



Math Saves Lives

How Formal Methods Keep Trains on Track and Software from Going Off the Rails



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Art mostly generated
with *ChatGPT or similar*



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CLEARSY

Safety Solutions Designer

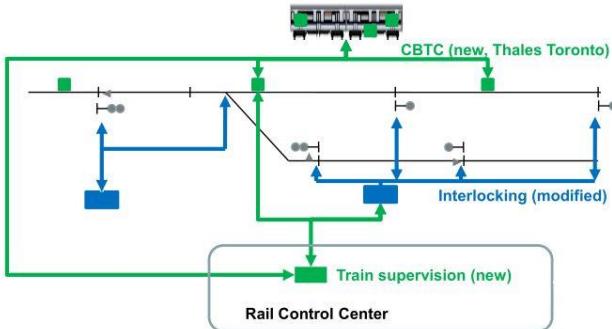


Metro automatic pilots



SIL2 - SIL4 safety critical systems

SME created in 2001
150+ engineers
~120 MR\$ turnover 2022
Aix, Lyon, Paris,
Strasbourg



System formal engineering,
RAMS, services



Editor



SIL4 products

<http://www.clearsy.com/>

My CV

Development of Atelier B and its proof tools

Applied Formal methods

Teached at IMD 



Applications to industry

Smartcard certification

ProB for formal data validation

Railways signalling software

SP metrô: platform screen doors control 

R&D and education

28 EU and French R&D projects

CLEARSY Safety Platform

Courses in 30 universities and schools

MOOC on B 



The B Method - Marketing Video

This video explains why you should follow the MOOC on B and what its expected benefits on your career are.

Level: Basic

Video duration: 02:55



My Subjects

Safety

How not to kill people ?



Autonomous Mobility

How to remove control from human?

Cybersecurity

How to protect secrets ?

Artificial Intelligence

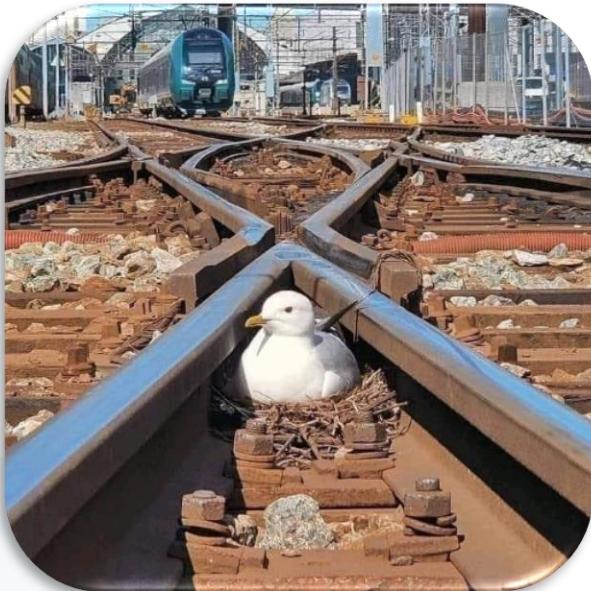
How to do smart things?

Common Thread



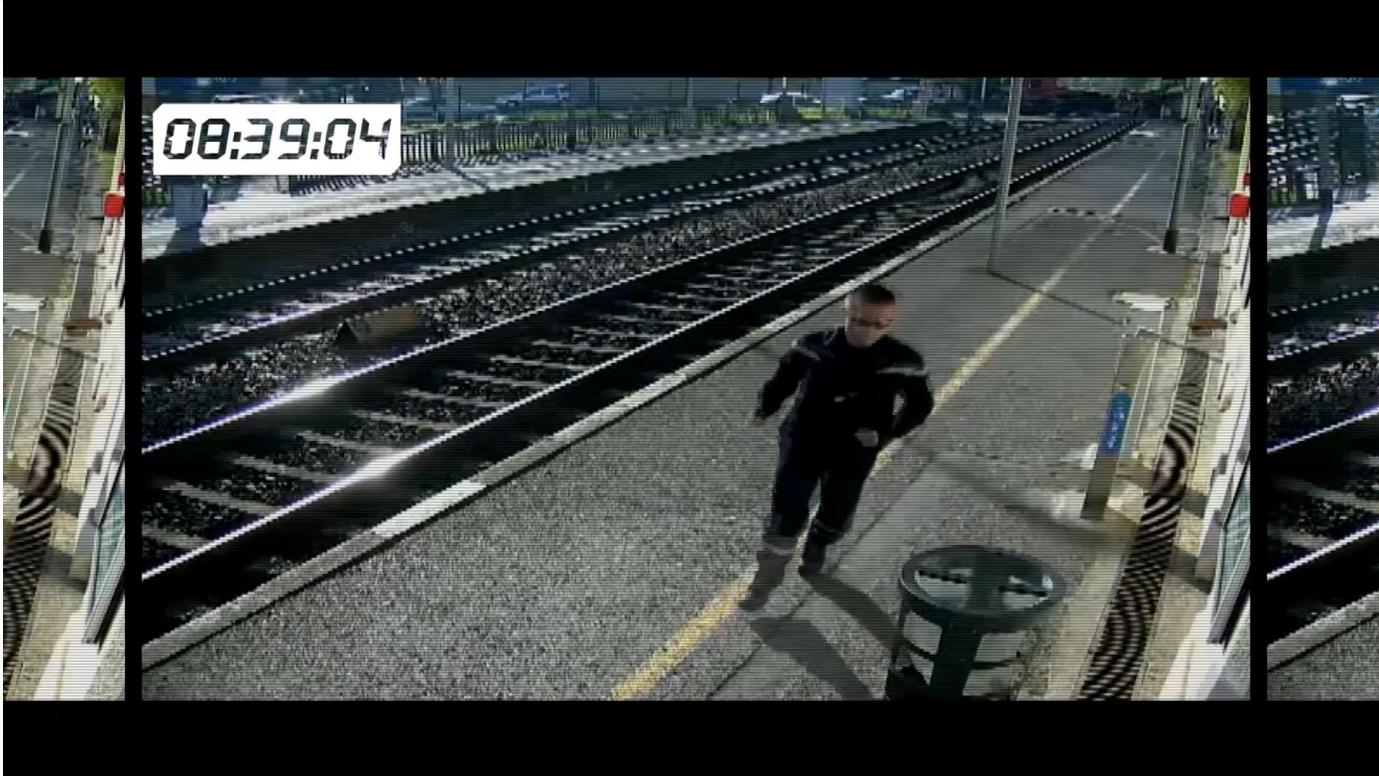
We are going to
design a train line
with different configurations
and see
how formal methods can help

SAFETY

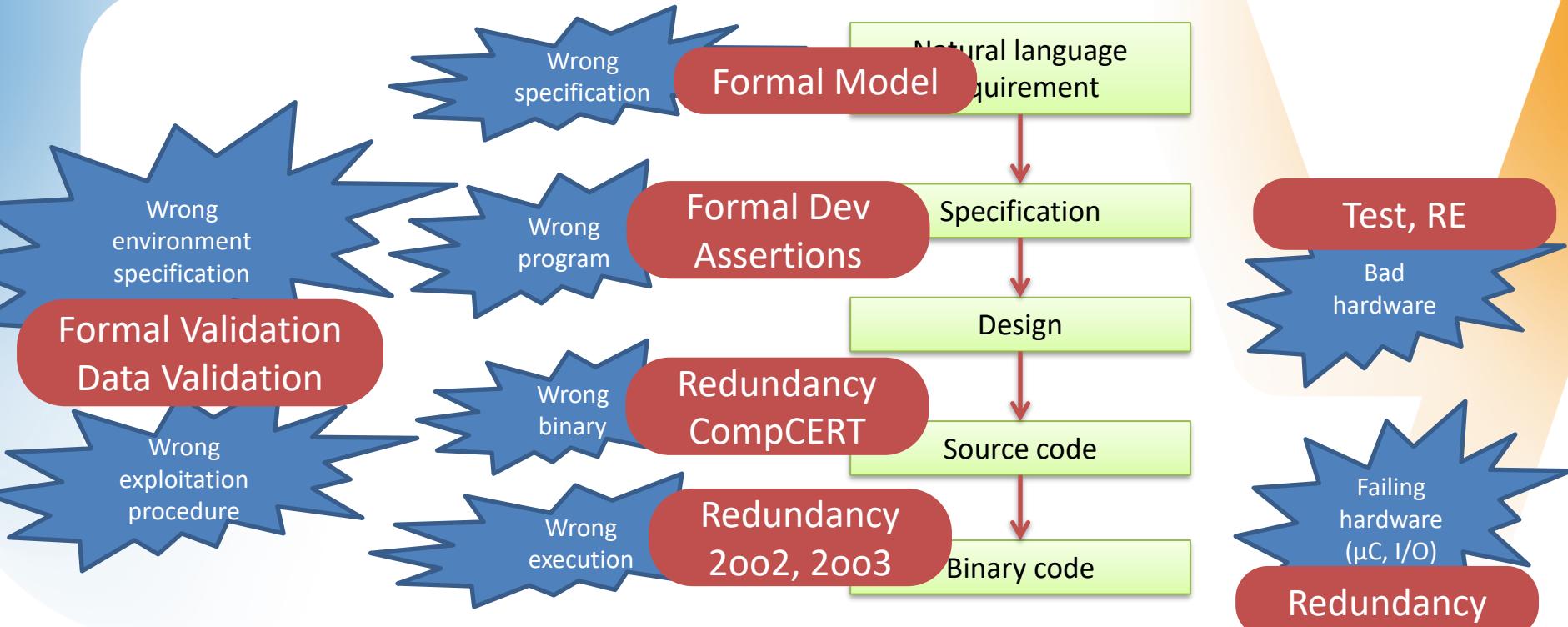


- Failing systems
- Safety critical
- Standards

Safety is about things that happen 1 in 1,000,000



Failing Software-Based Systems



Safety @ Railways

SAFETY INTEGRITY LEVELS

SIL3 : $10^{-7}/\text{h}$
SIL4 : $10^{-9}/\text{h}$

CATASTROPHIC
FAILURES

CERTIFICATION

NL safety demonstration
Convince responsible human expert
Formal methods **highly recommended**

STRONG STANDARDS

EN5012{6, 8, 9}

SYSTEMATIC FAILURES

Specification
Design
Implementation
Environment
Exploitation

RANDOM FAILURES

Execution machine
Entropic hardware

Railways in a Nutshell

- ▶ Move persons / goods from point A to B
 - ▷ using predefined paths
- ▶ Trams, Metros, Freight train, Passenger Trains, High Speed Train





