Inference in Bayes Nets

- What is $P(G|W)$? (i.e., the four probability values $P(g|w)$, $P(-g|w)$, $P(g|-w)$, and $P(-g|-w)$).

- Query variable is G.
- First run of VE, evidence is $W = w$.
- Second run of VE, evidence is $W = -w$.
- Use same ordering for both runs of VE: E, B, S, G.
- With same ordering some factors can be reused between the two runs of VE.

Inference in Bayes Nets

- What is $P(G|W)$? (i.e., the four probability values $P(g|w)$, $P(-g|w)$, $P(g|-w)$, and $P(-g|-w)$).
  1. E: $P(E)$, $P(S|E,B)$
  2. B: $P(B)$,
  3. S: $P(w|S)$, $\cancel{P(}\underline{S|}\cancel{G)}$
  4. G:

Inference in Bayes Nets

- What is $P(G|W)$? (i.e., the four probability values $P(g|w)$, $P(-g|w)$, $P(g|-w)$, and $P(-g|-w)$).
  1. E: $P(E)$, $P(S|E,B)$
  2. B: $P(B)$,
  3. S: $P(w|S)$, $\cancel{P(S|G)}$
  4. G:

$$
\begin{aligned}
F_1(S,B) &= \sum_E P(E) \times P(S|E,B) \\
&= P(e) \times P(S|e,B) + P(-e) \times P(S|-e,B)
\end{aligned}
$$

$$
\begin{aligned}
F_1(-s,-b) &= P(e)P(-s|e,-b) + P(-e)P(-s|-e,-b) \\
&= 0.1 \times 0.8 + 0.9 \times 1 = 0.98 \\
F_1(-s,b) &= P(e)P(-s|e,b) + P(-e)P(-s|-e,b) \\
&= 0.1 \times 0.1 + 0.9 \times 0.2 = 0.19 \\
F_1(s,-b) &= P(e)P(s|e,-b) + P(-e)P(s|-e,-b) \\
&= 0.1 \times 0.2 + 0.9 \times 0 = 0.02 \\
F_1(s,b) &= P(e)P(s|e,b) + P(-e)P(s|-e,b) \\
&= 0.1 \times 0.9 + 0.9 \times 0.8 = 0.81
\end{aligned}
$$

Inference in Bayes Nets

1. E: $P(E)$, $P(S|E,B)$
2. B: $P(B)$, $F_1(S,B)$
3. S: $P(w|S)$, $P\cancel{(}\underline{S|}\cancel{G})$
4. G:

$$
\begin{aligned}
F_2(S) &= \textstyle\sum_B P(B) \times F_1(S,B) \\
&= P(b)F_1(S,b) + P(-b)F_1(S,-b)
\end{aligned}
$$

$$
\begin{aligned}
F_2(-s) &= P(b)F_1(-s,b) + P(-b)F_1(-s,-b) \\
&= 0.1 \times 0.19 + 0.9 \times 0.98 = 0.901 \\
F_2(s) &= P(b)F_1(s,b) + P(-b)F_1(s,-b) \\
&= 0.1 \times 0.81 + 0.9 \times 0.02 = 0.099
\end{aligned}
$$

Inference in Bayes Nets

1. E: $P(E)$, $P(S|E,B)$
2. B: $P(B)$, $F_1(S,B)$
3. S: $P(w|S)$, $\cancel{P(\underline{S|}G)}$, $F_2(S)$
4. G:

$$
\begin{aligned}
F_3(G) &= \sum_S P(w|S) \times \cancel{P(\underline{S|}G)} \times F_2(S) \\
&= P(w|s)P(\cancel{\underline{s|}G})F_2(s) + P(w|-s)P(\cancel{\underline{-s|}G})F_2(-s)
\end{aligned}
$$

$$
\begin{aligned}
F_3(-g) &= P(w|s)P(s|-g)F_2(s) + P(w|-s)P(-s|-g)F_2(-s) \\
&= 0.8 \times 0.5 \times 0.099 + 0.2 \times 1 \times 0.901 = 0.2198 \\
F_3(g) &= P(w|s)P(s|g)F_2(s) + P(w|-s)P(-s|g)F_2(-s) \\
&= 0.8 \times 0.5 \times 0.099 + 0.2 \times 0 \times 0.901 = 0.0396
\end{aligned}
$$

Inference in Bayes Nets

1. E: $P(E)$, $P(S|E, B)$
2. B: $P(B)$, $F_1(S, B)$
3. S: $P(w|S)$, $\cancel{P(S|G)}$, $F_2(S)$
4. G: $F_3(G)$

Normalize $F_3(G)$:

$$P(-g|w) = \frac{0.2198}{0.2198+0.0396} = 0.8473$$

$$P(g|w) = \frac{0.0396}{0.2198+0.0396} = 0.1527$$

Inference in Bayes Nets

- Now $P(G| - w)$?
  1. E: $P(E)$, $P(S|E,B)$
  2. B: $P(B)$,
  3. S: $P(-w|S)$, $\cancel{P(}\underline{S}\cancel{|G)}$
  4. G:

Already computed as $F_1(S,B)$

Inference in Bayes Nets

1. E: $P(E)$, $P(S|E, B)$
2. B: $P(B)$, $F_1(S, B)$
3. S: $P(-w|S)$, $\cancel{P}(\underline{S}\cancel{|G})$
4. G:

Already computed as $F_2(S)$

Inference in Bayes Nets

1. E: $P(E)$, $P(S|E, B)$
2. B: $P(B)$, $F_1(S, B)$
3. S: $P(-w|S)$, $\cancel{P(\underline{S}|G)}$, $F_2(S)$
4. G:

$$
\begin{aligned}
F_3(G) &= \textstyle\sum_S P(-w|S) \times \cancel{P(\underline{S}|G)} \times F_2(S) \\
&= P(-w|s)P(\underline{s}|\cancel{G})F_2(s) + P(-w|-s)P(\underline{-s}|\cancel{G})F_2(-s)
\end{aligned}
$$

$$
\begin{aligned}
F_3(-g) &= P(-w|s)P(s|-g)F_2(s) + P(-w|-s)P(-s|-g)F_2(-s) \\
&= 0.2 \times 0.5 \times 0.099 + 0.8 \times 1 \times 0.901 = 0.7307 \\
F_3(g) &= P(-w|s)P(s|g)F_2(s) + P(-w|-s)P(-s|g)F_2(-s) \\
&= 0.2 \times 0.5 \times 0.099 + 0.8 \times 0 \times 0.901 = 0.0099
\end{aligned}
$$

Inference in Bayes Nets

1. E: $P(E)$, $P(S|E,B)$
2. B: $P(B)$, $F_1(S,B)$
3. S: $P(-w|S)$, $\cancel{P(\underline{S}|G)}$, $F_2(S)$
4. G: $F_3(G)$

Normalize $F_3(G)$:

$$P(-g|\cancel{w}) = \frac{0.7307}{0.7307+0.0099} = 0.9866$$

$$P(g|\cancel{w}) = \frac{0.0099}{0.7307+0.00099} = 0.0134$$

Inference in Bayes Nets

- What do these values tell us about the relationship between G and W, and why does this relationship differ when we know S?

Inference in Bayes Nets

- What do these values tell us about the relationship between G and W, and why does this relationship differ when we know S?

G and W are not independent of each other. But when S is known they become independent.