PERSISTENCE MODELS FOR CUSTOMER EQUITY

Lenka Vraná – Pavel Jašek

Abstract

Customer Equity is defined as a sum of Customer Lifetime Values of all the current and the future customers and is a priceless tool which enables us to measure the firm's performance.

One of the methods that can be used to examine Customer Equity is an application of a vector autoregressive model (VAR). The method captures dynamic relationships between the number of customers acquired by marketing actions, number of customers acquired by word of mouth and the firm's performance.

The estimated VAR model can then be used to define not only effects of acquisitions on performance, but also effects of firm's performance on new acquisitions, effects between marketing and word of mouth acquisitions or their behaviour in time, which can serve as valuable insights for managers.

In this paper, we will describe the VAR model and impulse response function in the context of Customer Equity analysis, and we will apply them to customer data sets from two companies.

Key words: customer equity, vector autoregressive model, impulse response function

JEL Code: C32, M21, M31

Introduction

One of the most severe problems that companies need to deal with is the customer churn. There are many statistic, data mining and machine learning methods that are used to identify customers, that are about to leave the company. These methods are usually accompanied by the computation of a Customer Lifetime Value (CLV), because the retention offer should be provided only to the customers that are worth it (to maximize the return of the investment). Therefore the experts focused on the CLV analysis and nowadays there are many different methods how to estimate it and it became one of the fundamental indicators which is used in dozens of areas.

CLV is defined as the net present value of all the profits that a specific customer brings to the firm (Berger and Nasr, 1998). It can serve as an indicator of profitable individuals. Customer Equity (CE) is then sum of CLV of all the current and the future customers and can therefore serve as a tool how to measure the firm's performance.

Gupta et al. (2006) classify the CLV (or CE) modeling techniques into six branches: 1) recency, frequency, monetary value (RFM) models, 2) probability models, 3) econometrics models, 4) persistence models, 5) computer science models and 6) diffusion/growth models. This paper is the extension of the last year's paper (Jašek and Vraná, 2014) which compared the RFM, profitability and persistence models. This paper describes the persistence models and their application in more detail. This paper covers the vector autoregressive (VAR) model and the impulse response function, which can be used to analyze the effects of different types of customer acquisition on Customer Equity and the company's performance. To illustrate these analyses we use real-world datasets from two on-line retailers.

1 Vector Autoregressive model of Persistence

The vector autoregressive model is one of the methods that are used most often for multivariate time series analysis. According to Tsay (2014), its usage has several advantages: 1) the parameters of the model can be easily estimated, 2) the model is well described in specialized literature and 3) the model is similar to multivariate multiple regression and therefore many inference methods can apply also to VAR model.

The *k*-variate time series $\mathbf{y}_t$, which is defined as

$$\mathbf{y}_t = \mathbf{c} + \sum_{l=1}^{p} \mathbf{a}_l \mathbf{y}_{t-l} + \mathbf{e}_t \ , \tag{1}$$

follows a VAR(*p*) model, where *p* stands for the number of lags, $\mathbf{c}$ is k-dimensional vector of constants, $\mathbf{a}_l$ are *k* x *k* matrices of parameters, and $\mathbf{e}_t$ is sequence of independent and identically distributed random *k*-dimensional vectors with zero mean and covariance matrix $\mathbf{\Sigma}_e$. For more information about the VAR model specification, see Tsay (2014).

1.1 Persistence model

In 2008 Villanueva et al. described application of VAR model to customer equity predictions. They researched impacts of customer acquisition on the company's performance. They examined the differences between customers gained by marketing activities and customers acquired spontaneously.

The model is designed as the classical VAR(p) model (1). It captures dynamic relationships between three time series: number of customers acquired by marketing actions (*MKT*), number of customers acquired by word of mouth (*WOM*) and the firm's performance (*VALUE*):

$$\begin{pmatrix} MKT_t \\ WOM_t \\ VALUE_t \end{pmatrix} = \begin{pmatrix} c_1 \\ c_2 \\ c_3 \end{pmatrix} + \sum_{l=1}^{p} \begin{pmatrix} a_{11,l} & a_{12,l} & a_{13,l} \\ a_{21,l} & a_{22,l} & a_{23,l} \\ a_{31,l} & a_{32,l} & a_{33,l} \end{pmatrix} \begin{pmatrix} MKT_{t-l} \\ WOM_{t-l} \\ VALUE_{t-l} \end{pmatrix} + \begin{pmatrix} e_{1,t} \\ e_{2,t} \\ e_{3,t} \end{pmatrix}, \tag{2}$$

where t stands for time, vector $\begin{pmatrix} c_1 & c_2 & c_3 \end{pmatrix}'$ contains the constant terms and vector $\begin{pmatrix} e_{1,t} & e_{2,t} & e_{3,t} \end{pmatrix}'$ contains the error terms with Gaussian white noise properties. The VAR(1) model in this form can describe the following relationships (Villanueva et al., 2008):