Safety Properties

- ▶ No collision
- ▶ No overspeed
- ▶ No derailment



Whatever the conditions

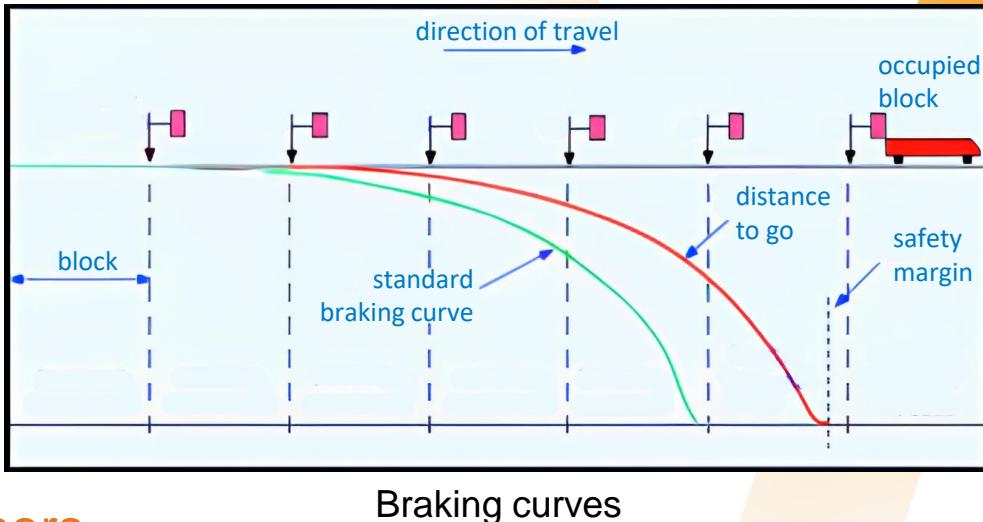
No collision

- ▶ Collision with trains
- ▶ Collision with obstacles
- ▶ Collision with animals



No collision

- ▶ Collision with trains
 - ▷ Braking curves
- ▶ Collision with obstacles
 - ▷ Driver, tunnels, sensors
- ▶ Collision with animals
 - ▷ Driver, fences, tunnels, sensors



No Overspeed

- ▶ Emergency braking if speed over speed limit
 - ▷ Shutdown engine, trigger break
 - ▷ Alarms (sound) when approaching speed limit
- ▶ Means to measure speed
 - ▷ Odometer, GNSS out
- ▶ Resume travel when at full stop

No Derailment

- ▶ Prevent switch to move when a train is passing by

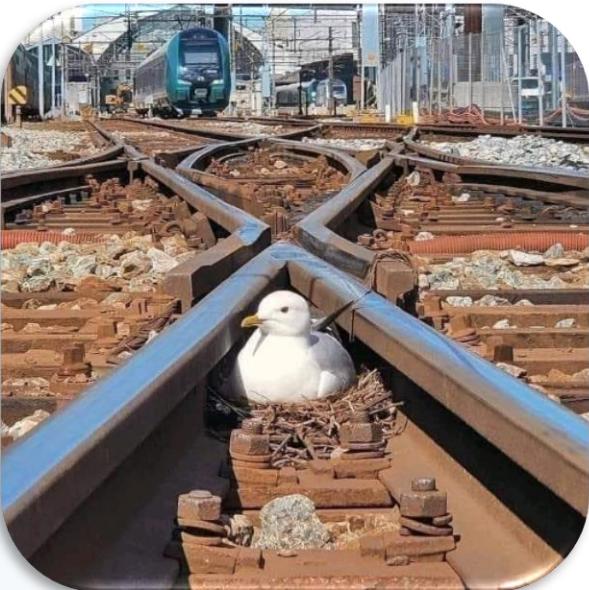


No Derailment

- ▶ Routes are allocated to trains by interlocking (IXL)
 - ▷ Switch fixed until the train leaves
- ▶ The absence of derailment is not always guaranteed
 - ▷ Ex: after an earthquake



CASE-STUDY



- Monocab
 - Configuration 1 (intro to B)
 - Configuration 2
 - Configuration 3
 - Data Validation
-
- Adding decisions, details
 - Perform some (simplified) verifications of different nature

MONOCAB

(<https://www.monocab-owl.de/english-language/>)

- ▶ Compact, autonomous monorail vehicle designed to revitalise disused railway lines
- ▶ Offers an innovative transport alternative in rural areas.
- ▶ Reuses existing German railway infrastructure



MONOCAB

(<https://www.monocab-owl.de/english-language/>)

- ▶ Uses advanced gyroscopic technology to stabilise itself on a single conventional track,
- ▶ Vehicles travelling in opposite directions pass on the same track.

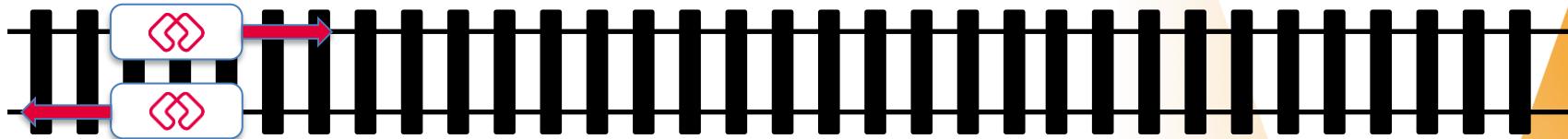


MONOCAB ao vivo



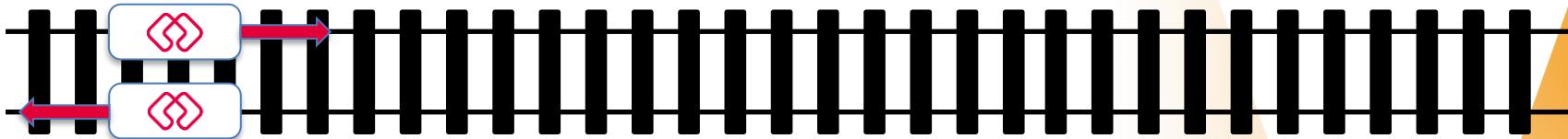
CONFIGURATION 1

Configuration 1

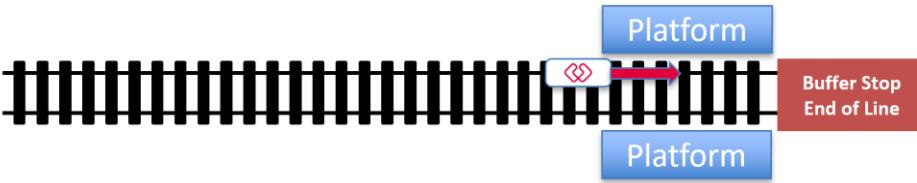


- ▶ 5 km line between only 2 stations
- ▶ Capacity: 8 passengers per car
- ▶ Operating speed: 50 km/h
- ▶ Slope: 0%
- ▶ No crossroads
- ▶ 2 trains in opposite phase on separate rail
- ▶ Hypotheses:
 - ▷ 6' for a ride;
 - ▷ 1' for passengers transfer
- ▶ Traffic:
 - ▷ 8.5 travels / h / train
 - ▷ 136 passengers / h both ways

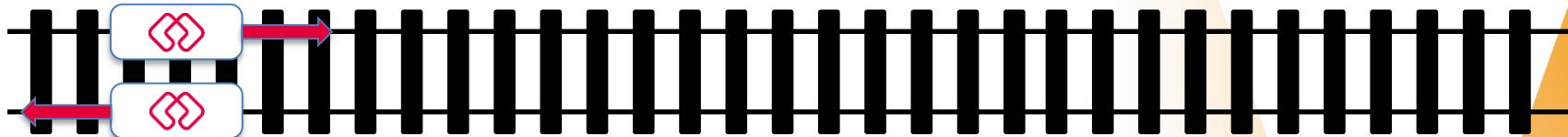
Configuration 1: safety



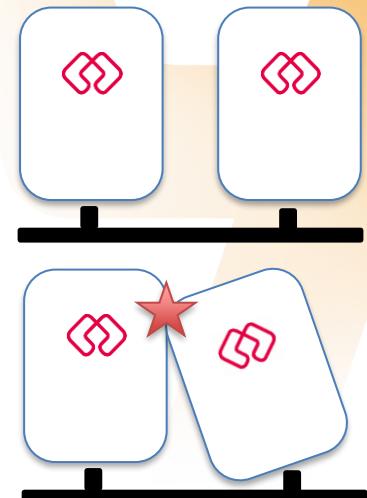
- ▶ No collision with train - on same rail
 - ▷ 1 train per rail
- ▶ No collision with train - on other rail ?
- ▶ No collision with obstacles
 - ▷ Flat ground, no slope, no trees around
 - ▷ Collision with buffer stop
- ▶ No collision with animals
 - ▷ Fences on both side, no crossroads
- ▶ No overspeed
 - ▷ Need reliable speedometer, minimum guaranteed braking capabilities
- ▶ ~~No derailment (no switch)~~
- ▶ NB: we are not controlling train doors opening but we could reduce speed if someone open a door during travel



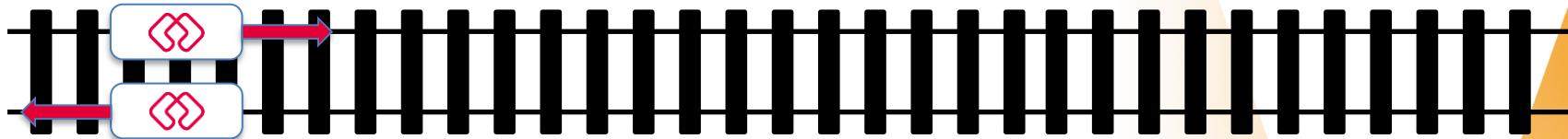
Configuration 1: safety



- ▶ No collision with train - on other rail ?
 - ▷ If gyroscopic mechanism fails
 - ▷ If unexpected conditions (shaky passengers, strong lateral wind)
- ▶ How to limit risks ?
 - ⇒ Reduce speed when trains “meet” (~~how ? when ?~~)
 - ▷ Reduce operation speed when windy

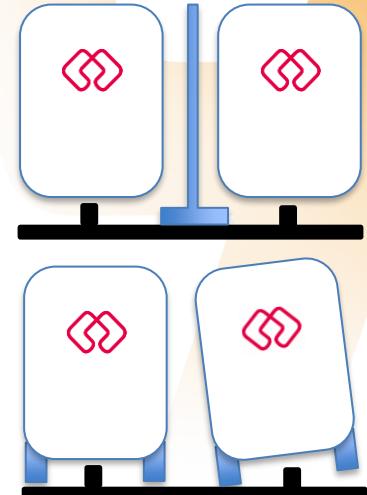


Configuration 1: safety



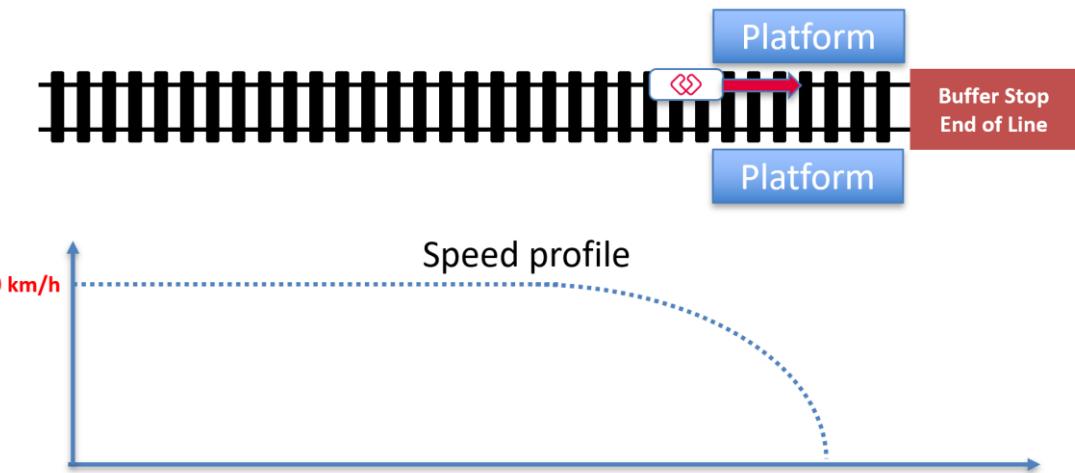
► How to avoid ?

