



Principles of Compiler Construction

线性文法不一定递归

match--shift

derive--reduce

key: 找到适合reduce的串

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Lecture 6. Operator Precedence Parsing (OPP)

1. Bottom-Up Parsing
2. Abstract Model for Shift-Reduce Parsing
3. Operator-Precedence Parser
4. What Have We discarded?
5. Does OPP Disappear?

1. Bottom-Up Parsing

- A motivating example

- Construct a parse tree beginning at the leaves and working up towards the root.
- For the grammar

$$E \rightarrow E + T \mid T$$

$$T \rightarrow T * F \mid F$$

$$F \rightarrow (E) \mid \mathbf{n}$$

-
- Begin with leaves: **$n * n$**



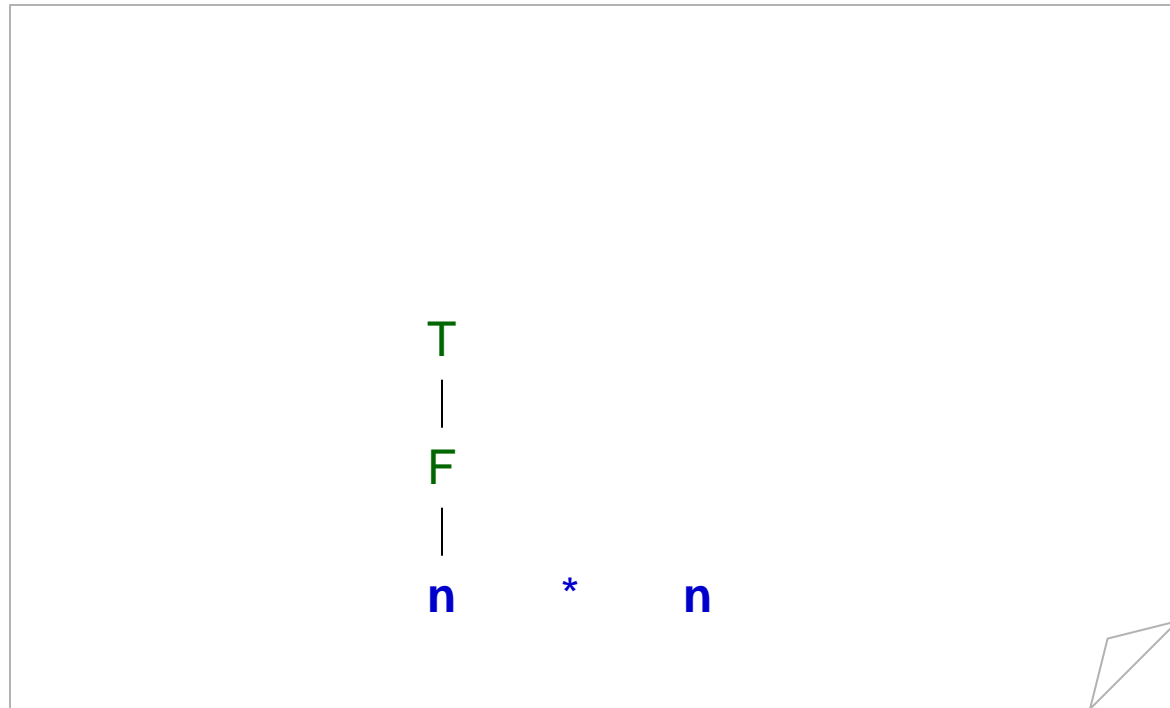
$n * n$

- The 1st **n** is the handle, after reduction we have the right sentential form: **F * n**

