SEB exercise, version 1.1

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The aim of this exercise to give you some feel of how the surface energy balance (SEB) over the ice sheet look like and how dependent the various processes are for changes.

The output you get, is output of the SEB model that is run to interpret and align the observations made by automated weather stations. These stations measure temperature, humidity and wind speed above the ground, the radiative fluxes, both short wave (SW) and long wave (LW), both upwards as downwards. Finally, the snow height and integrated ice melt is measured. The SEB-model combines these observations to close the SEB, sometimes with rectifying errors in the observations due to, for example, riming, station tilt and other issues that do occur.

Unluckily, the output we use has errors too. I rectified most of these errors during the reading phase of the python program.

1 The SEB

The SEB consists of the following terms

$$M_{\text{surf}} = SW_{\text{net}} + LW_{\text{down}} + LW_{\text{up}} + SHF + LHF + Gs. \quad (1)$$

The net radiative energy (R_{net}) is the sum of SW_{net} and LW_{down}. We use, unless stated otherwise, the convention that a positive flux value implies a flux towards the surface, and a negative value a flux away from the surface.

1.1 Shortwave radiation → SW_{net}

The net SW radiation is equal to

$$SW_{\text{net}} = (1 - \alpha)SW_{\text{down}} = SW_{\text{down}} + SW_{\text{up}} \quad (2)$$

The incoming radiation SW_{down} is depending on the solar zenith angle and cloudiness. The value to be used is the observation from the dataset, corrected for tilt of the AWS.

The albedo (α) is diagnosed from the ratio between downwelling and upwelling radiation. Because radiation measurements like these ones can get noisy for low solar angles (high zenith angles), the albedo is determined as a running weighted mean over one day of data.

If radiation penetration is taken into account, about half of SW_{net} is actually absorbed at the surface (SW_{surf}), and hence contribution to the SEB. The other rough half is absorbed internally (SW_{int}), so:

$$SW_{\text{net}} = SW_{\text{surf}} + SW_{\text{int}}. \quad (3)$$

1.2 Longwave radiation $\rightarrow LW_{\mathbf{net}}$

The downwelling longwave radiation is observed by the weather station and if needed corrected for errors. The upwelling longwave radiation to be used is output of the SEB model, it relates to the surface temperature needed to close the SEB for the given atmospheric state and estimated subsurface state. This modelled upwelling longwave radiation is compared to the measure upwelling long wave radiation, and by tuning of the unknown parameters (like surface roughness, initial snow density, and snow compaction rate) the differences between the modelled and measured upwelling longwave radiation is minimised.

For this project, you could approximate the apparent temperature of the free atmosphere (T_{Atm}) assuming perfect emissivity, thus

$$LW_{\mathrm{down}} = \sigma T_{\mathrm{Atm}}^4 \qquad \rightarrow \qquad T_{\mathrm{Atm}} = \sqrt[4]{\frac{LW_{\mathrm{down}}}{\sigma}}. \tag{4}$$

Alternatively, you could define this atmospheric temperature as depending on the observed 2 m temperature, thus $F(T_{\mathrm{2m}})$, and estimate the emissivity of the atmosphere (ϵ):

$$LW_{\mathrm{down}} = \epsilon\sigma F(T_{\mathrm{2m}})^4 \qquad \rightarrow \qquad \epsilon = \frac{LW_{\mathrm{down}}}{\sigma F(T_{\mathrm{2m}})^4}. \tag{5}$$

The upwelling longwave emissivity relates to the surface temperature. An surface emissivity of one is assumed. If you alter the fluxes, the surface temperature will alter as well, and hence the longwave emission. So

$$LW_{\mathrm{up}} = -\sigma T_{\mathrm{surf}}^4. \tag{6}$$

To solve the system you might end up, it is good to linearise LW_{up} as function of T for the estimated solution $T_{\mathrm{surf}}^{\mathrm{est}}$:

$$LW_{\mathrm{up}}(T_{\mathrm{surf}}^{\mathrm{adj}}) = -\sigma {T_{\mathrm{surf}}^{\mathrm{est}}}^4 - 4\sigma {T_{\mathrm{surf}}^{\mathrm{est}}}^3 (T_{\mathrm{surf}}^{\mathrm{adj}} - T_{\mathrm{surf}}^{\mathrm{estimated}}). \tag{7}$$

