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Week 3: Derivatives Introduction

Class Lecture Notes

Summary

Week 4: Derivatives Introduction

Class Lecture Notes

Summary

Adjusting for Dividends

- If the dividend yield ($y = \text{dividends} / \text{Current value of the asset}$) of the underlying asset is expected to remain unchanged during the life of the option, the Black-Scholes model can be modified to take dividends into account.

$$C = S e^{-yt} N(d_1) - K e^{-rt} N(d_2)$$

$$d_1 = \frac{\ln\left(\frac{S}{K}\right) + (r - y + \frac{\sigma^2}{2}) t}{\sigma \sqrt{t}}$$

$$d_2 = d_1 - \sigma \sqrt{t}$$

- The value of a put can also be derived:

$$P = K e^{-rt} (1 - N(d_2)) - S e^{-yt} (1 - N(d_1))$$

Figlewski, S., 1989, Options Arbitrage in Imperfect Markets, Journal of Finance, 64, 1289-1311.

Summary

Week 5: Risk Management

Class Lecture Notes

Summary

Abbassia, P. and Schmidt, M, 2018, A comprehensive view on risk reporting: Evidence from supervisory data, Journal of Financial Intermediation 36 (2018) 74–85.

Summary

- The fallout from the financial crisis raised concerns for scholars and regulators about the discretionary selection of risk models by banks to assess risk and capital requirement. New simple rules inhibit manipulation, increase transparency, but decrease efficiency. Recent studies identified banks use internal ratings boards (IRB) approaches to economize on capital by systematically reporting lower risk within a specified asset category e.g., credit risk in the banking book or market risk in the trading book. This analysis assesses different bank risk dimensions and ask whether banks report lower risk in one asset category to cross-subsidize the risks in another asset category, complimenting prior literature. In essence, banks could cross-subsidy between banking and trading books with freed capital to insulate their official capital adequacy ratio, less capitalized than implied by the ratios, and increase the fragility of the banking system. There are three major implications. Firstly, regulatory discretion manages short-term adverse market risk fluctuations. Secondly, risk is optimized, prioritizing aggregate level weighting opposed to asset-specific weighting. Lastly, supervisors should include a comprehensive view of the different bank risk dimensions.
- A unique, proprietary dataset (Deutsche Bundesbank), which collects supervisory information on internal credit risk rating for the loan portfolio of all IRB banks. Each records the IRB banks' estimates of creditor's one year probability of default and creditor-specific-risk-weighted asset at the borrower-bank time level for 2008: Q1 to 2012: Q4. This granularity enables the examination of differential PD reporting by banks across borrowers. Quarterly supervisory data on market risk-weighted assets for each IRB bank, in combination with comprehensive balance sheet information, enables the assessment of credit risk ratings relation to market risk exposures. After data cleansing, the dataset includes 17,339 distinct borrowers and 38 IRB banks providing more than 45% of all credit of the total German banking system, and total assets accounting for half of total assets of all banks in Germany.
- The hypothesis is that IRB banks report lower credit risk for their loan portfolio when they have higher market risk exposure compared to banks with lower market risk exposure is tested using the probability of default over the next year (1), borrower-specific credit risk weight that a bank assigns to a

- borrower in a quarter as implied by the banks PD reporting (2), and credit risk weighted asset banks report for borrowers in a quarter (in addition to the individual PD) as a fraction of the respective borrower-specific loan exposure (3).
- (1): $PD_{i,j,t} = \beta \times mRWA_{i,t-1} + \gamma' controls_{i,t-1} + \delta_{i,j} + \nu_{j,t} + \epsilon_{i,j,t}$
 - (2): $RW(PD - implied)_{i,j,t} = (RWA_{i,j,t}^{credit}/Loan_{i,j,t})^{fitted} = \beta \times mRWA_{i,t-1} + \gamma' controls_{i,t-1} + \delta_{i,j} + \nu_{j,t} + \epsilon_{i,j,t}$
 - (3): $(RWA_{i,j,t}^{credit}/Loan_{i,j,t})^{fitted} = \alpha + \beta \times RW(PD)_{i,j,t}^{credit} + \epsilon_{i,j,t}$
 - Suggestions on the existence of incentive spillovers across the two risk categories. On average, an IRB bank with one SD higher market risk weighted asset reports lower PD's by 0.03%, equivalent to a reduction of risk weights by about 3.57%. Therefore, economically significant. Conditioning on Tier One regulatory capital ratios, the effect is more pronounced in banks with more binding capital constraints. The result only holds for Advanced-IRB banks but not the foundational approach. Incentive spillovers across categories are weaker when the market discipline is higher and stronger for less transparent borrowers with respect to fundamental information. More stringent regulatory supervision interferes with the use of IRB model discretion for some banks but not for institutions with stricter capital constraints.
 - Contribute to the investigation linking risk reporting and bank capital under current IRB regulation, showing the use of risk reporting to manage risk across asset categories and the optimisation of risk weight at the risk-comprehensive level opposed to the specific-risk level. Strategic risk management can have severe consequences for the existence of an institution from micro prudential perspectives. The results inform regulatory arbitrage, misreporting incentives in financial markets, the importance of regulatory design, and the reliability/credibility of risk weights.

Week 6: Risk Management

Class Lecture Notes

Summary

Galindo, J. and Tamayo, P, 2000, Credit Risk Assessment Using Statistical and Machine Learning: Basic Methodology and Risk Modeling Applications, Computational Economics 15: 107–143, 2000.

Summary

- The purpose of the article was to present how machine learning can provide a better assessment of credit risk than traditional statistical models
- The paper studies 4000 mortgage loan records from a large financial institution of Mexico to assess the chance of default, aiming to find the best approach to minimising testing error for default
- The paper was one of the first papers to present that machine learning techniques can serve as a better approximation of credit risk than traditional statistical models. Although these results have data limitations the methodology, they can be applied to other data sets, used to assess a number of alternative credit risks, and be aggregated between a number of financial institutions to find aggregate credit risk.
- Credit risk: the lender will not get paid all principal and interest on time as scheduled
- Default risk (component of credit risk): risk that a borrower will not follow the agreed loan terms. This includes stopping payment, paying less or failing to maintain agreed financial covenants.
- Several recent financial crisis highlight the importance of credit risk
- Early warning systems and risk decomposition and aggregation are the predominant ways to measure a financial institutions credit risk
- Machine Learning is the science (and art) of programming computers so they can learn from data
- The representation of patterns, regularities, or trends in financial or business data from statistical/mathematical models persists in finance.
- Interdisciplinary techniques combining statistics and machine learning algorithms required to inform analysis on complex real-world data.
- Algorithm selection relies on the nature and characteristics of the dataset e.g., Intrinsic noise, complexity, relationship-type.
- Models: Probit Linear Regression, k-Nearest Neighbors, CART, Feed Forward Neural Network.
- Inductive principles guide the development of the most parsimonious models/descriptions which captures the structural or functional relationship in the data. Model construction and computational cost closely intertwine
- Computational and machine learning methods generalize parameter estimation underlying statistics using computational-based, and data-driven methods without relying on statistical assumptions.
- Basic model parameter exploration: Preliminary,
- Analysis of importance/sensitivity: Not in this instance given number of variables in dataset
- Train/test/generalization and evaluation error analysis: Performance/confusion matrix for both testing and training (Right)
- Data set divided into training (2000), testing (1000), and evaluation (1000) subsets. 22000 required
- Analysis of learning curves and estimation of noise and complexity parameters
- Model selection and combination of results
- Builds 30 models with different random samples from the original dataset for each dataset and model size, averages training and testing error rates, and fits errors to an inverse power law.
- Prediction of default in home mortgage loans. Data provided by Mexico's security exchange and banking commission: Comision Nacional Bancaria y de Valores. Possible set of 900,000 mortgage loans; 4000 mortgage loan from a single financial institution. Average loan amount is 266,827 pesos (\$33,300 U.S.) as of June 1996. 24 attributes/variables (Credit_Amount, Overdue_Bal, Debt, Guarantee, DGuarantee1/2, Soc_Interest, Residential etc.). Target: Default (Binary; 0,1), considered only if no payments made in the last two months. Creates Default_Index as condenses information about payment history and probability of payment
- Total errors: The probit (15.8%), CART (8.31%), Neural Net (15.4% and a 19.5% default), K-NN (17.7%)
- Analyses the performance of four different algorithms and 9000 models
- The optimal model in terms of the lowest average error rate was found to be the CART – decision tree model at a rate of 8.3%
- Introduce a modelling method based on studying error curves
- Provide a framework for how future machine learning models can be used for risk assessment of a credit portfolio

Week 7: Risk Management

Bartram, S.M. and G.W. Brown, 2009, International evidence on financial derivatives usage, Financial Management, Spring, 185-206.

Introduction & literature review

- Growing use of financial derivative contracts by non-financial corporations (1990 – 2020)

- Little consensus on derivative application or reasoning
- Lack of comprehensive data non-US, non-financial firms outside who are major users. Paper expands dataset, examining the use of financial derivatives by 7,292 companies (48 countries)
- Firstly, seeks to document FX, IR, and CP derivatives usage and compare characteristics of users across countries and firms. Secondly, use a large and diverse sample of international firms to increase the power of tests examining the motivations for derivative use. Thirdly, examine the use of derivatives at the country level and determine what country-specific factors, if any, are important for explaining cross-sectional variation. Lastly, determine if derivative use associated with higher firm value.
- Many non-US countries use derivatives. Across all countries, more than half of the sample firms (59.8%) use some type of financial derivative. More precisely, 43.6% of the firms use FX derivatives, 32.5% interest rate derivatives, and 10.0% use commodity price derivatives. We find that the type of derivative used varies across the different classes of financial risk. The use of non-linear derivatives less variable across types of risk. Commodity price hedgers do not appear to have a preferred type of contract with forwards, futures, swaps and options all used in about the same proportion by hedgers.
- Financial theory suggests that corporate risk management is apt to increase firm value in the presence of capital market imperfections e.g., bankruptcy costs, a convex tax schedule, or underinvestment problems. Risk management may arise from principal-agent conflicts or earnings management & speculation. Derivatives use appears consistent with some of the predictions generated by theories of shareholder value maximization. Strong evidence derivatives use is risk management rather than speculation. E.g., firms that use FX derivatives have higher proportions of foreign assets, sales, and income and firms that use interest rate derivatives have higher leverage. Tests indicate that derivatives users have significantly higher leverage, income tax credits, and lower liquidity (Quick/coverage ratios), aligning with financial distress hypotheses. More profitable/fewer growth opportunities firms hedge more, counterintuitive to theory
- Country-specific factors less importance than firm-specific factors. Size of the local derivatives market (Daily turnover of OTC FX/IR in FIG) relevant, suggesting supply side constraints important to determine derivative use.
- Evidence derivative use is likely associated with higher firm value. Results for U.S. firms using FX derivatives, are fairly weak in sample period (absent in non-US firms). IR risk management is associated with higher firm value for both U.S. and international firms.
- US HQ company studies not sufficient for understanding the risk management practices of non-financial firms.

[Bartram, S.M., Brown, G.W. and J. Conrad, 2011, The effects of derivatives on firm risk and value, Journal of Financial and Quantitative Analysis 46, No. 4, 967-999.](#)

Introduction & literature review

- Examine derivative use effects on firms' risk and market values using a new, larger dataset that including 6,888 non-financial firms (HQ in 47 different countries), covering a wide range of derivative use and risk measures.
- Investigate the impact of the use of FX, IR and CP derivatives on cashflow volatility, the standard deviation of stock returns, and market betas, as well as market values. Dataset allows the measuring of the derivative use effect on firms during a period including a sharp market correction (00-01 global recession), examining extent to which firms, either through their use of derivative contracts or other methods, can mitigate a market-wide decline.

