

236609 - AI and Robotics - Fall 2022

Lesson 3: A slightly deeper dive into the three decision-making layers

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Task Planning

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Putting it All
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Components of a Robotic Agents

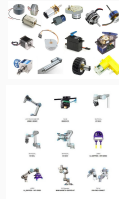
Sensors



Controllers



Actuators



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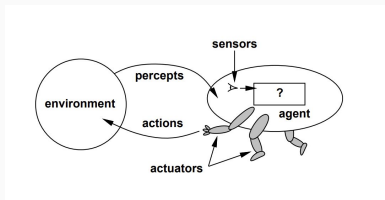
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Autonomy



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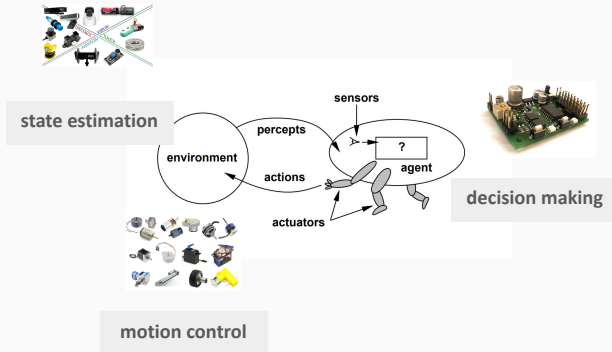
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Autonomy

State Estimation

$$\beta : \mathcal{S} \mapsto [0, 1]$$

Process incoming observations to maintain a *belief* as a probability distribution over states

Decision Making

$$\pi : \beta \mapsto \mathcal{A}$$

$$\pi : \beta \times \mathcal{A} \mapsto [0, 1]$$

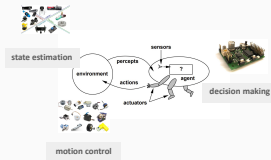
Find a policy - a mapping belief and objective into actions (probabilities)

Motion Control

$$\dot{x}(t) = f(x(t), u(t), t) + w(t)$$

Translate actions into low-level commands (and monitor their execution)

- $x(t) \in \mathbb{R}^n$ - state vector
- $u(t) \in \mathbb{R}^m$ - control input
- $f : \mathbb{R}^n \times \mathbb{R}^m \times \mathbb{R} \rightarrow \mathbb{R}^n$ - system dynamics
- $w(t)$ - noise or disturbance



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Layered Control Architecture (LCA)

Semantic Logic Discrete Planning	Decision Making	Flexible and Slow
Optimization Sampling Methods Continuous Planning	Trajectory Planning	Intermediate
PID Control CLFs/CBFs	Feedback Control	Rigid and Real Time



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Top-Down Flow:

- Goals → Plans → Commands
- Like company policies becoming specific actions

Bottom-Up Flow:

- Sensor data → Status updates → Results
- Like workers reporting to management

Different Speeds for Different Needs

- Planning: Seconds to minutes
- Behavioral: Fraction of seconds
- Execution: Milliseconds
- Control: Microseconds
- Hardware: Nanoseconds

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Task Planning

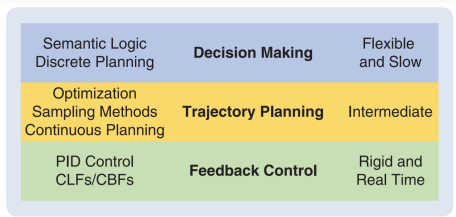
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Layered Control Architecture (LCA)



Matni, Nikolai, Aaron D. Ames, and John C. Doyle. "A Quantitative Framework for Layered Multirate Control: Toward a Theory of Control Architecture." IEEE Control Systems Magazine (2024)

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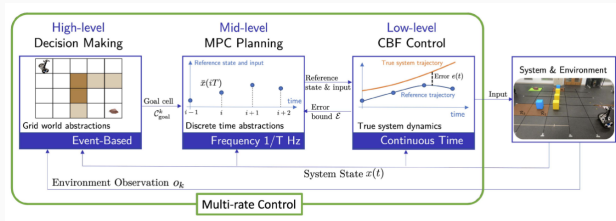
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Why Do We Need Layered Control?