- ▷ Physical obstacle between cars (wall, pole) – and a way to detect the situation (contact, angle, speed)
DECISION
- ▷ “Feets” (as in the video) to limit angular deviation (but risk of hitting a low obstacle on the track)
- ▷ Telescopic “feets” that are deployed when angular deviation is detected (such a sensor is needed) – and train is stopped



Configuration 1: safety

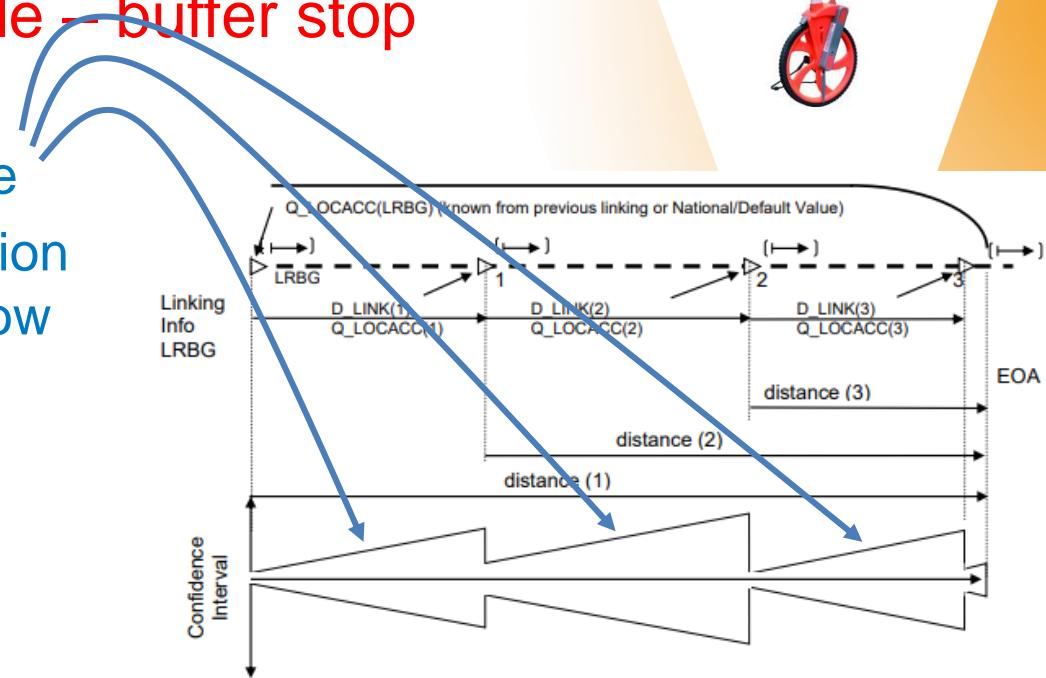
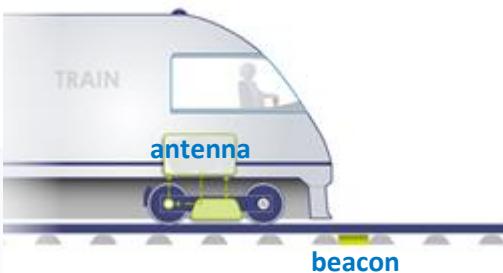
- ▶ No collision with obstacle – buffer stop
 - ▷ We need to know when the train is approaching the platform to slow down and avoid collision



Configuration 1: safety

► No collision with obstacle – buffer stop

- Use an odometer but increasing error over time
- Beacons to provide location when confidence is too low

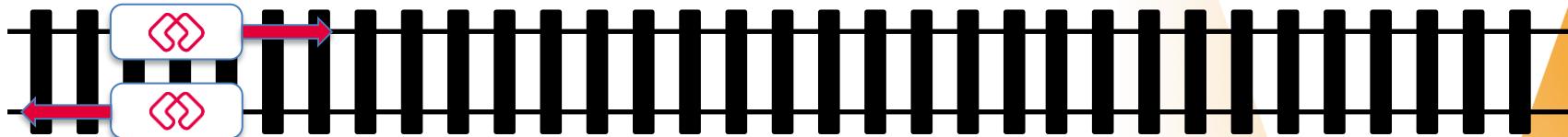


Configuration 1: safety

- ▶ No collision with obstacle – buffer stop
 - ▷ Beacons can be missed or misread (electronic failure, beacons moved, covered by object)
 - ▷ We need diversity to estimate if we are close to the end of the line
 - ▷ It could be a laser measuring the distance with the platform (but could be misaligned and function becomes critical) **DECISION**
 - ▷ An internal timer could activate a watchdog if 6' – 30" have passed since the start of the trip, the speed limit is decreased.
 - ▷ To stop the train, we could add gravels at the end of the line

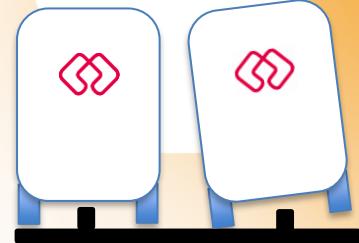


Configuration 1: safety

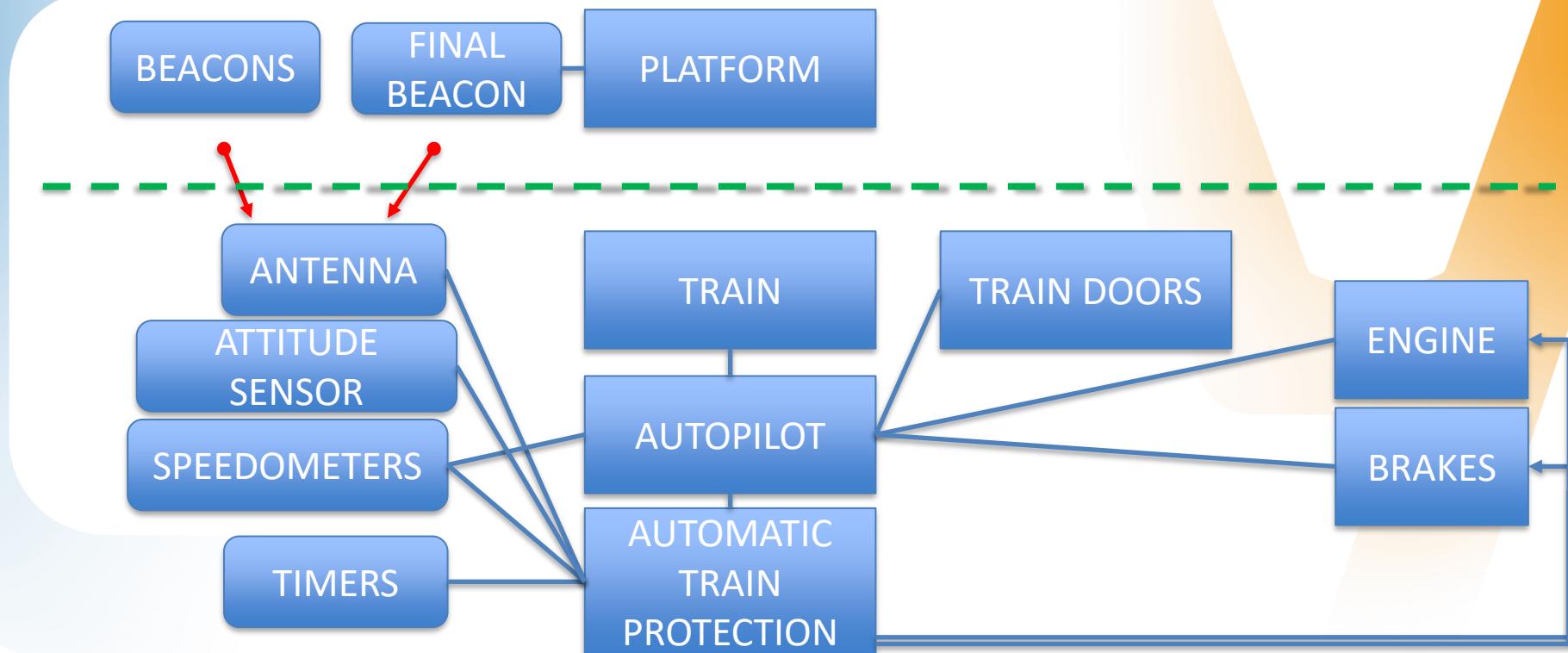


► Summary

- ▷ 1 train per rail, 2 trains total operated independently, no slope, no crossroads, fences on both sides
- ▷ Reduce speed operation when windy (experiments needed to define speed limits / wind speed)
- ▷ Fixed feet to limit angular deviation
- ▷ Use beacons and timer to determine if the train is approaching the end of line (engineering to determine number of beacons and locations, speed profile)
- ▷ Need reliable speedometer, minimum guaranteed braking capabilities



System Analysis



SHORT INTRODUCTION TO B



« Only inactive sequences can be added to the active sequences execution queue. »

```
activation_sequence = /* Activation d'une séquence non active */
PRE ¬(sequences = sequences_actives) THEN
  ANY sequ WHERE
    sequ ∈ sequences - sequences_actives
  THEN
    sequences_actives := sequences_actives ∪ {sequ}
  END
END;
```

```
activation_sequence = /* Activation d'une séquence non active */
VAR sequ IN
  sequ <-- indexSequenceInactive;
  activeSequence(sequ)
END;
```

```
void M0_activation_sequence(void)
{
  CTX_SEQUENCES sequ;

  sequence_manager_indexSequenceInactive(&sequ);
  sequence_manager_activeSequence(sequ);
}
```

0x01F970	FFFF 8B4C 2440 89C5 8D7D 0C8B 4110 89CE
0x01F980	83C6 0C8D 1485 0000 0000 8D42 0883 F807
0x01F990	7617 F7C7 0400 0000 740F 8B41 0C8D 7D10
0x01F9A0	83C6 0489 450C 8D42 04FC 89C1 C1E9 02F3

Natural language requirement

Behaviour + properties

B Specification

Behaviour + properties

B Implementation

C generated code

Binary code



« Only inactive sequences can be added to the active sequences execution queue. »

