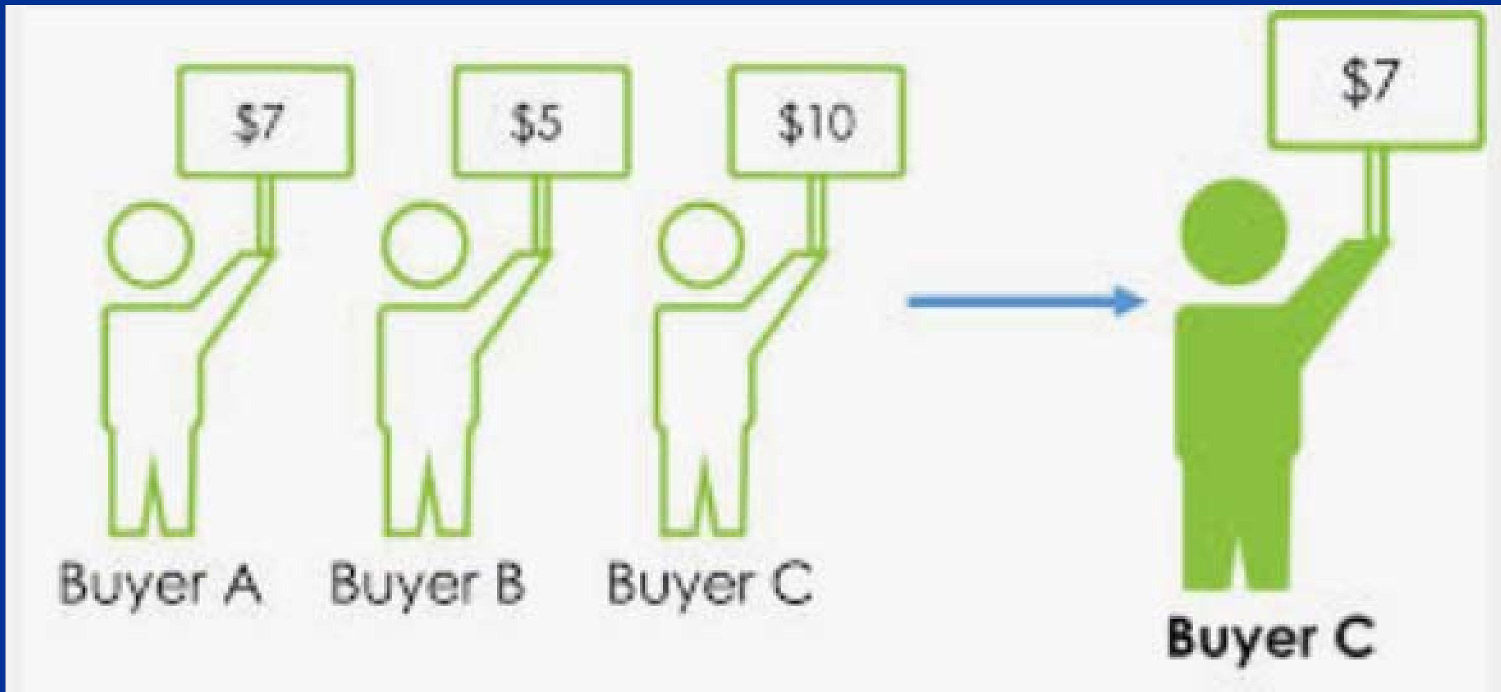
Second Price Auctions

The highest bidder wins but the price paid is the second-highest bid



Second Price Auctions

- *Players:* n bidders.
- *Types*: Each bidder i's type is her valuation is v_i.
- *Strategies*: bidder i's maximal bid is b_i.
- *Payoffs:* $u_i(b_1, \ldots, b_n, v) = \begin{cases} v_i - \underline{b}_i & \text{if } b_i > \underline{b}_i \\ 0 & \text{if } b_i \leq \underline{b}_i \end{cases}$

where, $\underline{b}_i = \max \{b_j : j \text{ different from } i\}$.

Weakly Dominant Solution

The strategy profile $(b^*_1, \ldots, b^*_n) = (v_1, \ldots, v_n)$ is the unique weakly dominant solution, and hence a Nash equilibirum.

	$\underline{b}_i < b_i$ or $\underline{b}_i = b_i$ & i wins	$b_i < \underline{b}_i < v_i$ or $\underline{b}_i = b_i$ & i loses	$\underline{b}_i > v_i$
$b_i < v_i$	$v_i - \underline{b}_i$	0	0
$b_i = v_i$	$v_i - \underline{b}_i$	$v_i - \underline{b}_i$	0

	$\underline{b}_i \leq v_i$	$v_i < \underline{b}_i < b_i$ or $\underline{b}_i = b_i$ & i wins	$\underline{b}_i > b_i$ or $\underline{b}_i = b_i$ & i loses
$b_i = v_i$	$v_i - \underline{b}_i$	0	0
$b_i > v_i$	$v_i - \underline{b}_i$	$v_i - \underline{b}_i$ (< 0)	0

In sum, bidding $b_i = v_i$ yields at least as high a payoff as bidding $b_i > v_i$ or $b_i < v_i$ for any opponents' bids.

Generalized Second Price Auctions were Adopted for Ads Bidding