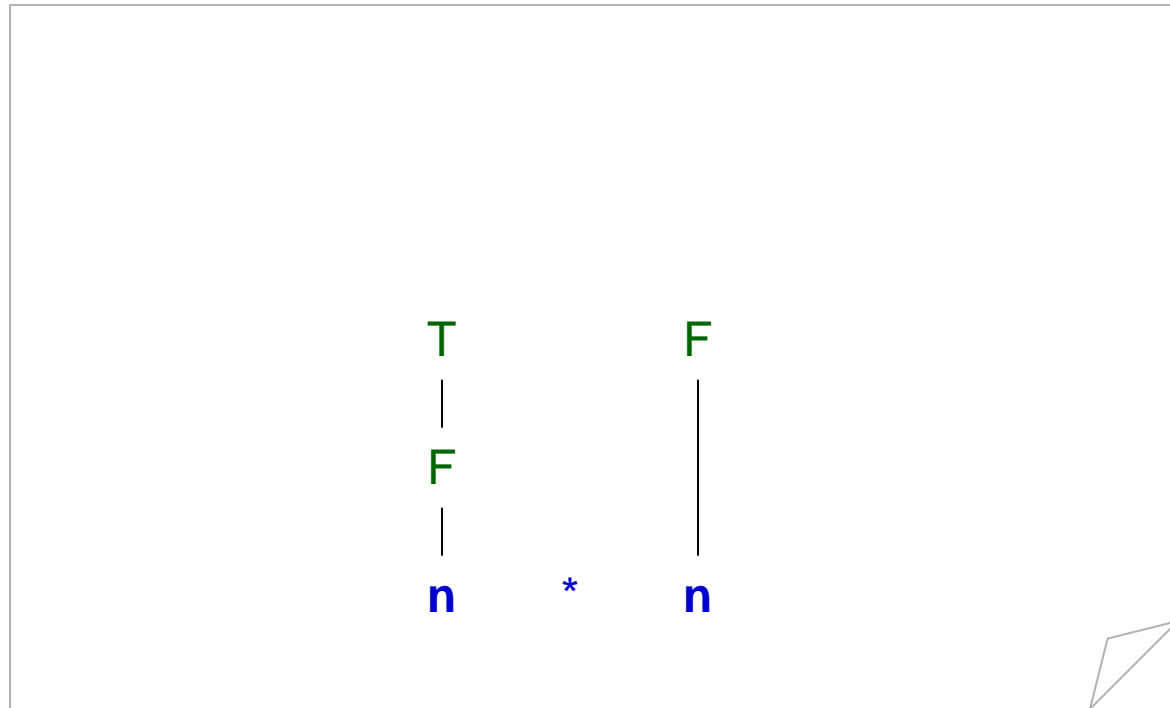


A diagram illustrating a sentential form. It consists of a large, empty rectangular box with a folded bottom-right corner. Inside the box, the symbol **F** is positioned above a vertical line, which is positioned above the symbol **n**. To the right of this **n** is an asterisk *****, and to the right of the asterisk is another **n**. The symbols **F** and ***** are green, while the symbols **n** and the vertical line are blue.

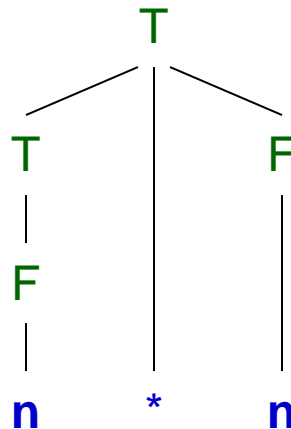
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- **F** is the handle and we have: **T * n**



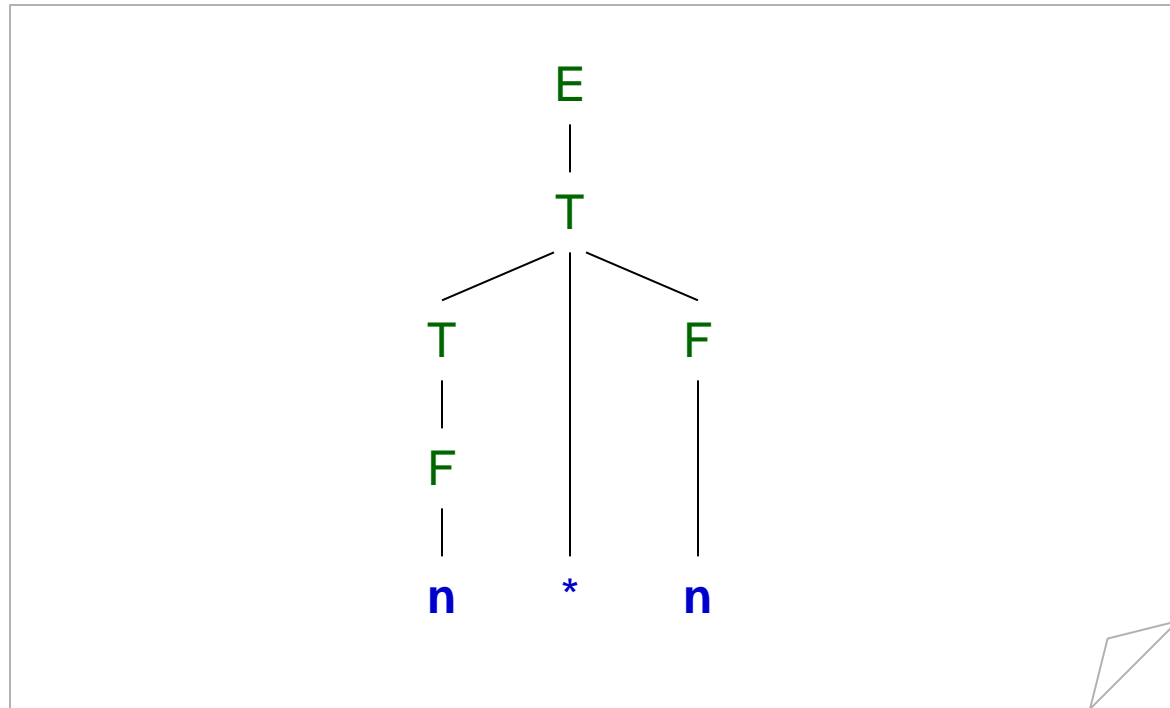
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- **n** is the handle in **T * n**, we have: **T * F**



