5 $A = \begin{bmatrix} 3 & 4 & 2 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix} \rightarrow \begin{pmatrix} 2 \\ -2 \\ 1 \end{pmatrix}, 0$

$0+1+3 = 3+1+0$ ✓ $\quad \begin{pmatrix} -2 \\ 1 \\ 0 \end{pmatrix}, 1$

$0 \times 1 \times 3 = 3 \times 1 \times 0$ ✓

$\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, 3$

$B = \begin{bmatrix} 0 & 0 & 2 \\ 0 & 2 & 0 \\ 2 & 0 & 0 \end{bmatrix} \rightarrow \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix}, -2$

$(-2)+2+2 = 0+2+0$ ✓ $\quad \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, 2$

$(-2) \times (2) \times (2) = -8$ ✓

$\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}, 2$

14 $A = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix} \rightarrow \begin{pmatrix} -1 \\ 0 \\ 0 \\ 0 \end{pmatrix}, 0$

$\text{rank}(A) = 1$ $\quad \begin{pmatrix} -1 \\ 0 \\ 1 \\ 0 \end{pmatrix}, 0$

$\begin{pmatrix} -1 \\ 0 \\ 0 \\ 1 \end{pmatrix}, 0$

$\boxed{\begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}, 4}$

$C = \begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix} \rightarrow \begin{pmatrix} -1 \\ 1 \\ 1 \\ 1 \end{pmatrix}, 0$

$\text{rank}(C) = 2$ $\quad \begin{pmatrix} 0 \\ -1 \\ 0 \\ 1 \end{pmatrix}, 0$

$\boxed{\begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}, 2}$

$\boxed{\begin{pmatrix} -1 \\ 1 \\ -1 \\ 1 \end{pmatrix}, -2}$

3. $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \rightarrow \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix}, 0$

$\begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix}, 0$

$\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, 3$

Examples: $S_1 = \begin{bmatrix} -1 & -1 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}$, $S_2 = \begin{bmatrix} 1 & -1 & -1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$

11. (a) (b) $A = \begin{bmatrix} 0.2 & 0.4 & 0.3 \\ 0.4 & 0.2 & 0.3 \\ 0.4 & 0.4 & 0.4 \end{bmatrix} \rightarrow \begin{pmatrix} 1 \\ 1 \\ -2 \end{pmatrix}, 0$ (a)

$\left.\begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix}, \frac{-1}{5} \right\}$ (b)

$\left.\begin{pmatrix} 3/4 \\ 3/4 \\ 1 \end{pmatrix}, 1 \right\}$

(c) $\begin{pmatrix} -1/2 & -1 & 3/4 \\ -1/2 & 1 & 3/4 \\ 1 & 0 & 1 \end{pmatrix} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} -2/5 & -2/5 & 3/5 \\ -1/2 & 1/2 & 0 \\ 2/5 & 2/5 & 2/5 \end{pmatrix} \begin{pmatrix} 0 \\ 10 \\ 0 \end{pmatrix}$

$= \frac{1}{10} \begin{pmatrix} 5 & 5 & 3 \\ 3 & 3 & 3 \\ 4 & 4 & 4 \end{pmatrix} \begin{pmatrix} 0 \\ 10 \\ 0 \end{pmatrix} = \begin{pmatrix} 3 \\ 3 \\ 4 \end{pmatrix}$

↳ eigenvector for 1

15- $u_1(t) = \frac{1}{2}\begin{pmatrix} \cos(2t) + \cos(\sqrt{6}t) \\ \cos(\sqrt{6}t) - \cos(2t) \end{pmatrix}$

21- $c_1 e^{4t}\begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad c_2 e^{t}\begin{pmatrix} -1 \\ 1 \end{pmatrix}$

$u(0) = (5, -2) \rightarrow c_1 = 3, \; c_2 = -2 \longrightarrow$

$3e^{4t}\begin{pmatrix} 1 \\ 0 \end{pmatrix} - 2e^{t}\begin{pmatrix} -1 \\ 1 \end{pmatrix}$

31- $P = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix} \longrightarrow P^2 = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$

$P^3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = I$

$P^{100} = (P^3)^{33} \cdot P = I^{33} P = P = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}$

$\lambda_1 = 1 \quad , \lambda_2 = \frac{1}{2}(-1 + i\sqrt{3}) \; , \lambda_3 = \frac{1}{2}(-1 - i\sqrt{3})$

22. $A = \begin{bmatrix} 5 & -3 \\ 4 & -2 \end{bmatrix} \rightarrow V = \begin{pmatrix} \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} \\ \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \end{pmatrix}$

$T = \begin{pmatrix} 2 & -3 \\ 0 & 1 \end{pmatrix}$

$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix} \longrightarrow V = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}$

$T = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}$