2.1.p76.#24

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Problems 21–30 are about column spaces $C(A)$ and the equation $Ax = b$.

24. For which vectors (b_1, b_2, b_3) do these systems have a solution?

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} \quad \text{and} \quad \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}.$$

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} \rightarrow \begin{aligned} x_1 + x_2 + x_3 &= b_1 \\ x_2 + x_3 &= b_2 \\ x_3 &= b_3 \end{aligned}$$

$x_3 = b_3 \rightarrow x_2 + x_3 = b_2$
$x_2 + b_3 = b_2$
$x_2 = b_2 - b_3$

$x_2 = b_2 - b_3 \rightarrow x_1 + x_2 + x_3 = b_1$
$x_1 + (b_2 - \cancel{b_3}) \cancel{+ b_3} = b_1$
$x_1 + b_2 = b_1$
$x_1 = b_1 - b_2$

$x_1 = b_1 - b_2$, $x_2 = b_2 - b_3$, $x_3 = b_3$

$b_1 = x_1 + b_2$, $b_2 = x_2 + b_3$, $b_3 = x_3$

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} \rightarrow \begin{aligned} x_1 + x_2 + x_3 &= b_1 \\ x_2 + x_3 &= b_2 \end{aligned}$$

$x_3 = 0 \rightarrow x_2 + x_3 = b_2$
$b_3 = 0 \qquad x_2 + 0 = b_2$
$x_2 = b_2$

$x_2 = b_2 \rightarrow x_1 + x_2 + x_3 = b_1$
$x_1 + b_2 + 0 = b_1$
$x_1 = b_1 - b_2$

$x_1 = b_1 - b_2$, $x_2 = b_2$, $x_3 = 0$

$b_1 = x_1 + b_2$, $b_2 = x_2$, $b_3 = 0$

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2/9 - PART 1

Problems 33–36 are about the solution of $Ax = b$. Follow the steps in the text to x_p and x_n. Reduce the augmented matrix $[A \;\; b]$.

33. Find the complete solutions of

$$\begin{aligned} x + 3y + 3z &= 1 \\ 2x + 6y + 9z &= 5 \\ -x - 3y + 3z &= 5 \end{aligned} \qquad \text{and} \qquad \begin{bmatrix} 1 & 3 & 1 & 2 \\ 2 & 6 & 4 & 8 \\ 0 & 0 & 2 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ t \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \\ 1 \end{bmatrix}.$$

$$\begin{aligned} x + 3y + 3z &= 1 \\ 2x + 6y + 9z &= 5 \\ -x - 3y + 3z &= 5 \end{aligned}$$

$$\begin{bmatrix} 1 & 3 & 3 \\ 2 & 6 & 9 \\ -1 & -3 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 5 \\ 5 \end{bmatrix}$$

$$\left[\begin{array}{ccc|c} 1 & 3 & 3 & 1 \\ 2 & 6 & 9 & 5 \\ -1 & -3 & 3 & 5 \end{array}\right] \xrightarrow{A_{12}(-2)} \left[\begin{array}{ccc|c} 1 & 3 & 3 & 1 \\ 1(-2)+2 & 3(-2)+6 & 3(-2)+9 & 1(-2)+5 \\ -1 & -3 & 3 & 5 \end{array}\right] = \left[\begin{array}{ccc|c} 1 & 3 & 3 & 1 \\ 0 & 0 & 3 & 3 \\ -1 & -3 & 3 & 5 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & 3 & 3 & 1 \\ 0 & 0 & 3 & 3 \\ -1 & -3 & 3 & 5 \end{array}\right] \xrightarrow{A_{13}(1)} \left[\begin{array}{ccc|c} 1 & 3 & 3 & 1 \\ 0 & 0 & 3 & 3 \\ 0 & 0 & 6 & 6 \end{array}\right] \longrightarrow \left[\begin{array}{ccc|c} \boxed{1} & 3 & 3 & 1 \\ 0 & 0 & \boxed{3} & 3 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$$Ux = c$$