Challenge: Robots need to:

- Make high-level decisions
- React to their environment
- Control precise movements
- Handle multiple tasks
- Respond at different speeds

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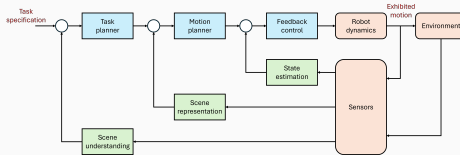
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Layered Control Architecture (LCA)



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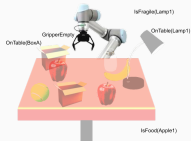
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Task Planning

Top Layer: High level planning



Example:

- Realize that the human needs a food item.
- Reason about the reachability of items and the ability to perceive them.
- Decide which food object to pick-up.
- Optional - Prepare for failure via a contingent plan
- ...

What is a good model?

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Markov Decision Process

A Markov Decision Process(MDP) is a tuple $\langle \mathcal{S}, \mathcal{A}, \mathcal{P}, \mathcal{R}, \gamma \rangle$ where

- \mathcal{S} is a finite set of states
- \mathcal{A} is a finite set of actions
- \mathcal{P} is a state transition probability matrix
$$\mathcal{P}_{s,s'}^a = \mathcal{P}[S_{t+1} = s' | S_t = s, A_t = a]$$
- \mathcal{R} is a reward function, $\mathcal{R}_s^a = \mathbb{E}[R_{t+1} | S_t = s, A_t = a]$, and
- **optional:** γ is a discount factor $\gamma \in [0, 1]$ that is used to favor immediate rewards over future rewards.

The Markov property: “The future is independent of the past given the present”.

Extensions: Infinite and continuous MDPs, partially observable MDPs, undiscounted, average reward MDPs. etc.

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Partially Observable Markov Decision Process (POMDP)

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A **Partially Observable Markov Decision Process**(POMDP) is a tuple $\langle \mathcal{S}, \mathcal{A}, \mathcal{P}, \mathcal{R}, \gamma, \Omega, \mathcal{O}, \beta_0 \rangle$ where

- $\mathcal{S}, \mathcal{A}, \mathcal{P}, \mathcal{R}$ and γ are as for an MDP.
- Ω is a set of observations (observation tokens),
- \mathcal{O} is a sensor function specifying the conditional observation probabilities $\mathcal{O}_{s,a}^o = \mathcal{P}[O_{t+1} = o | S_t = s, A_t = a]$ of receiving observation token $o \in \Omega$ in state s after applying action a ¹.
- β_0 the initial belief: a probability distribution over the states such that $\beta_0(s)$ stands for the probability of s being the true initial state.

¹alternatively: $\mathcal{O}_s^o = \mathcal{P}[o_t = o | S_t = s]$

Partially Observable Markov Decision Process (POMDP)

In its most general formulation, a belief β is represented as a probability distribution over the states \mathcal{S} , such that $\beta(s)$ stands for the probability of s being the true state.

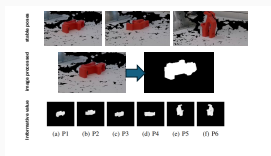


Figure 1: How to represent beliefs of robotic agents?

Typically, each agent is associated with a **belief update function** $\tau : \mathcal{B} \times \Omega \mapsto \mathcal{B}$ that represents how a belief changes when receiving a new observation.

POMDP for our example?

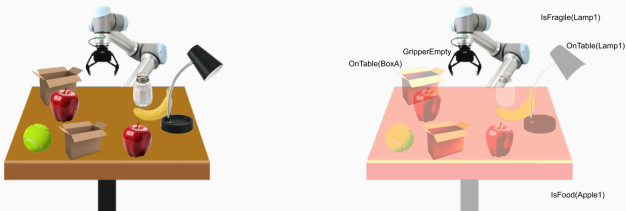


Figure 2: What is a good representation ?

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- The POMDP provides a principled and formal framework for long-term decision making in settings in which it is important to account for the probabilistic nature of the sensor function.
- Has been successfully used in many robotic applications, including localization and navigation, search and tracking, autonomous driving, multi-robot systems, manipulation, and human-robot interaction.
- A key benefit: many planning tools that have been developed (most based on sampling and approximations).