- **T * F** is the handle



- **T** is the handle

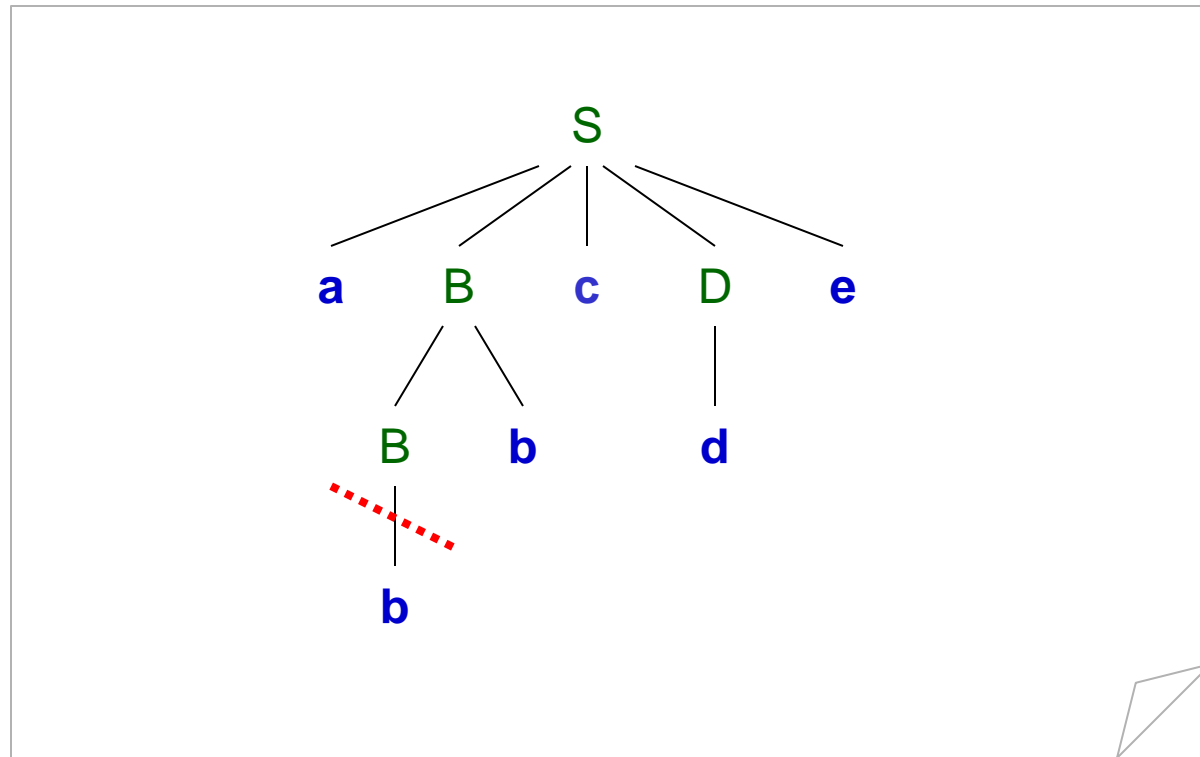


One More Example

- For the grammar

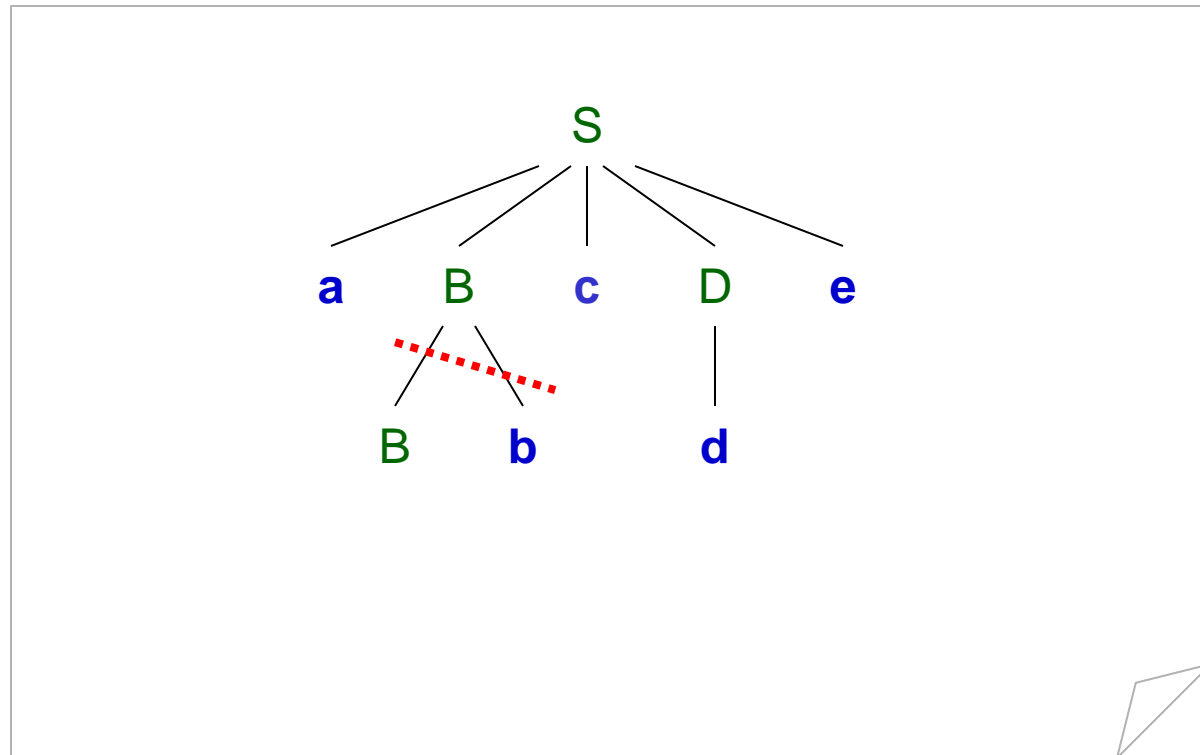
$$S \rightarrow a B c D e$$
$$B \rightarrow B b \mid b$$
$$D \rightarrow d$$

