MATH 511 HOMEWORK (ON GRADESCOPE)

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Homework # 1

Due on Jun 16, 2024, 11:59 PM

Section 1.3: Page17 #17, P19 #31,
Section 1.4: P28 #24,
Section 1.5: P43 #29, P44 #38,
Section 1.6: P55 #40, #41, P57 #56

1.3.p17.#17

[chapter].[section].[page].[problem]

Problems 10–19 study elimination on 3 by 3 systems (and possible failure).

17. Which number q makes this system singular and which right-hand side t gives it infinitely many solutions? Find the solution that has $z = 1$.

$$x + 4y - 2z = 1$$
$$x + 7y - 6z = 6$$
$$3y + qz = t.$$

$$A = \left[\begin{array}{ccc|c} 1 & 4 & -2 & 1 \\ 1 & 7 & -6 & 6 \\ \emptyset & 3 & q & t \end{array}\right] \xrightarrow{A_{12}(-1)} \left[\begin{array}{ccc|c} 1 & 4 & -2 & 1 \\ 1(-1)+1 & 4(-1)+7 & -2(-1)-6 & 1(-1)+6 \\ \emptyset & 3 & q & t \end{array}\right] = \left[\begin{array}{ccc|c} 1 & 4 & -2 & 1 \\ \emptyset & 3 & -4 & 5 \\ \emptyset & 3 & q & t \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & 4 & -2 & 1 \\ \emptyset & 3 & -4 & 5 \\ \emptyset & 3 & q & t \end{array}\right] \xrightarrow{A_{23}(-1)} \left[\begin{array}{ccc|c} 1 & 4 & -2 & 1 \\ \emptyset & 3 & -4 & 5 \\ \emptyset(-1)+\emptyset & 3(-1)+3 & -4(-1)+q & 5(-1)+t \end{array}\right] = \left[\begin{array}{ccc|c} 1 & 4 & -2 & 1 \\ \emptyset & 3 & -4 & 5 \\ \emptyset & \emptyset & q+4 & t-5 \end{array}\right]$$

THE SYSTEM IS SINGULAR WHEN $q+4=\emptyset \therefore q=-4$

THE SYSTEM HAS INFINITELY MANY SOLUTIONS WHEN $t-5=\emptyset \therefore t=5$

$$\begin{aligned} x + 4y - 2z &= 1 \\ x + 7y - 6z &= 6 \\ 3y + qz &= t \end{aligned} \quad \xrightarrow{z=1,\ q=-4,\ t=5} \quad \begin{aligned} x + 4y - 2 &= 1 \\ x + 7y - 6 &= 6 \\ 3y - 4 &= 5 \rightarrow 3y = 9 \end{aligned}$$

$$y = 3$$

$$\begin{aligned} x + 4(3) - 2 &= 1 \\ x + 7(3) - 6 &= 6 \end{aligned} \rightarrow \begin{aligned} x + 12 - 2 &= 1 \\ x + 21 - 6 &= 6 \end{aligned}$$

$$\begin{aligned} x + 12 - 2 &= 1 \\ x + 10 &= 1 \\ x &= -9 \end{aligned} \qquad \begin{aligned} x + 21 - 6 &= 6 \\ x + 15 &= 6 \\ x &= -9 \end{aligned} \qquad (-9, 3, 1)$$

<u>DEF.</u> IF A SYSTEM OF LIN. EQUATIONS HAS JUST UNIQUE SOLUTION IT IS CALLED <u>NON-SINGULAR.</u>
OTHERWISE, (IF IT HAS NO SOL. OR INFINITELY MANY) IT IS CALLED SINGULAR.