Results & discussion

- Hedgers' returns are less volatile than the returns of non-hedgers and also have a lower sensitivity to market returns than non-hedgers. Apparent in 2000-2001 when sharp sell-offs in the market lead to substantially larger declines for non-hedgers as compared to hedgers. 1998-2003 period shows stocks of non-hedgers perform slightly better than those of hedgers.
- Univariate results suggest that firms use derivatives to reduce risk with greater exposure to FX risk (due to more foreign sales, foreign income and foreign assets) and IR risk (due to higher leverage and lower quick ratios) before considering the potential effects of hedging. Likely to belong to commodity-based industries that are exposed to CP risk.
- Derivatives users exhibit unconditional average cashflow volatility that is almost 50% lower than non-users and stock return volatility that is on average 18% lower than the return volatility of non-users.
- Firms that use derivatives have market betas that are on average 6% lower than non-users. Find derivative users tend to be larger and older firms, on average. The unadjusted Tobin's Q of the average derivative user is approximately 17% lower than the average firm that does not use derivatives.
- Propensity score (measure to control for endogeneity concerns) matching allows to match firms on their estimated likelihood of using derivatives, rather than a large number of individual firm characteristics.
- Using a binary variable to measure derivative use, we directly estimate firms' propensity to hedge based on their characteristics, and then match hedging and non-hedging firms based on this propensity. Controlling for firms' likelihood to hedge, we find that derivative use is associated with lower cashflow volatility, lower standard deviation of returns, lower systematic risk and weakly higher market values.
- Hedging firms have 10% to 25% lower cashflow volatility, 3% to 10% lower standard deviation of returns, 6% to 22% lower betas, and 1% to 7% higher Tobin's q, than matching non-hedging firms, depending on the set of characteristics used to estimate the propensity to hedge. Able to estimate the extent to which our inferences may be driven by a hidden selection bias: related to an unobserved characteristic to affect our inferences regarding the effect of derivative use on risk, it would have to be large e.g., 2 SD leverage or 2x Market Cap. The results with respect to value appear sensitive to hidden selection bias. This sensitivity could explain why value results from previous studies are mixed.
- Overall, results suggest effect of derivative use in the cross-section is associated with a decline in both total and systematic risk; the effect on value is positive, but weaker. Cannot rule out the possibility other omitted control variables may explain estimated differences in value.
- Examine the differences in risk and value measures for hedgers and non-hedgers through time. Hedgers: lower total risk and betas.

Conclusion

- Some evidence hedging was more important for firm value during the global economic decline in 2001, caused by a change in the perceived value of risk management, with the value of firms that hedged increasing during the economic decline. Alternatively, may reflect the unstable nature of the value results. Analysis of alphas find hedgers significantly outperform non-hedgers in 2001 and 2002. Profit measures of hedgers (revenue, CF, ROA) more stable (six year sample period). Results suggest firms do reduce cashflow risk, total risk and systematic risk through financial risk management with derivatives (robust when controlling for differences in a large number of firm characteristics, country and industry-specific differences). Findings may be reassuring for policymakers, regulators and stakeholders regarding their concerns about widespread derivatives speculation by nonfinancial corporations that put the firm and thus shareholder value at undue risks. The value put on this risk reduction in the marketplace, however, is much less certain.

Week 8: Real Options Valuation

[Titman, S, 1985, "Urban Land Prices under Uncertainty." American Economic Review 75\(3\), 505-514.](#)

Summary

- The prevalence of underutilized and vacant urban land global, held by private investors whom seek to maximize their wealth, calls for an assessment of land valuation techniques, to explain vacancy.

- Traditional valuation methods assess market value of the building minus the lot preparation and construction costs. However, uncertainties persist from building types and future real estate prices.
- Paper develops a valuation equation for pricing vacant lots, providing rationality to postpone building until a future date.
- Optimality to delay irreversible investment decisions considered in environmental economics and capital investments literature.¹ Basic intuition states it may be advantageous to wait for additional information before deciding upon exact investment specifications. Not explicitly shown how option affects model of other related assets.
- Adapts methods used by Fisher Black and Myron Scholes (1973) and Robert Merton (1973), to value options/derivatives, to determine explicit values for vacant urban land, close to binomial option pricing models of John Cox, Stephen Ross, and Mark Rubinstein (1979), and Richard Rendleman and Brit Bartter (1979). The intuition being that a vacant lot can be viewed as an option to purchase one of a number of different possible buildings at exercise prices that are equal to their respective construction costs.
- Demonstrates that the amount of uncertainty about the type of building that will be optimal in the future is an important determinant of the value of vacant land. If there is a lot of uncertainty about future real estate prices, then the option to select the type of building in the future is very valuable, with vacant land relatively more valuable and current development less attractive (vice versa).
 - Enables us to address issues pertaining to the effect of uncertainty on real estate markets e.g., policy implications on construction stimulation, height restrictions

Trigeorgis, L., 1993, "Real Options and Interactions with Financial Flexibility," *Financial Management*, 22(3), 202-224.

Summary

- NPV rule and other DCF approaches to capital budgeting inadequate as cannot properly capture management's flexibility to adapt and revise later decisions in response to unexpected market developments. Traditional NPV makes implicit assumptions concerning expected CF scenarios and passive commitments to operating strategies, that don't consider realistic uncertainties in CF and market conditions. Managers may flex operating strategies in response e.g., defer, expand, contract, abandon, alter projects at different stages of operating life. Adaptation flexibility increases upside opportunity, mitigates downside, with active management to increase value.
- Expanded NPV analysis enables active management, i.e., Expanded (strategic) NPV = static (passive) NPV of expected cash flows + value of options from active management. Options approach quantifies active management, manifested as real (call or put) options embedded in capital investment opportunities, having as an underlying asset the gross project value of expected operating cash flows. Naturally occurring (to defer, contract, shut down or abandon) or planned/built-in at some extra cost (e.g., to expand capacity or build growth options, to default when investment is staged sequentially, or to switch between alternative inputs or outputs).
- Paper provides an overview of the existing real options literature and applications, and presents practical principles for quantifying the value of various real options, and extends real options literature to recognize interactions with financial flexibility.
 - Assuming all-equity financing, presents principles useful for valuing both upside-potential operating options (Investment deference/production expansion), as well as various downside-protection options (abandon for salvage value/switch use (inputs/outputs)/abandon project construction by defaulting on planned, staged future outlays).

McDonald, R., 2000, "Real Options and Rules of Thumb in Capital Budgeting." In *Project Flexibility, Agency, and Competition*, edited by Michael Brennan and Lenos Trigeorgis, 13-33. Oxford, UK: Oxford University Press.

Summary

- Common for firms to use investment criteria which do not strictly implement the NPV criterion e.g., \$5m for \$1000 NPV. Anecdotally, firms making capital budgeting decisions routinely do:
 1. Projects are taken based on whether or not internal rates of return exceed arbitrarily high discount rates (hurdle rates), sometimes higher for projects with greater idiosyncratic risk.
 2. Project selection is sometimes governed by a "profitability index", i.e., $NPV/(Investment\ Cost)$ must be sufficiently great, otherwise untaken (firms engage in capital rationing).
- Summers (1987) found 94% of reporting firms discounted all cash flows at the same rate, independently of risk; 23% used discount rates in excess of 19%, suggesting hurdle-rate rules, at odds with capital budgeting prescriptions.
- Article explores unconventional capital budgeting practices may serve as proxies for economic considerations not properly accounted for by the NPV rule e.g., companies adopted explicit option valuation methods, as these options could be economically important.
- Research Question: Is it possible that firms can make investment decisions that are close to optimal by following simple rules of thumb? Considers if observed investment decision-making behaviour justified as an informal way to account for real options considerations, and in particular, investment timing. Benchmark case the investment timing model (McDonald and Siegel (1986)). Firm should delay investing in a project until the NPV of the project is sufficiently positive, with the specific investment hurdle determined by inputs such as the volatility of the project, and the cash flows foregone by deferring investment. Focus on investment timing flexibility, asking whether simple investment decision rules can approximate the optimal investment deferral implied by the investment-timing model, and robust responses to changing project characteristics e.g., correct hurdle rate. Prior literature shows how to computations of hurdle rates, and equivalence of a payback rule.
- Experiments fix the investment rule, vary project characteristics (discount rate, expected growth rate of cash flows), finding for a wide range of project characteristics, fixed hurdle-rate rules and profitability index rules can provide a good approximation to optimal investment timing decisions in the sense that the ex-ante loss from following the suboptimal rule is small; it is possible to follow the wrong investment rule without losing much of the ex-ante value of the investment timing option.
- The investment timing option worth more and it becomes optimal to wait longer to invest, the option value becomes less sensitive to errors in investment rules. Considering the effect of permitting project abandonment shows permitting non-trivial reversibility, e.g., scrap the project for 50% of the investment cost, does not significantly alter the conclusions. Other value/optimality affecting options include multi-stage investments, options to shut-down production, strategic options and switching options.
- Focus on cash-flow uncertainty, a valuable investment timing option can also be generated by interest rate uncertainty. Findings do not suggest rules of thumb should be universally adopted. Paper helps assess the relative value of knowing different characteristics of a project, helping managers allocate time in investment decision-making.
- Section 2 explains basic investment timing problem and procedure for evaluating investment rules of thumb, finding an as investment option becomes more valuable, it also becomes less sensitive to errors in investment rules. Section 3 explores different investment rules, examining associated losses with different rules under various parameter values in the basic investment timing model. Paper deploys two arbitrary rules: 20% hurdle-rate rule and 1.5 profitability index. Section 4 examines robustness of the results to different assumptions about the evolution of project value, e.g., negative growth rate, possibility of a jump to zero in project value, impact of adding a scrapping option.

- Generally, the rules of thumb considered generally capture at least 50% of a project's option value, and often as much as 90%.

Week 9: Real Options & Credit Risk

Merton, R. C., 1974, "On the Pricing of Corporate Debt: The Risk Structure of Interest Rates", *Journal of Finance*, 29(2), 449 – 470.

Summary

- Corporate debt issue value depends on three factors. Firstly, the required rate of return on riskless debt (e.g., government bonds or very high-grade corporate bonds. Secondly, the provisions and restrictions in the indenture e.g., maturity date, coupon rate, call terms, default seniority, sinking fund, etc. Lastly, the probability to unable to satisfy a subset or all indenture requirements.
- There has been no systematic development of a pricing bond theory for significant probability of default. This paper presents a theory on the risk structure of interest rates, with "risk" referred to as the restriction of possible gains or losses to bondholders because of unanticipated changes in the probability of default, excluding gains or losses inherent to all bonds from general changes in interest rates.
- A given term structure is assumed, with differences in the probability of default driving price differentials among bonds. Black and Scholes presented a complete general equilibrium theory of option pricing with a form using observable variables, subjectable direct empirical tests. Merton clarified and extended the Black-Scholes model, as the application of a similar approach develops pricing theory for corporate liabilities in general. The basic equation for the pricing of financial instruments is developed along Black-Scholes lines. The model is applied to the simplest form of corporate debt, a discount bond with no coupon payments, and presents a formula for computing the risk structure of interest rates. Comparative statics develop graphs of the risk structure, questioning term premium as an adequate measure of the risk of a bond. The paper proves validity in the presence of bankruptcy of the famous Modigliani theorem, and the required return on debt as a function of the debt-to-equity ratio is deduced. The paper extends the analysis to include coupon and callable bonds.
- In summary, they develop a method for pricing corporate liabilities, grounded in economic analysis, requires observable inputs; useful for pricing almost any type of financial instrument. The method was applied to risky discount bonds to deduce a risk structure of interest rates. The Modigliani-Miller theorem was shown to obtain in the presence of bankruptcy provided, no differential tax benefits to corporations or transactions costs. The analysis was extended to include callable, coupon bonds.

Schwartz, E.S. and M. Moon, 2000, Rational Pricing of Internet Companies, *Financial Analysts Journal*, 56(3), 62-75.