1.3 Sensible heat flux SHF

In the SEB model, the sensible heat flux is estimated in the SEB model using

$$SHF = c_s U_{\mathrm{10m}}(T_{\mathrm{2m}} - T_{\mathrm{surf}}). \tag{8}$$

This constant c_s depends on the surface roughness, but is in reality also dependent on the atmospheric stability as turbulence is suppressed in a heavily stratified boundary layer.

If you update T_{surf} (or any other parameter in this relation), than you can choose to update SHF too. Next, you can take into account that T_{2m} is both related to T_{surf} as to the temperature at some height at the boundary layer (T_{BL}). You can approximate this T_{BL} with

$$T_{\mathrm{BL}} = T_{\mathrm{surf}} + \frac{T_{\mathrm{2m}} - T_{\mathrm{surf}}}{c_{\mathrm{BL}}}, \tag{9}$$

with c_{BL} representing the fraction of the (potential) temperature difference between the boundary layer and the surface (thus $T_{\mathrm{BL}} - T_{\mathrm{surf}}$) is situated in the lowermost 2 meters of the atmosphere.

In the end, you end up with a linear relationship like

$$SHF(T_{\mathrm{surf}}^{\mathrm{adj}}) = SHF^{\mathrm{SEB}} + c_{\mathrm{TBD}} U_{\mathrm{10m}}(T_{\mathrm{surf}}^{\mathrm{SEB}} - T_{\mathrm{surf}}^{\mathrm{adj}}), \tag{10}$$

in which c_{TBD} is a constant that needs to be determined with the information given above. Please note that SHF_{SEB} will change if you do an sensitivity experiment in which you take a different U_{10m}.

1.4 Latent heat flux LHF

Like the SHF, the latent heat flux is estimated in the SEB model with

$$LHF = c_l U_{10\mathrm{m}}(Q_{2\mathrm{m}} - Q_{\mathrm{surf}}), \tag{11}$$

in which c_l for a given moment is proportional to c_s (Eq. (8)) for that moment. The specific humidity (Q, kg/kg) is not directly measured, only the relative humidity (RH). Therefore, we rewrite this equation to

$$LHF = c_l U_{10\mathrm{m}}(RH_{2\mathrm{m}} Q_{\mathrm{sat}} - Q_{\mathrm{sat}}), \tag{12}$$

in which $RH_{2\mathrm{m}}$ ranges from 0 to 1 and the surface relative humidity, due to the omnipresence of water, is 1. The saturated specific humidity (Q_{sat}, kg/kg) is give by

$$Q_{\mathrm{sat}} = \frac{6.1121 * m_v}{P * m_{air}} \exp\left[\frac{c_1 T}{T + c_2}\right] \text{ with } \begin{cases} T \leq 0: & c_1 = 22,587 \quad c_2 = 273,86 \\ T \geq 0: & c_1 = 17,502 \quad c_2 = 240,97 \end{cases} \tag{13}$$

Here, m_v and m_{air} are the moleculair masses of water vapour (18.0153 g/mol) and air (28,9644 g/mol), respectively. The temperature T is in °C and P is the pressure in hPa.

Given that Q is measured only indirectly, adjusting the LHF is a bit more cumbersome, while the impact on the SEB is generally smaller than the impact of the SHF.

1.5 Ground heat flux Gs

The ground heat flux is not measured by the AWS, even not indirectly, but modelled by the SEB model. The ground heat flux depends both on the evolution of the surface temperature as on the near (< 5 cm) and deep ($5 - 50$ cm) snow and ice temperatures. Without proper snow model, it is hard to re-evaluate Gs. Hence, it is advised to leave Gs as is, even though surface temperatures or melt amounts change in the requested sensitivity experiments.