- Reduction = handle (**b**) pruning



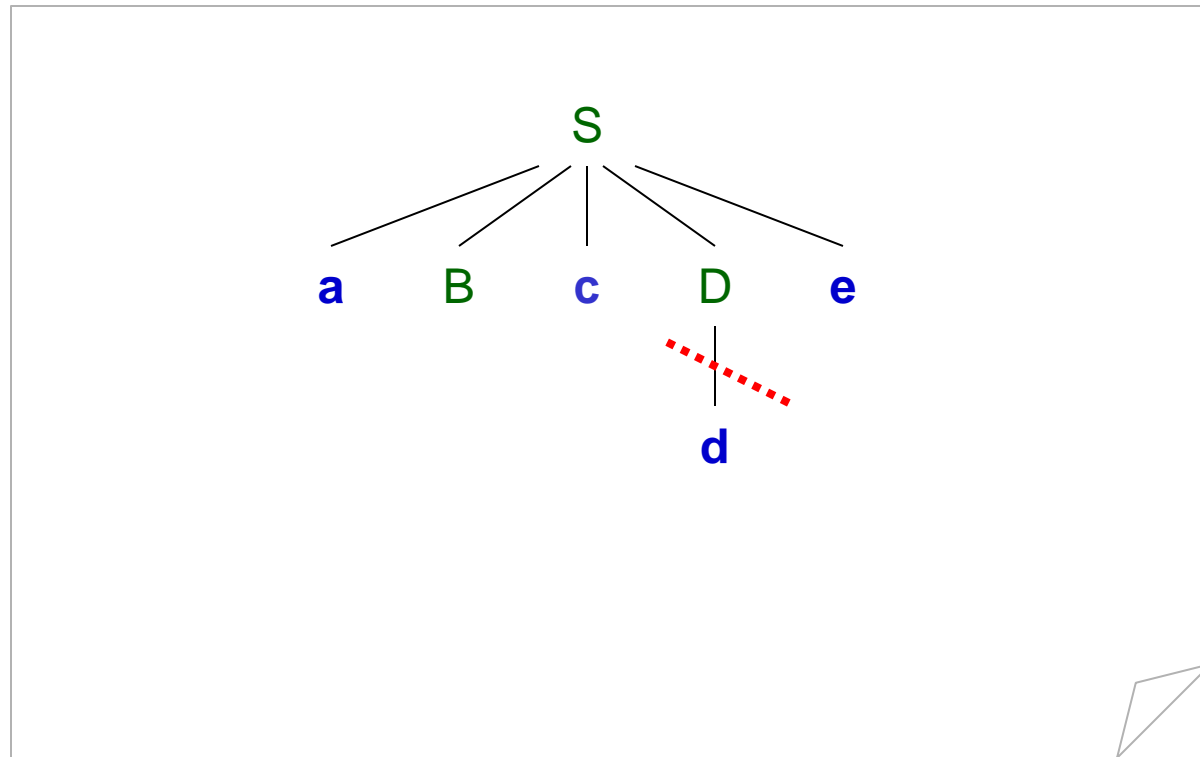
Right sentential form: a b b c d e

- **Bb** is the handle



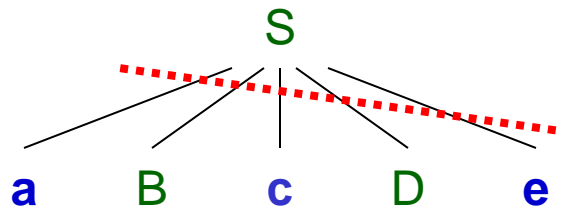
Right sentential form: a B b c d e

- **d** is the handle



Right sentential form: a B c d e

- **aBcDe** is the handle



Right sentential form: **a B c D e**

-
- Successfully reduced to the root



S

Right sentential form: **S**

Review

- Reducible string in a sentential form
 - Handle of a **right** sentential form
 - If $S \Rightarrow_{rm}^* \alpha A \omega \Rightarrow_{rm} \alpha \beta \omega$, then
 - production $A \rightarrow \beta$ in the position following α is a handle of $\alpha \beta \omega$.
 - Properties
 - The substring to the right of the handle (i.e. ω in the previous definition) must contain only terminals.
 - Note that it is a right-most (rm) derivation.