Summary

- Valuations created many millionaires and billionaires out of many Internet entrepreneurs while companies generated growing, losses. Trading capabilities improved with low costs. The consensus among traditional money managers leading to high valuations is upward, irrational bidding by individual day traders. Traditionalists fear negative consequences to the real economy after bubble bursts. There is a view the internet dramatically transforms the way in business is transacted. These investors believe some upstart Internet companies will rapidly grow to dominate and make bricks-and-mortar competitors obsolete.
- Application of real-options theory, and modern capital-budgeting techniques, informs valuing internet stocks. The paper formulates a model in continuous time, forms a discrete time approximation, estimates the model parameters, solves the model by simulation, and performs sensitivity analysis.
- The model to value internet stocks is based fundamentally on expected growth rate of revenues, and cost structure expectations of the company. The model generates company values and stock prices that are highly volatile from continuously changing assumption. Assumptions about future financing, future cash distributions to shareholders and bondholders, horizon of the estimation etc. inform implementation. Subject-matter experts on company required to make better assumptions.
- The paper concludes value of internet stocks may be rational, parameter and high-growth rate dependent. In addition, paper finds valuation has great sensitivity to initial conditions and exact specification of the parameters, a finding consistent across observations with returns of Internet stocks have been volatile.
- The paper examines the value of an exit option for Internet companies. A previous paper concludes firm value does increase in exit value after controlling for other variables. The abandonment option can be valuable, even if exit value deemed to be zero.
- The paper only uses data from one company, with judgment calls for the parameters (no data), where cross-sectional data from Internet companies improves parameter estimates and informs tests. Seasonality is not considered in revenue growth parameters.

Klobucnik, J. and S. Sievers, 2013, Valuing High Technology Growth Firms, *Journal of Business Economics*, 83(9), 947-984.

Summary

- The paper identifies social networks as one of the fastest growing industries, attracting investors' attention. Consistently high IPOs, eclipsing mature internet companies, persist regardless of losses. Schwartz & Moon (2000, 2001) develop and extend a theoretical model explicitly focusing on the value generating process in high technology growth stocks, based on expected growth rate of revenues and cost structure to derive valuations.
- The Schwartz-Moon model simulates a growing technology firm's possible paths of development using Monte Carlo techniques and short term historical accounting data. It calculates a fundamental firm value by averaging all discounted, risk-adjusted outcomes of the simulated enterprise values. The model provides investors not only with a value estimate but also with a long term probability of bankruptcy. It does not require market data, suited for unlisted assets. High technology firms experience losses and do not have analyst coverage. Must consider in DCF and P/E multiples.
- Keiber et al. (2002) operationalise the model using 46 German technology firms during the dotcom bubble. The issue arises whether the Schwartz-Moon model can fill this gap in the valuation literature, despite the difficulty that many of the model's input parameters need to be estimated ex-ante. Research questions are (1) How does an economic reasonable, but at the same time feasible implementation of the Schwartz-Moon model look like? (2) How accurate is the valuation from proposed model? (3) Is it possible to indicate market mis-valuation in the technology sector?
- The paper presents an easily applicable configuration of the Schwartz-Moon model, suitable for large scale valuations purposes on a sample of around 30,000 technology firm quarter observations from 1992 to 2009 using realized accounting data. The paper compares the fundamentals based Schwartz-Moon model to the Enterprise-Value Sales method, finding comparably performance accurate with regard to deviations from market values, with smaller deviations for firms in the chemicals and computer industries and for smaller companies. The Schwartz-Moon model shows the ability to indicate severe over or undervaluation in each quarter from 1992 until 2009, producing reasonable estimates for the probability of default.
- The paper demonstrates a trading strategy based on the Schwartz-Moon model has significant investment value, both before and after transaction costs, contributing to the literature on company valuation, with findings offering promising results on how to accurately value small firms.
- Analyst forecasts frequently are overoptimistic, contradicting the effort to detect mis-valuation. The paper introduce a sensible implementation, which is only based on major items from the income statement and the balance sheet and information about firms in the same industry, applicable to private firms without market prices. The paper draws on the benefits and costs of the Schwartz-Moon model. The benchmark for the model's

accuracy to market values, the Enterprise-Value/ Sales multiple (EV-Sales), should naturally yield smaller deviations as it captures the market sentiment, a materially factor in initial valuations. They hypothesize market valuations can be unjustified during bubble times, adding to the literature which indicates that the financial accounting data can serve as an anchor for rational pricing during these times. This is especially true for technology growth firms subjective valuations, strongly affected by investor sentiment. Previous literature shows sentiment plays an important role for hard to value stocks, demonstrating a sentiment peak during the dotcom bubble.

- They provide additional evidence a trading strategy, based on our model implementation of Schwartz-Moon, has economic and statistically significant investment value, both before and after transaction costs. Risk adjusted abnormal returns before transaction costs are as high as 1.5% per month.

Week 10: Equity as a Call Options & Implied Volatility

Galai, D. and R. Masulis. 1976, "The Option Pricing Model and the Risk Factor of Stock." Journal of Financial Economics, 3, 53- 81.

Key Notes

- Combines CAPM with OPM and application to deriving equity value and systematic risk
- Develop two models, present newly found properties of the options pricing model, and effect on security holders of a firm with less than perfect me first rules.
- **Unanticipated changes in asset structures can differentially affect debt and equity, theoretical and empirical implications such as investment policy, non-stationarity (cause/effect) in the systematic risk of levered equity and risky debt.**
- Paper seeks to understand corporate stock risk and how OPM contributes to understanding.
- **Differences in the variance of the rate of return of firms causes differences in firm equity/debt market values (Higher variance, higher S, Lower D)**
- **Differences in expected rate of returns. Expect lower rates of return on higher rate of return variances. Value of equity will be higher while expected rate of return is lower.**
- **Acquisition is trading riskless assets for risky asset, increases rate of return variance, increase stock, decrease value of debt, value of firm constant. Divestment is the opposite.**
- **Changes in Scale of a Firm:**
 - Both Galai and Merton hypothesize the corporation has two classes of claims. A single homogenous class of debt, functioning as the exercise price of a call options, with residual claim in equity as a call options. Galai et al., assess the changes in the scale of the a firm.
 - The systematic risk of an individual firm, externally finance. The value of equity and debt are first degree homogenous function of V & C with $S = SvV + ScC$, $D = DvV + DcC$.
 - Systematic Risk of equity and debt are zero homogenous functions of equity and debt.
 - The financing policy is devoid of redistribution effects, therefore the firm can issue debt until change in C/C = change in V/V, then raise the remaining debt, an equivalent to raising the firm's debt and equity proportionally with rise in the firm scale. If raised differently, there would be a watering down effect with one of other class of security. Therefore, supports pecking order theory.
- Conglomerate Mergers:
 - Merger of firms with no economic synergies decrease rate of return variation if not perfectly correlated. Therefore, value of equity of the combined firm decreases, while combined value of debt increases with the risk of ruin decreasing. This is because the equity holders of both firms protect the claims of both bondholders, weakening limited liability. However, equity holders protected by issuing new debt with the same levels of seniority and retire a portion the combined equity to increase debt capacity and D/E ratio.
- Spinoffs
 - Lead to equity holders stealing a proportion of bondholders collateral. Assume $V_0 = V_a + V_b$ (economic independence)
 - Leverage increased, the rate of return variance will change in line with changes in scale of the firm. The value of equity for the two separate companies combined will be larger than the combined firm firm.
- Corporate Investment Decisions
 - Assume no side payments, perfect 'first me rules' or transaction costs. Levered firm may not maximise value with imperfect 'me first rules' where equity holders control investment decisions. E.g., choice between two mutually exclusive, equally profitable projects, but one has a higher rate of return variance. Equity holders may reject a more profitable project in favour of one with higher returns. An externality facing security holders prevents the maximisation of firm value. Unexpected change in rate of return variance will increase value of equity and decrease value of bond, with a rise and fall in systematic risk borne by equity and bondholders respectively.
- Implications for empirical studies
 - Systematic risk and rate of return variance are no-stationary, therefore so are rate of return distributions of equity and debt.

Summary

- Presentation – Review Package

S. Mayhew, 1995, Implied Volatility, Financial Analysts Journal, 51(4), 8-20.

Summary

- Literature review summarising option-implied volatility
- Describes algorithms for calculating implied volatility, various weighting schemes used to derive a single volatility estimate from the prices of multiple options, summarizes evidence in the debate on whether to use historical data or implied volatility in forecasting, and reviews several other papers on the various uses of implied volatility, including market efficiency studies and event studies.
- Suggests implied volatility is being widely misused in practice, describes the schizophrenic notion of the volatility smile, and reviews various methods for calculating a risk-neutral density function consistent with option prices and the new generation of option pricing models (such as Rubenstein's implied binomial tree method) based on these implied volatilities.

C.J. Neely, 2005, Using Implied Volatility to Measure Uncertainty About Interest Rates, Federal Reserve Bank of St. Louis Review, 87(3), 407-25.

Key Notes

- Option prices infer the levels of uncertainty in future asset prices.
- This paper explains implied volatility, difference from market's expectation of volatility, and evaluate implied volatility from Eurodollar to predict released volatility. IV shows uncertainty about short-term interest rates fallen for 20 years, changes in volatility co-incident with major news, real economy, and monetary policy.
- Option prices depend on perceived volatility of an underlying asset so can quantify volatility of an asset price.

- Article explains pricing options, Bi-nomial options pricing and the Black-Scholes Model.
- The black-scholes formula is frequently inverted to find the implied volatility of log asset returns in terms of the observed quantities, where BS assumes IV is constant. If volatility evolves independently or underlying asset price and no priced risk, the expectation of the Black-Scholes price correctly prices the option, with variance the average variance until expiry.
- **Realised volatility is the square root of average variance.**
- Since the expectation is taking in regards to variance, not standard deviation, therefore cannot justify using the linearity of BS with variance. A series of second order taylor series expansions shows BS IV is approximately the expected value of variance until expiry.
- **Volatility Smile**
 - Constant in BS (does not vary with moneyness), where stochastic volatilities with jumps create smirking. IV computed from near the money options
 - BS is most sensitive to IV at-the-money
 - Near the money options are usually the most heavily traded, therefore, smaller pricing errors.
 - IV from at-the-money options provide best estimates for realised volatility.
- Constructs IV from options data on pg 412
- IV should be an unbiased estimator of RV and no other forecast should improve the forecast from IV. However, IV is a significantly biased predictor of RV. Tests for informational efficiency suffer from 'Data Snooping'
- **Price of Volatility Risk**
 - Two sources of risk for the options writer: Volatility and price of underlying asset over time, Therefore, would need to delta (position in underlying asset) and vega (position in another option) hedge. If hedge for the former, return is uncertain. If volatility fluxuation create systemic risk, investors must be compensated for them.
- Application of IV of options on ST interest rates may help understand economic forces which drive monetary policy.
- Paper considers settlement data on three-month euro-dollar futures and options contract to explore IV behaviours of short-term interest rates incorporating data splicing.
- Table 1: IV less volatile than RV, expected given the former predicts the latter. Mean of RV lower than IV, indicating volatility premium.
- Figure 2: Shows right skewness of IV and Changes in IV – Must be positive and explains the skewness. Large positive changes more common than negative.
- Figure 3: Three-month euro-dollar more variable than federal funds, the two closely track eachother. The IV on TMED likely tracks the FF.
- Measure the uncertainty with expectations of future interest rates from IV from euro-dollar option prices since option prices depend on the volatility of underlying assets.
- Risk neutral lead to the assumption all risk associated can be arbitrated away.
- Table 2
 - IV being an overly volatile predictor of RV, with the main reason for this bias stochastic volatility introducing risk to delta hedging. The flatness in the regression of figure 5 also provides evidence for the volatile nature.
- Decreasing inflation, as well as seasonality in contract expiration, may explain increasing certainty of short-term interest rates in figures 3 and 4.

Summary

- Option prices used to infer the level of uncertainty about future asset prices.
- Article explain implied volatility, how they can differ from the market's true expectation of uncertainty, estimates the implied volatility of three month euro dollar interest rates from 1985 to 2001, evaluating its ability to predict realized volatility.
- Implied volatility shows uncertainty about short-term interest rates falling for almost 20 years, as interest rates and inflation fell.
- Changes in implied volatility are usually coincident with major news about the stock market, the real economy, and monetary policy.
- Authors explain concept of IV, applied it to measure uncertainty about three-month euro dollar rates.
- IVs associated with three-month eurodollars can be interpreted to reflect uncertainty about the Federal Reserve's primary monetary policy instrument, the federal funds target rate. As with IV in most financial markets, the IV of the three-month euro dollar rate has been an overly volatile predictor of RV. IV on the three month euro dollar rates has been declining since 1985, as inflation and interest rates have fallen and the Fed has gained credibility with financial markets.
- Largest changes in IV were coincident with important economic events such as the stock market crash of 1987, fears of war in the Persian Gulf, and the Russian debt crisis.
- Most of the rest of the largest changes in IV have similarly been associated with important news about the real economy or the stock market or revisions to expected monetary policy.