1.6 Melt M, or no melt

As long as the surface temperature is below 273.16 K, the SEB (Eq. (1)) closes with $M = 0$. This is because LW_{up} (being negative), SHF, LHF, G_s all decrease if T_{surf} increases. However, T_{surf} cannot rise above 273.16 K, hence if above melting point temperatures are required to close the SEB, the surface temperature is set to the melting point and the remaining energy is used to melt snow and ice.

1.7 Non-closure of the SEB model (R)

Although the SEB model, of which we use output, is designed to 'close' the SEB. Thus, it tries to find a surface temperature (T_{surf}) for which Equation (1) holds. At the other hand, one does not want that the modelled T_{surf} deviates too much from the observed surface temperature, measured by the upwelling longwave radiation. Hence, sometimes the SEB model cannot find a T_{surf} that satisfy all requirements, and the model has residual energy 'going nowhere'. This happens typically at the start of the data series when the initial snow pack of the model state differs quite from the snow pack as happened.

2 Methods to find adjusted surface temperature.

In order to find the adjusted surface temperature for which the SEB closes again, you have to solve iteratively the following equation:

$$M_{\text{surf}}^{\text{adj}} = SW_{\text{net}}^{\text{adj}} + LW_{\text{down}}^{\text{adj}} - \sigma {T_{\text{surf}}^{\text{est}}}^4 - 4\sigma {T_{\text{surf}}^{\text{est}}}^3 (T_{\text{surf}}^{\text{adj}} - T_{\text{surf}}^{\text{est}}) + SHF^{\text{SEB}} + c_{\text{TBD}} U_{10\text{m}} (T_{\text{surf}}^{\text{SEB}} - T_{\text{surf}}^{\text{adj}}) + \ldots, \tag{14}$$

which is a linearly disturbed SEB, Equation (1)[1] To solve this, assume initially that $M_{\text{surf}}^{\text{adj}} = 0$, and rewrite this equation to

$$c^{\text{adj}} T_{\text{surf}}^{\text{adj}} = c^{\text{fixed}} \qquad \rightarrow \qquad T_{\text{surf}}^{\text{adj}} = \frac{c^{\text{fixed}}}{c^{\text{adj}}}, \tag{15}$$

in which

$$\begin{aligned} c^{\text{adj}} &= 4\sigma {T_{\text{surf}}^{\text{est}}}^3 + c_{\text{TBD}} U_{10\text{m}} + \ldots \\ c^{\text{fixed}} &= SW_{\text{net}}^{\text{adj}} + LW_{\text{down}}^{\text{adj}} + 3\sigma {T_{\text{surf}}^{\text{est}}}^4 + SHF^{\text{SEB}} + c_{\text{TBD}} U_{10\text{m}} T_{\text{surf}} + \ldots. \end{aligned}$$

If this leads to $T_{\text{surf}}^{\text{adj}} > 273.16$ K, there is melting. In that case, $M_{\text{surf}}^{\text{adj}}$ is derived with Equation (14) using that $T_{\text{surf}}^{\text{adj}} = 273.16$ K. Otherwise, it is wise to repeat (a couple times) solving Equation (14) while updating $T_{\text{surf}}^{\text{est}}$ with the latest estimated $T_{\text{surf}}^{\text{adj}}$. Please note that updating $T_{\text{surf}}^{\text{est}}$ leads to different c^{adj} and c^{fixed}.

3 Adjusting the SEB for radiation penetration

At station S6, the SEB model has employed radiation penetration as this is a relevant proces over ice. However, this leads to a shift of relevance of processes which complicates the sensitivity analyses requested here. Therefore, you could opt to 'rectify' radiation penetration by including the energy fluxes of internal SW absorption (SW_{int}) and melt (M_{int}) and subsequently adjust Gs.