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1.3.p19.#31

[chapter].[section].[page].[problem]

31. For which three numbers a will elimination fail to give three pivots?

$$\begin{aligned} ax + 2y + 3z &= b_1 \\ ax + ay + 4z &= b_2 \\ ax + ay + az &= b_3. \end{aligned}$$

$$\left[\begin{array}{ccc|c} a & 2 & 3 & b_1 \\ a & a & 4 & b_2 \\ a & a & a & b_3 \end{array}\right] \xrightarrow{A_{12}(-1)} \left[\begin{array}{ccc|c} a & 2 & 3 & b_1 \\ a(-1)+a & 2(-1)+a & 3(-1)+4 & b_1(-1)+b_2 \\ a & a & a & b_3 \end{array}\right] = \left[\begin{array}{ccc|c} a & 2 & 3 & b_1 \\ 0 & a-2 & 1 & b_2-b_1 \\ a & a & a & b_3 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} a & 2 & 3 & b_1 \\ 0 & a-2 & 1 & b_2-b_1 \\ a & a & a & b_3 \end{array}\right] \xrightarrow{A_{13}(-1)} \left[\begin{array}{ccc|c} a & 2 & 3 & b_1 \\ 0 & a-2 & 1 & b_2-b_1 \\ a(-1)+a & 2(-1)+a & 3(-1)+a & b_1(-1)+b_3 \end{array}\right] = \left[\begin{array}{ccc|c} a & 2 & 3 & b_1 \\ 0 & a-2 & 1 & b_2-b_1 \\ 0 & a-2 & a-3 & b_3-b_1 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} a & 2 & 3 & b_1 \\ 0 & a-2 & 1 & b_2-b_1 \\ 0 & a-2 & a-3 & b_3-b_1 \end{array}\right] \xrightarrow{A_{23}(-1)} \left[\begin{array}{ccc|c} a & 2 & 3 & b_1 \\ 0 & a-2 & 1 & b_2-b_1 \\ 0(-1)+0 & (a-2)(-1)+a-2 & 1(-1)+a-3 & (b_2-b_1)(-1)+b_3-b_1 \end{array}\right] = \swarrow$$

$$\left[\begin{array}{ccc|c} a & 2 & 3 & b_1 \\ 0 & a-2 & 1 & b_2-b_1 \\ 0 & 0 & a-4 & b_3-b_2 \end{array}\right] \qquad \begin{aligned} a &= 0 \\ a-2 &= 0 \\ a-4 &= 0 \end{aligned} \longrightarrow \begin{aligned} a &= 0 \\ a &= 2 \\ a &= 4 \end{aligned} \begin{aligned} &\xrightarrow{\text{PIVOT COLUMNS}} 2+3 \\ &\rightarrow 1+3 \\ &\rightarrow 1+2 \end{aligned}$$

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1.4.p28.#24

[chapter].[section].[page].[problem]

Problems 22–31 are about elimination matrices.

24. Which three matrices E_{21}, E_{31}, E_{32} put A into triangular form U?

$$A = \begin{bmatrix} 1 & 1 & 0 \\ 4 & 6 & 1 \\ -2 & 2 & 0 \end{bmatrix} \quad \text{and} \quad E_{32}E_{31}E_{21}A = U.$$

Multiply those E's to get one matrix M that does elimination: $MA = U$.

$$A = \begin{bmatrix} 1 & 1 & 0 \\ 4 & 6 & 1 \\ -2 & 2 & 0 \end{bmatrix} \xrightarrow{A_{12}(-4)} \begin{bmatrix} 1 & 1 & 0 \\ 1(-4)+4 & 1(-4)+6 & 0(-4)+1 \\ -2 & 2 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 1 \\ -2 & 2 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 1 \\ -2 & 2 & 0 \end{bmatrix} \xrightarrow{A_{13}(2)} \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 1 \\ 1(2)-2 & 1(2)+2 & 0(2)+0 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 1 \\ 0 & 4 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 1 \\ 0 & 4 & 0 \end{bmatrix} \xrightarrow{A_{23}(-2)} \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 1 \\ 0(-2)+0 & 2(-2)+4 & 1(-2)+0 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 1 \\ 0 & 0 & -2 \end{bmatrix} = U$$