Natural language requirement

```
activation_sequence = /* Activation d'une séquence non active */  
PRE ¬(sequences = sequences_actives) THEN  
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    sequences_actives := sequences_actives ∪ {sequ}  
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END;
```

```
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{  
  CTX_SEQUENCES sequ;  
  
  sequence_manager_indexSequenceInactive(&sequ);  
  sequence_manager_activeSequence(sequ);  
}
```

0x01F970	FFFF 8B4C 2440 89C5 8D7D 0C8B 4110 89CE
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0x01F9A0	83C6 0489 450C 8D42 04FC 89C1 C1E9 02F3

Cyclic software single-thread

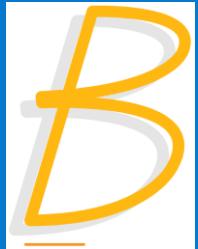
B Specification

B Implementation

C generated code

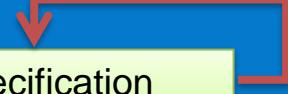
Binary code

Proof (coherence)

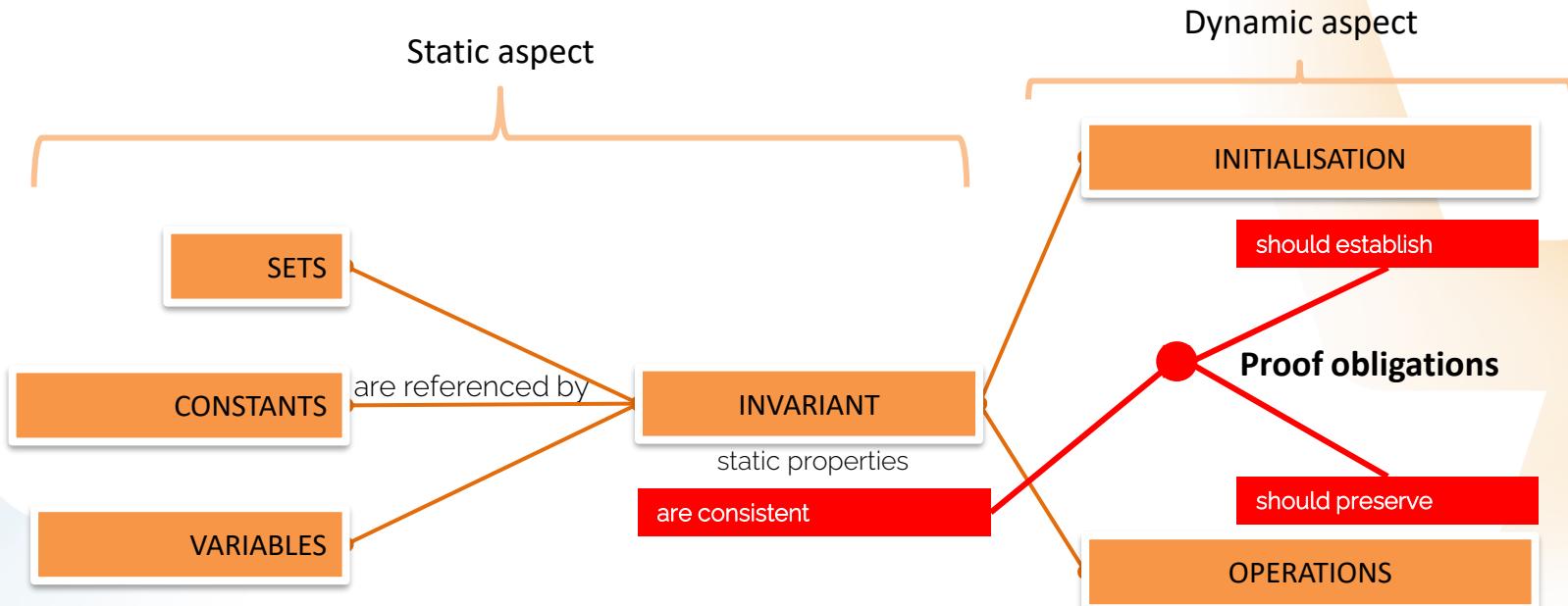


Proof (refinement)

Proof (coherence)



Proof Obligations from B Models

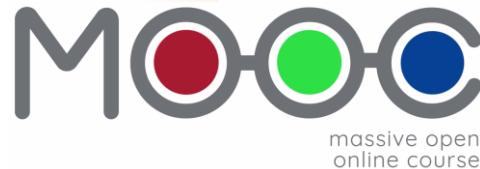


B Code Generation

- ≡ The software code is generated from the model
Code is readable, very close to the model and is easily checked

```
M0.mch M0_i.imp
1 IMPLEMENTATION M0_i
2 REFINES M0
3
4 CONCRETE_VARIABLES
5 5/5    xx
6 INVARIANT
7 3/3    xx: INT
8 INITIALISATION
9 1/1    xx := 0
10
11 OPERATIONS
12 4/4    inc = IF xx = MAXINT THEN xx:=0 ELSE xx := xx +1 END
13 END
```

```
11 static int32_t M0_xx;
12 /* Clause INITIALISATION */
13 void M0_INITIALISATION(void)
14 {
15     M0_xx = 0;
16 }
17
18 /* Clause OPERATIONS */
19
20 void M0_inc(void)
21 {
22     if(M0_xx == 2147483647)
23     {
24         M0_xx = 0;
25     }
26     else
27     {
28         M0_xx = M0_xx+1;
29     }
30 }
31 }
```



massive open
online course

<https://mooc.imd.ufrn.br/>



Lecture 0: Marketing video

This video explains why you should follow the MOOC on B and what its expected benefits on your career are.

Level: Basic

Video duration: 02:55



Lecture 1: Course Introduction

This video presents the structure of the course, provides an overview of the different kinds of formal methods and specification styles, and tells us some myths on formal methods

Level: Basic

Video duration: 08:43

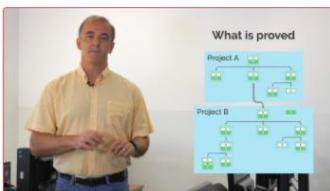


Lecture 2: Overview of the B method

This video briefly introduces the tool Atelier-B, the B and Event-B languages, and some industrial references. The main concepts of B are exposed.

Level: Basic

Video duration: 15:03



Lecture 3: The concepts of B

This video presents the founding notions of B: projects, libraries, modules, components, abstract machine, refinement, implementation, and proof.

Level: Basic

Video duration: 09:29



Lecture 4 : introduction to Abstract Machines

This video introduces the notion of abstract machines, based on an example that is verified, animated and for which C source code is generated.

Level: Basic

Video duration: 16:31



Software Formal Development

1998

Paris L14 Automatic Train Protection (ATP)
Emergency braking in case of danger (86 kloc B, 110 kloc Ada)



2000-2024

Used by ~30% radio-based control metro worldwide
CDGVAL shuttle (500 kloc / automatic refinement)

2006-2024

Used for Paris L1, L4, L13, L14 (Olympics)

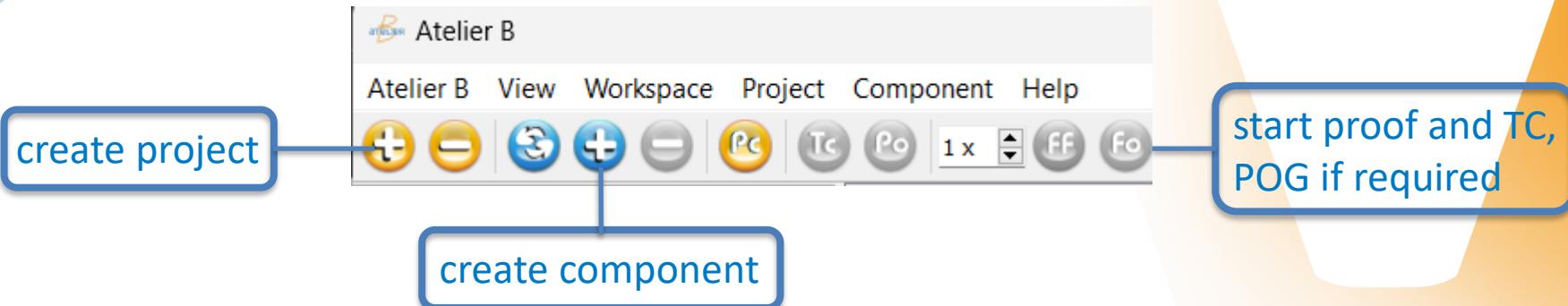
2024-2030

To be used for Paris L15, L16, L17, L18

Formal IDE for B

- ▶ **ATELIER B** <https://www.atelierb.eu/en/atelier-b-support-maintenance/download-atelier-b/>
 - ▷ Model software (B) and systems (Event-B)
 - ▷ Automatic and interactive proof
 - ▷ Code generation
 - ▷ Simple installer
- ▶ **PROB** <https://prob.hhu.de/w/index.php?title=Download>
 - ▷ Same support but simple B models architecture
 - ▷ Model checking, verification
 - ▷ Animation
 - ▷ Requires Java, Tcl/Tk

ATELIERB



- ▶ If asked, relocate your projects in a directory where you can write
- ▶ Create a project (software development)
- ▶ Add component with « + » blue button or drag/drop file from Windows explorer (Classical view)

Component	TypeChecked	POs Generated	Proof
M CTX	OK	OK	2
M M0	OK	OK	6

PROB

Load file

```
ProB 1.13.0-final: [beacons.mch]
File Edit Animate Verify Analyse Visualize Preferences Debug Files Help
SETS
Enregistré dans ce PC | NS = {b0, b1, b2, b3, b4, b5}
CONSTANTS
nextB,
lengthTC,
kB,
lastB
PROPERTIES
nextB: BEACONS --> BEACONS &
nextB = {
    b0 -> b1,
    b1 -> b2,
    b2 -> b3,
    b3 -> b4,
    b4 -> b5,
    b5 -> b0
} &
dom(nextB) V ran(nextB) = BEACONS &
lengthTC: BEACONS --> INTEGER &
States 3/3 processed, Ln 40, Col 3, Mode b
State Properties (3)
MAXINT = 3
MININT = -1
card(BEACONS) = 6
Enabled operations (1)
SETUP_CONSTANTS(nextB={(b0->b1),(b1->b2),(b2->b3),(b3->b4),(b4->b5),(b5->b0)})
```

```

1- MACHINE
2-   CTX      Context machine, No variable, Only sets and constants
3- SETS
4-   BEACONS = {b0_stop, b1_leave, b2_approach, b3_approach, b4_enter, b5_stop}
5- CONSTANTS
6-   S_MANOEUVER, /* minimum speed limit - guaranteed not to injure passengers */
7-   S_MAX, /* maximum speed limit */
8-   S_BEACONS, /* associate to each beacon a speed limit */
9-   DELAY_TRAVEL_APPROACH, /* delay where we consider that
                           first approach beacon has been missed */
10-  NEXT_BEACONS /* What is the order of reading beacons - for simulation */
11-
12- PROPERTIES
13-   S_MANOEUVER: INTEGER &          Typing + properties
14-   S_MAX : INTEGER &
15-   S_MANOEUVER > 0 &
16-   S_MAX >= S_MANOEUVER &
17-   S_BEACONS : BEACONS --> INTEGER & Total function
18-   ran(S_BEACONS) <: S_MANOEUVER..S_MAX & Codomain is included in interval
19-   S_BEACONS = {
20-     b0_stop |-> S_MANOEUVER,
21-     b1_leave |-> S_MAX,
22-     b2_approach |-> S_MAX,
23-     b3_approach |-> S_MAX,
24-     b4_enter |-> S_MANOEUVER,
25-     b5_stop |-> S_MANOEUVER
26-   } &

