MATH 511 HOMEWORK (ON GRADESCOPE)

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Homework # 5

Due on July 14, 2024, 11:59 PM

Section 5.1: P241 #5, P242 #14,
Section 5.2: P250 #3,
Section 5.3: P264 #11,
Section 5.4: P277 #15, P278 #21
Section 5.5: P291 #31,
Section 5.6: P303 #22

5.1.p241.#05

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5. Find the eigenvalues and eigenvectors of

$$A = \begin{bmatrix} 3 & 4 & 2 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} 0 & 0 & 2 \\ 0 & 2 & 0 \\ 2 & 0 & 0 \end{bmatrix}.$$

Check that $\lambda_1 + \lambda_2 + \lambda_3$ equals the trace and $\lambda_1\lambda_2\lambda_3$ equals the determinant.

5.1.p242.#14

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14. Find the rank and all four eigenvalues for both the matrix of ones and the checkerboard matrix:

$$A = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix} \quad \text{and} \quad C = \begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix}.$$

Which eigenvectors correspond to nonzero eigenvalues?

5.2.p250.#03

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3. Find *all* the eigenvalues and eigenvectors of

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

and write two different diagonalizing matrices S.

5.3.p264.#11

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11. (a) From the fact that column 1 + column 2 = 2(column 3), so the columns are linearly dependent, find one eigenvalue and one eigenvector of A:

$$A = \begin{bmatrix} .2 & .4 & .3 \\ .4 & .2 & .3 \\ .4 & .4 & .4 \end{bmatrix}.$$

(b) Find the other eigenvalues of A (it is Markov).

(c) If $u_0 = (0, 10, 0)$, find the limit of $A^k u_0$ as $k \to \infty$.

5.4.p277.#15

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15. Solve the second-order equation

$$\frac{d^2u}{dt^2} = \begin{bmatrix} -5 & -1 \\ -1 & -5 \end{bmatrix} u \quad \text{with} \quad u(0) = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \quad \text{and} \quad u'(0) = \begin{bmatrix} 0 \\ 0 \end{bmatrix}.$$

5.4.p278.#21

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21. Find λ's and x's so that $u = e^{\lambda t} x$ solves

$$\frac{du}{dt} = \begin{bmatrix} 4 & 3 \\ 0 & 1 \end{bmatrix} u.$$

What combination $u = c_1 e^{\lambda_1 t} x_1 + c_2 e^{\lambda_2 t} x_2$ starts from $u(0) = (5, -2)$?

5.5.p291.#31

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30. Which classes of matrices does P belong to: orthogonal, invertible, Hermitian, unitary, factorizable into LU, factorizable into QR?

$$P = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}.$$

31. Compute P^2, P^3, and P^{100} in Problem 30. What are the eigenvalues of P?

5.6.p303.#22

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22. Find a unitary U and triangular T so that $U^{-1}AU = T$, for

$$A = \begin{bmatrix} 5 & -3 \\ 4 & -2 \end{bmatrix} \quad \text{and} \quad A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}.$$