Series Notes

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Special Series

Geometric Series

- definition: A geometric series is any series that can be written in the form $\sum_{n=0}^{\infty} ar^n$.
- value: $a\frac{1-r^n}{1-r}$ and $\frac{a}{1-r}$ when $n \to \infty$
- convergence: converges when $|r| < 1$

Telescoping Series

- definition: In mathematics, a telescoping series is a series whose general term t_n is of the form $t_n = a_{n+1} + a_n$.
- value: consider the partial sum and calculate by cancelling some parts
- convergence: decide with its limit after cancelling all parts that can be cancelled

Harmonic Series

- definition: A Harmonic Series is any series that can be written in the form $\sum_{n=1}^{\infty} \frac{1}{n}$.
- value: use Integral Test to decide
- convergence: use Integral Test to decide

Integral Test

Integral Test is to decide a series' convergence with improper integral.

The infinite series $\sum_{n=N}^{\infty} f(n)$ converges to a real number if and only if $\int_N^{\infty} f(x)dx$ is finite. In particular, if the integral diverges, then the series diverges as well.

If the improper integral is finite, then the proof also gives the lower and upper bounds

$$\int_N^{\infty} f(x)dx \leq \sum_{n=N}^{\infty} f(n) \leq f(N) + \int_N^{\infty} f(n)dx$$

for the infinite series.