MATH 511 HOMEWORK (ON GRADESCOPE)

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Homework # 1

Due on Jun 16, 2024, 11:59 PM

Section 1.3: Page17 #17, P19 #31,
Section 1.4: P28 #24,
Section 1.5: P43 #29, P44 #38,
Section 1.6: P55 #40, #41, P57 #56

1.3.p17.#17

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Problems 10–19 study elimination on 3 by 3 systems (and possible failure).

17. Which number q makes this system singular and which right-hand side t gives it infinitely many solutions? Find the solution that has $z = 1$.

$$\begin{aligned} x + 4y - 2z &= 1 \\ x + 7y - 6z &= 6 \\ 3y + qz &= t. \end{aligned}$$

1.3.p19.#31

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31. For which three numbers a will elimination fail to give three pivots?

$$
\begin{aligned}
ax + 2y + 3z &= b_1 \\
ax + ay + 4z &= b_2 \\
ax + ay + az &= b_3.
\end{aligned}
$$

1.4.p28.#24

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Problems 22–31 are about elimination matrices.

24. Which three matrices E_{21}, E_{31}, E_{32} put A into triangular form U?

$$A = \begin{bmatrix} 1 & 1 & 0 \\ 4 & 6 & 1 \\ -2 & 2 & 0 \end{bmatrix} \quad \text{and} \quad E_{32}E_{31}E_{21}A = U.$$

Multiply those E's to get one matrix M that does elimination: $MA = U$.

1.5.p43.#29

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Problems 20–31 compute the factorization $A = LU$ (and also $A = LDU$).

29. (Recommended) Compute L and U for the symmetric matrix

$$A = \begin{bmatrix} a & a & a & a \\ a & b & b & b \\ a & b & c & c \\ a & b & c & d \end{bmatrix}.$$

Find four conditions on a, b, c, d to get $A = LU$ with four pivots.

1.5.p44.#38

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38. (Review) For which numbers c is $A = LU$ impossible—with three pivots?

$$A = \begin{bmatrix} 1 & 2 & 0 \\ 3 & c & 1 \\ 0 & 1 & 1 \end{bmatrix}.$$

1.6.p55.#40

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40. True or false (with a counterexample if false and a reason if true):

(a) A 4 by 4 matrix with a row of zeros is not invertible.

(b) A matrix with 1s down the main diagonal is invertible.

(c) If A is invertible then A^{-1} is invertible.

(d) If A^{T} is invertible then A is invertible.

1.6.p55.#41

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41. For which three numbers c is this matrix not invertible, and why not?

$$A = \begin{bmatrix} 2 & c & c \\ c & c & c \\ 8 & 7 & c \end{bmatrix}.$$

1.6.p57.#56

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Problems 56–60 are about symmetric matrices and their factorizations.

56. If $A = A^{\mathrm{T}}$ and $B = B^{\mathrm{T}}$, which of these matrices are certainly symmetric?
(a) $A^2 - B^2$ (b) $(A + B)(A - B)$ (c) ABA (d) $ABAB$.