Week 11: Implied Volatility

Bollen, N.P.B. and R.E. Whaley, 2004, Does Net Buying Pressure Affect the Shape of Implied Volatility Functions? Journal of Finance, 59(2), 711 – 753.

Key Notes

- Examines the relationship between net buying pressure and the shape of the implied volatility curve (index and stock option)
- Changes in IVF directly related to net buying pressure from public order flow. S&P500 options most strongly affected by buying pressure for index puts, changes in IV of stock options dominated by call option demand.
- Delta-neutral option-writing trading strategies generate abnormal returns, matching derivations of IVFs above realised historical return volatilities.
- IVF decreases monotonically across exercise prices but should be flat, constant under BS assumptions.
- Valuing options with stochastic volatility assumptions may explain the shape of the IVF e.g., Stochastic volatility generate downward sloping IVF if innovations to volatility and underlying asset returns are negatively correlated.
- Levels of implied volatility depend on public net demand for an option series e.g., index puts for S&P500 index options for portfolio insurance.
- Upward sloping supply curve, IV > RV, and market makers compensation for hedging costs/exposure to volatility risk making up the difference.
- Paper explores supply and demand in the options market, assesses relationship between net buying pressure, shape and movement of the IVF of S&P500 and options on 20 individual stocks.
- Net Buying Pressure is the difference in number of buyer and seller motivated contracts traded daily. Trades above and below midpoint of bid-ask spreads are buyer and seller-motivated trades, respectively. The difference computed series-by-series, multiplied by the absolute value of option delta for the expression of demand in stock/index equivalent units.
- Most trading in index and stock options involves puts and calls, respectively.
- Hypothetically, different demands for index and stock options imply that the shape of index IVF differs from typical stock IVF if net buying pressure plays an important role, calculating IVF across multiple moneyness categories.
- Index IVF more negatively correlated with net buying pressure than stock IVF.

- Assess time series relationship between net-buying pressure and shape of the IVF, finding changes in levels of option implied volatility positively correlated to time variation in demand for the option with transient changes.
- Simulates abnormal returns of delta-neutral trading strategy systematically selling options.
 - Vega-Neutral Hedge has options sold at 3pm (CST) ask price. ATM call options purchased at midpoint of bid-ask spread, hedging vega while rebalanced daily.
 - Delta - Units of S&P 500 manage delta risk through absolute delta units of underlying security (S&P500, sold in put circumstances, bought in call circumstances revised daily during trade life by changing number of units in underlying asset. **How determine the weights?**
 - Index - Positive abnormal returns selling options with lowest K most profitable.
 - Stock - AR smaller and symmetric across varying K.
 - Profitability consistent with deviations in IVF from RV.
- Empirical Cumulative Distribution of S&P500 vs Analytical CDF of Standard Normal
 - Mostly follow the same cumulative distribution of standardised returns except for in the left tail where S&P500 has evidence of leptokurtosis.
 - There is a translation of leptokurtosis (condition of a probability density function to have fatter tails) in the empirical distributions to a smiling of the implied volatility functions, not present in analytical CDF of a standard normal, an assumption under Black Scholes model.
 - The changes in implied volatility for S&P500 options are from net buying pressure for index puts, leading to this leptokurtosis, and explanation for differences in figure 3.
- In summary, Net Buying Pressure (NBP) plays significant role in shape of IVF (S&P500 index greatest as imbalance largest).

Summary

- Prior to the October 1987 market crash, the relation between the Black and Scholes (1973) implied volatility of S&P 500 index options and exercise price gave the appearance of a smile. Since, index implied volatility function (IVF) decreases monotonically across exercise prices. Under the assumptions of the Black-Scholes model, the IVF should be flat and constant through time. Most attempts to explain the shape of the IVF focus on relaxing the Black-Scholes assumption of constant volatility by allowing the local volatility rate of underlying security returns to evolve deterministically or stochastically through time.
- Prior literature explores multiple explanations of IVF smile with studying of option market participants' supply and demand for different option series in different option markets. IVF could be a series of market clearing option prices quoted in terms of Black-Scholes implied volatilities. Under dynamic replication, the supply curve for each option series is a horizontal line. The level of implied volatility will be higher or lower depending upon whether net public demand for a particular option series is to buy or to sell. In the S&P 500 index option market, for example, it is well known that institutional investors buy index puts as portfolio insurance. Unfortunately, there are no natural counter-parties to these trades, and market makers must step in to absorb the imbalance. With an upward sloping supply curve, implied volatility will exceed actual return volatility, with the difference being the market maker's compensation for hedging costs and/or exposure to volatility risk. If institutional demand tends to be focused in a particular option series the IVF will be downward sloping.
- Paper investigates the role of supply and demand in the options market, assessing the relation between net buying pressure and the movement and shape of the IVF of S&P 500 index options and options on 20 individual stocks.
- Define net buying pressure as the difference between the number of buyer-motivated contracts traded each day and the number of seller-motivated contracts traded. Trades executed at a price above (below) the prevailing bid/ask midpoint are categorized as buyer-motivated (seller-motivated).
- The difference is computed on a series-by-series basis, and is multiplied by the absolute value of the option's delta to express demand in stock/index equivalent units. The empirical test design is motivated by the different demands for index options and stock options.
- Document most trading in index options involves puts, whereas most trading in stock options involves calls. If net buying pressure plays an important role in the options market, the different demands for index options and stock options implies that the shape of the index IVF should differ from the shape of the typical stock IVF.
- Characterize the shape of the index and individual stock IVFs by calculating the implied volatilities of options in five moneyness categories. Consistent with the results of Bakshi, Kapadia, and Madan (2003), and Dennis and Mayhew (2002), find that the index IVF is significantly more negatively sloped than individual equity option IVFs. Bakshi, Kapadia, and Madan characterize the structure of the IVF by estimating the risk-neutral skewness of the return distributions of the index and of individual stocks implicit in option prices. They then explain the difference between the risk-neutral skewness of the index and that of individual stocks in the context of an asset-pricing model. In contrast, ascribe the difference between the index IVF and individual equity option IVFs to differential demands for index options vis-a-vis stock options.
- Empirical evidence supporting the net buying pressure hypothesis first assesses the time-series relation between net buying pressure and the shape of the IVF, finding changes in the level of an option's implied volatility are positively related to time variation in demand for the option, and that these changes are transitory. Second, simulate the abnormal returns of a delta-neutral trading strategy that systematically sells options. For index options, find significantly positive abnormal returns when selling options across the range of exercise prices, with the lowest exercise prices (e.g., out-of-the-money puts) being most profitable. In contrast, abnormal returns from selling stock options are smaller and symmetric across exercise prices. Interestingly, these patterns of profitability are consistent with the respective deviations of the IVFs from realized volatility and with known demands of investors for different options.
- Overall, results suggest that net buying pressure plays an important role in determining the shape of IVFs, particularly for options on the S&P 500 index where public order imbalances are greatest. The results support the hypothesis that the IVF reflects a series of supply and demand equilibria.

Driessen, J., P.J. Maenhout, and G. Vilkov, 2009, The Price of Correlation Risk: Evidence from Equity Options, Journal of Finance, 64(3), 1377 – 1406.

Key Notes

- Market-wide increases in correlations decrease diversification benefits to investors and increases market volatility.
- Paper analyses whether cross-sectional differences in market exposures to correlation risk account for cross sectional differences in expected returns.
 - Differential pricing of index and stock options contain price correlation risk information.
 - Individual options do not embed a negative variance risk premium
 - Index options are expensive, earn low returns, as offer hedge against correlation risk, and insure against diversification losses.
- Evidence on correlation risk provided in three separate ways.
 - General decomposition of index variance risk (individual variance and correlations), finding large negative index variance risk premium in index options (no evidence in stock options). However, findings only relevant in models where exposure to correlation shocks priced.
 - Derive simple options trading strategy to exploit price correlation risk (sell index straddle, buy individual straddle/stocks) to hedge individual variance risk and stock market risk, respectively. Higher sharpe ratio and excess return,. Correlation strategy gets gains for CRRA investor
 - Estimated correlation risk premium (CRP) from cross-section of index and individual option returns. Differences in exposure to correlation risk factor account for 70% of cross-sectional variation in CAPM residuals if both options, therefore CRP is high and significant, with no pricing of individual variance risk.
- Decomposition of index Risk Premium: Individual Variance Premium and Correlation Premium
 - Individual Variance Premium reflects the combination of individual risk premia and correlation risk premia.
$$E_t^Q[d\phi_i^2] - E_t^P[d\phi_i^2] = \sum_{i=1}^N \iota_i \{E_t^Q[d\phi_i^2] - E_t^P[d\phi_i^2]\} + \sum_{i=1}^N \sum_{j \neq i} w_i w_j \phi_i \phi_j \{E_t^Q[d\rho_{ij}] - E_t^P[d\rho_{ij}]\}. \quad (3)$$
 - The authors test for the variance risk premia in index options, cross-section of individual variance swap returns, and individual options on all constituent stocks, conducted using model-free implied variance to isolate individual risk premia.
 - Test for correlation risk (Right of + in equation) through analysis of a correlation risk trading strategy, using empirical analysis to explore whether a correlation risk factor, and common individual variance risk factor, can account for option returns cross-sectional variation.
- Correlation Trading Strategy

- Portfolio Weight is the portfolio weight optimised empirically for a CRRA investor, using derivatives-based trading strategies, obtained through the maximisation of in-sample expected utility.
- Cert. Equity is the monthly certainty equivalent, a percentage measure of the initial wealth demanded by a CRRA investor for the inability to invest in a particular derivatives strategy and must invest in the equity index and the risk-free asset.
- The table considers three strategies (short index straddle, short index put, correlation strategy) across several risk aversions where 1.8 hold the market (100% equity), showing different combinations of portfolio weights, with equity and risk free asset, showing the required compensation for investing in a derivatives based-strategy, instead of the market and risk free asset.
- All combinations of optimal portfolio weights, dependant of risk aversion, require compensation for derivatives strategies when assessing return-risk tradeoffs (most prominently for correlation risk strategies). Therefore, this highlights the value of using derivatives in generating returns in compensating for risk, incentivising the adoption of these strategies in trading.
- Summary
 - Correlation risk (CR) priced, ICAPM by Merton supports, or theory on mispricing of index options from investor irrationality and lacking arbitrage. Subsequently analyse transaction costs and margin requirements on profitability, with large impacts from transaction cost on profitability.