```

Speed limit per beacon

<https://github.com/CLEARSY/ETMF2024/>

Directory: Configuration1

Model: CTX.mch

```
28      DELAY_TRAVEL_APPROACH : NATURAL1 & Value only to shorten the simulation
29      DELAY_TRAVEL_APPROACH <= 10 &
30      NEXT_BEACONS : BEACONS --> POW(BEACONS) &
31      NEXT_BEACONS = {
32          b0_stop |-> {b0_stop, b1_leave, b2_approach, b3_approach, b4_enter, b5_stop},
33          b1_leave |-> {b1_leave, b2_approach, b3_approach, b4_enter, b5_stop},
34          b2_approach |-> {b2_approach, b3_approach, b4_enter, b5_stop},
35          b3_approach |-> {b3_approach, b4_enter, b5_stop},
36          b4_enter |-> {b4_enter, b5_stop},
37          b5_stop |-> {b5_stop}
38      }
39      END
40 
```

The order of beacon reading,
Only for simulation

```

1  MACHINE M0
2  SEES CTX
3
4  VARIABLES
5      current_speed, /* current speed - input */
6      last_beacon_read, /* id of last beacon read - input */
7      current_speed_limit, /* current speed limit computed */
8      emergency_braking, /* trigger for emergency braking - output */
9      travel_time, /* time (cycles) measured since the beginning of travel */
10     travel_completed
11  INVARIANT
12  1/1    current_speed : NATURAL &
13  1/1    last_beacon_read : BEACONS &
14  1/1    current_speed_limit : S_MANOEUVRE..S_MAX &
15    emergency_braking : BOOL &
16    travel_time : NATURAL &
17    travel_completed : BOOL &
18    /* Safety p1: overspeed implis emergency braking */
19    (current_speed > current_speed_limit => emergency_braking = TRUE)
20
21  INITIALISATION
22  1/1    current_speed := 0 ||
23  1/1    last_beacon_read := b0_stop ||
24  1/1    current_speed_limit := S_MANOEUVRE ||
25    emergency_braking := TRUE || /* safety position
26    we leave this state if conditions met */
27    travel_time := 0 ||
28    travel_completed := FALSE

```

Variables**Typing +properties****Initial values all set in //**<https://github.com/CLEARSY/ETMF2024/>

Directory: Configuration1

Model: M0.mch

```

30-
31  OPERATIONS
32    cycle_b0_b5 =
33      PRE travel_completed = FALSE THEN           Precondition
34        current_speed,
35        emergency_braking,
36        last_beacon_read,
37        travel_time: (
38          current_speed : NATURAL & /* input */
39          last_beacon_read : BEACONS & /* input */
40          /* to ensure that past beacons are read */
41          last_beacon_read : NEXT_BEACONS(last_beacon_read$0) &
42          emergency_braking : BOOL & /* output */
43          travel_time : NATURAL &
44          travel_time > travel_time$0 & Time is passing
45
46          (travel_time > DELAY_TRAVEL_APPROACH =>
47            current_speed_limit = S_MANOEUVRE) &
48          (travel_time <= DELAY_TRAVEL_APPROACH
49            => current_speed_limit = S_BEACONS(last_beacon_read)) &
50
51          (emergency_braking = TRUE => current_speed
52            <= current_speed$0 & current_speed >= 0) &
53          /* safety p1 */
54          (current_speed > current_speed_limit
55            => emergency_braking = TRUE) &
56          /* safety p2 */
57          (emergency_braking$0 = TRUE & current_speed = 0
58            => emergency_braking = FALSE) &
59          /* safety p2 */
60          (emergency_braking$0 = TRUE & current_speed > 0
61            => emergency_braking = TRUE)
62
63      )
64  END;

```

Precondition

2/2

Substitution: variables become such as ...

2/2

Var\$0 is the value of Var before the exécution of the operation

2/2

Once the approach delay is over, speed limit is set to minimum

Logic for the emergency braking



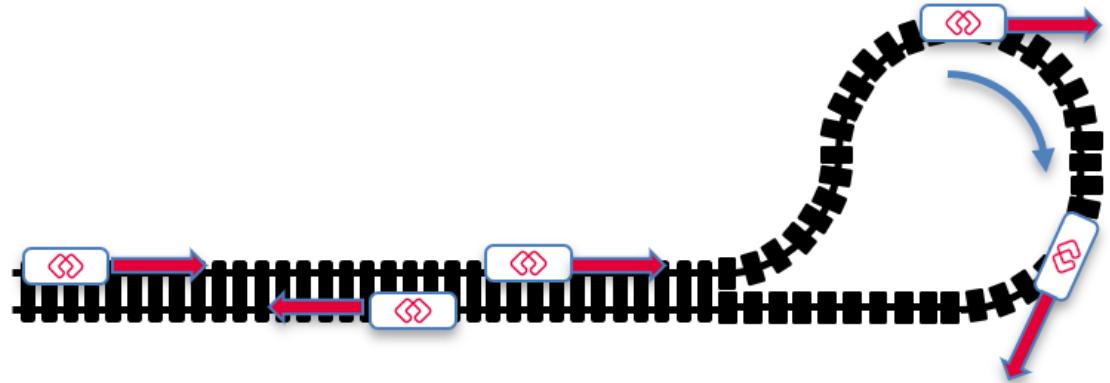
```
64 end_travel =  
65 PRE  
66     travel_completed = FALSE &  
67     last_beacon_read = b5_stop &  
68     current_speed = 0  
69 THEN  
70     travel_completed := TRUE  
71 END  
72  
73 END
```

For simulation

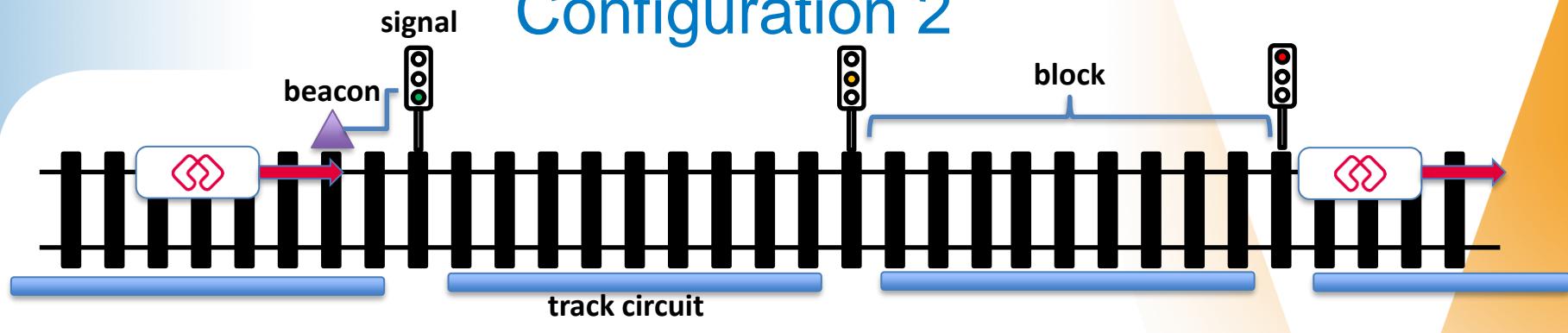
CONFIGURATION 2

Configuration 2

- ▶ Higher capacity
- ▶ Several trains
- ▶ Looped lines
- ▶ To avoid collisions
 - ▷ Signals
 - ▷ Interlocking

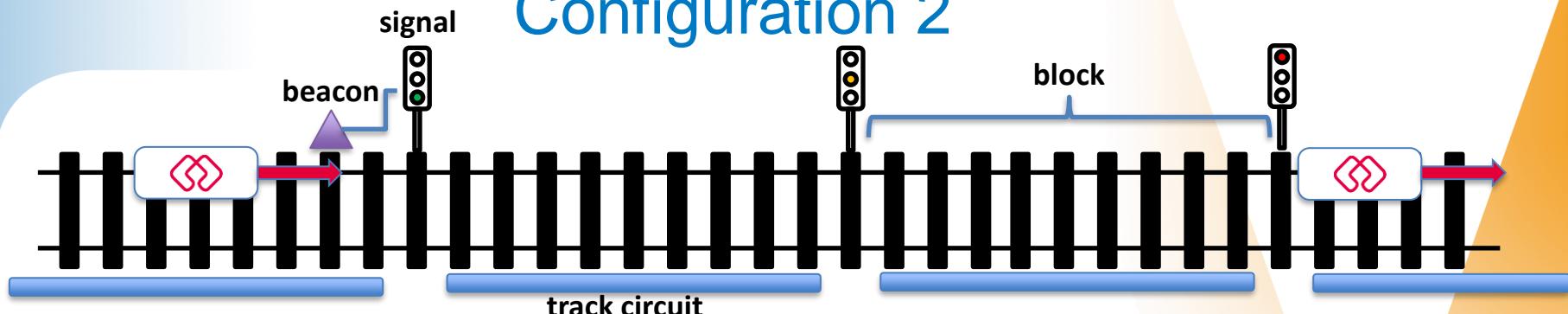


Configuration 2



- ▶ Definition of fixed blocks
- ▶ Track circuit to detect if there is a train in the block
- ▶ One train maximum per block
- ▶ Beacon: similar to position beacon, but message is variable

Configuration 2



► Signal (virtual)

- ▷ Red: next block is occupied, stop before next signal
- ▷ Orange: next next block is occupied, slow down, prepare to stop
- ▷ Green: next two blocks are free, continue

► Interlocking:

- ▷ Detect train with track circuit, position signal and beacon message
- ▷ When a train goes over a beacon, reads the status of the virtual signal
- ▷ ATO to slow down, ATP to check that the train is able to stop before red signal

```
1- MACHINE
2-     CTX
3- SETS
4-     TRACK_CIRCUITS = {tc1, tc2, tc3, tc4, tc5, tc6, tc7, tc8, tc9};
5-     SIGNALS = {s1, s2, s3, s4, s5, s6, s7, s8, s9};
6-     STATUS = {GREEN, RED}
7- CONSTANTS
8-     IS_PROTECTED_BY
9- PROPERTIES
10-    IS_PROTECTED_BY : TRACK_CIRCUITS +-> SIGNALS &
11-    IS_PROTECTED_BY = {
12-        tc1 |-> s1,
13-        tc2 |-> s2,
14-        tc3 |-> s3,
15-        tc4 |-> s4,
16-        tc5 |-> s5,
17-        tc6 |-> s6,
18-        tc7 |-> s7,
19-        tc8 |-> s8,
20-        tc9 |-> s9
21-    }
22- END
```

Static configuration
As many signals as TC
Each signal protects a TC

```

1- MACHINE
2-   IXL
3- SEES CTX
4- VARIABLES
5-   is_occupied,      TC occupancy (input)
6-   signal_status     Signal status (output)
7- INVARIANT
8-   1/1  is_occupied <: TRACK_CIRCUITS &
9-   1/1  signal_status : SIGNALS --> STATUS
10- INITIALISATION
11-  1/1  is_occupied :: POW(TRACK_CIRCUITS) ||
12-  1/1  signal_status := SIGNALS * {RED}
13- OPERATIONS
14-   update_protection =
15- BEGIN
16-   signal_status: (
17-     signal_status : SIGNALS --> STATUS &
18-     /* all signals protecting an occupied TC should be red */
19-     signal_status[IS_PROTECTED_BY[is_occupied]] = {RED}
20-   )
21- END
22- END

```

Set of occupied TC

Signals state

Any subset of TRACK_CIRCUITS

All signals are RED

Not deterministic specification

All signals may remain RED all the time

No problem: it is safe !