- *direct effects* of acquisition on the firm's performance (coefficients $a_{31,1}$ and $a_{32,1}$),
- *cross-effects* between two types of customer acquisition (coefficients $a_{12,1}$ and $a_{21,1}$),
- *feedback effects*, which states how the firm's performance affects the acquisition in the next time periods (coefficients $a_{13,1}$ and $a_{23,1}$),
- *reinforcement effects*, when value of series in time t affects its future values, e. g. customers acquired by word of mouth would spread the positive information about the firm which would lead to more acquisitions (coefficients $a_{11,1}$, $a_{22,1}$ and $a_{33,1}$).

Villanueva et al. discovered that customers gained by marketing promotions generate higher value in short term. However, customers acquired spontaneously had greater impact in long-term evaluation. We try to apply their approach to online retailers' data and compare the results.

1.2 Impulse response function

There are several ways how to analyze the relationships between the time series in a VAR model like Granger Causality or impulse response function. The impulse response function (sometimes called the multiplier analysis) quantifies the effects of changes in one series on the values of the others.

The principle is simple: the shock is sent to one of the time series in the moving-average representation of the VAR(p) model. Then this impulse spreads through the whole system of the time series and we can estimate the changes in their values caused by the initial shock. The impulse response function shows these changes for each time lag.

As the series are usually correlated ($\boldsymbol{\Sigma}_e$ is not diagonal matrix), it is in fact unlikely to observe the shock only in one of the time series. Therefore the whole system can be transformed – orthogonalized. Then we can get the impulse response function with orthogonal innovations, which can describe the real world situations better than the original impulse response function.

Pesaran and Shin (1998) suggested the generalized impulse response analysis, which doesn't require the orthogonalization of shocks.

2 Application of persistence model: Company A

The first company whose data we analyze in this paper is a fashion online retailer focused primarily on mid-aged women. The historical log contains 77 289 logged-in visits to the e-commerce website and 33 613 online purchases made by 29 589 different customers from the time period of September 1, 2011 to March 31, 2014.

As we are missing more detailed data, we define newly acquired customers in time t as customers, who make their first purchase. Therefore we use first year data as a purchase history and we don't include them in the analysis. We work with weekly time series from September 1, 2012 to December 31, 2013. We keep 2014 data as the validation set.

We add data from Google Analytics to distinguish between *MKT* and *WOM* customers. Villanueva et al. (2008) used number of log-ins as *VALUE* series as they were working with data from internet firm that provided Web hosting. We tried to use income as firm's performance indicator, but there was no significant dependency of income on number of acquired customers, so we used number of purchases. Because the new customer is not identified until she makes her first purchase, the analysis focuses on any subsequent purchases.

Possibly, we could also use the number of websites visits as the *VALUE* series, as it was proved that this indicator can significantly improve the CLV models (Jašek, 2014).

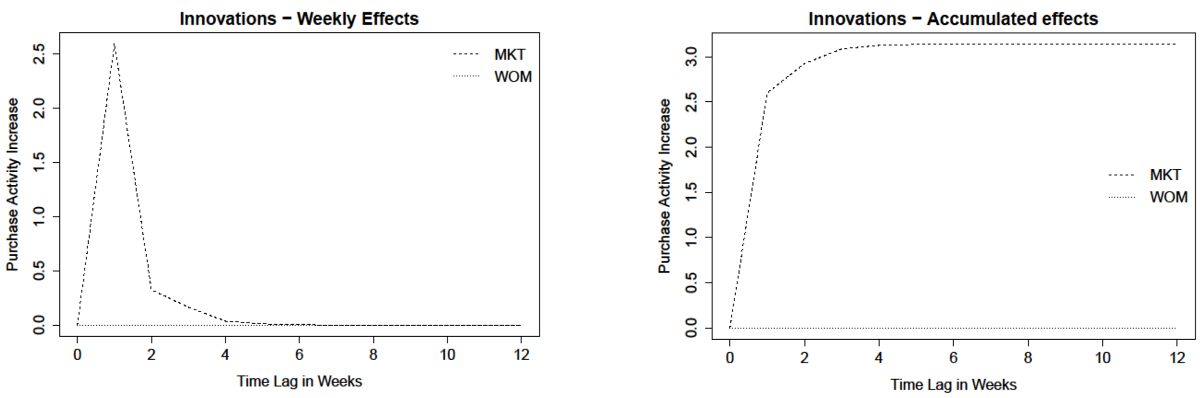
All the series are tested for unit root by augmented Dickey-Fuller (ADF) test and Kwiatkowski–Phillips–Schmidt–Shin (KPSS) test and are recognized as stationary (their means and variances are time invariant). To minimize the Akaike information criterion we fitted VAR(1) model:

$$\begin{pmatrix} MKT_t \\ WOM_t \\ VALUE_t \end{pmatrix} = \begin{pmatrix} 68.51 \\ 40.82 \\ 237.96 \end{pmatrix} + \begin{pmatrix} 0.92 & 0.00 & -0.27 \\ 0.75 & 0.59 & -0.32 \\ 2.60 & 0.00 & -0.80 \end{pmatrix} \begin{pmatrix} MKT_{t-1} \\ WOM_{t-1} \\ VALUE_{t-1} \end{pmatrix}. \tag{3}$$