POMDPs are hard! (and not always needed)

- Need to address the continuous nature of the environment through approximation and discretization techniques or through specialized algorithms designed for continuous domains.
- Computationally intractable - especially true for the high-dimensional robotic settings.

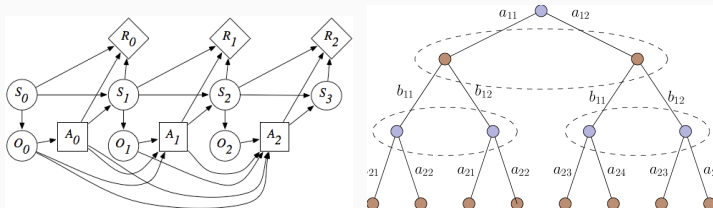


Figure 3: POMDP planning

Ideas?

- Use a simpler model when possible (e.g., MDP)
- Function approximations (AKA as Deep Learning)
- Sampling based approaches.
- Adopt a modular approach that decomposes the system into modules for state estimation, planning, and control

More on this later in the course !

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Motion Planning

- Task planning focuses on achieving high-level goals and on finding sequences of high-level actions to achieve them
- Motion planning ensures the physical feasibility of these actions by generating **collision-free paths** or trajectories that respect the robot's **kinematic and dynamic constraints**.

Happens in a high-dimensional space!

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Motion Planning

From Oren Salzman's course slides*

(Formal) definition of the basic **motion-planning**** problem

Let \mathcal{R} be a robot system with d **degrees of freedom**, moving in a **known environment** cluttered with obstacles. Given start and target **configurations** q_0 and q_g for \mathcal{R} , decide whether there is a collision-free, continuous path $\tau : [0, 1] \rightarrow \mathcal{C}_{free}$ such that $\tau(0) = q_0$ and $\tau(1) = q_g$ and if so, plan such a motion.

Alternatively, we can consider a *motion path* from initial configuration $q_0 \in \mathcal{C}$ to goal configuration $q_g \in \mathcal{C}$ as a sequence of configurations $(q_0, q_1, \dots, q_m = q_g)$ where for all i , $\{\alpha q_{i-1} + (1 - \alpha)q_i | \alpha \in [0, 1]\} \subseteq \mathcal{C}_{free}$.

*Consider taking course 236901 to learn about algorithms for robotic motion planning.

<https://arxiv.org/pdf/2209.14471.pdf>

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Motion Planning for our example?

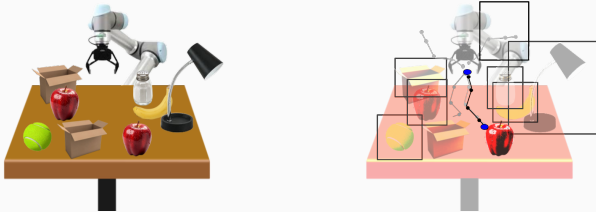


Figure 4: What is a good representation ?

Happens in a high-dimensional space!

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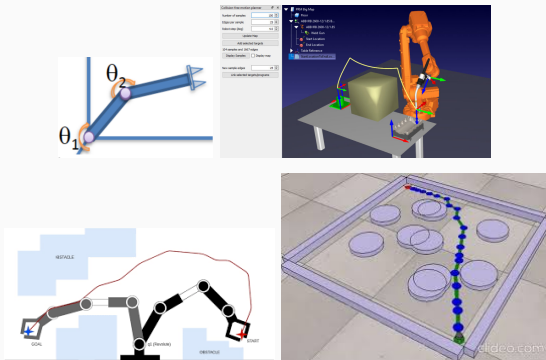


Figure 5: Motion Planning Examples

Happens in a high-dimensional space!

Motion Planning

- Typically, in order to support computational feasibility, the path that the motion planner generates is a discretized abstraction of a path
- A path $\tau = \{w_i\}_{i=0}^N$ is a sequence of waypoints w_i , where N denotes the total number of waypoints in τ , and w_0 and w_N represent the initial and goal configurations q_0 and q_g , respectively.