Summary

- Correlation play a central role in financial markets, changing over time and stock return correlations increase when returns are low. A marketwide increase in correlations negatively affects investor welfare by lowering diversification benefits and by increasing market volatility, so that states of nature with unusually high correlations may be expensive. Ask whether marketwide correlation risk is priced that assets that pay off well when marketwide correlations are higher than expected earn lower returns than can be justified by their exposure to other priced risk factors.
- This is the first paper to analyze whether cross-sectional differences in exposure to marketwide correlation risk can account for cross-sectional differences in expected returns. Our first contribution is to provide evidence of a large correlation risk premium.
- Show differential pricing of index and individual stock options contains unique information on the price of correlation risk with analysis of the cross-section of index and individual option returns, as well as the study of variance risk premia in index and individual options, highlights an important tension between index and individual option prices. Demonstrating this tension and offering a risk-based explanation for it forms the second contribution of this paper.
- Add formal evidence that individual options, unlike index options, do not embed a negative variance risk premium, nor earn economically significant returns in excess of a one-factor model. Analysis emphasizes that a challenge in option pricing concerns explaining the difference between expected index and individual option returns since the index process is the weighted average of the individual processes. A risk-based explanation for the contrast between index and individual options requires that aggregated individual processes be exposed to a risk factor that is lacking from the individual processes. Priced correlation risk makes this possible. Intuitively, index options are expensive and earn low returns, unlike individual options, because they offer a valuable hedge against correlation increases and insure against the risk of a loss in diversification benefits.
- Results offer a novel view on the source of the large volatility risk premium that recent work on index options has disclosed.
- Use data on S&P100 index options and on individual options on all the S&P100 index components, combined with prices of the underlying stocks from January 1996 until the end of December 2003, provide evidence for a correlation risk premium in three different ways. (General decomposition of index variance risk. Index variance changes are due to changes in individual variances and changes in correlations, so that index variance risk is priced to the extent that individual variance risk and correlation risk are priced. Find a large negative index variance risk premium, in line with results in the recent literature, estimate variance risk premia in all individual options on all S&P100 components and find no evidence of a negative risk premium on individual variance risk.
- Second, derive a simple option-based trading strategy aimed at exploiting priced correlation risk, selling index straddles and buys individual straddles and stocks in order to hedge individual variance risk and stock market risk, respectively. Correcting for standard risk factors, find a large excess return of more than 10% per month. Demonstrate strategy has more attractive risk-return properties than the option-based trading strategies that have been suggested in the literature. In a portfolio choice setting find that the correlation strategy generates a utility gain for a CRRA investor that is substantially larger than what can be obtained with existing option-based strategies (selling market variance or selling index puts).
- Finally, estimate the correlation risk premium from the cross-section of index and individual option returns. Because of the large dispersion in their sensitivities to marketwide correlation shocks, these assets constitute a particularly well-chosen cross-section. Show differences in exposure to the correlation risk factor account for 70% of the cross-sectional variation in CAPM residuals of index and individual option returns. The estimated correlation risk premium is large and highly significant. Exposure to individual variance risk is not priced in this cross-section, in line with our other results.
- Findings strongly suggest that correlation risk is priced. Merton's ICAPM (1973b) may provide a theoretical explanation for this finding to the extent that marketwide correlation levels have predictive power for market variance. As an alternative hypothesis, the large correlation risk premium we document may be interpreted as reflecting mispricing of index options due to investor irrationality and lack of arbitrage. To explore this limits-to-arbitrage hypothesis, analyze the impact on the profitability and feasibility of our correlation trading strategy of market frictions in the form of transaction costs and margin requirements. Show that transaction costs have an important impact on the profitability of the trading strategy. Its Sharpe ratio no longer exceeds the equity Sharpe ratio and the optimal portfolio weight for the correlation strategy becomes statistically insignificant. The impact of transaction costs on the correlation strategy is large because of the high bid-ask spreads for individual options. Margin requirements make the correlation trading strategy infeasible for risk-tolerant investors, who stand to gain most from the strategy. Thus, if the large correlation risk premium reflects mispricing of index options, rational investors facing realistic market frictions cannot arbitrage the mispricing away and cannot exploit the correlation risk premium. Very few papers have studied trading strategies based on individual options. A notable exception is Goyal and Saretto (2007), who analyze trading strategies using the cross-section of individual options and obtain very high Sharpe ratios. Their paper is complementary, since they study in detail the cross-sectional predictability of individual option returns, while this paper focuses on the difference between index and individual option returns.
- The negative correlation risk premium found implies higher expected correlation paths under the risk-neutral measure than under the actual measure. This divergence in expected correlations under the two measures can explain why option-implied correlations exceed average realized correlations.

Week 12: Link Between Options Markets & Equity Markets

Ofek, E., M. Richardson, and R.F. Whitelaw, 2004, Limited Arbitrage and Short Sales Restrictions: Evidence from the Options Markets, Journal of Financial Economics, 74(2), 305 – 342.

Key Notes

- Investigate put-call parity relation in presence of short-selling constraints, showing asymmetrical violations in P-C parity, directionally aligned to the constraints with magnitudes-related to cost and difficulty. Violations relate to option maturity and stock market valuations. Additionally, significant predictors of stock returns are put-call parity violations and short-selling costs.
- Arbitrage theory informs assets with the same payoff should be priced the same. However, violated limits to arbitrage to prevent price convergence, or there is a reason for the diverging prices.
- Paper provides comprehensive put-call parity analysis in context of short-sale restrictions, constructing matched pairs from two novel datasets with measures for shorting costs of each underlying stock via the rebate rate.
- Assymetric violations persist after incorporating shorting costs and extreme shorting costs e.g., 13.63% of stock still exceed implied UB by options market compared to 4.36% below the LB. Mean difference between the option-implied stock price and actual stock price for violations.
- Assume a one-to-one mapping of the rebate rate with shorting difficulties, find strong violations of arbitrage and short selling restrictions. Strong statistically significant evidence a decrease in rebate rate implies an increase in price deviations between options and stocks, suggesting relative prices of assets with identical payoffs can deviate substantially when arbitrage is not possible.
- Examine equity market segmentation and less rationality than options market. The framework allows an interpretable difference between stock value and option implied value by options pricing as mispricing in the equity market.
- Put-call parity variations are increasing are increasing in both maturity of the options and potential equity mispricing.
- **Rebate rate**

- The interest rate on the cash deposit equal to the proceeds of the shorted stock. The easier the shorting, the more closely the rebate rate reflects prevailing market rates. Tight supply leads to low rebate rates.
- The rebate rate spread is either the actual cost of borrowing the stock, or as a signal of the difficulty of shorting.

Summary

- No arbitrage is at the core of our beliefs about finance theory. Two assets with the same payoffs should have the same price. If this restriction is violated, then at least two conditions must be met. First, limits to arbitrage prevent the convergence of prices. Second, a reason for diverging prices.
- The paper analyzes the two conditions in a no-arbitrage framework through the case of redundant assets e.g. stocks and options on these stocks with put-call parity, a condition assuming investors can short the underlying securities. If short sales are not allowed, then this no-arbitrage relation may no longer hold. The condition does not necessarily fail without short sales. Suppose the stock is priced too high on a relative basis. Therefore, one could form a portfolio buying a call, writing an equivalent put, and owning a bond; the return on this portfolio would exceed the return on the stock in all possible circumstances, a difficult phenomenon to explain in rational equilibrium asset pricing models. There has been much less attention paid to understanding the direct links between short sales and the options market. Lamont and Thaler (2003) document severe violations of put-call parity for a small sample of three stocks that have gone through an equity carve-out, and the parent sells for less than its ownership stake in the carve-out, consistent with high costs to short these stocks.
- This paper provides a comprehensive analysis of put-call parity in the context of short sales restrictions, employing two novel databases constructing matched pairs of call and put options across the universe of equities, as well as a direct measure of the shorting costs of each of the underlying stocks, namely their rebate rate, the interest rate investors earn on the required cash deposit equal to the proceeds of the short sale.
- Firstly, consistent with the theory of limited arbitrage, find the violations of the put-call parity no-arbitrage restriction are directionally asymmetric of short sales restrictions, persisting after incorporating shorting costs and/or extreme assumptions about transactions costs (i.e., all options transactions take place at ask and bid prices). E.g., after shorting costs, 13.63% of stock prices still exceed the upper bound implied by the options market while only 4.36% are below the lower bound. Moreover, the mean difference between the option-implied stock price and the actual stock price for these violations is 2.71%.
- Second, under the assumption that the rebate rate maps one-to-one with the difficulty of shorting, finding a strong general relation between violations of no arbitrage and short sales restrictions. In particular, both the probability and magnitude of the violations linked directly to the magnitude of the rebate rate. In a regression context, a one standard deviation decrease in the rebate rate spread implies a 0.67% increase in the deviation between the prices in the stock and options markets, robust to the inclusion of additional variables to control for effects such as liquidity, in either the equity or options markets, stock and option characteristics, and transactions costs. The relative prices of similar assets can deviate from each other when arbitrage is not possible. If markets are sufficiently incomplete, and there is diversity across agents, then it may be the case that these securities offer benefits beyond their risk-return profiles. Alternatively, if markets are segmented such that the marginal investors across these markets are different, it is possible that prices can differ. Of course, in the absence of some friction that prevents trading in both markets, this segmentation will not be rational.
- Third, examining a simple framework in which the stock and options markets are segmented and the equity markets are “less rational” than the options markets, allowing interpretations on the difference between a stock’s market value and its value implied by the options market as mispricing in the equity market. It also generates predictions about the relation between put-call parity violations, short sales constraints, maturity, valuation levels, and future stock returns. Consistent with the theory, find that put-call parity violations are increasing in both the maturity of the options and the potential level of mispricing of the stocks. Also evaluate the model’s ability to forecast future movements in stock returns. Filtering on rebate rate spreads and put-call parity violations yields average returns on the stock over the life of the option that are as low as 12.6%. In addition, cumulative abnormal returns, net of borrowing costs, on portfolios that are long the industry and short stocks chosen using similar filters are as high as 65% over our sample period.

J. Pan and A.M. Potoshman, 2006, *The Information in Option Volume for Future Stock Prices*,
Review of Financial Studies, 19(3), 871 – 908.

Key Notes

- Presents evidence option trading contains information about future stock price movements through the construction of put-call ratios from option volume buyers from new positions.
- On risk adjusted basis, stocks with low put-call ratios outperform stocks with high put-call ratios by more than 40 basis points on the next day and more than 1% over the next week.
- Partition into public and non-public sets, with non-public being the source of predictability opposed to market inefficiency. Greater predictability exists from option signals with higher concentrations of informed traders and or greater leverage in options contracts.
- Most predictable are option prices constructed from DOTM option (highly levered), with low leverage providing very few levels predictability.

Summary

- This paper examines the informational content of option trading for future movements in underlying stock prices, addressing the fundamental economic question of how information gets incorporated into asset prices. Goals are to establish the presence of informed trading in the option market. The capital markets have experienced an impressive proliferation of derivative securities, ranging from equity options to fixed-income derivatives to, more recently, credit derivatives. The view that informed investors might choose to trade derivatives because of the higher leverage offered by such instruments has long been entertained by academics and can often be found in the popular press. A formal treatment of this issue is provided by Easley, O’Hara, and Srinivas (1998), who allow the participation of informed traders in the option market to be decided endogenously in an equilibrium framework. In their model, informed investors choose to trade in both the option and the stock market – in a “pooling equilibrium” – when the leverage implicit in options is large, when the liquidity in the stock market is low, or when the overall fraction of informed traders is high.
- The main empirical result directly tests whether the stock and option market are in the pooling equilibrium of Easley, O’Hara, and Srinivas (1998). Using option trades that are initiated by buyers to open new positions, form put-call ratios to examine the predictability of option trading for future stock price movements, finding predictability that is strong in both magnitude and statistical significance. Stocks with positive option signals (i.e., those with lowest quintile put-call ratios) outperform those with negative option signals (i.e., those with highest quintile put-call ratios) by over 40 basis points per day and 1 percent per week on a risk-adjusted basis in the 1990 to 2001 sample. When the stock returns are tracked for several weeks, the level of predictability gradually dies out, indicating that the information contained in the option volume eventually gets incorporated into the underlying stock prices. Main empirical result clearly documents that there is informed trading in the option market, it does not necessarily imply that there is any market inefficiency, because the option volume used in the main test – which is initiated by buyers to open new positions – is not publicly observable. Indeed, information-based models imply that prices adjust at once to the public information contained in the trading process but may adjust slowly to the private information possessed by informed traders. The predictability captured in the main test may well correspond to the process of stock prices gradually adjusting to the private component of information in option trading.
- Motivated by the differing theoretical predictions about the speed at which prices adjust to public versus private information, explore the predictability of publicly versus nonpublicly observable option volume. Following previous empirical studies in this area, we use the Lee and Ready (1991) algorithm to back out buyer-initiated put and call option volume from publicly observable trade and quote records from the Chicago Board Options Exchange (CBOE), finding the resulting publicly observable option signals are able to predict stock returns for only the next 1 or 2 trade days.
- Moreover, the stock prices subsequently reverse which raises the question of whether the predictability from the public signal is a manifestation of price pressure rather than informed trading. In a bivariate analysis which includes both the public and non-public signals, the non-public signal has the same pattern of information-based predictability as when it is used alone, but there is no predictability at all from the public signal. This set of findings underscores the distinction between public/nonpublic signals and respective roles in price discovery. Further, the weak predictability exhibited by the public signal suggests that the economic source of the main result is valuable private information in the option volume rather than an inefficiency across the stock and option market.
- Central to all information-based models are the roles of informed and uninformed traders. In particular, the concentration of informed traders is a key variable in such models with important implications for the informativeness of trading volume. Using the PIN variable proposed by Easley, Kiefer, and O’Hara (1997) and Easley, Hvidkjaer, and O’Hara (2002) as a measure of the prevalence of informed traders, they investigate how the predictability from option volume varies across underlying stocks with different concentrations of informed traders, finding a higher level of predictability from the option signals of stocks with a higher