规范句型

Review (cont')

- Handle in a parse tree
 - A left-most two-level subtree.
 - Not practical. Only helpful to understand the concept.
- Critical problem in bottom-up parsing
 - How to find a reducible string in a sentential form?
 - 最左直接短语
 - LR parsing: handle (left-most simple phrase)
 - OPP: left-most prime phrase ← 子树可能好几层
 - 最左素短语
 - 必须含有终结符号

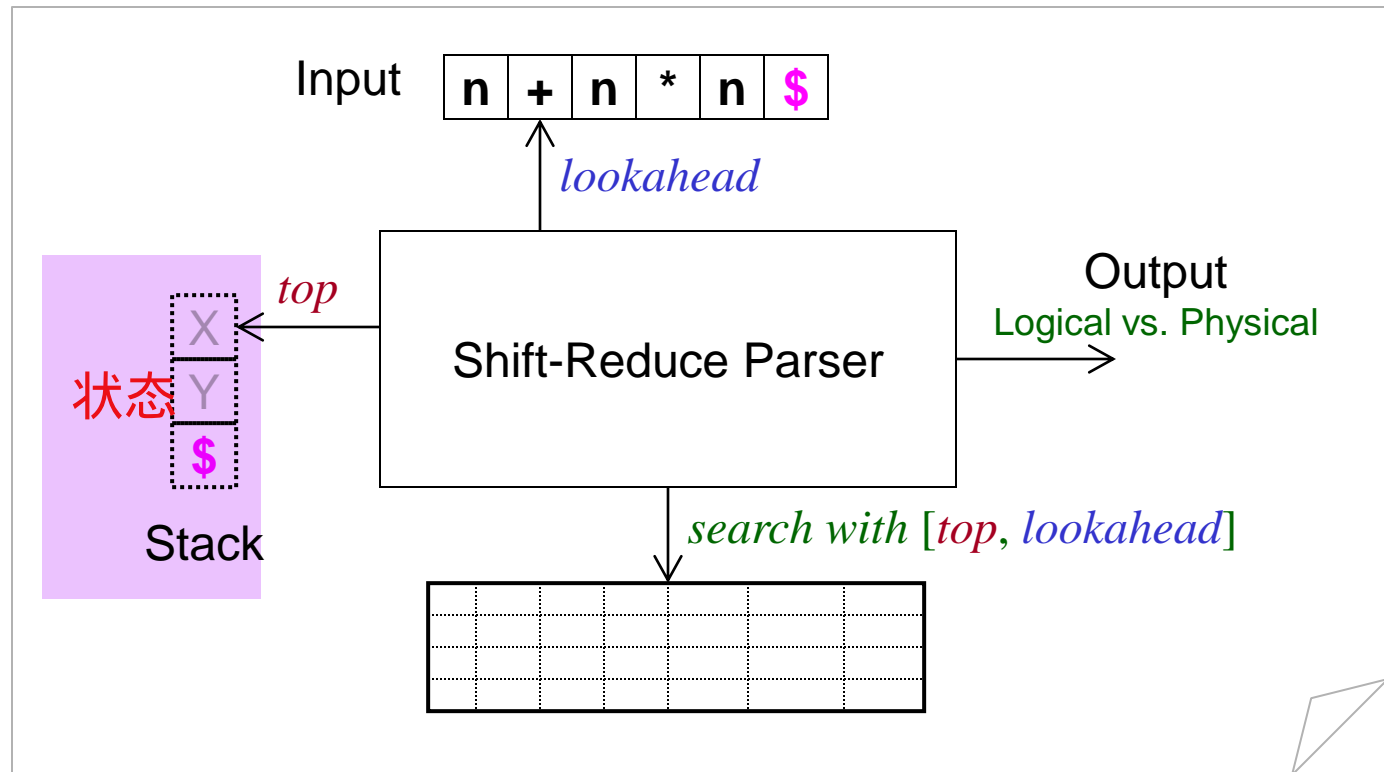
2. Abstract Model for Shift-Reduce Parsing

match-derive parser

- Symmetric to LL(1) parsing
 - Table-driven approach
 - Parsing decision making based on a table
 - LL(1) parsing table vs.
 - Operator-precedence relation table
 - LR(0)/SLR(1)/LALR(1)/LR(1) parsing table
 - Explicit stack
 - Top-down parsing: $[\$S, \omega\$] \Rightarrow [\$, \$]$
 - Bottom-up parsing: $[\$, \omega\$] \Rightarrow [\$S, \$]$
 - Actions
 - Top-down parsing: match-derive
 - Bottom-up parsing: shift-reduce

An Abstract Model

- A shift-reduce parser



A Motivating Example

- Given the previous grammar

$S \rightarrow a B c D e$

$B \rightarrow B b \mid b$

$D \rightarrow d$

- For the previous sentence

$a b b c d e$

A Motivating Example (cont')

Step	Stack	Input	Action	Output
1	\$	a b b c d e \$	shift	
2	\$ a	b b c d e \$	shift	
3	\$ a b	b c d e \$	reduce	B → b
4	\$ a B	b c d e \$	shift	
5	\$ a B b	c d e \$	reduce	B → B b
6	\$ a B	c d e \$	shift	
7	\$ a B c	d e \$	shift	
8	\$ a B c d	e \$	reduce	D → d
9	\$ a B c D	e \$	shift	
10	\$ a B c D e	\$	reduce	S → a B c D e
11	\$ S	\$	accept	

More Motivating Examples

- Given the **ambiguous** grammar for expressions

$$E \rightarrow E + E \mid E * E \mid (E) \mid n$$

- For the following sentence

$$n + n * n$$

More Motivating Examples (cont')