$$E_{21} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \xrightarrow{A_{12}(-4)} \begin{bmatrix} 1 & 0 & 0 \\ -4 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad \Big| \quad E_{31} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \xrightarrow{A_{13}(2)} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 2 & 0 & 1 \end{bmatrix} \quad \Big| \quad E_{32} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \xrightarrow{A_{23}(-2)} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -2 & 1 \end{bmatrix}$$

$$E_{32}E_{31}E_{21}A = U$$

$$\begin{array}{cccccc} E_{32} & E_{31} & E_{21} & A & & U \\ \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -2 & 1 \end{bmatrix} & \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 2 & 0 & 1 \end{bmatrix} & \begin{bmatrix} 1 & 0 & 0 \\ -4 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} & \begin{bmatrix} 1 & 1 & 0 \\ 4 & 6 & 1 \\ -2 & 2 & 0 \end{bmatrix} & = & \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 1 \\ 0 & 0 & -2 \end{bmatrix} \end{array}$$

$\times$

$$\begin{bmatrix} 1+0+0 & 0+0+0 & 0+0+0 \\ 0+0+0 & 0+1+0 & 0+0+0 \\ 0+0+2 & 0-2+0 & 0+0+1 \end{bmatrix} = \begin{array}{ccccc} E_{32}\cdot E_{31} & E_{21} & A & & U \\ \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 2 & -2 & 1 \end{bmatrix} & \begin{bmatrix} 1 & 0 & 0 \\ -4 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} & \begin{bmatrix} 1 & 1 & 0 \\ 4 & 6 & 1 \\ -2 & 2 & 0 \end{bmatrix} & = & \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 1 \\ 0 & 0 & -2 \end{bmatrix} \end{array}$$

$\times$

$$\begin{bmatrix} 1+0+0 & 0+0+0 & 0+0+0 \\ 0-4+0 & 0+1+0 & 0+0+0 \\ 2+8+0 & 0-2+0 & 0+0+1 \end{bmatrix} = \begin{array}{cccc} M & A & & U \\ \begin{bmatrix} 1 & 0 & 0 \\ -4 & 1 & 0 \\ 10 & -2 & 1 \end{bmatrix} & \begin{bmatrix} 1 & 1 & 0 \\ 4 & 6 & 1 \\ -2 & 2 & 0 \end{bmatrix} & = & \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 1 \\ 0 & 0 & -2 \end{bmatrix} \end{array}$$

$\times$ $\qquad = \checkmark$

$$\begin{bmatrix} 1+0+0 & 1+0+0 & 0+0+0 \\ -4+4+0 & -4+6+0 & 0+1+0 \\ 10-8-2 & 10-12+2 & 0-2+0 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 1 \\ 0 & 0 & -2 \end{bmatrix}$$

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1.5.p43.#29

[chapter].[section].[page].[problem]

Problems 20–31 compute the factorization $A = LU$ (and also $A = LDU$).

29. (Recommended) Compute L and U for the symmetric matrix

$$A = \begin{bmatrix} a & a & a & a \\ a & b & b & b \\ a & b & c & c \\ a & b & c & d \end{bmatrix}.$$

Find four conditions on a, b, c, d to get $A = LU$ with four pivots.

$$A = \begin{bmatrix} a & a & a & a \\ a & b & b & b \\ a & b & c & c \\ a & b & c & d \end{bmatrix} \xrightarrow{A_{12}(-1)} \begin{bmatrix} a & a & a & a \\ a(-1)+a & a(-1)+b & a(-1)+b & a(-1)+b \\ a & b & c & c \\ a & b & c & d \end{bmatrix} = \begin{bmatrix} a & a & a & a \\ 0 & b-a & b-a & b-a \\ a & b & c & c \\ a & b & c & d \end{bmatrix} \xrightarrow{A_{13}(-1)} \begin{bmatrix} a & a & a & a \\ 0 & b-a & b-a & b-a \\ a(-1)+a & a(-1)+b & a(-1)+c & a(-1)+c \\ a & b & c & d \end{bmatrix}$$