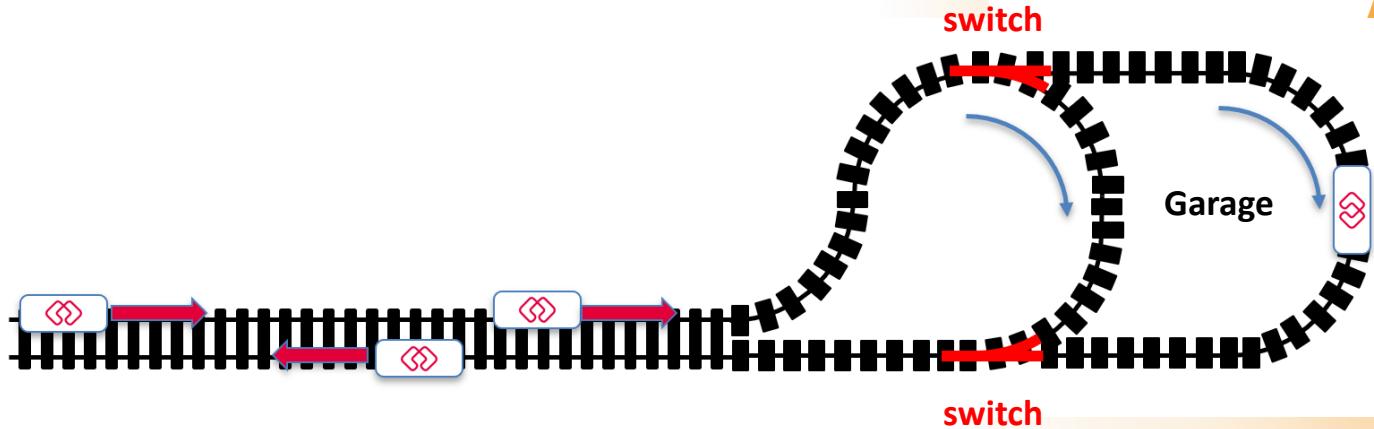
Controlling Signals

- ▶ We cannot specify where the trains are as we do not have this information
- ▶ Sensor information is made redundant and diverse
 - ▷ track circuits,
 - ▷ border detectors
- ▶ Real processing more complex with
 - ▷ history from cycle to another
 - ▷ 3 colors: Red, Orange, Green for optimisation

CONFIGURATION 3

Configuration 3

- ▶ Garage
- ▶ Switches



- ▶ Interlocking control switches position (**left, right, unknown**)
- ▶ Redundant sensors to detect switch position
- ▶ Switch cannot move when associated track circuit is occupied (=> derailment) (**orientation is ignored here**)

Detecting the position of a switch

- ▶ With 3 contact sensors !
- ▶ Each sensor returns a measure:
 - ▷ Right,
 - ▷ Left,
 - ▷ Unknow (failing sensor, switch moving during measurement)
- ▶ The position of the switch is
 - ▷ Right, at least one right, no left
 - ▷ Left, at least one left, no right
 - ▷ Unknown otherwise

<https://github.com/CLEARSY/ETMF2024/> Directory: Configuration3 Model: BLADE*

```
1 - MACHINE      Specification component
2 -   BLADE
3 - SETS
4 -   POSITION = {Left, Right, Unknown}
5 - OPERATIONS
6 -   pos <-- estimate (s1, s2, s3) =
7 -     PRE
8 -       s1 : POSITION &
9 -       s2 : POSITION &
10 -      s3 : POSITION
11 -    THEN
12 -      IF Right: {s1, s2, s3} THEN
13 -        IF Left: {s1, s2, s3} THEN
14 -          pos := Unknown
15 -        ELSE
16 -          pos := Right
17 -        END
18 -      ELSIF Left: {s1, s2, s3} THEN
19 -        pos := Left
20 -      ELSE
21 -        pos := Unknown
22 -      END
23 -    END
24 - END
```

Fully proved

Stateless component
No variable

Parameters typing
in preconditions

Set theory based specification

IMPLEMENTATION BLADE2_i Implementation component**REFINES BLADE**
OPERATIONS

```
1 24/24
2 5/5
3 5/5
4 pos <-- estimate (s1, s2, s3) =
5 IF s1 = Left THEN
6   IF s2 = Right or s3 = Right
7     THEN pos := Unknown ELSE pos := Left
8   END
9 ELSIF s1 = Right THEN
10   IF s2 = Left or s3 = Left
11     THEN pos := Unknown ELSE pos := Right
12   END
13 ELSE
14   IF s2 = Left
15     THEN
16     IF s3 = Right THEN pos := Unknown
17     ELSE pos := s2
18     END
19   ELSIF s2 = Right
20     THEN
21     IF s3 = Left THEN pos := Unknown
22     ELSE pos := s2 END
23   ELSE
24     pos := s3
25   END
26 END
27 END
```

Fully proved

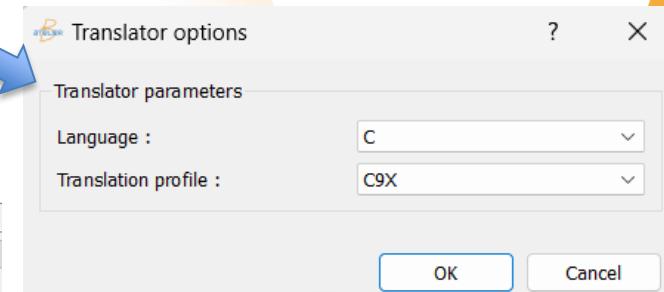
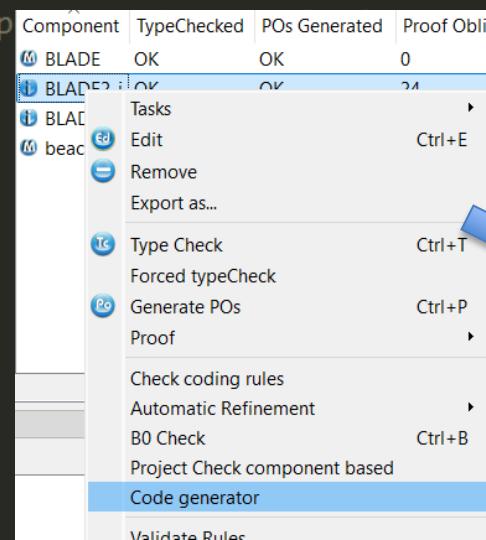
```
1 |IMPLEMENTATION BLADE_i      Other implementation component
2 |REFINES BLADE
3 |LOCAL_OPERATIONS
4 |    res <- has_pos (pos, s1, s2, s3) =  Local operation specification
5 |    PRE
6 |        pos : POSITION &
7 |        s1 : POSITION &
8 |        s2 : POSITION &
9 |        s3 : POSITION
10 |    THEN
11 |        1/1      res := bool (pos : {s1, s2, s3})
12 |    END
13 |    OPERATIONS
14 |        1/1      res <- has_pos (pos, s1, s2, s3) =  Local operation implementation
15 |        BEGIN
16 |            1/1      res := bool (s1 = pos or s2 = pos or s3
17 |            = pos)
18 |            END
19 |            ;
```

```
19      2/2    pos <- estimate (s1, s2, s3) = Operation implementation
20-     VAR
21       has_left, has_right           Using local operation
22-
23-     IN
24       has_left <- has_pos (Left, s1, s2, s3);
25       has_right <- has_pos (Right, s1, s2, s3);
26-     IF
27       has_left = TRUE & has_right = FALSE
28-   THEN
29       pos := Left
30-   ELSIF
31       has_left = FALSE & has_right = TRUE
32-   THEN
33       pos := Right
34-   ELSE
35       pos := Unknown
36- END
37- END
```

```

1 /* WARNING if type checker is not p
2
3 #include "BLADE.h"
4
5 /* Clause CONCRETE_CONSTANTS */
6 /* Basic constants */
7
8 /* Array and record constants */
9 /* Clause CONCRETE_VARIABLES */
10
11 /* Clause INITIALISATION */
12 void BLADE__INITIALISATION(void)
13 {
14
15 }
16
17 /* Clause OPERATIONS */
18
19 void BLADE__estimate(BLADE__POSITION s1, BLADE__POSITION s2,
20 {
21     if(s1 == BLADE__Left)
22     {
23         if((s2 == BLADE__Right) ||
24             (s3 == BLADE__Right))
25         {
26             (*pos) = BLADE__Unknown;
27         }

```



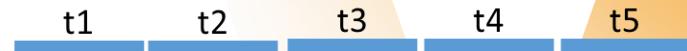
- To access the code
- Select the project
 - Click open folder
 - Go to lang/C directory

DATA VALIDATION

Properties with the B Mathematical Language

≡ Modelling language based on set theory and first order predicates logic (B mathematical language)

Let the set TrackCircuit = {t1, t2, t3, t4, t5}



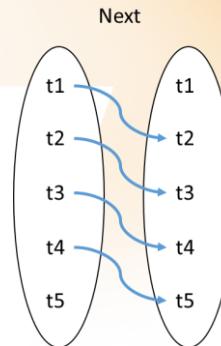
Let the function Next \in TrackCircuit \rightarrow TrackCircuit

Example: Next(t1) = t2, Next(t2) = t3, Next(t3) = t4, Next(t4) = t5

Next = {t1 \mapsto t2, t2 \mapsto t3, t3 \mapsto t4, t4 \mapsto t5}

Let the function KpAbs : TrackCircuit \rightarrow N

$\forall x. (x \in \text{TrackCircuit} \wedge x \in \text{dom}(\text{Next}) \Rightarrow \text{KpAbs}(\text{Next}(x)) > \text{KpAbs}(x))$