x + z = PIVOTS
y = FREE

$$\begin{aligned} x + 3y + 3z &= 1 \\ 3z &= 3 \end{aligned} \longrightarrow \begin{aligned} z &= 1 \\ y &= 0 \end{aligned} \longrightarrow \begin{aligned} x + 3(0) + 3(1) &= 1 \\ x + 0 + 3 &= 1 \\ x + 3 &= 1 \\ x &= -2 \end{aligned} \implies x_p = \begin{bmatrix} -2 \\ 0 \\ 1 \end{bmatrix}$$

$$\begin{aligned} x + 3y + 3z &= 0 \\ 3z &= 0 \end{aligned} \longrightarrow z = 0 \longrightarrow \begin{aligned} x + 3y + 3(0) &= 0 \\ x + 3y + 0 &= 0 \\ x + 3y &= 0 \\ x &= -3y \end{aligned} \implies x_n = \begin{bmatrix} -3y \\ y \\ 0 \end{bmatrix} \overset{y=1}{\implies} \begin{bmatrix} -3 \\ 1 \\ 0 \end{bmatrix}$$

$$x_c = x_p + x_n$$

$$x_c = \begin{bmatrix} -2 \\ 0 \\ 1 \end{bmatrix} + \begin{bmatrix} -3 \\ 1 \\ 0 \end{bmatrix} x_2$$

$$\left[\begin{array}{ccc|c} 1 & 3 & 3 & 1 \\ 0 & 0 & 3 & 3 \\ 0 & 0 & 0 & 0 \end{array}\right] \xrightarrow{M_2(1/3)} \left[\begin{array}{ccc|c} 1 & 3 & 3 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{array}\right] \xrightarrow{A_{21}(-3)} \left[\begin{array}{ccc|c} 1 & 3 & 0 & -2 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$$Rx = d$$

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2/9 – PART 2

Problems 33–36 are about the solution of $Ax = b$. Follow the steps in the text to x_p and x_n. Reduce the augmented matrix $[A \;\; b]$.

33. Find the complete solutions of

$$\begin{aligned} x + 3y + 3z &= 1 \\ 2x + 6y + 9z &= 5 \\ -x - 3y + 3z &= 5 \end{aligned} \qquad \text{and} \qquad \begin{bmatrix} 1 & 3 & 1 & 2 \\ 2 & 6 & 4 & 8 \\ 0 & 0 & 2 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ t \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \\ 1 \end{bmatrix}.$$

$$\left[\begin{array}{cccc|c} 1 & 3 & 1 & 2 & 1 \\ 2 & 6 & 4 & 8 & 3 \\ 0 & 0 & 2 & 4 & 1 \end{array}\right] \xrightarrow{A_{12}(-2)} \left[\begin{array}{cccc|c} 1 & 3 & 1 & 2 & 1 \\ 0 & 0 & 2 & 4 & 1 \\ 0 & 0 & 2 & 4 & 1 \end{array}\right] \xrightarrow{A_{23}(-1)} \begin{array}{c} \begin{array}{cccc} x & y & z & t \end{array} \\ \left[\begin{array}{cccc|c} \boxed{1} & 3 & 1 & 2 & 1 \\ 0 & 0 & \boxed{2} & 4 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right] \end{array}$$

$x + z =$ PIVOT
$y + t =$ FREE

$Ux = c$

$$x + 3y + z + 2t = 1$$
$$2z + 4t = 1 \longrightarrow 2z + 4(0) = 1$$
$$y + t = 0$$
$$2z = 1$$
$$z = 1/2$$

$$x + 3(0) + 1/2 + 2(0) = 1$$
$$x + 0 + 1/2 + 0 = 1$$
$$x + 1/2 = 1$$
$$x = 1/2$$

$$x_p = \begin{bmatrix} 1/2 \\ 0 \\ 1/2 \\ 0 \end{bmatrix}$$

$$x + 3y + z + 2t = 0 \longrightarrow x + 3y(\cancel{-2t}) + \cancel{2t} = 0$$
$$2z + 4t = 0$$
$$2z = -4t$$
$$z = -2t$$
$$x + 3y = 0$$
$$x = -3y$$