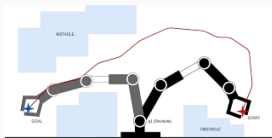


Figure 6: Motion Planning

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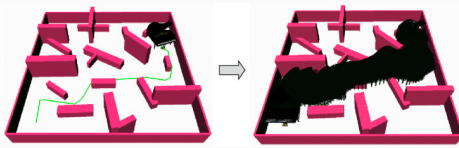
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Motion Planning is Hard



Motion planning approaches are diverse: optimization-based methods (e.g., trajectory optimization, gradient descent) sampling-based methods (e.g., Rapidly-exploring Random Trees (RRT) and Probabilistic Roadmaps (PRM)), physics-based planners and hybrid approaches.

More on this later on ...

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Task Planning

Motion Planning

Feedback
Control - Motion
Control

Putting it All
Together

Feedback Control - Motion Control

- Once a motion plan is found, there is a need to execute it.
- While motion planning aims to find the best path for the robot to follow,
- focus on ensuring that the robot follows this trajectory in the presence of real-world factors like sensor noise, dynamic disturbances, or actuator inaccuracies.

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**Feedback
Control - Motion
Control**

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Feedback Control for our example?

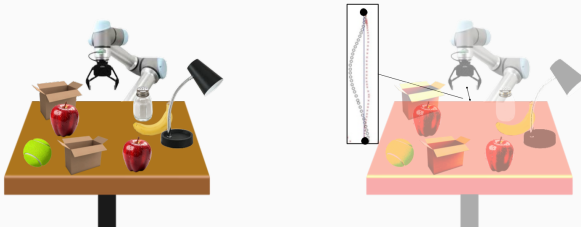


Figure 7: What is a good representation ?

Happens in a high-dimensional space!

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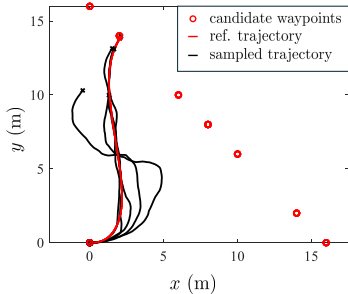
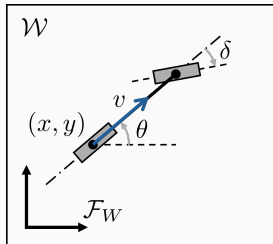
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Feedback Control



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Putting it All
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- To describe the control episode between consecutive waypoints w_{j-1} and w_j denoted by γ_j , we use a *dynamic model*.
- Let $\mathcal{F}_{\mathcal{W}}$ denote the inertial frame of reference embedded in \mathcal{W} , with origin $\mathcal{O}_{\mathcal{W}}$. The state $\mathbf{x}(k) \in \mathcal{R} \subset \mathbb{R}^d$ of the mobile agent is governed by the following difference equation,

$$\mathbf{x}(k+1) = f[\mathbf{x}(k) + \mathbf{B}[\mathbf{x}(k)]\mathbf{u}(k) + \mathbf{p}(k) \quad (1)$$

where $k = 0, \dots, T$ is the index of time steps in the control episode, $\mathbf{u}(k) \in \mathcal{U} \subset \mathbb{R}^m$ is the control input, $\mathbf{B}[\mathbf{x}(k)]$ is the control effect matrix, and the vector $\mathbf{p}(k)$ represents process noise.

- We refer to the deterministic system with $\mathbf{p}(k) \equiv 0$ as the **nominal model**.

- Given the dynamic model, the challenge is to design a control system to set the behavior of a dynamic system in a desired way.
- This involves determining a sequence of control inputs $\mathbf{u}(k)$ to achieve specified states while satisfying system constraints, and dynamically adjusting them based on the current state.
- Many model-based approaches (e.g., PID, LQR, MPC) and many emerging techniques based on neural network-based control.

Putting it All Together

3-tiered State Estimation

- The account so far considers external or internal factors that affect the system dynamics but are not directly controlled or predicted by the model.
- In some settings, it is beneficial to explicitly model sensor noise, and the mapping between states and observations.
- State estimation needs to be performed for the 3 layers of abstraction:
 - What is the value of POMDP features ?
 - What is the position of the robot w.r.t its motion plan?
 - What is the position of the robot w.r.t its nominal path to the next waypoint?

