Value of Assistance for Mobile Agents - Supplementary Material

We provide here additional results and details on our computations. Videos demonstrating our lab experiments can be found online. The video at **https://vimeo.com/791863135** demonstrates a single run without assistance and the video at **https://vimeo.com/791864921** shows the performance when assistance is provided at waypoint 5.

I. Localization Cordiality

As stated in section 4, here we present in Figure 5b the percentage of map-path pairs for which the AECD value was within the 1, 2, or 3 maximal VOA values for the different value.

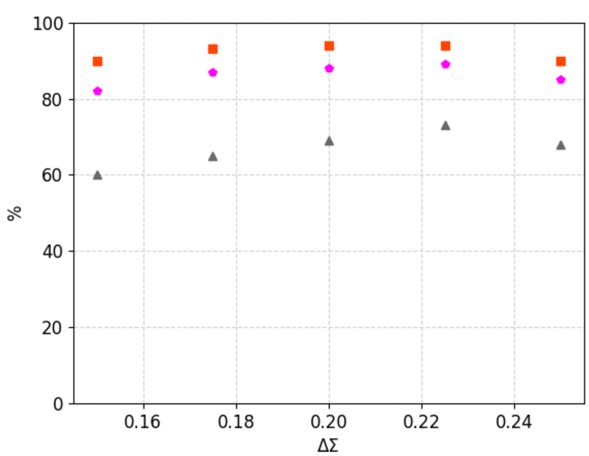


Fig. 1: Localization assistance: The percentage of map-path instances in which the waypoint with maximal AECD was within the sets of 1, 2, and 3 waypoints with maximal VOA values.

II. Numerical Calculations for Localization

In section 4.1, we calculated the values of AECD and VOA for localization in simulation. Below are the details of the numerical computation of VOA in this scenario.

According to Equation 20, VOA for localization is computed as follows.

$$\mathcal{V}\left(\mathcal{M}, \Pi^{P}, \tau = l/m\right) = C_1(l) - C_2(l) - C_3(l), \quad (1)$$

With the costs $C_1(l), C_2(l)$ and $C_3(l)$ as the path before assistance, during the correction and after the correction, respectively. The difference between $C_1(l)$ and $C_3(l)$ was calculated similarly to how we numericly calculated VOA for relocation as seen in equation 21.

$$C_1(l) - C_3(l) = \sum_{k=l}^{m} \sum_{i=1}^{100} \sum_{j=1}^{100} \mathcal{K}_{l,k}\left(\boldsymbol{p}_k, k\Delta\Sigma I_2\right)_{i,j} \mathcal{M}_{i,j}, \quad (2)$$

The difference between location uncertainties with and without assistance is calculated according to Equation 8

$$\mathcal{K}_{l,k} = \mathcal{K}\left(\boldsymbol{p}_k, k\Delta\Sigma I_2\right) - \mathcal{K}\left(\boldsymbol{p}_k, \Sigma_l + (k-l)\,\Delta\Sigma I_2\right) \quad (3)$$

The cost of the corrective path $\Pi^{P,2}$ computed according to Equation 14.

$$C_2(l) = \sum_{a=1}^{100} \sum_{b=1}^{100} \mathcal{K}\left(\boldsymbol{p}_s + l\boldsymbol{v}, l\Delta\Sigma I_2\right)_{a,b} C_{a,b} \quad (4)$$

Location uncertainty after the corrective path is calculated according to Equation 15.

$$\Sigma_l = \sum_{a=1}^{100} \sum_{b=1}^{100} \mathcal{K}\left(\boldsymbol{p}_s + l\boldsymbol{v}, l\Delta\Sigma I_2\right)_{a,b} m_1 \Delta\Sigma I_2 \quad (5)$$

The cost of $\Pi_i^{P,2}$ is computed according to Equation 13

$$C_{a,b} = \sum_{k=1}^{m_1} \sum_{i=1}^{100} \sum_{j=1}^{100} \mathcal{K}\left(\begin{bmatrix} a \\ b \end{bmatrix} + k\boldsymbol{u}, k\Delta\Sigma I_2\right)_{i,j} \mathcal{M}_{i,j} \quad (6)$$

III. Conditions for Validity of our VOA Localization

Conditions under which the expected cost of the corrective path can be computed using a Gaussian process as stated in section 3.2. These conditions can hold if the path planner used generates similar corrective paths to adjacent points. Formally, these conditions hold if for any two initial points $\boldsymbol{x}_i$, $\boldsymbol{x}_j$ and their corresponding corrective paths $\Pi_i^{P,2}$, $\Pi_j^{P,2}$ the following holds: $\forall \varepsilon > 0$ s.t. $\|\boldsymbol{x}_j - \boldsymbol{x}_i\| < \varepsilon$, $\exists \delta_C > 0$, $\exists \delta_\Sigma > 0$, s.t.