$$\begin{bmatrix} a & a & a & a \\ 0 & b-a & b-a & b-a \\ 0 & b-a & c-a & c-a \\ a & b & c & d \end{bmatrix} \xrightarrow{A_{14}(-1)} \begin{bmatrix} a & a & a & a \\ 0 & b-a & b-a & b-a \\ 0 & b-a & c-a & c-a \\ a(-1)+a & a(-1)+b & a(-1)+c & a(-1)+d \end{bmatrix} = \begin{bmatrix} a & a & a & a \\ 0 & b-a & b-a & b-a \\ 0 & b-a & c-a & c-a \\ 0 & b-a & c-a & d-a \end{bmatrix} \xrightarrow{A_{23}(-1)} \begin{bmatrix} a & a & a & a \\ 0 & b-a & b-a & b-a \\ 0(-1)+0 & (b-a)(-1)+b-a & (b-a)(-1)+c-a & (b-a)(-1)+c-a \\ 0 & b-a & c-a & d-a \end{bmatrix}$$

$$\begin{bmatrix} a & a & a & a \\ 0 & b-a & b-a & b-a \\ 0 & 0 & c-b & c-b \\ 0 & b-a & c-a & d-a \end{bmatrix} \xrightarrow{A_{24}(-1)} \begin{bmatrix} a & a & a & a \\ 0 & b-a & b-a & b-a \\ 0 & 0 & c-b & c-b \\ 0(-1)+0 & (b-a)(-1)+(b-a) & (b-a)(-1)+c-a & (b-a)(-1)+d-a \end{bmatrix} = \begin{bmatrix} a & a & a & a \\ 0 & b-a & b-a & b-a \\ 0 & 0 & c-b & c-b \\ 0 & 0 & c-b & d-b \end{bmatrix} \xrightarrow{A_{34}(-1)} \begin{bmatrix} a & a & a & a \\ 0 & b-a & b-a & b-a \\ 0 & 0 & c-b & c-b \\ 0(-1)+0 & 0(-1)+0 & (c-b)(-1)+c-b & (c-b)(-1)+d-b \end{bmatrix}$$

$$U = \begin{bmatrix} a & a & a & a \\ 0 & b-a & b-a & b-a \\ 0 & 0 & c-b & c-b \\ 0 & 0 & 0 & d-c \end{bmatrix} \qquad A = (E_{21})^{-1}(E_{31})^{-1}(E_{41})^{-1}(E_{32})^{-1}(E_{42})^{-1}(E_{43})^{-1}\, U$$

$$A = \underset{(E_{21})^{-1}}{\begin{bmatrix} 1 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}} \underset{(E_{31})^{-1}}{\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}} \underset{(E_{41})^{-1}}{\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -1 & 0 & 0 & 1 \end{bmatrix}} \underset{(E_{32})^{-1}}{\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}} \underset{(E_{42})^{-1}}{\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 1 \end{bmatrix}} \underset{(E_{43})^{-1}}{\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & -1 & 1 \end{bmatrix}} U$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -1 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & -1 & 1 \end{bmatrix} U$$

$$L = \begin{bmatrix} 1 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 \\ -1 & -1 & 1 & 0 \\ -1 & -1 & -1 & 1 \end{bmatrix}$$

$$\underset{A}{\begin{bmatrix} a & a & a & a \\ a & b & b & b \\ a & b & c & c \\ a & b & c & d \end{bmatrix}} = \underset{L}{\begin{bmatrix} 1 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 \\ -1 & -1 & 1 & 0 \\ -1 & -1 & -1 & 1 \end{bmatrix}} \underset{U}{\begin{bmatrix} a & a & a & a \\ 0 & b-a & b-a & b-a \\ 0 & 0 & c-b & c-b \\ 0 & 0 & 0 & d-c \end{bmatrix}}$$