- prevalence of informed traders. While the theoretical models define informed and uninformed traders strictly in terms of information sets, speculate outside of the models about who the informed and uninformed traders might be.
- Dataset is unique in that in addition to recording whether the initiator of volume is a buyer or a seller opening or closing a position, it also identifies the investor class of the initiator, finding that option signals from investors who trade through full service brokerage houses provide much stronger predictability than the signals from those who trade through discount brokerage houses. Given that the option volume from full service brokerages includes that from hedge funds, this result is hardly surprising. The option signals from firm proprietary traders contain no information at all about future stock price movements. In the framework of the information-based models, this result suggests that firm proprietary traders are uninformed investors who come to the option market primarily for hedging purposes. Finally, a unique feature of the multitariff stock and option setting is the availability of securities with differing leverage. Black (1975) asserts that leverage is the key variable which determines whether informed investors choose to trade in the option market, and Easley, O'Hara, and Srinivas (1998) demonstrate under a natural set of assumptions this is indeed the case.
 - Motivated by these considerations, investigate how the predictability documented in main test varies across option contracts with differing degrees of leverage, finding that option signals constructed from deep out-of-the-money options, which are highly leveraged contracts, exhibit the greatest level of predictability, while the signals from contracts with low leverage provide very little, if any, predictability.

Exam Practise

2020

Question 1

- Replicating portfolio to value a put option is a short position in the stock plus the purchase of the bond. Therefore, use the following equations to derive the alpha (h) and Beta (V) $S = 15$, $S_u = 18$, $S_d = 12$, $r_f = 10\%$, $K = 21$
 - $P = \text{Max}(K-S, 0)$
 1. $P_u = \text{Max}(21-18, 0) = 3$
 2. $P_d = \text{Max}(21-12, 0) = 9$
 - $P_u = hS_u + V^*(1.10)$, $P_d = hS_d + V(1.10)$, $h = (P_d - P_u)/(S_d - S_u)$
 - $h = (9-3)/(12-18) = -1$
 - $P_u = -S_u + 1.1V$
 - $3 = -18 + 1.1V$
 - $V = 21/1.1 = 19.0909091$
 - $P = -15 + 19.0909091$
 - $P = 4.0909091$
 - The Arrow-Debreu security price must exist for a complete market with no arbitrage. This implies a unique risk probability measure exists. There are prices stated for both up and down states, therefore a complete market persists.
- Table III shows reports the standard deviation of excess returns across 250 price series, varying strike prices (in/at/out/deep-out of the money), incorrect volatilities from 15% (hedging ratios) and indivisibilities (rounding to nearest integer to correct hedge ratio) when compared to the base case.
 - Little variation with incorrect volatility with in/at the money call options + short underlying stock, but increasing variation in SD of ER in out and deep-out of the money, however greater variation at smaller volatility (counterintuitive to pricing models). Underestimating volatility has greater consequences, therefore best to overestimate volatility in out/deep out of the money options.
 - SD of ER increases as indivisibilities increases for all strike prices, but extreme for $K = 0, 1$ at out/deep-out of the money. Informs should hedge in the money ($K = 0, 1$) and leave unhedged out of the money.
 - In summary, table III informs trading strategies to take advantage of market imperfections when forming portfolios with call options, shorting the underlying assets.

Question 2

- Firm may use derivatives for the following reasons from Bartram et al (2009, 2011):
 - Less volatile returns than non-hedgers with lower sensitivity to market returns than non-hedgers.
 - Univariate results suggest that firms use derivatives to reduce risk with greater exposure to FX risk from higher foreign sales, foreign income and foreign assets and interest rate risk from higher leverage and lower quick ratios before considering the potential effects of hedging.
 - Average cashflow volatility is almost 50% lower than non-users. Stock return volatility that is on average 18% lower than the return volatility of non-users.
 - Use derivatives to lower market betas (6% lower) than non-users. Derivative users tend to be larger and older firms, on average. The unadjusted Tobin's Q of the average derivative user is approximately 17% lower than the average firm that does not use derivatives, aligning market value closer to intrinsic value, providing more accurate valuations when assessing market opportunities.
 - Overall, results suggest effect of derivative use in the cross-section is associated with a decline in both total and systematic risk. Therefore, these reasons highlight why firms would use derivatives.
- Derivative use may increase firm value for the following reasons
 - Increase firm value in the presence of capital market imperfections e.g., bankruptcy costs, a convex tax schedule, or underinvestment problems. Principal-agent conflicts may arise or earnings management & speculation. Derivatives use appears consistent with some of the predictions generated by theories of shareholder value maximization.
 - Strong evidence derivatives use is risk management rather than speculation. Firms with FX derivatives have higher proportions of foreign assets, sales, and income and firms that use interest rate derivatives have higher leverage. Derivatives users have significantly higher leverage, income tax credits, and lower liquidity (Quick/coverage ratios), aligning with the financial distress hypotheses.
 - Firms do reduce cashflow risk, total risk and systematic risk through financial risk management with derivatives, reassuring for policymakers, regulators and stakeholders regarding concerns about derivatives speculation by non-financial corporations that put the firm and thus shareholder value at undue risks.

Question 3

- The authors look at both Debt Maturity and operational FX Hedging, proxied by the difference in percentage of sales that are foreign and the percent of foreign assets as the FX exposure. Carrying debt maturities would vary cash flows and interest rate risks associated with bonds. They also look at endogenous factors proxying for dividend exposure dividends and liquid assets (Quick ratio) as alternatives to derivatives, as can adjust both according to balance risk. These methods would be considered in conjunction with derivatives strategies.
- They test this relationship using a generalised methods of moments (firm and country-level instruments) calculation on the above endogenous variables. The results of the table are intuitive (derivative use has a positive effect of leverage as upward sloping yield curve, firms trade off higher interest payments, debt maturity negative as lower interest, negative of quick ratio implies firms able to hold lower liquid assets). The results are all statistically significant, showing firms able to undertake stronger value-enhancing activities, validating the simultaneous assumption requirements.
- Test derivative use is more valuable in market downturns. Two portfolios are formed, long derivative users combined with shorted matched non-derivatives users, in domestic upturn and downturn respectively. The only statistical significant findings are in the downturn portfolio, where the

market beta is -0.074 with <0.001 pvalue, an annualised alpha of 0.055 (5.5%) (excess return). However, both values for the upturn portfolio are statistical insignificant. We can confirm derivatives are good in valuable in a downturn but cannot confirm if more valuable compared to an upturn.

Question 4

- h. Traditional NPV: $V_0 - I_0$. $I_0 = 103$, $u = 80\%$, $d = 40\%$, $q = 50\%$, $R_w = 20\%$, $r_f = 9\%$
 - i. $V_0 - I_0 = ((103 \times 1.80) \times 0.5 + 103 \times (0.6) \times 0.5) / 1.09 - 103 = 18.8990826$
- i. $E^+ = \max(V^+ - I_1, 0)$, $E^- = \max(V^- - I_1, 0)$, $E_0 = (qE^+ - (1-q)E^-) / (1+r_f)$
 - i. $I_1 = 103 \times 1.09 = 112.27$
 - ii. $V^+ = 103 \times 1.80 = 185.4$
 - iii. $V^- = 103 \times (0.60) = 61.8$
 - iv. $E^+ = \max(V^+ - I_1, 0) = 185.4 - 112.27 = 73.13$
 - v. $E^- = \max(V^- - I_1, 0) = (61.8 - 112.27, 0) = 0$
 - vi. $E_0 = (qE^+ - (1-q)E^-) / (1+r_f) = (0.5 \times 73.13 + 0.5 \times 0) / 1.09 = 33.5458716$
 - vii. **Option to defer = 33.5458716 - 18.8990826 = 14.646789**
 - viii. **Value of Trump = Static NPV + Option to Defer = 18.8990826 + 14.646789 = 33.5458716**

Question 5

- j. Rate Sensitive Assets
 - i. $RSA = 5 + 10 + 15 + 10 \times 0.20 = \$32m$
- k. Rate Sensitive Liabilities
 - i. $RSL = 5 + 10 + 15 + 5 + 10 + 15 \times 0.1 + 10 \times 0.2 = \$49.5m$
- l. Income Change with 4% Increase
 - i. Change in Income = $4\% \times RSA - 4\% \times RSL = 4\% \times 32 - 4\% \times 49.5 = -0.7s$
- m. Net Income when $RSL > RSA$
 - i. Net Income would decrease as the interest rate payments would exceed interest rate income.

Question 6

- n. Find derivatives of a dividend-paying underlying asset, $F = Se^{(r-\delta)t}$ Note: $r_f = r$
 - i. $df/dt = S(r-\delta)e^{(r-\delta)t}$
 - ii. $df/ds = e^{(r-\delta)t}$
 - iii. $d^2f/ds^2 = 0$
- o. Modify the PDE with the above derivatives
 - i. $df/dt + (1/2)(d^2f/ds^2)(\sigma^2)(s^2) + r(df/ds)s - rf = 0$
 - ii. $S(r-\delta)e^{(r-\delta)t} + (1/2)(0)(\sigma^2)(s^2) + r(e^{(r-\delta)t})s - r = 0$
 - iii. $S(r-\delta)e^{(r-\delta)t} + r(e^{(r-\delta)t})s - r = 0$
 - iv. $e^{(r-\delta)t}x(sr-s\delta+r) - r = 0$

Question 7

- a. Model (Help with this one)
 - a. Galan et al, combine the CAPM and Option pricing model to value the derivation of equity value and systematic risk.
 - b. Model hypothesises $d\text{Beta}(s)/d\sigma^2 < 0$,
 - c. Therefore, increase in volatility leads to a decrease in systematic risk, leading to a reduction of expected return via CAPM in the same period (Month t)
 - d. Use volatility (t) to predict next month's returns, therefore partially explaining the phenomena without the other factors mentioned in question.
 - e. **Galai et al., combine CAPM with OPM. Show systematic risk of equity is a decreasing function of a firms total variation of return. Firms with higher volatilities will have lower expected returns as the expected return is a positive function of the firms systematic risk using CAPM.**
- b. M&A
 - a. The model assumes an acquisition is an exchange of riskless assets for risky, physical assets.
 - b. Hypothesize different variance of rate of return lead to different capital structure in market value terms.
 - c. The Debt/Equity ratio will be greater with the firm with lower variance and vice versa.
 - d. Hypothesize $d\text{Beta}(s)/d\sigma^2 < 0$,
 - e. Other case assumptions hypothesise, value of firm's equity will be higher while expected return will be lower as a rate of return variance dependent on the former. The opposite is true.
 - f. Therefore, an exchange of riskless for risky assets by the acquirer increases return variance, increases expectation for rate of return, decreases value of firm's equity, decreasing the stock price of the firm.
- c. Winners vs Losers
 - v. Draws on above analysis. Lower returns would lead to lower expectation of returns, decreasing the value of equity and therefore the stock price. However, the asset may deem to be mispriced/undervalued due to all the perfect market assumptions. Therefore, attract investor attention leading to upward prices pressure, demand for the stock, therefore leading to outperformance of winners as the opposite true for past winners with overvaluation.