1) $\left|C\left(\Pi_j^{P,2}\right) - C\left(\Pi_i^{P,2}\right)\right| < \delta_C$ and $\lim_{\varepsilon \to 0} \delta_C = 0$;
2) $\left|\sigma_{j,k}\left(\Pi_j^{P,2}(1), 1\right) - \sigma_{i,k}\left(\Pi_i^{P,2}(1), 1\right)\right| < \delta_\Sigma$, $\forall k \in \left\{1, \cdots, n^2\right\}$ and $\lim_{\varepsilon \to 0} \delta_\Sigma = 0$, where $\sigma_{i,k}$ is the k element in Σ_i;

IV. Extra Results

In Section 4.1 describing our simulation, we showed in figures 4.a and 4.b results of VOA and AECD for single map-path pairs. We provide here additional results for simulation with localization and relocation and real-world settings with relocation in figures 2, 3 and 4. The results here are akin to those shown in the paper, demonstrating the similarity between the VOA and AECD values.

V. VOA Analysis

In the empirical section (Section 4 in the main paper) we showed in figure 5 the percentage of map-path instances in which the waypoint with maximal AECD was within the sets of 1, 2, and 3 waypoints with maximal VOA values. The waypoint with the maximal AECD value was within the 1,2 or 3 maximal VOA values 60%, 85%, and 92%, respectively. We also showed in Figure 4a of the main paper that VOA values are close to AECD (within 0.1 STD). To examine these results more deeply, we compare the magnitude of the difference between the top 2 and the top and third VOA values.

Tables I and II show the average percentage of the difference between the maximal VOA value and the second and third maximal value divided by the maximal difference of VOA for a given map-path pair, for different $\Delta\Sigma$ values. Results show that differences are small, with a maximal difference of 5.94 and 12.85 for the difference between the first and second, and first and third, respectively.

$\Delta\Sigma$	VOA1 to VOA2	VOA1 to VOA3
0.15	5.77%	12.49%
0.175	5.88%	12.60%
0.20	5.94%	12.85%
0.225	5.90%	12.69%
0.25	5.87%	12.43%

TABLE I: The average percentage of the difference between the maximal VOA value and the second and third maximal value divided by the maximal difference of VOA for a given map-path pair, for different $\Delta\Sigma$ values with relocation

$\Delta\Sigma$	VOA1 to VOA2	VOA1 to VOA3
0.15	3.20%	6.95%
0.175	3.01%	6.74%
0.20	2.78%	6.62%
0.225	2.78%	6.63%
0.25	2.60%	6.62%

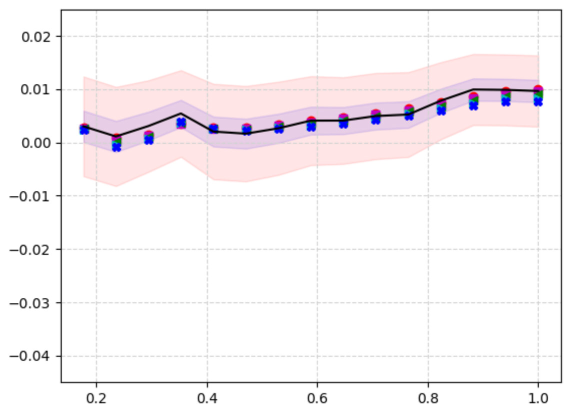
TABLE II: The average percentage of the difference between the maximal VOA value and the second and third maximal value divided by the maximal difference of VOA for a given map-path pair, for different $\Delta\Sigma$ values with localization

Let us denote τ_{voa} as the best point of assistance designated by the VOA, and τ_{AECD} as the true best point of assistance according to the AECD. Figure 5 shows results for settings in which AECD at τ_{VOA} is up to 20% smaller than AECD at τ_{AECD}.