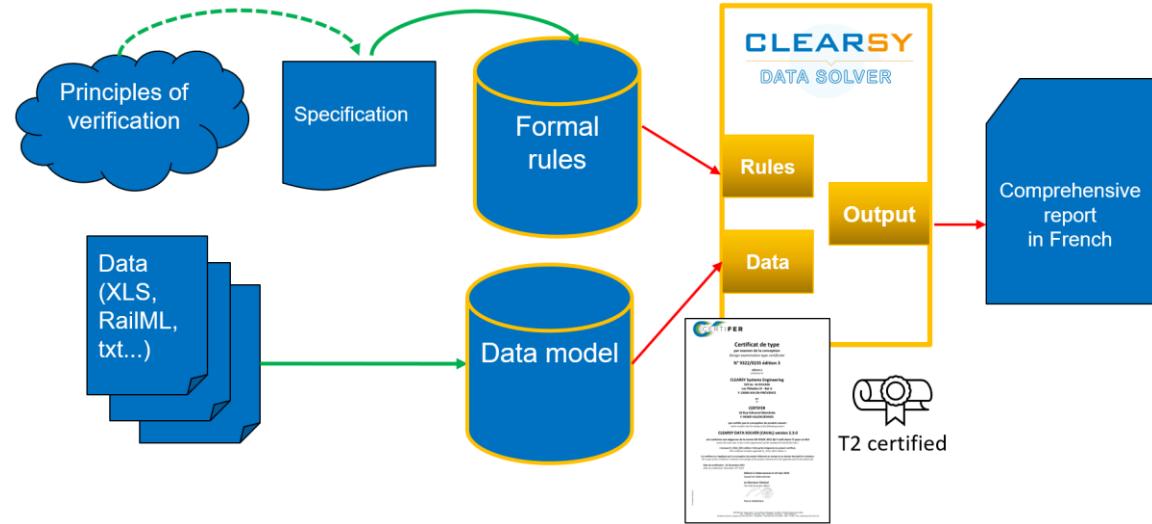
Formal Data Validation

Safety critical constant data
formally specified & model-checked

100k data chunk, up to 2k rules
Human errors avoided

SET THEORY
FIRST ORDER LOGIC
INTEGER
BOOLEAN
GRAPHS

J-R Abrial
The B-Book
Abusing programs to reason



References:

- *Formally Checking Large Data Sets in the Railways*, ICFEM, 2012
- *ProB*, <https://prob.hhu.de/>

Achievements

2003

First tool to verify embedded topology data
For Certification

2012

First tool integrated into CBTC metro dev process

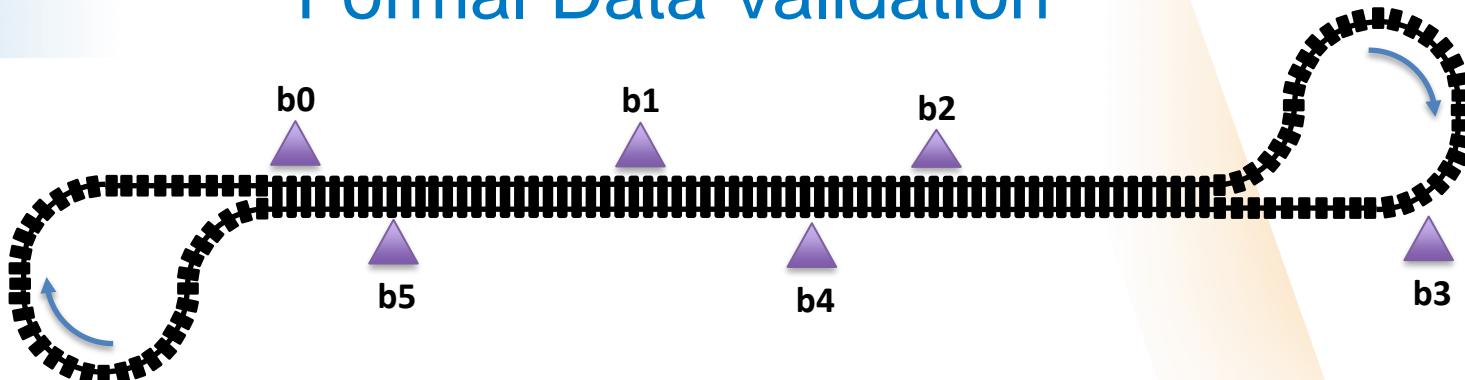
2018

First application to ERTMS (beacons)

2024

Core tool certified 50128 T2
Applied by major train manufacturers and metros
Call for tenders requiring formal data validation

Formal Data Validation



- ▶ Beacons should be known from the onboard software
 - ▷ Location, attached information (speed limit, target distance)
 - ▷ Specific properties have to be checked
 - ▷ A formal model and a model-checker to obtain this

<https://github.com/CLEARSY/ETMF2024/> Directory: DataValidation Model: beacons.mch

```
1- MACHINE beacons
2
3- SETS
4-     BEACONS = {b0, b1, b2, b3, b4, b5}      Set of beacon ids, no order
5- CONSTANTS
6-     nextB,
7-     lenghtTC,
8-     kpB,
9-     lastB
10- PROPERTIES
11-     nextB: BEACONS --> BEACONS &           Beacon successor, total function
12-     nextB = {
13-         b0 |-> b1,
14-         b1 |-> b2,
15-         b2 |-> b3,
16-         b3 |-> b4,
17-         b4 |-> b5,
18-         b5 |-> b0
19-     } &
20-     dom(nextB) \wedge ran(nextB) = BEACONS & All beacon ids used, no « dead code »
```

```

22 lenghtTC: BEACONS --> INTEGER & TC length
23 - lenghtTC = {
24     b0 |-> 1000,
25     b1 |-> 1000,
26     b2 |-> 2000,
27     b3 |-> 2000,
28     b4 |-> 1000,
29     b5 |-> 1000
30 } &
31
32 kpB : BEACONS --> INTEGER &
33 - !bc. (bc: BEACONS =>
34     bc = b0 => kpB(bc) = 0) &
35     (not(bc=b0) => kpB(bc) = lenghtTC(nextB~(bc)) + kpB(nextB~(bc)))
36 ) &
37 lastB: BEACONS &
38 lastB = b5 &
39 !bc. (bc: BEACONS - {lastB} => kpB(bc) <= kpB(nextB(bc)))
40 END

```

Not fully proved
With Atelier B

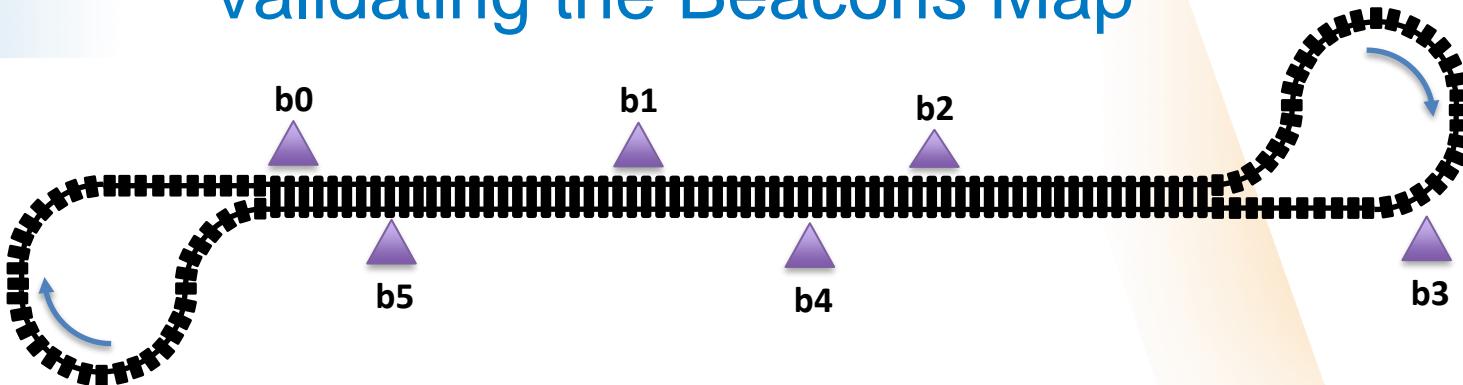
Fully calculated
with ProB

OK	State Properties (20)
lengthTC(b0) = 1000	
lengthTC(b1) = 1000	
lengthTC(b2) = 2000	
lengthTC(b3) = 2000	
lengthTC(b4) = 1000	
lengthTC(b5) = 1000	
kpB(b0) = 0	
kpB(b1) = 1000	
kpB(b2) = 2000	
kpB(b3) = 4000	
kpB(b4) = 6000	
kpB(b5) = 7000	
lastB = b5	

Calculation of TC position

Verification of location increasing with successors

Validating the Beacons Map



- ▶ This is not all !
- ▶ We have validated an Excel file (or similar)
- ▶ We also need to verify the real distances on the track (manually, with a robot, a drone)

clearsy
Safety Solutions Designer

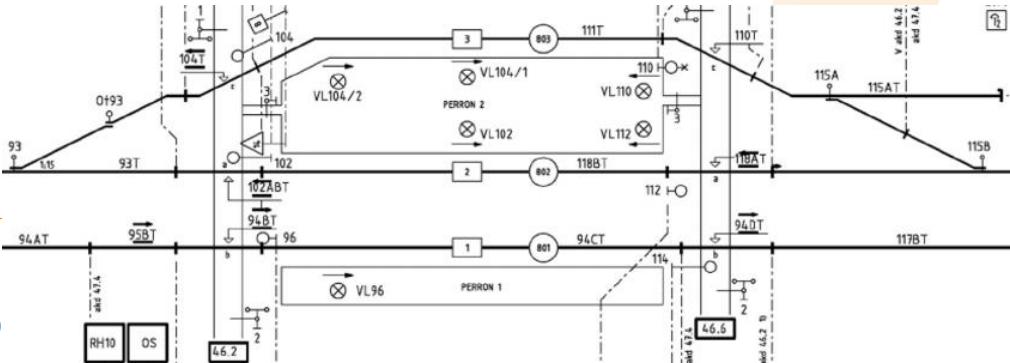
READY FOR REAL WORLD?



Towards the limits ...