$y = 1, \; t = 0 \qquad y = 0, \; t = 1$

$$x_n = \begin{bmatrix} -3y \\ y \\ -2t \\ t \end{bmatrix} \qquad \begin{bmatrix} -3 \\ 1 \\ 0 \\ 0 \end{bmatrix} y \qquad \begin{bmatrix} 0 \\ 0 \\ -2 \\ 1 \end{bmatrix} t$$

$x_c = x_p + x_n$

$$x_c = \begin{bmatrix} 1/2 \\ 0 \\ 1/2 \\ 0 \end{bmatrix} + \begin{bmatrix} -3 \\ 1 \\ 0 \\ 0 \end{bmatrix} x^2 + \begin{bmatrix} 0 \\ 0 \\ -2 \\ 1 \end{bmatrix} x^4$$

$$\left[\begin{array}{cccc|c} 1 & 3 & 1 & 2 & 1 \\ 0 & 0 & 2 & 4 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right] \xrightarrow{M_2(1/2)} \left[\begin{array}{cccc|c} 1 & 3 & 1 & 2 & 1 \\ 0 & 0 & 1 & 2 & 1/2 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right] \xrightarrow{A_{21}(-1)} \left[\begin{array}{cccc|c} 1 & 3 & 0 & 0 & 1/2 \\ 0 & 0 & 1 & 2 & 1/2 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

$Rx = d$

2.2.p89.#43

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43. Choose the number q so that (if possible) the ranks are (a) 1, (b) 2, (c) 3:

$$A = \begin{bmatrix} 6 & 4 & 2 \\ -3 & -2 & -1 \\ 9 & 6 & q \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} 3 & 1 & 3 \\ q & 2 & q \end{bmatrix}.$$

$$A = \begin{bmatrix} 6 & 4 & 2 \\ -3 & -2 & -1 \\ 9 & 6 & q \end{bmatrix} \xrightarrow{A_{21}(2)} \begin{bmatrix} 0 & 0 & 0 \\ -3 & -2 & -1 \\ 9 & 6 & q \end{bmatrix} \xrightarrow{A_{23}(3)} \begin{bmatrix} 0 & 0 & 0 \\ -3 & -2 & -1 \\ 0 & 0 & q-3 \end{bmatrix}$$

a) $q = 3$ → 1 PIVOT
b) $q \mathrel{!}= 3$ → 2 PIVOTS
c) NOT POSSIBLE → X

$$B = \begin{bmatrix} 3 & 1 & 3 \\ q & 2 & q \end{bmatrix} \xrightarrow{A_{12}(-2)} \begin{bmatrix} 3 & 1 & 3 \\ q-6 & 0 & q-6 \end{bmatrix}$$

a) $q = 6$ → 1 PIVOT
b) $q \mathrel{!}= 6$ → 2 PIVOTS
c) NOT POSSIBLE → X

2.2.p91.#68

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68. Show by example that these three statements are generally *false*:

(a) A and A^{T} have the same nullspace.

(b) A and A^{T} have the same free variables.

(c) If R is the reduced form rref(A) then R^{T} is rref(A^{T}).

a) $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$ $\quad A^T = \begin{bmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{bmatrix}$

$$Ax = 0 \neq A^T x = 0$$

b) $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$ $\quad A^T = \begin{bmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{bmatrix}$

$A_{12}(-4)$

$\begin{bmatrix} 1 & 2 & 3 \\ 0 & -3 & -6 \end{bmatrix}$

$M_2(-3)$

$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \end{bmatrix}$

$A_{21}(-2)$

$\begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 2 \end{bmatrix}$

$A_{12}(-2)$
$A_{13}(-3)$

$\begin{bmatrix} 1 & 4 \\ 0 & -3 \\ 0 & -6 \end{bmatrix}$

$A_{21}(-2)$
$M_2(-3)$

$\begin{bmatrix} 1 & 4 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$

c) $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$ $\quad A^T = \begin{bmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{bmatrix}$

$\mathrm{RREF}(A) = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 2 \end{bmatrix}$ $\quad \mathrm{RREF}(A^T) = \begin{bmatrix} 1 & 4 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$

$$\mathrm{RREF}(A)^T = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ -1 & 2 \end{bmatrix} \neq \mathrm{RREF}(A^T) \begin{bmatrix} 1 & 4 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$$

2.3.p98.#02

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Problems 1–10 are about linear independence and linear dependence.