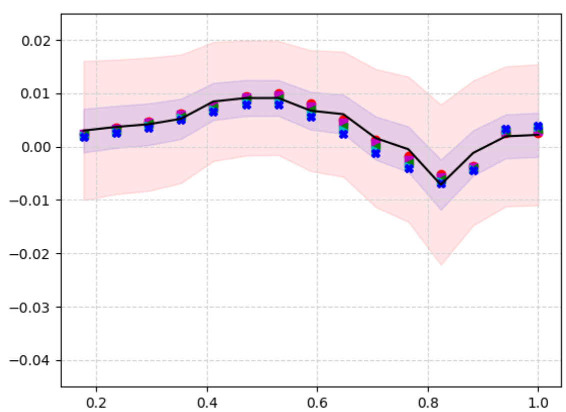
Results show that when allowing up to 20% error, the percentage of map-path instances in which the waypoint with maximal AECD was within the sets of 1, 2, and 3 waypoints with maximal VOA values increases, especially for cordiality 1 for which it increased from 60% to $\sim 85\%$. This indicates that even if VOA does not assign the highest value to the waypoint with the highest AECD value, its error is bounded, especially when considering instances for which the difference between the AECD values is non-negligible.

VI. Results for Example 1

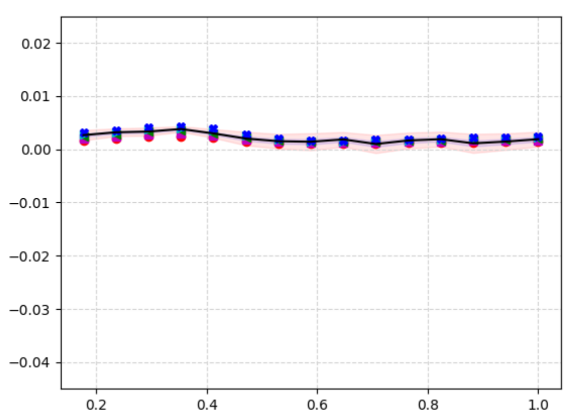
We demonstrate our measures for the setting described in Example 1, in which the agent has a sequence of waypoints it needs to follow (modeled using high rewards near the current waypoint while avoiding high-risk areas). The map for this settings is given in Figure 6. Results can be seen in Figure 7.



(a)

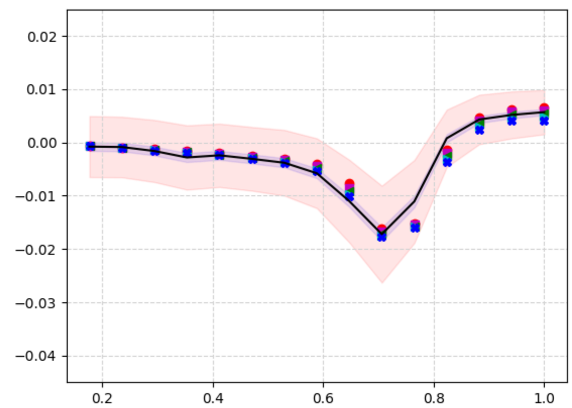


(b)

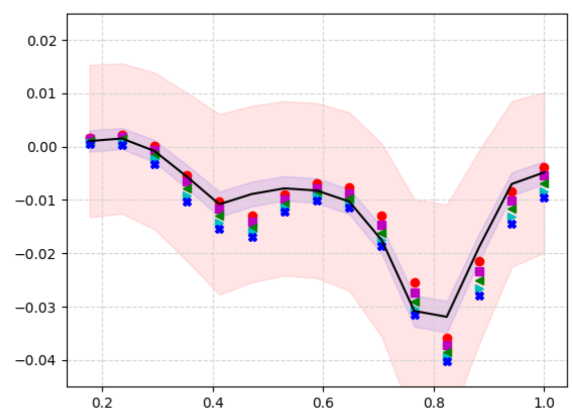


(c)