The coefficients $a_{12,1}$ and $a_{32,1}$ are insignificant and are set to zero, thus there is no direct effect of *WOM* customers on firm's performance and no cross-effect of *WOM* customers on *MKT* acquisitions.

Based on the fitted model we also create impulse response functions (Fig. 1), that show the response of *VALUE* series to newly acquired customer via marketing promotion or word of mouth. The effect includes not only purchases made by the new customer, but also purchase activity of others which could have been encouraged by the newcomer (Villanueva et al., 2008).

Fig. 1: Direct effects of customer acquired through marketing promotions (*MKT*) and customer acquired spontaneously (*WOM*) on number of purchases (*VALUE*) in Company A



As the model doesn't find any direct effects of *WOM* on firm's *VALUE*, the impulse response function (weekly effects as well as accumulated) is constant and equal to zero. This means that the *WOM* customers usually make only one purchase (the one when they are identified as new customers) and no more.

The function of weekly effects shows that each unexpected acquisition made through the marketing channel generates 2.60 additional purchases during the first week and then the effect fades. The new *MKT* customer causes 3.14 additional purchases during her whole lifetime.

The results of this analysis are opposite to the results of Villanueva et al. (2008). Their company's value is affected mostly by *WOM* customers; our model suggests that the *WOM* customers don't have any significant impact on the firm's performance after their first purchase.

This zero effect of *WOM* is also in contrast with the study done by Smutný et al. (2013), where customers of a studied telecommunications company influenced their own interactions more than communications activities of the studied brand itself, thus impacting positively *WOM* channels.

3 Application of persistence model: Company B

The second company is a health and beauty online retailer from the Czech Republic. The dataset consists of 48 435 orders from October 17, 2010 to March 29, 2015. Again, we keep the first year data (before October 17, 2011) as the purchase history to be able to identify new customers. Therefore the customer would be considered as newly acquired even if she placed any orders before the start date of the dataset, but no orders in the first year. The 2015 data are kept as the validation set.

The data are enriched with the source of each transaction from Google Analytics, so we can divide new customers into two groups: those acquired by the marketing activities and those acquired by the word of mouth. We aggregate the data into weekly time series. We use number of purchases as *VALUE* indicator as well as in the previous example, so the results are comparable.

According to the ADF and KPSS tests, the *MKT* time series (number or new customers acquired through marketing channels) is not stationary so we use the first difference of *MKT* instead, which will unfortunately make the interpretation of the results difficult.

If more than one of the series would be non-stationary, we could take the co-integration analysis into consideration. Then the VAR(p) model could be written in a form of error correction model. For more detail about co-integration and error correction, see Engle and Granger (1987) or Arlt (1997).

To minimize the Akaike information criterion, the VAR(4) model was fitted – the model of order 4 is quite complex[1] so we show only the impulse response function (Fig. 2), not the fitted model.

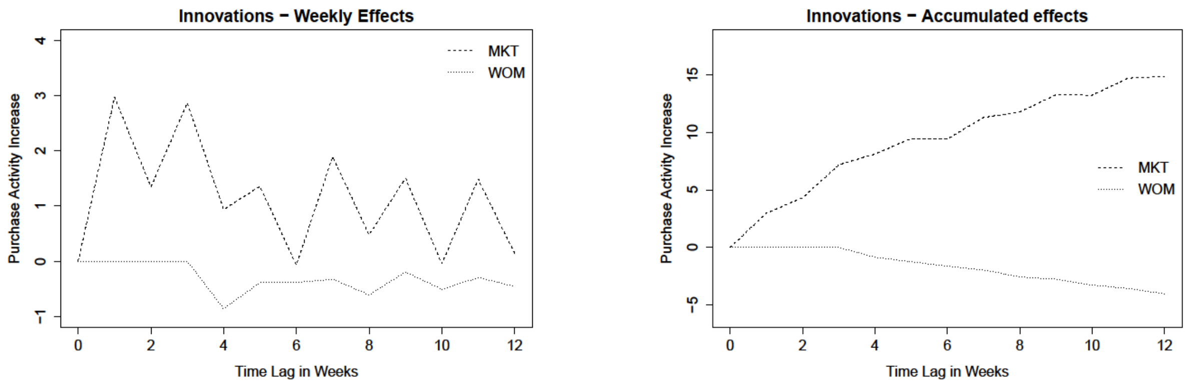
According to the model, the customers acquired by the word of mouth rather reduce the expected performance of the company. Gupta et al. (2006) state that these declines may be caused by the hidden market mechanisms, e. g., the success of the first company may provoke its competitors to start their own acquisition campaigns and therefore it may seem that the new customers cause the firm's performance to decline. The *WOM* customer in the first 12 weeks

[1] The VAR(4) model contains 39 parameters. In this case, 22 parameters were set to zero as they were insignificant in the model.

causes the decline in the firm's performance by 4.03 purchases and the cumulative response function tends to decrease even further.