2. Find the largest possible number of independent vectors among

$$v_1 = \begin{bmatrix} 1 \\ -1 \\ 0 \\ 0 \end{bmatrix} \quad v_2 = \begin{bmatrix} 1 \\ 0 \\ -1 \\ 0 \end{bmatrix} \quad v_3 = \begin{bmatrix} 1 \\ 0 \\ 0 \\ -1 \end{bmatrix} \quad v_4 = \begin{bmatrix} 0 \\ 1 \\ -1 \\ 0 \end{bmatrix} \quad v_5 = \begin{bmatrix} 0 \\ 1 \\ 0 \\ -1 \end{bmatrix} \quad v_6 = \begin{bmatrix} 0 \\ 0 \\ 1 \\ -1 \end{bmatrix}.$$

This number is the ______ of the space spanned by the v's.

$$\begin{bmatrix} 1 & 1 & 1 & 0 & 0 & 0 \\ -1 & 0 & 0 & 1 & 1 & 0 \\ 0 & -1 & 0 & -1 & 0 & 1 \\ 0 & 0 & -1 & 0 & -1 & -1 \end{bmatrix} \xrightarrow[A_{13}(1)]{A_{12}(1)} \begin{bmatrix} 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & -1 & 0 & 1 \\ 0 & 0 & -1 & 0 & -1 & -1 \end{bmatrix} \xrightarrow[A_{23}(1)]{A_{14}(1)} \begin{bmatrix} 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & -1 & -1 \end{bmatrix} \xrightarrow{A_{24}(1)} \begin{bmatrix} 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & -1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & -1 \end{bmatrix} \xrightarrow{A_{34}(1)} \begin{bmatrix} \boxed{1} & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & \boxed{1} & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & \boxed{1} \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \Rightarrow$$

3 PIVOTS
3 FREE VARIABLES

2.3.p99.#13

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Problems 11–18 are about the space *spanned* by a set of vectors. Take all linear combinations of the vectors.

13. Find the dimensions of (a) the column space of A, (b) the column space of U, (c) the row space of A, (d) the row space of U. Which two of the spaces are the same?

$$A = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 3 & 1 \\ 3 & 1 & -1 \end{bmatrix} \quad \text{and} \quad U = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 1 \\ 0 & 0 & 0 \end{bmatrix}.$$

$$A = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 3 & 1 \\ 3 & 1 & -1 \end{bmatrix} \xrightarrow{A_{12}(-1)} \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 1 \\ 3 & 1 & -1 \end{bmatrix} \xrightarrow{A_{13}(-3)} \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 1 \\ 0 & -2 & -1 \end{bmatrix} \xrightarrow{A_{23}(1)} \begin{bmatrix} \boxed{1} & 1 & 0 \\ 0 & \boxed{2} & 1 \\ 0 & 0 & 0 \end{bmatrix} = U$$

a) $\mathrm{COL}(A) = 2 = \left\{ \begin{bmatrix} 1 \\ 1 \\ 3 \end{bmatrix}, \begin{bmatrix} 1 \\ 3 \\ 1 \end{bmatrix} \right\}$

b) $\mathrm{COL}(U) = 2 = \left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix} \right\}$

c) $\mathrm{ROW}(A) = 2 = \{ [1\ 1\ 0], [0\ 2\ 1] \}$

d) $\mathrm{ROW}(U) = 2 = \{ [1\ 1\ 0], [0\ 2\ 1] \}$

THE ROW SPACES OF A AND U ARE THE SAME.

2.3.p101.#29

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Problems 19–37 are about the requirements for a basis.