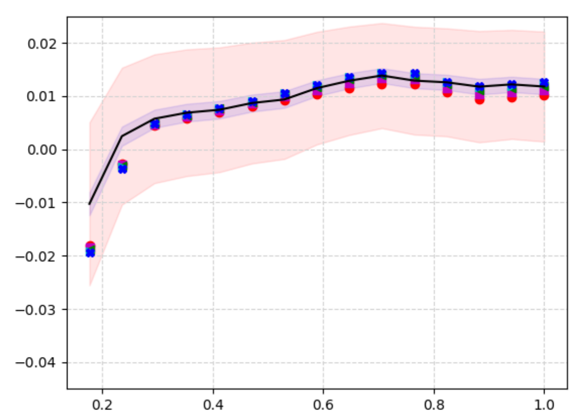
Fig. 2: AECD and VOA (y-axis) for each path progression τ (x-axis) for random map-path pair with **relocation** in simulation. AECD is plotted with its 99% confidence interval and 0.1 standard deviations.



(a)

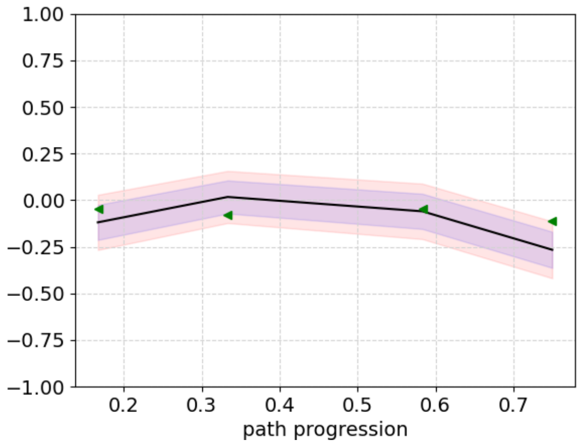


(b)

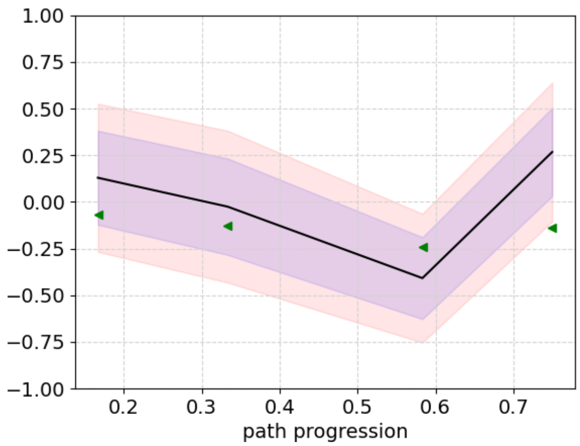


(c)

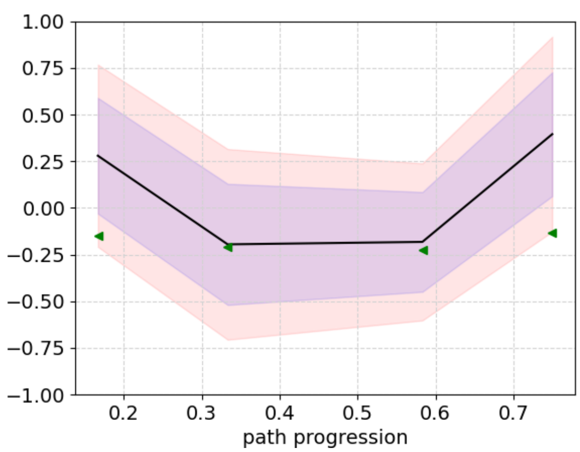
Fig. 3: AECD and VOA (y-axis) for each path progression τ (x-axis) for random map-path pair with **localization** in simulation. AECD is plotted with its 99% confidence interval and 0.1 standard deviations.