Step	Stack	Input	Action	Output
1	\$	n + n * n \$	shift	
2	\$ n	+ n * n \$	reduce	E → n
3	\$ E	+ n * n \$	shift	
4	\$ E +	n * n \$	shift	
5	\$ E + n	* n \$	reduce	E → n
6	\$ E + E	* n \$	shift	
7	\$ E + E *	n \$	shift	
8	\$ E + E * n	\$	reduce	E → n
9	\$ E + E * E	\$	reduce	E → E * E
10	\$ E + E	\$	reduce	E → E + E
11	\$ E	\$	accept	

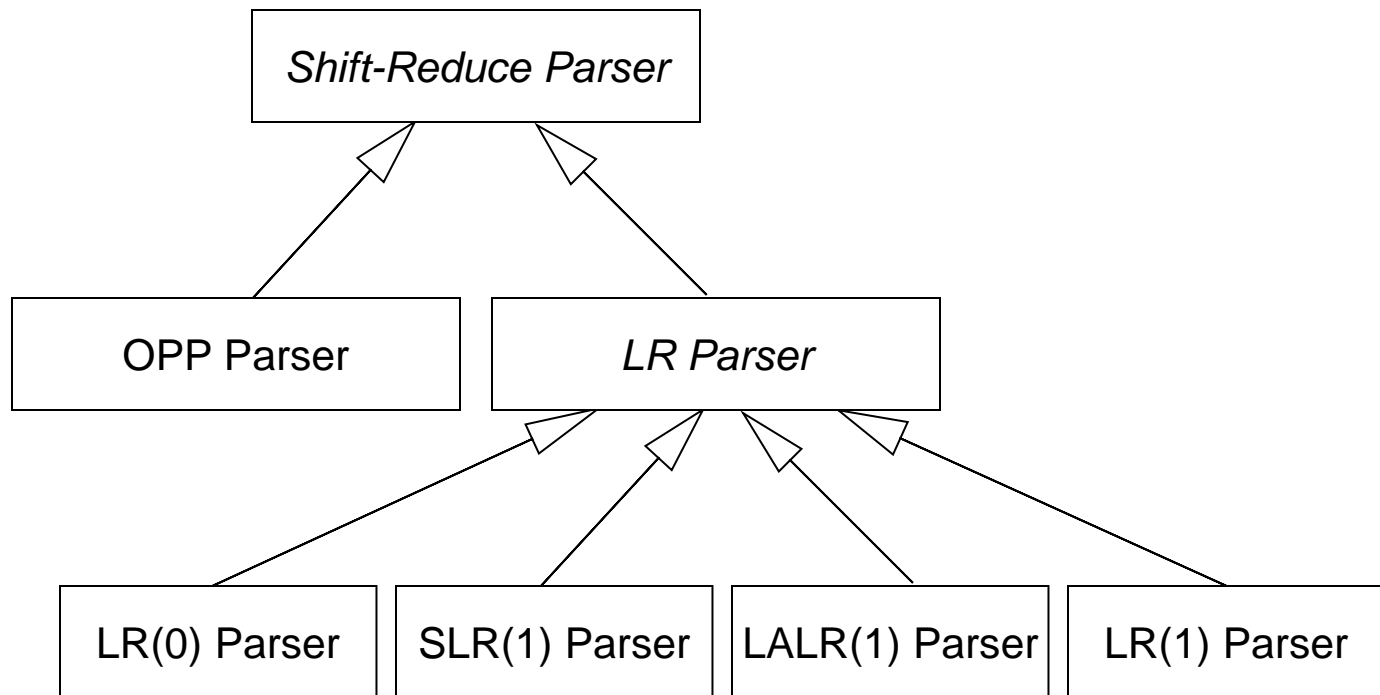
What Is Abstract in the Model?

- What is the form of the parsing table?
 - Operator-precedence relation table
 - LR parsing table
- What is the contents of the stack?
 - Operators *opp*
 - Parsing states 实质是状态的转换 LR
- How to determine a reducible substring on the top of the stack?
 - Left-most prime phrase *opp*
 - Handles LR