In case of radiation penetration, the SEB is

$$M_{\text{surf}} = SW_{\text{surf}} + LW_{\text{down}} + LW_{\text{up}} + SHF + LHF + Gs, \tag{16}$$

while the internal energy balance is

$$M_{\text{int}} = SW_{\text{int}} + G_{\text{int}}. \tag{17}$$

This internal ground heat flux (G_{int}) cannot be estimated, as the layer over which radiation penetration or internal melt is not well defined nor known. So, with $M_{\text{tot}} = M_{\text{surf}} + M_{\text{int}}$, we defined the rectified SEB as

$$M_{\text{tot}} = SW_{\text{new}} + LW_{\text{down}} + LW_{\text{up}} + SHF + LHF + Gs^{\text{rec}}, \tag{18}$$

in which

$$Gs^{\text{rec}} = Gs - SW_{\text{int}} + M_{\text{int}}. \tag{19}$$

In practice it leads to a strong reduction of the ground heat flux for sunning non melting conditions, which brings the SEB more in line with the SEB derived without radiation penetration. For conditions with internal melting, $Gs \sim Gs^{\text{rec}}$ as the internally absorbed radiation drives this heat and the upper part of the snow/ice pack is isothermal.

[1] The LHF and G_s terms are left out in Equations (14) and following for brevity, but should not be forgotten.

4 Work plan

1. Load the data into python, using the python code provided.
    - The file 'SEB_functions.py' provides the modules and routines to load and adjust the SEB model data. Data is provided in one variable of a class defined in 'SEB_functions.py'. The class (and thus variable) includes functions to extract the relevant data out of the variable. Call 'help(<*variable name*>)' for more information, or look into the python code.
    - The file 'AnalyseSEBdata.py' shows how the SEB data file can be read, how data can be extracted from the class, and how figures can be made. Furthermore, it shows a very simple SEB adjustment, namely getting adjusted surface temperatures and surface melt by increasing LW_{down} only.
    - Please note that the date and time are registered using the datetime type. See https://docs.python.org/3/library/datetime.html#datetime.datetime for more info on this type.
2. (Re-)construct and plot the SEB as modelled by the SEB model. Hence, adjust the code in 'AnalyseSEBdata.py' to your liking.
3. Select a research question, listed below.
4. Adjust the SEB component according the research question, and consider to adjust also other fluxes if you think these will be affected by your research question. Write code that "closes" the SEB again, thus a code that finds a surface temperature or melt energy for which $SEB = M$ holds.
5. Write a brief report on this project, containing
    - The research question, and how you implemented that in the code. Please also mention which AWS location has been chosen.
    - List which other fluxes have been adjusted and discuss the equations and rationales of these adjustments.
    - Show, he original and adjusted SEB (e.g. typical monthly cycle) and surface climate, and discuss what these changes learn you about the sensitivity of the surface climate and snow melt to 'your change'.

5 Research questions

How will the near surface climate, SEB and melt change if:

1. atmospheric temperature rises / falls with 1 K?
2. the wind speed would be always half / double the wind speed now observed?
3. the snow/ice albedo always would be 0.85 / 0.7 / 0.3?
4. it always would be cloudy at this site?
5. there never would be clouds at this site?
6. the air at 2 m height would always be completely dry ($RH = 0\%$) or saturated ($RH = 100\%$)?

6 Variable look-up table

The variable names in the data files are not always easy to interpret, and some variables are, with variations, multiple times available in the files. Therefore, please use Table 1 and 'Data_description.pdf' to use the right data. Otherwise the SEB budget won't close (that nicely).

Table 1: Non-complete look-up table of suitable variables.

Variable	name in file	sign (convention followed)
SW_{down}	SWin_corr	always positive (yes)
SW_{up}	SWout	always positive (no)
SW_{net}	SWnet_corr	always positive (yes)
SW_{int}	SumDivQ	always positive (no)
SW_{surf}	-	
LW_{down}	LWin	always positive (yes)
LW_{up}	LWout_corr	always positive (no)
LW_{net}	LWnet_model	always positive (yes)
R_{net}	Rnet_model	varies (yes)
SHF	Hsen	varies (yes)
LHF	Hlat	varies (yes)
Gs	Gs	varies (yes)
Gs^{rec}	-	
M_{surf}	melt_energy	positive (no)
M_{tot}	totm_nrg	positive (no)
M_{int}	-	
T_{surf}	Tsurf_calc	
T_{2m}	Tair_2m	
Q_{2m}	RH	
U_{10m}	FF_10m	