(a)

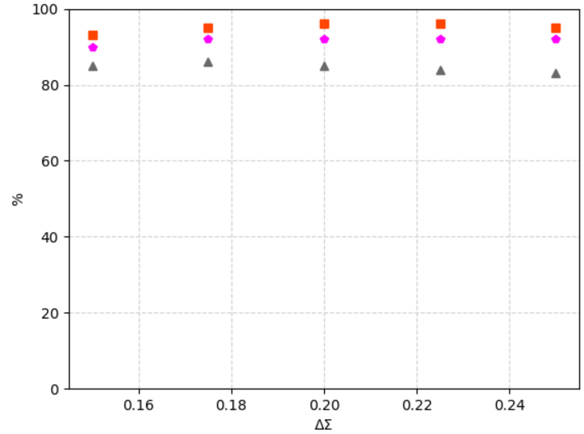


(b)

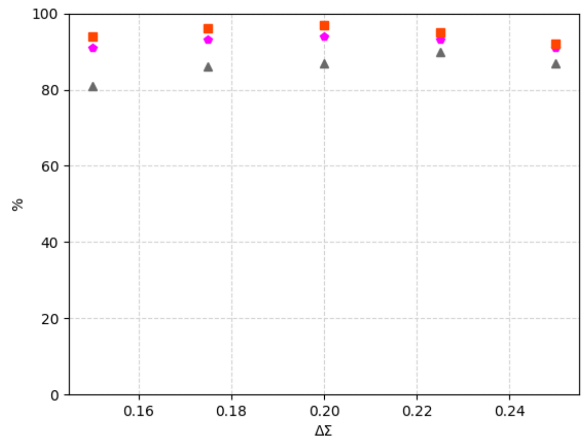


(c)

Fig. 4: AECD and VOA (y-axis) for each path progression τ (x-axis) for random map-path pairs for relocation real-world setting. AECD is plotted with its 99% confidence interval and 0.3 standard deviations.



(a) Relocation assistance



(b) Localization assistance

Fig. 5: The percentage of map-path pairs in which the waypoint with maximal AECD was within 20% of the value of the top 1, 2, and 3 waypoints with maximal VOA values.

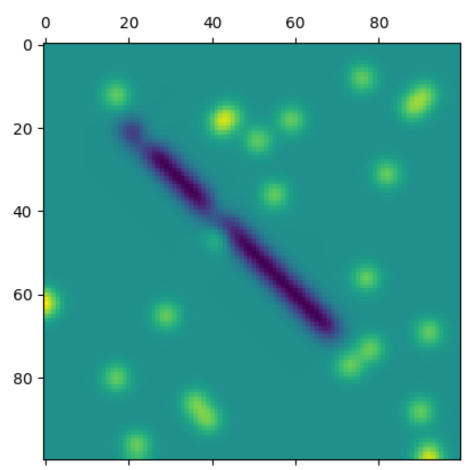
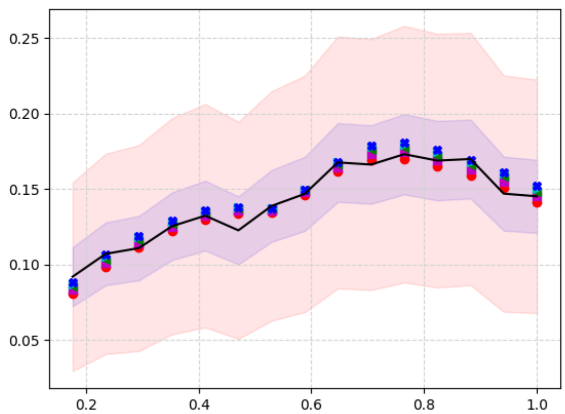
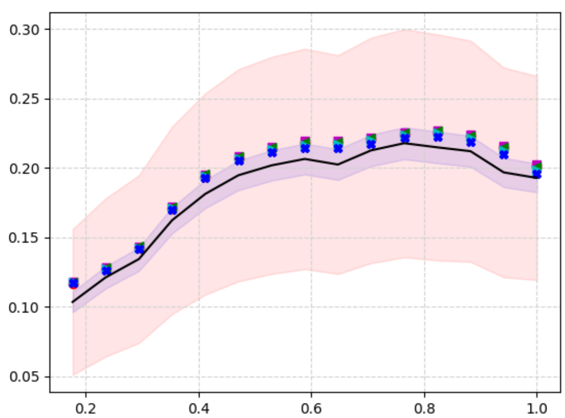


Fig. 6: Map Example 1



(a) Relocation assistance



(b) Localization assistance

Fig. 7: AECD and VOA (y-axis) for each path progression τ (x-axis) for random map-path pair with **reward for being close to the path**. AECD is plotted with its 99% confidence interval and 0.1 standard deviations.