- « There is overEnergy iff I can find a track section starting at X2M, complying with the dynamic chaining of blocks, on which I can
 - either find a restriction belonging to a block such as the energy on that restriction, computed by summing deltas of energy of all restrictions located between X2MRes and this restriction, is greater than the energy associated to this restriction,
 - or find 2 restrictions belonging to the EOA block, one being before the track section under consideration, the other after the track section, such as the energy associated to the EOA by using these restrictions is positive. »

[Extract from Automatic Train Protection specification]



Towards the limits

```
p_over := bool (#(over_track).((over_track : seq(t_block * t_direction) & over_track /= {} & first(over_track) = p_X2MBlock |> p_X2MDir & !ii .(ii : 1 .. size(over_track)-1 => (over_track)(ii) : dom(sidb_nextBlock)) & !ii .(ii : 1 .. size(over_track) => sidb_nextBlock((over_track)(ii)) = (over_track)(ii+1))) & #(over_res).((over_res : sidb_restrictionApplicable & (#ii .(ii : dom(over_track) & ((prj2(t_block, t_direction)(over_track(ii)) = c_up => over_res : ran(sgd_blockUpRestrictionSeq((prj1(t_block, t_direction)(over_track(ii)))))) & ((prj2(t_block, t_direction)(over_track(ii)) = c_down => over_res : ran(sgd_blockDownRestrictionSeq((prj1(t_block, t_direction)(over_track(ii)))))) & (ii = 1 => not(over_res <= p_X2MRes)) & p_X2MSSWorst + p_X2MDSS + (SIGMA(jj).(jj : 1 .. ii | SIGMA(pre_res).((pre_res : t_restriction & ((prj2(t_block, t_direction)(over_track(jj)) = c_up => pre_res : ran(sgd_blockUpRestrictionSeq((prj1(t_block, t_direction)(over_track(jj)))))) & ((prj2(t_block, t_direction)(over_track(jj)) = c_down => pre_res : ran(sgd_blockDownRestrictionSeq((prj1(t_block, t_direction)(over_track(jj)))))) & (jj = 1 => not(pre_res <= p_X2MRes)) & (jj = ii => not(pre_res >= over_res)) | sgd_restrictionDeltaSqSpeed(pre_res))) > sgd_restrictionSquareSpeed(over_res) & (over_res : sgd_restrictionFront => p_X2MResDist + ((SIGMA(ti).(ti : 1 .. ii | sgd_blockLength((prj1(t_block, t_direction)((over_track)(ti)))))) {c_down |> sgd_blockLength(p_X2MBlock) sgd_restrictionAbs(p_X2MRes), c_up |> sgd_restrictionAbs(p_X2MDir)) {c_down |> sgd_restrictionAbs(over_res), c_up |> sgd_blockLength((prj1(t_block, t_direction)((over_track)(ii)))) sgd_restrictionAbs(over_res) {((prj2(t_block, t_direction)((over_track)(ii))))} + sgd_restrictionLength(over_res) > loc_locationUncertainty + c_trainLength))) or (#(eoares, res_after_eoa, ii).(eoares : t_restriction & res_after_eoa : t_restriction & ii : dom(over_track) & p_EOABlock = (prj1(t_block, t_direction)(over_track(ii))) & (ii = 1 => p_X2MRes <= eoares) & ((prj2(t_block, t_direction)(over_track(ii)) = c_up => eoares : ran(sgd_blockUpRestrictionSeq(p_EOABlock)) & res_after_eoa : ran(sgd_blockUpRestrictionSeq(p_EOABlock)) & sgd_restrictionAbs(eoares) <= p_EOAabs & p_EOAabs < sgd_restrictionAbs(res_after_eoa) & !ri.(ri : ran(sgd_blockUpRestrictionSeq(p_EOABlock)) => ri <= eoares or res_after_eoa <= ri)) & ((prj2(t_block, t_direction)(over_track(ii)) = c_down => eoares : ran(sgd_blockDownRestrictionSeq(p_EOABlock)) & res_after_eoa : ran(sgd_blockDownRestrictionSeq(p_EOABlock)) & sgd_restrictionAbs(eoares) >= p_EOAabs & p_EOAabs > sgd_restrictionAbs(res_after_eoa) & !ri.(ri : ran(sgd_blockDownRestrictionSeq(p_EOABlock)) => ri <= eoares or res_after_eoa <= ri)) & p_X2MSSWorst + p_X2MDSS + (SIGMA(jj).(jj : 1 .. ii | SIGMA(pre_res).((pre_res : t_restriction & ((prj2(t_block, t_direction)(over_track(jj)) = c_up => pre_res : ran(sgd_blockUpRestrictionSeq((prj1(t_block, t_direction)(over_track(jj)))))) & ((prj2(t_block, t_direction)(over_track(jj)) = c_down => pre_res : ran(sgd_blockDownRestrictionSeq((prj1(t_block, t_direction)(over_track(jj)))))) & (jj = 1 => not(pre_res <= p_X2MRes)) & (jj = ii => pre_res <= eoares) | sgd_restrictionDeltaSqSpeed(pre_res))) {c_up |> (sgd_restrictionAccel(eoares) * ((sgd_restrictionAbs(res_after_eoa) p_EOAabs)/1024))/2, c_down |> (sgd_restrictionAccel(eoares) * ((p_EOAabs sgd_restrictionAbs(res_after_eoa))/1024))/2} {((prj2(t_block, t_direction)(over_track(ii)))) > 0) or (#(eoares, ii).(eoares : t_restriction & ii : dom(over_track) & (ii = 1 => not(eoares <= p_X2MRes)) & p_EOABlock = (prj1(t_block, t_direction)(over_track(ii))) & ((prj2(t_block, t_direction)(over_track(ii)) = c_up => eoares : ran(sgd_blockUpRestrictionSeq(p_EOABlock)) & eoares = last(sgd_blockUpRestrictionSeq(p_EOABlock)) & sgd_restrictionAbs(eoares) <= p_EOAabs) & ((prj2(t_block, t_direction)(over_track(ii)) = c_down => eoares : ran(sgd_blockDownRestrictionSeq(p_EOABlock)) & eoares = last(sgd_blockDownRestrictionSeq(p_EOABlock)) & sgd_restrictionAbs(eoares) >= p_EOAabs) & p_X2MSSWorst + p_X2MDSS + (SIGMA(jj).(jj : 1 .. ii | SIGMA(pre_res).((pre_res : t_restriction & ((prj2(t_block, t_direction)(over_track(jj)) = c_up => pre_res : ran(sgd_blockUpRestrictionSeq((prj1(t_block, t_direction)(over_track(jj)))))) & ((prj2(t_block, t_direction)(over_track(jj)) = c_down => pre_res : ran(sgd_blockDownRestrictionSeq((prj1(t_block, t_direction)(over_track(jj)))))) & (jj = 1 => not(pre_res <= p_X2MRes)) & (jj = ii => not(pre_res >= eoares)) | sgd_restrictionDeltaSqSpeed(pre_res))) + ({c_up |> (sgd_restrictionAccel(eoares) * ((p_EOAabs sgd_restrictionAbs(eoares))/1024))/2, c_down |> (sgd_restrictionAccel(eoares) * ((sgd_restrictionAbs(eoares) p_EOAabs)/1024))/2} {((prj2(t_block, t_direction)(over_track(ii)))) > 0)))
```

Towards the limits again

For each GradientTopology (GradientTopology.BOT-Zone) totally included in a segment, a Gradient (Gradient.BOT-Zone) is created with the same attributes.

For GradientTopology intersecting different segments, several Gradients (Gradient.BOT-Zone) are created so that each of them is located in only one segment.

When the gradient is constant (GradientTopology.isConstant = Yes):

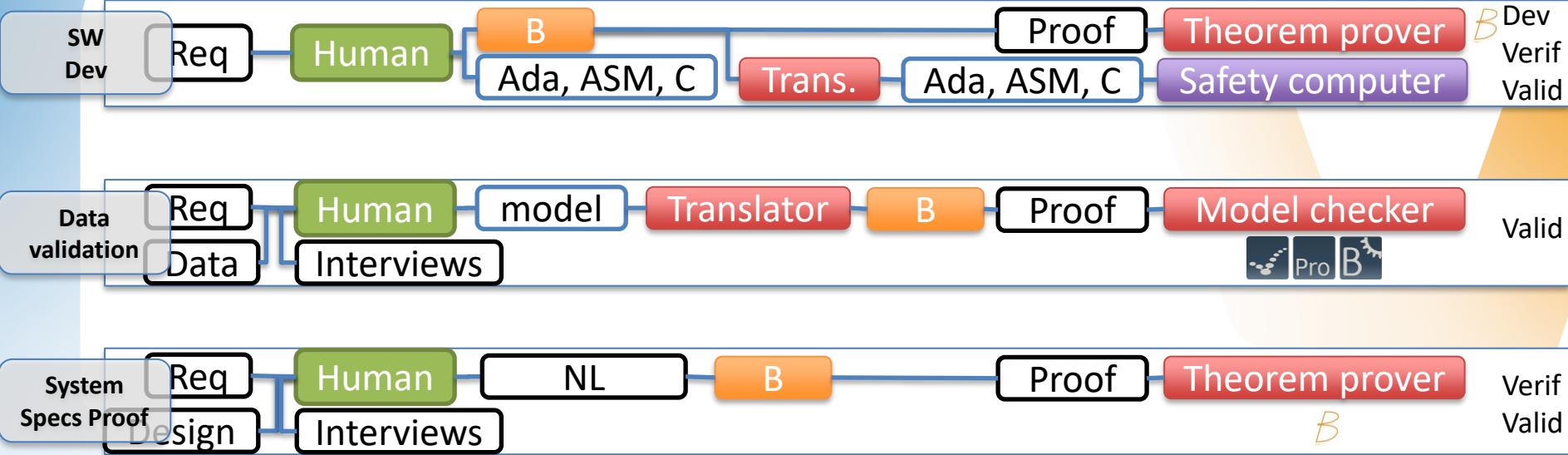
- the variable gradient information (Gradient.VariableGradient) is not set.
- the constant gradient information is set with the same information of GradientTopology for both parts.
- the elevationDifference.elevationEnd of the part1 and elevationDifference.elevationStart of the part2 (reference to the above figure) are equal to elevationStart + gradient*Length1.
- the information isConstant is set to Yes for both parts.

When the gradient is not constant (GradientTopology.isConstant = No):

- the constant gradient information (ConstantGradient) is not set.
- the elevationDifference.elevationEnd of the part1 and elevationDifference.elevationStart of the part2 (reference to the above figure) are equal to elevationStart + $2 \cdot \text{radius} \cdot \sin(\text{Length1} / (2 \cdot \text{radius})) \cdot \sin(\text{gradientStart} + \text{Length1} / (2 \cdot \text{radius}))$.
- the information radius and transitionCurveType of the variableGradient information are the same for both parts (as initial GradientTopology information).
- the information gradientEnd for part1 and gradientStart of part2 for variableGradient information are set to $(\text{gradientEnd} - \text{gradientStart}) / (\text{Length1} + \text{Length2}) \cdot \text{Length1} + \text{gradientStart}$.
- the information isConstant is set to No for both Part.



Formal Methods in Action



Conclusion

- ▶ FM can complement engineering when designing a “system”
- ▶ **Safety is by design**
 - ▷ There is no ultimate design
 - ▷ Only consequences of decisions and a reasoning
- ▶ Particularly useful when dealing with cyber-physical systems
- ▶ **Handling industry strength systems is a challenge, even with FM**

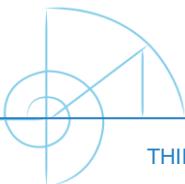


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Thank you
for your attention



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ETMF
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<https://mooc.imd.ufrn.br/>



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online course



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