FOR $A=LU$ TO HAVE FOUR POSITIONS, THE FOLLOWING CONDITION MUST BE TRUE: $a \neq 0$, $b \neq a$, $c \neq b$, $d \neq c$

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1.5.p44.#38

[chapter].[section].[page].[problem]

38. (Review) For which numbers c is $A = LU$ impossible—with three pivots?

$$A = \begin{bmatrix} 1 & 2 & 0 \\ 3 & c & 1 \\ 0 & 1 & 1 \end{bmatrix}.$$

$$A = \begin{bmatrix} 1 & 2 & 0 \\ 3 & C & 1 \\ 0 & 1 & 1 \end{bmatrix} \xrightarrow{A_{12}(-3)} \begin{bmatrix} 1 & 2 & 0 \\ 1(-3)+3 & 2(-3)+C & 0(-3)+1 \\ 0 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 0 \\ 0 & C-6 & 1 \\ 0 & 1 & 1 \end{bmatrix} \xrightarrow{A_{23}\left(\frac{-1}{C-6}\right)} \begin{bmatrix} 1 & 2 & 0 \\ 0 & C-6 & 1 \\ 0\left(\frac{1}{C-6}\right)+0 & (C-6)\left(\frac{-1}{C-6}\right)+1 & 1\left(-\frac{1}{C-6}\right)+1 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 0 \\ 0 & C-6 & 1 \\ 0 & 0 & 1-\frac{1}{C-6} \end{bmatrix} = U$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \xrightarrow{A_{12}(3)} \begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \xrightarrow{A_{23}\left(\frac{1}{C-6}\right)} \begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 0 & C-6 & 1 \end{bmatrix} \therefore \overset{A}{\begin{bmatrix} 1 & 2 & 0 \\ 3 & C & 1 \\ 0 & 1 & 1 \end{bmatrix}} = \overset{L}{\begin{bmatrix} 1 & 2 & 0 \\ 0 & C-6 & 1 \\ 0 & 0 & 1-\frac{1}{C-6} \end{bmatrix}} \overset{U}{\begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 0 & C-6 & 1 \end{bmatrix}}$$

FOR U TO HAVE 3 PIVOTS AND FAIL A = LU, C MUST NOT EQUAL 6 OR 7.

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1.6.p55.#40

[chapter].[section].[page].[problem]

40. True or false (with a counterexample if false and a reason if true):

(a) A 4 by 4 matrix with a row of zeros is not invertible.
(b) A matrix with 1s down the main diagonal is invertible.
(c) If A is invertible then A^{-1} is invertible.
(d) If A^{T} is invertible then A is invertible.

A) TRUE AN INVERTIBLE MATRIX MUST HAVE ALL FOUR PIVOTS.

B) FALSE, RREF MATRIX MUST BE I. NOT ENOUGH INFO ABOUT ROWS.

C) TRUE, $(A^{-1})^{-1} = A$

D) TRUE, $(A^{T})^{-1} = (A^{-1})^{T}$

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1.6.p55.#41

[chapter].[section].[page].[problem]

