

$$\text{area}\,(ABC) = \frac{\mathbf{1}}{\mathbf{2}}\,\mathbf{det}\begin{bmatrix} x_1 & y_1 & \mathbf{1} \\ x_2 & y_2 & \mathbf{1} \\ x_3 & y_3 & \mathbf{1} \end{bmatrix} = \frac{1}{2}\det\begin{bmatrix} 2 & 2 & 1 \\ -1 & 3 & 1 \\ 1 & -4 & 1 \end{bmatrix}.$$

Hint: Subtracting the last row from each of the others leaves

$$\det\begin{bmatrix} 2 & 2 & 1 \\ -1 & 3 & 1 \\ 1 & -4 & 1 \end{bmatrix} = \det\begin{bmatrix} 1 & 6 & 0 \\ -2 & 7 & 0 \\ 1 & -4 & 1 \end{bmatrix} = \det\begin{bmatrix} 1 & 6 \\ -2 & 7 \end{bmatrix}.$$

Sketch $A' = (1, 6)$, $B' = (-2, 7)$, $C' = (0, 0)$ and their relation to A, B, C.

4.4.p227.#13

[chapter].[section].[page].[problem]

Problems 13–17 are about Cramer's Rule for $\boldsymbol{x} = A^{-1}\boldsymbol{b}$.

13. Solve these linear equations by Cramer's Rule $x_j = \det B_j / \det A$:

(a) $$\begin{aligned} 2x_1 + 5x_2 &= 1 \\ x_1 + 4x_2 &= 2. \end{aligned}$$

(b) $$\begin{aligned} 2x_1 + x_2 &= 1 \\ x_1 + 2x_2 + x_3 &= 0 \\ x_2 + 2x_3 &= 0. \end{aligned}$$

4.4.p228.#31

[chapter].[section].[page].[problem]

Problems 27–36 are about area and volume by determinants.

31. The Hadamard matrix H has orthogonal rows. The box is a hypercube!

$$\text{What is } \det H = \begin{vmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \\ 1 & -1 & 1 & -1 \end{vmatrix} = \text{volume of a hypercube in } \mathbf{R}^4?$$