可能有多层，不止两层

Concrete Implementations

- Implementations of the abstract model for shift-reduce parsing



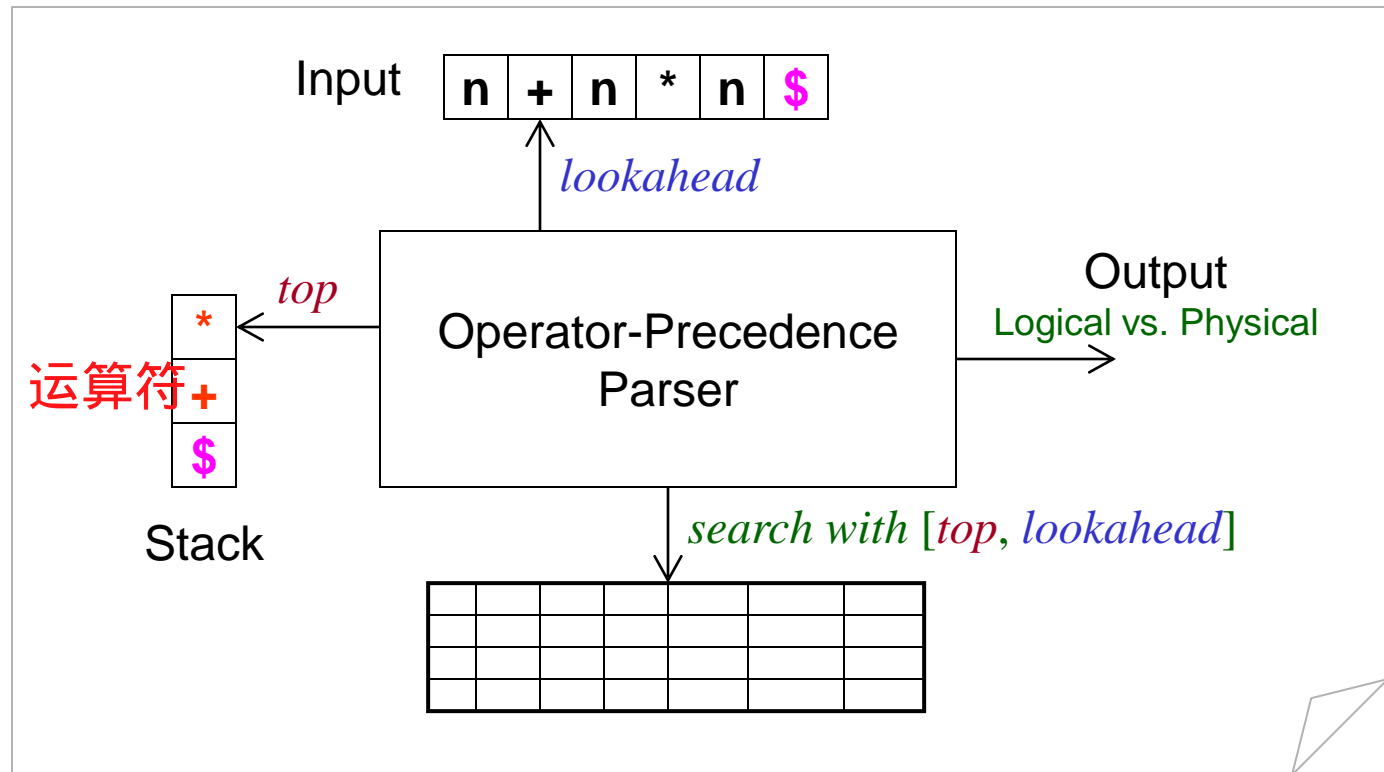
simple

3. Operator-Precedence Parser

- Concrete implementations
 - Parsing table
 - Operator-precedence relations
 - Explicit stack
 - Operators
 - Reducable substring
 - Left-most prime phrases

A Concrete Model for OPP Parser

○ Operator Precedence Parser



Operator Grammar

- A small but important class of grammars
 - There is no ε -production.
 - No production has two adjacent nonterminals.

- For example

- Not an operator grammar

$$E \rightarrow \underline{E A E} \mid (E) \mid -E \mid \mathbf{id}$$

$$A \rightarrow + \mid - \mid * \mid / \mid ^$$

- An operator grammar

$$\begin{aligned} E \rightarrow E + E \mid E - E \mid E * E \mid E / E \mid E ^ E \\ \mid (E) \mid -E \mid \mathbf{id} \end{aligned}$$

Operator-Precedence

○ Three precedence relations

- $a \prec b$ 表示运算的顺序
- $a \equiv b$
- $a \succ b$

○ Properties of the binary relation

- Non-reflexive: “+ \equiv +” is not true. 无自反性
- Non-symmetric: “+ \succ -”, but “- \succ +”. 无对称性
- Non-transitive: “> \succ <” & “< \succ =”, but “= \succ >”
无传递性
- A partial relation 部分是不定义的

Operator-Precedence Relation Table

优先级最高

	id	+	*	\$
id		\prec	\prec	\prec
+	\prec	\prec	\prec	\prec
*	\prec	\prec	\prec	\prec
\$	\prec	\prec	\prec	

先计算后面的

说明是左结合

OPP

Parsing Algorithm

```
initialize();  
for (;;) {  
    if (top == $ && lookahead == $) accept();  
    topOp = stack.topMostTerminal();  
    if (topOp < lookahead || topOp == lookahead) { // shift  
        stack.push(lookahead);  
        lookahead = scanner.getNextToken();  
    } else if (topOp > lookahead) { // reduce  
        do {  
            topOp = stack.pop();  
        } while (stack.topMostTerminal() > || == topOp);  
    } else error();  
}
```

(与)一起pop

Parsing a Sentence

查前面优先级表

Step	Stack	Input	Reference	Action	Output
1	\$	n + n * n \$	\$ < n	shift	
2	\$ n	+ n * n \$	n > +	reduce	E → n
3	\$ E	+ n * n \$	\$ < +	shift	
4	\$ E +	n * n \$	+ < n	shift	
5	\$ E + n	* n \$	n > *	reduce	E → n
6	\$ E + E	* n \$	+ < *	shift	
7	\$ E + E *	n \$	* < n	shift	
8	\$ E + E * n	\$	n > \$	reduce	E → n
9	\$ E + E * E	\$	* < \$	reduce	E → E * E
10	\$ E + E	\$	+ < \$	reduce	E → E + E
11	\$ E	\$		accept	

