

Stackelberg ILQGames Writeup

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In this document, we describe the alterations made to the ILQGames algorithm described in *Efficient Iterative Linear-Quadratic Approximations for Nonlinear Multi-Player General-Sum Differential Games* (Fridovich-Keil et al. 2020) made to adapt it to solve nonlinear, non-quadratic 2-player Stackelberg games.

We assume no affine terms in this initial draft. **TODO: adjust to include affine terms.**

1 Steps

The algorithm accepts a number of inputs:

- initial state \mathbf{x}_0 ,
- a horizon T over which the game is played,
- cost functions g_t^i , $i \in \{1, 2\}$ at each time step t ,
- initial control strategies $\gamma_{k=0}^i$ described by the gain matrix $S_{t,k=0}^i$ and the recursive cost matrix $L_{t,k=0}^i$, $i \in \{1, 2\}$.

We pre-select arbitrary reference state and controls to be

$$\mathbf{x}_t = 0, \quad \mathbf{u}_t^i = 0 \quad \forall i \in \{1, 2\}$$

and $\eta \in (0, 1]$.

At each iteration $k = 1, 2, \dots$ until convergence condition $\|\Delta\gamma_k^i\| < \text{threshold } \epsilon$,

1. Propagate γ_{k-1}^i forward in time from \mathbf{x}_0 to get $\xi_k \equiv \{\hat{\mathbf{x}}_{1:T}, \hat{\mathbf{u}}_{1:T}^i\}$.
2. Compute errors

$$\delta\mathbf{x}_t = \mathbf{x}_t - \hat{\mathbf{x}}_{1:T}, \quad \delta\mathbf{u}_t^i = \mathbf{u}_t^i - \hat{\mathbf{u}}_{1:T}^i.$$

3. Linearize the dynamics and quadraticize about the error state and controls.

$$A_t = \frac{\partial f_t(\hat{\mathbf{x}}_t, \hat{\mathbf{u}}_t^i)}{\partial \hat{\mathbf{x}}_t}, \quad B_t^i = \frac{\partial f_t(\hat{\mathbf{x}}_t, \hat{\mathbf{u}}_t^i)}{\partial \hat{\mathbf{u}}_t^i}$$
$$Q_t^i = \nabla_{\hat{\mathbf{x}}} \hat{\mathbf{x}} g_t^i, \quad R_t^{ii} = \nabla_{\hat{\mathbf{u}}^i} \hat{\mathbf{u}}^i g_t^i$$

4. Solve LQ Stackelberg game $\gamma_k^i = \mathcal{S}_t^1(A_t, B_t, \{Q_t^i\}, \{R_t^{ij}\}, T; \mathbf{x}_0) = \{S_{t,k}^i, L_{t,k}^i\}$.
5. Adjust the control strategies based on this approximate solution:

$$\gamma_k^i = \hat{\mathbf{u}}_t^i - S_{t,k}^i \delta \mathbf{x}_t.$$