Question 8

- a. The Price of Correlation Risk
 - vi. Portfolio Weight is the portfolio weight optimised empirically for a CRRA investor, using derivatives-based trading strategies, obtained through the maximisation of in-sample expected utility.
 - vii. Cert. Equity is the monthly certainty equivalent, a percentage measure of the initial wealth demanded by a CRRA investor for the inability to invest in a particular derivatives strategy and must invest in the equity index and the risk-free asset.
 - viii. The table considers three strategies (short index straddle, short index put, correlation strategy) across several risk aversions where 1.8 hold the market (100% equity), showing different combinations of portfolio weights, with equity and risk free asset, showing the required compensation for investing in a derivatives based-strategy, instead of the market and risk free asset.
 - ix. All combinations of optimal portfolio weights, dependant of risk aversion, require compensation for derivatives strategies when assessing return-risk trade-offs (most prominently for correlation risk strategies). Therefore, this highlights the value of using derivatives in generating returns in compensating for risk, incentivising the adoption of these strategies in trading.
- p. Exploiting return predictability (Help with this question!)
 - i. Hypothetically, non-public option/informed traders can capitalise on information, shown through the partitioning our option signals into components that are publicly and non-publicly observable. The paper finds predictability is non-public information possessed by option traders rather than market inefficiencies.
 - ii. Public observable Option Volume
 - 1. The slope co-efficient of adjusted stock returns is significantly negative until the 95% Fama-Macbeth range is positive until 16th – 19th trade day relative to option observation for open-buy Put-Call ratio. However, bivariate shows very little predictability (negative from price pressure), accountable for predictability.
 - iii. Public & Non-public Option Volumes

1. Several tests in the bivariate analysis, after orthogonalization, private information is slow to adopt return information, therefore exploit opportunities taking advantage or private information, as linked to predictability, before opportunities close.
- iv. In light of these results a trading strategy that could be used would be buying stocks with 0 put-call ratios and shorting stocks with high put-call ratios. If you have only public information then this position would have to held only for a day, while with private information you would hold this position for around 18 days. If you had access to both public and private then you would just use the same strategy as with private only as the public information has no incremental value.

2019

Question 1

- a. Option Pricing with Market Imperfections
 - a. Arbitrage costs with transaction costs: shows the bounds on option prices and implied volatilities suitable for an arbitrageur to bid and offer options, with 50/75% of recovering costs with base case and no rebalance assumptions, for in/at/out/deep-out of the money options.
 - b. The spread differentiate from theory (breakdown) with no rebalancing with in, out, and deep-out of the money options.
 - c. The table summarises arbitrageur would focus on at the money spread, but no reflective of market observations as standard arbitrage and 50%/75% recovering costs not the only objectives for traders/investors. A rational market maker would look at other objectives, met profit requirements/margins, or engage in several other strategies.
 - d. Table again highlights how market imperfections affect option pricing, informing trading strategies.
- b. Derivatives, market-book, leverage
 - a. Derivative usage related to leverage, as higher leverage leads to different levels of debt, with higher levels of debt related to higher interest rate payments, leading to requirements for interest rate derivatives. This follows the financial distress and tax hypotheses.
 - v. Derivatives related to market-book ratios, with hedger tending to have lower market-to-book ratios, as tend to be older, more mature firms. Derivative use maybe used with foreign assets, currency fluxuations, related to market value when valuing offshore assets. Use derivatives to lower market betas (6% lower) than non-users. Derivative users tend to be larger and older firms, on average. The unadjusted Tobin's Q of the average derivative user is approximately 17% lower than the average firm that does not use derivatives, aligning market value closer to intrinsic value, providing more accurate valuations when assessing market opportunities. Therefore, expect market-to-book ratios related as alternative measure for tobin's Q.
 - vi. Use the interaction to capture the overall relationship as market to book can be positively related to FX derivative, leading to higher market-to-book ratios. Leverage and market value also related from studies on capital structure informing valuation. Therefore, use the interaction variations to remove endogenous variables and capture overall impact of derivative use without cancelling out market to book ratio impact.

Question 2

- a. Risk neutrality is a measure where each security price is exactly equivalent to the discounted expectation.
- b. $Io = 4.8m$, $CoC = 10\%$, $rf = 4\%$, Risk adverse $p = 50\%$, $I+ = 6m$, $I- = 4m$, Salvage = $5m$
 - a. Firstly, work out the Vo using the CoC and risk adverse p
 - i. $Vo = (0.5x6 + 0.5x4)/(1.10) - 4.8 = -0.2545455$ (Rejected under traditional NPV Analysis)
 - b. Calculate the risk neutral probability
 - i. $p = ((1+r)/Io - I-)/(I+ - I-) = ((1.04)4.8 - 4)/(6 - 4) = 50\%$
 - c. Calculate $E+ = \text{Max}(V+, A+) = \text{Max}(6, 5) = 6$, $E- = \text{Max}(V-, A-) = \text{Max}(4, 5) = 5$
 - d. Calculate $Eo = (pE+ + (1-p)E-)/(1+rf) = (0.5x6 + (1-0.5)x5)/(1+0.04) = 5.28461538$
 - e. Real option = $Eo - Vo - Io = 5.28461538 - (-0.2545455) - 4.8 = 0.73916084$

Question 3

- a. Propensity Score Matching (PSM)
 - a. Used to match user of derivatives with non-users of derivatives.
 - b. The PSM is a measure of a firms propensity to use derivatives based on unique characteristics (Z-Altman Score, leverage, quick ratio for liquidity, some dummy variables for foreign debt, Gross-FX-exposure, country vs industry dummy variables etc.)
 - c. Firm size, leverage, multiple share class, stock options dummy, exchange rate exposure, foreign debt dummy positively related to derivative use while z-Altman and quick ratio (liquidity) negative, with results being positive.
 - d. Matching analysis used a single nearest neighbour approach or weighted average to construct with/without replacement.
 - e. Account for selection bias and examine results from all six methods, reporting only matching with replacement as greatly limits bias.
 - f. Find low values for cash flow volatility, standard deviation of returns, and beta risk for firms who use derivatives, providing significant differentials between user and non-user.
 - g. These three variables affect the risk, and the cost of capital, and therefore the value of the firm.
 - h. Use of derivatives lowers cost of capital, leading to higher valuations.
- b. Market Downturns
 - a. Test the hypothesis derivative use is more valuable in market downturns. Two portfolios are formed, long derivative users combined with shorted matched non-derivatives users, in domestic upturn and downturn respectively. The only statistical significant findings are in the downturn portfolio, where the market beta is -0.074 with <0.001 pvalue, an annualised alpha of 0.055 (5.5%) (excess return). However, both values for the upturn portfolio are statistical insignificant. We can confirm derivatives are good in valuable in a downturn but cannot confirm if more valuable compared to an upturn.

Question 4

- a. Rate Sensitive Assets
 - vii. $RSA = 5 + 10 + 15 + 10 * 0.20 = \$32m$
- b. Rate Sensitive Liabilities
 - viii. $RSL = 5 + 10 + 15 + 5 + 10 + 15 * 0.1 + 10 * 0.2 = \$49.5m$
- c. Income Change with 4% Increase
 - ix. $\text{Change in Income} = 5\% \times RSA - 5\% \times RSL = 4\% \times 32 - 5\% \times 49.5 = -0.875m$
- d. Net Income when $RSL > RSA$
 - a. Net Income would decrease as the interest rate payments would exceed interest rate income.

Question 5

- a. Ito's Lemma
 - a. Same as lecture notes, but comes to $(u-r+q)Fdt + oFdZt$
- b. European Call Option (Check this notes)
 - a. Formula
 - i. Delivery Price = K ,
 - ii. Payoff = $(F-K)e^{-(r(T-t))} = \text{Max}(S-K, 0)$

- iii. $C = SN(d_1) - Ke^{-(rT)}N(d_2)$
- iv. $d_1 = (\ln(S/K) + (r + (\sigma^2)/2)T)/(oT^{(1/2)})$
- v. $d_2 = oT^{(1/2)}$
- b. Explanation
 - i. Yes, the option price can be written on the forward contract.
 - ii. Run with assumption Payoff will be S-K
 - iii. Rearrange to find $S = (F-K)e^{-(r(T-t))} - K$
 - iv. Substitute S into all the formula to use black scholes

Question 6

- a. Momentum Effect (Galai et al., 1974)
 - i. Firms equity being a call option on the firms value leads to higher past returns increasing the value of the firms equity. Equity is less sensitive to future drops in total assets with decreasing elasticity. The return distribution of future returns are skewed upwards and so positive past returns are likely to lead to positive future returns and vice versa as the opposite true for past winners with overvaluation.
- b. Acquirers Share Price (Galai et al., 1974)
 - i. The model assumes an acquisition is an exchange of riskless assets for risky, physical assets.
 - ii. Hypothesize different variance of rate of return lead to different capital structure in market value terms.
 - iii. The Debt/Equity ratio will be greater with the firm with lower variance and vice versa.
 - iv. Hypothesize $d\text{Beta}(s)/d \sigma^2 < 0$,
 - v. Other case assumptions hypothesise, value of firm's equity will be higher while expected return will be lower as a rate of return variance dependent on the former. The opposite is true.
 - vi. Therefore, an exchange of riskless for risky assets by the acquirer increases return variance, increases expectation for rate of return, decreases value of firm's equity, decreasing the stock price of the firm.
- b. Idiosyncratic Volatility Puzzle (Merton, 1974)
 - i. Firms equity is a call option on value, stocks with high volatility have higher bankruptcy value, raising financial distress costs, decreasing firm value, and as equity is the residual claim after bond holders, the market value of equity decreases.
- c. Pecking Order Theory
 - i. Both Galai and Merton hypothesises the corporation has two classes of claims. A single homogenous class of debt, functioning as the exercise price of a call options, with residual claim in equity as a call options. Galai et al., assess the changes in the scale of the a firm.
 - ii. The systematic risk of an individual firm, externally finance. The value of equity and debt are first degree homogenous function of V & C with $S = SvV + ScC$, $D = DvV + DcC$.
 - iii. Systematic Risk of equity and debt are zero homogenous functions of equity and debt.
 - iv. The financing policy is devoid of redistribution effects, therefore the firm can issue debt until change in C/C = change in V/V, then raise the remaining debt, an equivalent to raising the firm's debt and equity proportionally with rise in the firm scale. If raised differently, there would be a watering down effect with one of other class of security. Therefore, adds supporting the pecking order theory.

Question 7

- a. One way to prove that the Black Scholes formula correctly calculates fair options prices is to reverse engineer observed options prices.
 - a. The Black Scholes model does not correctly price option prices,
 - b. You can assess the market imperfections inform realistic departures from options pricing
 - c. You can assess the implied volatilities to determine how the option prices depart from the pricing
 - d. There are a number of variables which determine option pricing. Reverse engineering option prices leads to making choices on which variable to optimize, so assumptions must be made on which to vary (most commonly
- b. If the CAPM correctly prices stocks, then it should also correctly price options written on stocks so one way to test the CAPM is to run a single index model on the cross-section of option returns.
 - a. Call in the support
- c. Under the Black and Scholes assumptions, a call option written on an underlying asset with a high expected return is more likely to pay off and thus, the option price should be high, too.
 - a. Call in the support

2018

Question 1

- a. Stock Price Process $dS = uSdt + oSdZ$
 - a. Stock price process models the price diffusion using a Geometric brownian motion (stochastic process with stationary, independent increments, with increment $Z(t+s) - Z(s)$ normally distributed), combining a modelling of deterministic trends ($uSdt$) with the stochastic trends ($oSdZ$). Overall, models the stochastic process of dollar changes.
 - b. S is the stock price at t, dS is change in stock price, u is the constant percentage drift, Z is the Brownian Motion, dZ increment in Brownian motion, o is the constant percentage volatility.
- b. Yes, as company earnings are dependant variables/directly related to the share price. Therefore, share price and earnings diffusion may be modelled using the same process.
- c. Table V in Options Arbitrage in Imperfect Markets
 - a. Arbitrage costs with transaction costs: shows the bounds on option prices and implied volatilities suitable for an arbitrager to bid and offer options, with 50/75% of recovering costs with base case and no rebalance assumptions, for in/at/out/deep-out of the money options.
 - b. The spread differentiate from theory (breakdown) with no rebalancing with in, out, and deep-out of the money options.
 - c. The table summarises arbitrager would focus on at the money spread, but no reflective of market observations as standard arbitrage and 50%/75% recovering costs not the only objectives for traders/investors.
 - d. Table again highlights how market imperfections affect option pricing, informing trading strategies.
 - e. A rational market maker would look at other objectives, meet profit requirements/margins, or engage in several other strategies. Rather than just breaking even. Therefore, bids not observed in reality.