实际上，栈里只有操作符和\$

一个运算有两个operator是两个是=

More Practical OPP Table

	+	-	*	/	^	id	()	\$
+	⌞	⌞	⌞	⌞	⌞	⌞	⌞	⌞	⌞
-	⌞	⌞	⌞	⌞	⌞	⌞	⌞	⌞	⌞
*	⌞	⌞	⌞	⌞	⌞	⌞	⌞	⌞	⌞
/	⌞	⌞	⌞	⌞	⌞	⌞	⌞	⌞	⌞
^	⌞	⌞	⌞	⌞	⌞	⌞	⌞	⌞	⌞
id	⌞	⌞	⌞	⌞	⌞			⌞	⌞
(⌞	⌞	⌞	⌞	⌞	⌞	⌞	=	
)	⌞	⌞	⌞	⌞	⌞			⌞	⌞
\$	⌞	⌞	⌞	⌞	⌞	⌞	⌞		

直接代替为操作

Another Form of OPP Table

	+	−	*	/	^	id	()	\$
+	reduce	reduce	shift	shift	shift	shift	shift	reduce	reduce
−	reduce	reduce	shift	shift	shift	shift	shift	reduce	reduce
*	reduce	reduce	reduce	reduce	shift	shift	shift	reduce	reduce
/	reduce	reduce	reduce	reduce	shift	shift	shift	reduce	reduce
^	reduce	reduce	reduce	reduce	shift	shift	shift	reduce	reduce
id	reduce	reduce	reduce	reduce	reduce			reduce	reduce
(shift	shift	shift	shift	shift	shift	shift	shift	
)	reduce	reduce	reduce	reduce	reduce			reduce	reduce
\$	shift	shift	shift	shift	shift	shift	shift		

Error Recovery in OPP

Phrase-Level Recovery

	+	−	*	/	^	id	()	\$
+	reduce	reduce	shift	shift	shift	shift	shift	reduce	reduce
−	reduce	reduce	shift	shift	shift	shift	shift	reduce	reduce
*	reduce	reduce	reduce	reduce	shift	shift	shift	reduce	reduce
/	reduce	reduce	reduce	reduce	shift	shift	shift	reduce	reduce
^	reduce	reduce	reduce	reduce	shift	shift	shift	reduce	reduce
id	reduce	reduce	reduce	reduce	reduce	error1	error1	reduce	reduce
(shift	shift	shift	shift	shift	shift	shift	shift	error2
)	reduce	reduce	reduce	reduce	reduce	error1	error1	reduce	reduce
\$	shift	shift	shift	shift	shift	shift	shift	error3	accept

Handling Overloading Operators

- Overloading unary and binary operators
 - **id – id * id**
 - **id * – id**
- Distinguish or resolve **overloading** by the scanner
 - **Remember the previous processed token.**

4. What Have We Discarded?

- Automatically table construction
 - Construct an operator-precedence relation table from an **unambiguous** grammar of expressions.
 - Trade-off: cost vs. cost
 - Writing an unambiguous grammar
 - Constructing an precedence relation table

Saving Table Space

- Precedence functions
 - Pack an $n \times n$ matrix into a $2 \times n$ matrix.
 - Save **some** space to store the parsing table.
 - But something is missing.
 - Each pair of operators must have a relation.

	+	−	*	/	^	id	()	\$
<i>f</i>	2	2	4	4	4	6	0	6	0
<i>g</i>	1	1	3	3	5	5	5	0	0

5. Does OPP Disappear?

- OPP has disappeared from many modern compiler textbooks, e.g.
 - DBv2 (while DBv1 concerned with OPP)
 - Tiger book
 - The Art of Compiler Design
 - etc.

Reasons for Ignoring OPP

- Some reasons are: 数据结构已经有使用
 - OPP only handles very simple languages.
 - OPP isn't very good at discovering when a string isn't in the language.
 - Operator precedences have been incorporated into LR parsers, so writing an LR parser for expressions with many precedence levels isn't very hard.
 - Space isn't nearly as much an issue as it used to be.
 - OPP is often taught in introductory Data Structures courses.

可能是只有表达式是使用OPP

But OPP Still Works

- Even in commercial applications, specially combination of OPP and recursive parsing
 - Some Sun **javac** compiler
 - Recursive parser parses expressions coarsely.
 - OPP parses the binary operators correctly.
 - Some open source Verilog simulator
 - OPP parses Verilog expressions as a separate section of a hand coded recursive descent parser.
 - Verilog expressions are complex and language is irregular and non context-free.
 - The early Symantec's THINK C compiler
 - The original C compiler at Bell Labs (reported)

Exercise 6.1

- Consider the grammar

$$S \rightarrow (L) \mid a$$

$$L \rightarrow L, S \mid S$$

- We have the following operator-precedence relations for the grammar.
 - Show the detailed process of the parsing of the sentence $(a, (a, a))$, follow the style in the previous slides.

	a	()	,	\$
a			\prec	\prec	\prec
(\prec	\prec	\equiv	\prec	
)			\succ	\succ	\succ
,	\prec	\prec	\succ	\succ	
\$	\prec	\prec			

Further Reading

- Dragon Book, 2nd Edition (DBv2)
 - Comprehensive Reading:
 - Section 4.5 for the abstract model for shift-reduce parsing.
- Dragon Book, 1st Edition (DBv1)
 - Comprehensive Reading:
 - Section 4.6 for the OPP details.
 - Skip Reading:
 - Section 4.6 on precedence functions.

Enjoy the Course!

