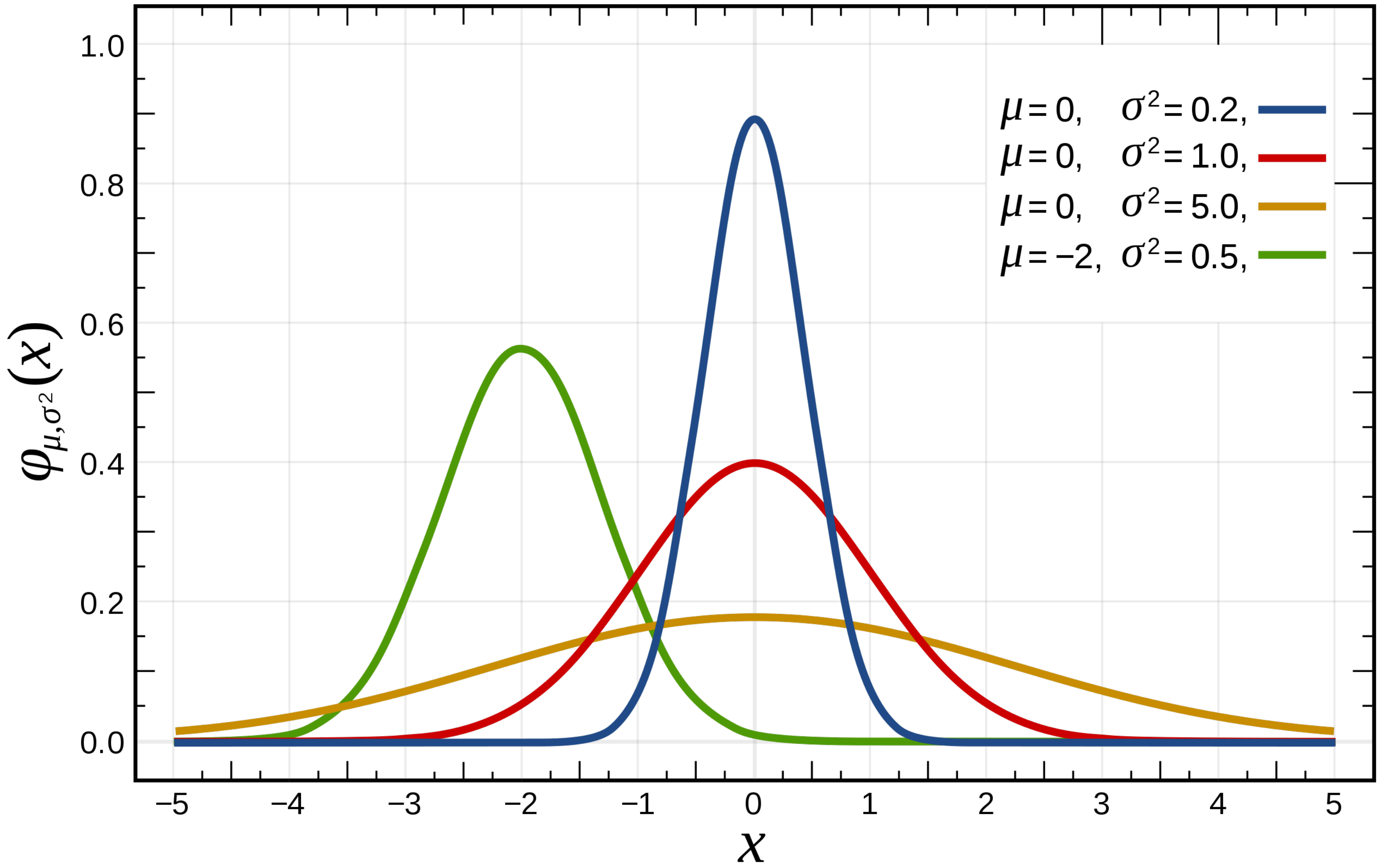

Introduction to Mathematical Analysis

Math 521

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Mathematical Analysis is as extensive as nature herself.

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Chapter 1 Basics

1.1 Set

Definition 1.1

A set is a collection of objects called elements.

Definition 1.2

The Cartesian product $A \times B$ *is the set whose elements are odered pairs* (a, b) *with* $a \in A$*,* $b \in B$*. Particularly,* $A \times A$*:* $\{(ai, aj) \mid ai, aj \in A\}$*. Often* A^2

Henri Poincaré: Mathematics is the art of giving the same name to different things.

Definition 1.3

A function f *from A to B is a subset* $f \subset A \times B$*, with the properties if* $(a, b) \in f$ *and* $(a, b') \in f$ *then* $b = b'$*. We write* $f(a) = b$ *for* $(a, b) \in f$ *means A function* f *from A to B.*

$$f : A \to B \qquad\qquad A \xrightarrow{f} B$$

Definition 1.4

We say a function $f : A \to B$ *is injective if* $f(a) = f(a') \Rightarrow a = a'$*. We say it is surjective if* $\forall b, \exists a \mid f(a) = b$*.*

Corollary 1.1

Injective: if for every $b \in B$*, there is at most one* $a \in A$ *with* $f(a) = b$

Surjective: if for every $b \in B$*, there is at least one* $a \in A$ *with* $f(a) = b$

Note *If f is both injective and surjective for every* $b \in B$*. There is exactly one* $a \in A$ *such that* $f(a) = b$*, and we called this function is bijective*

Definition 1.5

If $G \subset B$*, then the inverse image* $f^{-1}(G)$ *of* G *under* f *is defined as:*

$$\{x \in A : f(x) \in G\}$$

If f is bijective then there is a new function $f^{-1} : B \to A$

Chapter 2 Functions

2.1 Connectness

Definition 2.1 (Topologist Definition)

X is connected if there are no 2 non-empty open stes $U_1, U_2 \subseteq X$ with $U_1 \cap U_2 = \emptyset$ and $U_1 \cup U_2 = X$ ♣

Note *When $U_2 = U_1^c$, and they satisfy the above condition, U_2 is closed and open*

Definition 2.2 (Analyst Definition)

X is connected if there is no continuous surjective function $f : X \to \{0, 1\}$ ♣

Proof : If $f : X \to \{0, 1\}$ surjective and continuous. Let $U_1 = f^{-1}(0)$, $U_2 = f^{-1}(1)$ are non-empty and open since f is surjective and continuous.

As a result, U_1 and U_2 are disjoint and cover X.

$$\text{Hello} \tag{2.1}$$

Corollary 2.1

Cantor set is disconnected, $\mathbb{Q}$ is disconnected. ♡

Proof Map first $\frac{1}{3}$ of Cantor set to $\{0\}$, last $\frac{1}{3}$ to $\{1\}$. $\mathbb{Q}$ is sepaeated by $\sqrt{2}$.

2.2 Sequence of function