SIA flow model exercise

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The aim of this exercise to give some hands-on experience in simple ice flow modelling. Along with this task sheet, a python outline-code is given as starting point. You are not obliged to use this code.

In the lectures, we have derived the following equation if the shallow ice approximation is applied and Weertman sliding is used (See table 1 for the symbol definitions):

$$\frac{\partial H}{\partial t} = -\frac{\partial F}{\partial x} + \dot{b} \tag{1}$$

$$F = HU \tag{2}$$

$$U = U_d + U_0 = \overset{\text{deformation}}{\frac{2}{5} f_d H \tau_d^3} + \overset{\text{sliding}}{f_s H^{-1} \tau_d^3}$$

$$\tau_d = \rho g H \frac{\partial h}{\partial x} \quad \text{driving stress}$$

$$\text{with} \quad \dot{b} = \beta(h - h_{\mathrm{ELA}}) \quad \text{or more complicated relations.}$$

Typical values of the constant are given below. Use a notation like $H^n_{i+1/2}$, which denotes the value of H for time step n and between grid boxes i and $i+1$.

1a: Discretise Eq. (2), thus the ice flux (F), as function of H, h and constants. Discuss whether F needs to be derived on grid points or between grid points.

1b: Discretise Eq. (1), thus $\frac{\partial H}{\partial t}$, as function of F and $\dot{b}$. You may use Euler forward for the time stepping.

1c: Update the given code accordingly the results above - or write a model from scratch.

2: Analyse, of the questions listed in below, the one assigned to your group. Do this in a systematic manner and test if a growing glacier responds equally to a growing glacier, or not. In all cases, present a figure in which you plot the response time as function of the ELA - for the various cases you investigate - and a figure with the total glacier mass (M) change as function of the ELA, thus $\partial M/\partial \mathrm{ELA}$.

The report should be brief, thus typically 2-3 pages. It should include

- The derived equation of (1), with short discussion.
- The derived equation of (2).
- A discussion of (4) using at max 4 figures and at max 2 movies.

Along with the report, at max 2 movies (saved as .mp4) and the model code (saved as .py) must be handed in on Blackboard. Item 1a-c of the tasks is 30% of the grade, item 2 the remaining 70% of the grade.

Table 1: Typical physical values

Symbol	Value	Unit	Physical description
ρ	917.	kg m^{-3}	Ice density
g	9.81	m s^{-2}	Gravitational acceleration
f_d	$1.9 \cdot 10^{-24}$	Pa^{-3} s^{-1}	Deformation parameter
f_s	$5.7 \cdot 10^{-20}$	Pa^{-3} m^2 s^{-1}	Sliding parameter

Response time and research strategy

In the research questions, the concept 'response time' is used. It is measure of the time scale a glacier needs to reach a new equilibrium. Of course, mathematically an equilibrium is never reached, and also practically it is hard to define when a new equilibrium is sufficiently reached. Therefore, the response time is defined as the time elapsed to the point in time that the glacier has undergone $1 - 1/e$ ($\sim 63\%$) of the total change.

Next, how do you model the impact of changing ELA on the glacier response? There are, of course, many good approaches. To speed up the work, you could use the provided function that defines the ELA as function of the model year. This function changes the ELA every (say) century, and during that century the glacier can get into balance again. By making a graph or list of the integrated ice volume or glacier length, you can estimate the response time. Please note that in some situations the response time can be (way) longer than a century. And of course, you are not obliged to use the provided methodology to vary the ELA over time.

Research directions

1. Investigate the effect of the bedrock slope on the glacier mass and response time to changes in the ELA.
2. Investigate the effect of a decreasing bedrock slope with x on the glacier mass and response time to changes in the ELA. Compare, for example, your 'decreasing bedrock slope glacier' with a glacier on a bed with constant slope.
3. Investigate the effect of $\dot{b}$ on the glacier mass and response time to changes in the ELA.
4. Investigate the effect of local bedrock bump on the glacier mass and response time to changes in the ELA. Vary, for example, also the height or length of the bump.
5. Investigate the effect of local bedrock depression on the glacier mass and response time to changes in the ELA. Vary, for example, also the depth or length of the depression.
6. Investigate the effect of the chosen dx on the glacier mass and response time to changes in the ELA.
7. Approximate a the flow line of real-world Alpine glacier and estimate the sensitivity of the glacier mass and response time to realistic changes in the ELA.
8. Approximate a the flow line of real-world Arctic glacier and estimate the sensitivity of the glacier mass and response time to realistic changes in the ELA.
9. Investigate the effect of excluding sliding on the glacier shape for different conditions.
10. Investigate the effect of excluding internal deformation on the glacier shape for different conditions.