Midterm Exam - Results

Attempt 1 of 1

Written Jul 8, 2024 9:55 AM - Jul 8, 2024 11:02 AM

Attempt Score	**100 / 100 - 100 %**
Overall Grade (Highest Attempt)	**100 / 100 - 100 %**

Instructions

1. You are allotted **1 hour and 15 minutes** for the midterm exam.
    - The exam time is 1 hour and the last 15 minutes are for the purposes of uploading the exam.
    - You must upload the exam within the time limit.
    - If you have trouble uploading the exam to Gradescope, you must email me within the time limit of 1 hour and 10 minutes.
    - Any exam that is not received either from Gradescope or my email within the time limit (1 hour and 15minutes unless you have other accommodations) will not be graded.
2. The exam consists of 10 multiple-choice questions and five short-answer questions. Note that each short-answer question may have multiple parts.
    - For the short-answer questions, **any answers without justification or work will receive no credit.** For example, if the short answer question asks you to row-reduce a matrix to reduced echelon form, and you only write down the reduced echelon form, no credit will be awarded.
3. I will do my best to be reachable via email during the scheduled class time 8:40 AM–9:40 AM EDT, and outside of this scheduled class time,

from 10 AM EDT to 5 PM EDT on July 08, 2024. However, I may not be able to respond to your emails while you take the exam.

4. When using facts and theorems from class, you can just say, “From class, we know that...,” but please make sure I can tell what fact or theorem you are using instead of just the conclusion you are drawing from said fact or theorem. You can also cite the relevant facts from the course texts instead if you know where it is.
5. This exam is closed notes and closed book. No calculators.
6. Clearly designate each problem number and your solution.
7. **Examity must remain running until you finish submitting your work to Gradescope AND the exam must be submitted to Gradescope BEFORE the Brightspace exam timer runs out.**

Question 1

Problem 1 (10 points)

Let $u = \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix}$, $v = \begin{bmatrix} 3 \\ 1 \\ 0 \end{bmatrix}$, and $w = \begin{bmatrix} 1 \\ -1 \\ c \end{bmatrix}$ where c is a real number.

The set $\{u, v, w\}$ is a basis for $\mathbb{R}^3$, provided that c is not equal to

A. -1

B. 1

C. -2

D. 2

E. 3

Problem 2 (10 points)

Let A be an $m \times n$ matrix. Which of the following statements must be true?

(i) If the equation $Ax = 0$ has only the trivial solution, then $Ax = b$ has at most one solution.

(ii) If A has m pivot columns, then columns of A span $\mathbb{R}^m$.

(iii) If $Ax = b$ is consistent for each $b \in \mathbb{R}^m$ then $rank(A) = n$.

(iv) If $rank(A) = n$, then the rows of A form a linearly dependent set.

(v) If the dimension of $Col(A) = t$, then $Null(A)$ is a subspace of $\mathbb{R}^{m-t}$

A. (i), (ii)

B. (i), (ii), (iii)

C. (ii), (iii), (iv)

D. (i), (iii), (v)

E. none of the above

Problem 3 (10 points)

Find a least-squares solution of an inconsistent system $Ax = b$ where $A = \begin{bmatrix} 1 & 2 \\ -1 & 4 \\ 1 & 2 \end{bmatrix}$ and $b = \begin{bmatrix} 3 \\ -1 \\ 5 \end{bmatrix}$

A. $\begin{bmatrix} 3 \\ 1 \end{bmatrix}$

B. $\begin{bmatrix} 3 \\ 1/2 \end{bmatrix}$

C. $\begin{bmatrix} -3 \\ 1/2 \end{bmatrix}$

D. $\begin{bmatrix} 3 \\ -1/2 \end{bmatrix}$

E. $\begin{bmatrix} 3 \\ -1 \end{bmatrix}$

Problem 4 (10 points)

Let $C[-1,1]$ be the space of all continuous functions over $[-1,1]$ with the inner product

$$< f(t), g(t) >= \int_{-1}^{1} f(t)g(t)dt$$

for any $f(t), g(t) \in C[-1,1]$.

which of the following set is an orthogonal basis of $Span\{1, t+1, t^2-t\}$

A. $\{1, t, t^2-1\}$

B. $\{1, t+1, t^2\}$

C. $\{1, t-1, t^2-4/3\}$

D. $\{1, t, t^2\}$

E. $\{1, t, t^2-1/3\}$

Problem 5 (10 points)