29. For which numbers c and d do these matrices have rank 2?

$$A = \begin{bmatrix} 1 & 2 & 5 & 0 & 5 \\ 0 & 0 & c & 2 & 2 \\ 0 & 0 & 0 & d & 2 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} c & d \\ d & c \end{bmatrix}.$$

$$A = \begin{bmatrix} 1 & 2 & 5 & 0 & 5 \\ 0 & 0 & c & 2 & 2 \\ 0 & 0 & 0 & d & 2 \end{bmatrix} \qquad B = \begin{bmatrix} c & d \\ d & c \end{bmatrix}$$

RANK = 2 WHEN
C = 0 + d = 2

RANK = 2 WHEN
C != d + C != -d

$$\begin{bmatrix} \boxed{1} & 2 & 5 & 0 & 5 \\ 0 & 0 & 0 & \boxed{2} & 2 \\ 0 & 0 & 0 & 2 & 2 \end{bmatrix} \qquad \begin{bmatrix} \boxed{1} & 0 \\ 0 & \boxed{1} \end{bmatrix}$$

2.4.p113.#32

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32. Describe the four subspaces of $\mathbf{R}^3$ associated with

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \quad \text{and} \quad I + A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}.$$

4 SUBSPACES OF R^3
ROW(A), COL(A), NULL(A), NULL(A^T)

A)

ROW(A) $= \text{SPAN}\{[0\ 1\ 0], [0\ 0\ 1]\}$

COL(A) $= \text{SPAN}\left\{\begin{bmatrix}1\\0\\0\end{bmatrix}, \begin{bmatrix}0\\1\\0\end{bmatrix}\right\}$

NULL (A) $= \begin{bmatrix}0 & 1 & 0\\0 & 0 & 1\\0 & 0 & 0\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix} = \begin{bmatrix}0\\0\\0\end{bmatrix} \Rightarrow \begin{matrix}x_2 = 0\\x_3 = 0\end{matrix} \Rightarrow$ NULL(A) $= \text{SPAN}\left\{\begin{bmatrix}1\\0\\0\end{bmatrix}\right\}$

$Ax = 0$

NULL(A^T) $= \begin{bmatrix}0 & 0 & 0\\1 & 0 & 0\\0 & 1 & 0\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix} = \begin{bmatrix}0\\0\\0\end{bmatrix} \Rightarrow \begin{matrix}x_1 = 0\\x_2 = 0\end{matrix} \Rightarrow$ NULL(A^T) $= \text{SPAN}\left\{\begin{bmatrix}0\\0\\1\end{bmatrix}\right\}$

$I+A \begin{bmatrix}1 & 1 & 0\\0 & 1 & 1\\0 & 0 & 1\end{bmatrix} \quad (I+A)^T \begin{bmatrix}1 & 0 & 0\\1 & 1 & 0\\0 & 1 & 1\end{bmatrix}$

ROW (I+A) $= \text{SPAN}\{[1\ 1\ 0], [0\ 1\ 1], [0\ 0\ 1]\}$

COL(I+A) $= \text{SPAN}\left\{\begin{bmatrix}1\\0\\0\end{bmatrix}, \begin{bmatrix}1\\1\\0\end{bmatrix}, \begin{bmatrix}0\\1\\1\end{bmatrix}\right\}$

NUL(I+A) $= 0$
NUL$(I+A)^T = 0$ $= \text{SPAN}\left\{\begin{bmatrix}0\\0\\0\end{bmatrix}\right\}$

2.4.p113.#37

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37. True or false (with a reason or a counterexample)?

T (a) A and A^{T} have the same number of pivots.

F (b) A and A^{T} have the same left nullspace.

F (c) If the row space equals the column space then $A^{\mathrm{T}} = A$.

T (d) If $A^{\mathrm{T}} = -A$ then the row space of A equals the column space.

a) $A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ $A^T = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ **TRUE**

b) $A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$ $A^T = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$

$NULL(A) = SPAN\left\{\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}\right\}$ $NULL(A^T) = SPAN\left\{\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}\right\}$ **FALSE**

c) $A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ $A^T = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$ **FALSE**

$ROW(A) = SPAN\{[1\ 1], [0\ 1]\}$

$COL(A) = SPAN\left\{\begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \end{bmatrix}\right\}$

d) $A = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$ $A^T = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$ $RREF = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$ROW(A) = SPAN\{[1\ 0], [0\ 1]\}$

$COL(A^T) = SPAN\left\{\begin{bmatrix} 0 \\ 1 \end{bmatrix}, \begin{bmatrix} -1 \\ 0 \end{bmatrix}\right\}$

TRUE