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State Estimation at the CLAIR Lab

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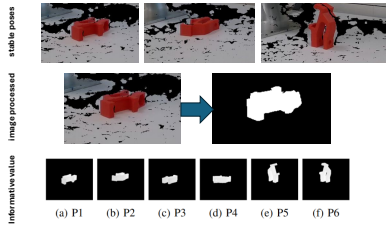
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in-table-section(green-mug,blue): False
in-table-section(green-mug,white): True
in-table-section(mineral-water-bottle,blue): False
in-table-section(mineral-water-bottle,white): True
in-table-section(red-can,blue): True
in-table-section(red-can,white): False
in-table-section(spray-bottle,blue): **True**
in-table-section(spray-bottle,white): False
robot-gripper-empty(): **True**
robot-holding-in-air(green-mug): False
robot-holding-in-air(mineral-water-bottle): False
robot-holding-in-air(red-can): False
robot-holding-in-air(spray-bottle): **False**

(a) Setup.



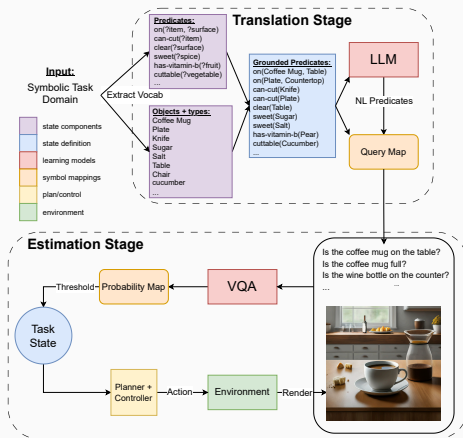
pick-up
spray-bottle



in-table-section(green-mug,blue): False
in-table-section(green-mug,white): True
in-table-section(mineral-water-bottle,blue): False
in-table-section(mineral-water-bottle,white): True
in-table-section(red-can,blue): True
in-table-section(red-can,white): False
in-table-section(spray-bottle,blue): **False**
in-table-section(spray-bottle,white): False
robot-gripper-empty(): **False**
robot-holding-in-air(green-mug): False
robot-holding-in-air(mineral-water-bottle): False
robot-holding-in-air(red-can): False
robot-holding-in-air(spray-bottle): **True**

(b) Example transition annotated with S3E. State changes highlighted.

State Estimation at the CLAIR Lab



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3-tiered State Estimation

For task planning, we have a probabilistic sensor function where Ω is a set of observations and \mathcal{O} is a sensor function specifying the conditional observation probabilities

$\mathcal{O}_{s,a}^o = \mathcal{P}[O_{t+1} = o | S_t = s, A_t = a]$ of receiving observation token $o \in \Omega$ in state s after applying action a .

At the lower levels, a *sensor function* is used to record the position of the mobile agent in the inertial frame \mathcal{F}_W as

$$\mathbf{z}(k) = h[\psi(k)] + \mathbf{q}(k). \quad (2)$$

where, h is the expected observation of the system state $\psi(k)$ and $\mathbf{q}(k) \sim \mathcal{N}(\mathbf{0}, \mathbf{R})$ is the zero-mean i.i.d. Gaussian sensor noise.

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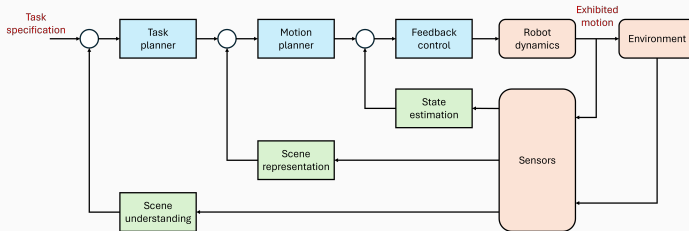


Figure 9: Full Process

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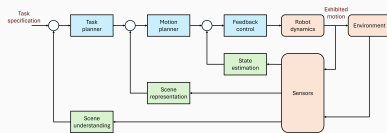


Figure 10: Caption

Different Time Scales for Different Needs

- Planning: Seconds to minutes
- Behavioral: Fraction of seconds
- Execution: Milliseconds
- Control: Microseconds
- Hardware: Nanoseconds

Information Flow:

- **Top-Down Flow:** Goals → Plans → Commands
- **Bottom-Up Flow:** Sensor data → Status updates → Results

Any ideas for information flow?

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Summary:

- We looked at the different layers of decision-making

What next ?

- Take a closer look at feedback control