41. For which three numbers c is this matrix not invertible, and why not?

$$A = \begin{bmatrix} 2 & c & c \\ c & c & c \\ 8 & 7 & c \end{bmatrix}.$$

$$A = \begin{bmatrix} 2 & c & c \\ c & c & c \\ 8 & 7 & c \end{bmatrix} \xrightarrow{A_{12}\left(-\frac{c}{2}\right)} \begin{bmatrix} 2 & c & c \\ 2\left(-\frac{c}{2}\right)+c & c\left(-\frac{c}{2}\right)+c & c\left(-\frac{c}{2}\right)+c \\ 8 & 7 & c \end{bmatrix} = \begin{bmatrix} 2 & c & c \\ 0 & c-\frac{c^2}{2} & c-\frac{c^2}{2} \\ 8 & 7 & c \end{bmatrix} \xrightarrow{A_{13}(-4)} \begin{bmatrix} 2 & c & c \\ 0 & c-\frac{c^2}{2} & c-\frac{c^2}{2} \\ 2(-4)+8 & c(-4)+7 & c(-4)+c \end{bmatrix} = \begin{bmatrix} 2 & c & c \\ 0 & c-\frac{c^2}{2} & c-\frac{c^2}{2} \\ 0 & 7-4c & -3c \end{bmatrix} \xrightarrow{M\left(\frac{1}{c-\frac{c^2}{2}}\right)} \begin{bmatrix} 2 & c & c \\ 0 & 1 & 1 \\ 0 & 7-4c & -3c \end{bmatrix} \xrightarrow{A_{12}^{-}(7-4c)} \begin{bmatrix} 2 & c & c \\ 0 & & \\ & & \end{bmatrix}$$

$$\begin{bmatrix} 2 & c & c \\ 0 & 1 & 1 \\ 0 & 7-4c & -3c \end{bmatrix} \xrightarrow{A_{23}^{-}(7-4c)} \begin{bmatrix} 2 & c & c \\ 0 & 1 & 1 \\ 0(-7+4c)+0 & 1(-7+4c)+7-4c & 1(-7+4c)-3c \end{bmatrix} = \begin{bmatrix} 2 & c & c \\ 0 & 1 & 1 \\ 0 & 0 & c-7 \end{bmatrix}$$

THE FOLLOWING THREE C'S MAKE THE MATRIX INVERTIBLE.
C = 0, MAKES THE MIDDLE ROW 0, NO PIVOT
C = 2, MAKES ROWS 1 + 2 THE SAME
C = 7, MAKES ROW THREE OF U ALL 0'S.

$$c - \frac{c^2}{2} \text{ w/ } c = 7 \neq 0$$

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1.6.p57.#56

[chapter].[section].[page].[problem]

Problems 56–60 are about symmetric matrices and their factorizations.

56. If $A = A^{\mathrm{T}}$ and $B = B^{\mathrm{T}}$, which of these matrices are certainly symmetric?
(a) $A^2 - B^2$ (b) $(A+B)(A-B)$ (c) ABA (d) $ABAB$.

A SYMMETRIC MATRIX IS A MATRIX THAT IS EQUAL TO IT'S OWN TRANSPOSE.

A) $A^2 - B^2 = AA - BB$

$$\begin{bmatrix} 1 & a & a \\ a & 1 & a \\ a & a & 1 \end{bmatrix} \begin{bmatrix} 1 & a & a \\ a & 1 & a \\ a & a & 1 \end{bmatrix}$$

SINCE $AA = A^2$ IS SYMMETRIC $A^2 - B^2$ IS SYMMETRIC

X

$$\begin{bmatrix} 2a^2+1 & a^2+2a & a^2+2a \\ a^2+2a & 2a^2+1 & a^2+2a \\ a^2+2a & a^2+2a & 2a^2+1 \end{bmatrix}$$ SYMMETRIC ✓

B) $(A+B)(A-B)$
$= A^2 - AB + BA - B^2$
$A^2 - B^2 - AB + BA$
ALTHOUGH $A^2 - B^2$ IS SYMMETRIC
$BA - AB$ CAN NOT BE CERTAINLY SYMMETRIC.

C) ABA
$A = A^T \quad B = B^T$
$ABA = A^T B^T A^T = (ABA)^T$
SINCE A + B ARE SYMMETRIC
C IS CERTAINLY SYMMETRIC

D) $ABAB = (ABAB)^T = B^T A^T B^T A^T = BABA$
AB DOES NOT NECESSARILY EQUAL BA
NOT SYMMETRIC

A) $A^2 - B^2$
$AA - BB$
SYMMETRIC

B) $(A+B)(A-B)$
$AA - AB + BA - BB^2$
$AB \neq BA$
NOT SYMMETRIC

C) ABA

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