Find dimension of the following subspace of 2×3 matrix

$$H = \left\{ \begin{bmatrix} a+3b+c & -2b+2c & 2a+8c \\ -3a+2b-14c & a-3b+7c & 5a+3b+17c \end{bmatrix} \quad a,b,c \in \mathbb{R} \right\}$$

A. 1

B. 2

C. 3

D. 4

E. 5

Problem 6 (10 points)

Consider the basis S for $\mathbb{R}^3$ given by

$$\left\{ \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \right\}$$

If we apply the Gram-Schmidt process to S to obtain an orthonormal basis, we obtain

A.

$$\left\{ 1/\sqrt{2} \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, 1/3\sqrt{2} \begin{bmatrix} 1 \\ -1 \\ 4 \end{bmatrix}, 1/3 \begin{bmatrix} 2 \\ -2 \\ -1 \end{bmatrix} \right\}$$

B.

$$\left\{ 1/\sqrt{2} \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, 1/3\sqrt{2} \begin{bmatrix} -1 \\ 1 \\ 4 \end{bmatrix}, 1/3 \begin{bmatrix} 2 \\ -2 \\ 1 \end{bmatrix} \right\}$$

C.

$$\left\{ 1/\sqrt{2} \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, 1/\sqrt{3} \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}, 1/\sqrt{6} \begin{bmatrix} -1 \\ 1 \\ 2 \end{bmatrix} \right\}$$

D.

$$\left\{ 1/\sqrt{2} \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, 1/\sqrt{6} \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix}, 1/\sqrt{3} \begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix} \right\}$$

E.

$$\left\{ 1/\sqrt{2} \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, 1/\sqrt{2} \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \right\}$$

Short Answer Questions Below - Any answers without justification or work will receive no credit.

Problem 7 (10 points)

Consider the system of linear equation:

$$\begin{array}{rrrr} x & - & z & = 1 \\ x & +y+(h-1) & z & = 3 \\ & +2y+(h^2-3) & z & = h+1 \end{array}$$

(a) (4 point) Find a row echelon form for the augmented matrix of the system.

(b) (2 point) For which value(s) of h does the system have an infinitely many solutions?

(c) (2 point) For which value(s) of h does the system have no solution?

(d) (2 point) For which value(s) of h does the system have a unique solution?

Problem 8 (10 points)

Let $A = \begin{bmatrix} 1 & 0 & 2 & 4 & 11 \\ 1 & 0 & 5 & 13 & 20 \\ 2 & 0 & 4 & 12 & 22 \\ 3 & 0 & 2 & 0 & 21 \end{bmatrix}$

(a) (4 points) Find the reduced row echelon form for the matrix A.

(b) (3 points) Find a basis for the column space of A.

(c) (3 points) Find a basis for the null space of A.

Problem 9 (10 points)

Let $T : \mathbb{R}^2 \to \mathbb{R}^3$ be the linear transformation for which $T(\begin{bmatrix} 1 \\ 1 \end{bmatrix}) = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$ and $T(\begin{bmatrix} 1 \\ 2 \end{bmatrix}) = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$.

(a) (4 points) Find the standard matrix A of T.

(b) (3 point) Find the image of the vector $u = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$.

(c) (3 points) Is the vector $b = \begin{bmatrix} -1 \\ 2 \\ -1 \end{bmatrix}$ in the range of T? If so, find a vector x in $\mathbb{R}^2$ such that $T(x) = b$.

Problem 10 (10 points)

Let $A = \begin{bmatrix} 1 & 2 & 2 \\ -6 & -11 & -12 \\ 1 & -4 & 3 \end{bmatrix}$.

(a) (5 points) Factor A to LDU.

(b) (5 points) Find A^{-1} if it does exist.

Congratulations, you are almost done with this exam.

DO NOT end the Examity session until you have submitted your work.

Do the following:

1. Use your phone to scan your answer sheet and save it as a PDF.
2. Submit your PDF:
    - Email the PDF to yourself or save it to the cloud (Google Drive, etc.).
    - Click this link to submit your work: **Gradescope**
3. Return to this window and click the button below to agree to the honor statement.
4. Click **Submit Quiz** to end the exam, and then end the Examity session.

☑ **Honor Pledge:** As a Boilermaker pursuing academic excellence, I pledge to be honest and true in all that I do. Accountable together - we are Purdue. I certify that I have neither given nor received unauthorized aid on this exam.

Done