We need to keep in mind, that we used the first difference of *MKT* instead of the original *MKT* series. That means that a shock in this series doesn't mean one newly acquired customer through the marketing channels, but increase of acquired customers by additional one when compared to the previous week.

Fig. 2: Direct effects of customer acquired through marketing promotions (first difference of *MKT*) and customer acquired spontaneously (*WOM*) on number of purchases (*VALUE*) in Company B



This increase of acquired customers will bring the company 2.97 additional purchases during the first week, 2.86 purchases during the third week and the impact tends to lower slowly (with biweekly peaks). According to the model, this acquisition generates additional 14.85 purchases during the first twelve weeks and there is obvious increasing trend (Fig. 2). These purchases may be produced directly by the acquired customer, or by other customers that were affected by this person (thus caused by reinforcement or cross-effects).

Again, these results do not support the hypothesis that the firm's performance is pulled mostly by the *WOM* customers as Villanueva et al. (2008) stated.

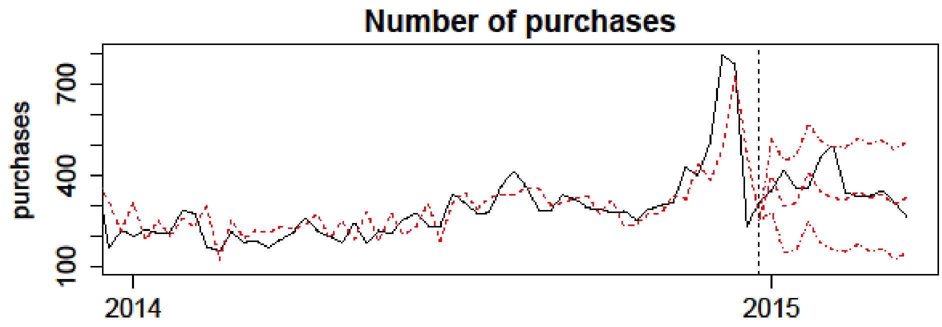
4 Value predictions

One of the biggest advantages of the persistence models is their ability to predict future values of all the included time series (*MKT*, *WOM* and *VALUE*). We show the interpolation and extrapolation of the number of purchases (*VALUE*) for the Company B as an example (Fig. 3).

We held the 2015 data as the validation sample and we computed the 95% confidence intervals for these observations.

However, the description of the construction of the predictions and their confidence intervals and the evaluation of their quality exceed the focus of this paper. For more information on these topics, see Tsay (2014).

Fig. 3: The number of purchases (*VALUE*) from January 1, 2014 to March 29, 2015 and its interpolation and extrapolation by the VAR(4) model



Conclusion

In 2008 Villanueva et al. showed how to use vector autoregressive models for customer equity analysis. They analyzed data from Web hosting company and used impulse response function to prove, that the customers acquired by word of mouth generate greater number of log-ins than the customers gained by some marketing activity.

We tried to use their approach and applied the vector autoregressive model to the real world datasets from Czech companies. The impulse response functions showed the expected increase in firms' performances (measured here by the number of purchases) caused by the *MKT* customers, however, the impact of *WOM* customers was not proved. Actually, the model for the Company B shows that the acquisition of new *WOM* customer has negative effect on the number of purchases. This can be caused by some market mechanisms (e.g., competitive reaction) which are not included in the model.

The next possible extension of this analysis would be to observe the source of the marketing acquisitions – to have separate time series for each type of marketing campaign. This could help us to identify the outperforming campaigns, to better understand the factors that affect the firm's performance or to detect if the campaign's output meets the expectations.

Persistence models brings interesting insights about the inner workings of companies and can be priceless tools for their managers.

Acknowledgment

This paper was processed with the financial support of IGA grant F4/18/2014 and contribution of long term institutional support of research activities by Faculty of Informatics and Statistics, University of Economics, Prague.

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Contact

Lenka Vraná
University of Economics, Prague
W. Churchill Sq. 4, Prague, Czech Republic
lenka.vrana@vse.cz

Pavel Jašek
University of Economics, Prague
W. Churchill Sq. 4, Prague, Czech Republic
pavel.jasek@vse.cz