Question 2

- a. Simultaneous relationship between derivative use and other variables
 - a. The authors look at both Debt Maturity and operational FX Hedging, proxied by the difference in percentage of sales that are foreign and the percent of foreign assets as the FX exposure. Carrying debt maturities would vary cash flows and interest rate risks associated with bonds. They also look at endogenous factors proxies for dividend exposure dividends and liquid assets (Quick ratio) as alternatives to derivatives, as can adjust both according to balance risk. These methods would be considered in conjunction with derivatives strategies.
 - b. They test this relationship using a generalised methods of moments (firm and country-level instruments) calculation on the above endogenous variables. The results of the table are intuitive (derivative use has a positive affect of leverage as upward sloping yield

curve, firms trade off higher interest payments, debt maturity negative as lower interest, negative of quick ratio implies firms able to hold lower liquid assets). The results are all statistically significant, showing firms able to undertake stronger value-enhancing activities, validating the simultaneous assumption requirements.

- b. Paper related to this question absence from question set

Question 3

- a. Paper related to this question absence from question set
- b. Paper related to this question absence from question set

Question 4

The image shows a handwritten derivation of the Black-Scholes formula for a call option price. It starts with the stochastic differential equation for a stock price S_t :

$$\frac{dS}{S} = \mu S dt + \sigma S dz$$

where $F(S,t) = S e^{(r-\delta)t}$. The derivation then proceeds through several steps:

- Ito's lemma application leads to the equation:

$$\left\{ \frac{\partial F}{\partial t} + \frac{\partial F}{\partial S} \mu S + \frac{1}{2} \frac{\partial^2 F}{\partial S^2} \sigma^2 S^2 \right\} dt + \frac{\partial F}{\partial S} \sigma S dz$$
- The term $\frac{\partial F}{\partial S}$ is shown to be $e^{(r-\delta)t}$.
- The term $\frac{\partial^2 F}{\partial S^2}$ is shown to be 0.
- The term $\frac{\partial F}{\partial t}$ is shown to be $-rSe^{(r-\delta)t} - rSe^{(r-\delta)t}$.
- The final result is the Black-Scholes formula for a call option price:

$$F = (r - \delta)Se^{(r-\delta)t} dt + \sigma Se^{(r-\delta)t} dz$$
- A note indicates that the drift term is $(\mu - r)e^{(r-\delta)t}$, volatility is $\sigma e^{(r-\delta)t}$.
- It is noted that since $F(S,t) = S e^{(r-\delta)t}$, the formula becomes $F = (\mu - r)F dt + \sigma F dz$, following Geometric Brownian motion.

- a.
- b. Merton (1974) and Galai stipulate the value of a firm can be separated into two component, the value of debt functioning as the strike price for an option, and the residual claim to equity holders as the value of equity. **Yes, use put-call parity**
- c. **Merton Model is N(-d1) for probability of default**
- d. Agree, the value of equity can be written as a call option. The volatility of the call options underlying asset affect the value of the call options, with volatilities being a prominent determinant in valuation. Therefore, may stipulate the volatility of the call option (leveraged equity) proportional to the value of the S&P 500.

Question 5

- a. Equilibrium Land Price
 - a. Variables
 - i. Phc = 150,000, Plc = 120,000, Phl = 128,000
 - ii. Cost per unit for 6 and 9 units is \$80,000 and \$90,000 respectively.
 - iii. Rf = 12%
 - iv. Rental rate = \$8000 per unit
 - b. Formula
 - i. $P_o = ShPh + SIPl + Rt(Sh + Sl)$
 - ii. $1/(1+Rf) = Sl + Sh$
 - iii. $Sh = (Po - (Pl + Rt/(1 + Rf)))/(Ph - Pl)$
 - iv. $Sl = ((Pl + Rt/(1 + Rf)) - Po)/(Ph - Pl)$
 - v. $Vo = T(Ph)Sh + T(Pl)Sl$
 - c. Workings
 - i. $Tih = MAX((150,000 - 80,000)*6, ((150,000 - 90,000)*9) = \$540,000$
 - ii. $T1l = MAX((90,000 - 80,000)*6, ((90,000 - 90,000)*9) = \$60,000$
 - iii. $Sh = (100,000 - (150,000 + 8000/(1.04)))/(150,000 - 90,000) = 0.208333..$
 - iv. $Sl = ((150,000 + 8000/(1.04)) - 100000)/(150,000 - 90,000) = 0.6845$
 - v. $Vo = 540,000 * 0.203833... + 60,000 * 0.6845 = 153,571.43$
 - vi. Excel Workings

<u>Land Valuation Under Uncertainty</u>		
O1	6	6 \$ 420,000.00
C1	80000	9 \$ 540,000.00
O2	9	t=1
C2	90000	π1h \$ 540,000.00
Land	128000	p1h \$ 150,000.00
t = 0		
π0	\$ 120,000.00	Rt1h \$ 8,000.00
p0	\$ 100,000.00	Rf 12%
Rt0	\$ 8,000.00	sh1h -0.69444444
Rf	12%	s1h 1.58730159
sh0	0.208333333	V1h \$ -
s10	0.6845	π1l \$ 60,000.00
V0	\$ 153,571.43	p1l \$ 90,000.00
		Rt1l \$ 8,000.00
		Rf 12%
		sh1l -0.21190476
		s1l 1.10476190
		V1h \$ -
		6 \$ 60,000.00
		9 \$ -

- vii.
- b. Disagree as use risk neutral probabilities, which derive certainty equivalent measures to assess upside and downside probabilities, where the measures such that each security price is exactly equal to the discounted expectation using this measure. The state pricing measures use the theory the derivatives security (building units) can be priced by forming a portfolio between the risk free asset and privative asset closely related to the derivative product. This methodologies aligns valuation methodologies across multiple investor risk appetites. Therefore, the probabilities are fair.
 - c. \$153,571.43 (expected value of waiting and building in the next period) is greater than \$128,000 land value. Assume any number of units above first threshold will assume cost of the next threshold. Since the land sell less than expected profit, arbitrage by:
 - a. Purchasing the land for \$128,000
 - b. Hedge the risk by short-selling 8 condo units and investing net proceeds from transactions into the risk free asset.
 - i. Difference in positive and negative states in land value is \$480,000 (540K – 60K),
 - ii. Difference in unit value is (150K – 90K) x 8 = \$480k
 - iii. Therefore, invest difference in \$(153,571.43 – 128,000), to get \$28,640 \$(153,571.43 – 128,000)*(1.12) in riskless profit at T=1.
 - d. Yes, vacant land in option. Greater volatility increase likelihood of being in the money, increasing the value of vacant land. The difference would lead to further departure from selling price, leading to greater opportunity to invest discrepancy and get greater profit from arbitrage.

Question 6

- a. Empirical Cumulative Distribution of S&P500 vs Analytical CDF of Standard Normal
 - a. Mostly follow the same cumulative distribution of standardised returns except for in the left tail where S&P500 has evidence of leptokurtosis.
 - b. There is a translation of leptokurtosis (condition of a probability density function to have fatter tails) in the empirical distributions to a smiling of the implied volatility functions, not present in analytical CDF of a standard normal, an assumption under Black Scholes model.
 - c. The changes in implied volatility for S&P500 options are from net buying pressure for index puts, leading to this leptokurtosis, and explanation for differences in figure 3.
- b. Delta/Vega Neutral Strategies
 - a. Delta/Vega- Neutral Hedge has options sold at 3pm (CST) ask price. ATM call options purchased at midpoint of bid-ask spread, hedging vega while rebalanced daily. Units of S&P 500 manage delta risk through absolute delta units of underlying security (S&P500, sold in put circumstances, bought in call circumstances revised daily during trade life by changing number of units in underlying asset. **How determine the weights?**
- c. Portfolio Weight & Cert. Equiv
 - i. Portfolio Weight is the portfolio weight optimised empirically for a CRRA investor, using derivatives-based trading strategies, obtained through the maximisation of in-sample expected utility.
 - ii. Cert. Equity is the monthly certainty equivalent, a percentage measure of the initial wealth demanded by a CRRA investor for the inability to invest in a particular derivatives strategy and must invest in the equity index and the risk-free asset.
- d. Decomposition of Index Risk Premium: Individual Variance Premium and Correlation Premium
 - a. Individual Variance Premium reflects the combination of individual risk premia and correlation risk premia.
$$E_t^Q[d\phi_I^2] - E_t^P[d\phi_I^2] = \sum_{i=1}^N \iota_i \{E_t^Q[d\phi_i^2] - E_t^P[d\phi_i^2]\} + \sum_{i=1}^N \sum_{j \neq i} w_i w_j \phi_i \phi_j \{E_t^Q[d\rho_{ij}] - E_t^P[d\rho_{ij}]\}. \quad (3)$$
- b. The authors test for the variance risk premia in index options, cross-section of individual variance swap returns, and individual options on all constituent stocks, conducted using model-free implied variance to isolate individual risk premia.
- c. Test for correlation risk (Right of + in equation) through analysis of a correlation risk trading strategy, using empirical analysis to explore whether a correlation risk factor, and common individual variance risk factor, can account for option returns cross-sectional variation.

2017

Question 1

- a. Assumptions of Risk-Neutral pricing
 - a. Market is complete with no arbitrage opportunities
 - b. Log-normal distributions – no presence of leptokurtosis
 - c. Relies on risk-neutral probabilities where each security prices is exactly equal to the discounted expectation.
 - d. An Arrow-Debreu security price persists for each state.
- b. Risk-neutral probabilities are certainty equivalent measures, adjusting probabilities to consider the risk-averseness of the market.
 - i. However, should yield the same result when using market-based discount measures if complexities do not exist.
 - ii. E.g., Considering the following example from Trigeorgis (1993)
 - iii. P = 0.4 (risk neutral probability), q = risk-averse probability, rf = 8%, r = 20%

- iv. $E = (pE+ + (1-p)E-) / (1+r) = (qE+ + (1-q)E-) / (1+r)$,
 v. $E = (0.4 * 180 + 0.6 * 60) / (1.08) = 100$
 vi. $E = (0.5 * 180 + 0.5 * 60) / (1.20) = 100$
 vii. When assessing traditional NPV, will be the same. However, must use risk-neutral when considering real options due to complexities in discounting opportunities.

Question 5

- a. Risk Neutral Probabilities
- a. $P = ((1+r)S - S+) / (S+ - S-) = ((1.10)20 - 10) / (40 - 10) = 0.47$
 - b. Passive NPV analysis unable to capture: value of embedded options due to discretionary, asymmetric nature and dependence on future events. The claims are not symmetric or proportional and discount rates vary in a complex way over time. Contingent claims analysis relies on valuing them, relying on a backward risk-neutral valuation process, where the current value of contingent claim could be obtained from its expected future values. Physical probabilities are from risk-adverse perspectives, not conducive for CCA.
- b. Calculate Fair interest Rate ($E_0 = 70$, $D_0 = 35$)
- a. Contingent Claims Analysis
 - i. $D_2 = 74.370741$
 - ii. $D_2 = D_0 * (1+r)^2$
 - iii. $R_d = (D_2/D_0)^{0.5} - 1 = (74.370741/35)^{0.5} - 1 = 45.7696\% (4.d.p)$
- c. Portfolio Construction & Sell and Scrap

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$D_0 = 35, E_0 = 70, F_0 = 35$

 $E_{2+} = \max(400 - D_2, 0) = 400 - D_2$

 $E_{2-} = \max(100 - D_2, 0) = 100 - D_2$

 $E_2^{\pm} = \max(25 - D_2, 0) = 0 \quad (D_2 > 35 > 25)$

 $E_{1+} = \rho(pE_{2+} + (1-p)E_{2-}) / (1+r)$

$$= (0.47 \times (400 - D_2) + (0.53 \times 100 - D_2)) / 1.10$$

$$= (188 - 0.47D_2 + 53 - 0.53D_2) / 1.10$$

$$= (241 - D_2) / 1.10$$

 $E_{1-} = (\rho E_{2+} + (1-\rho)E_{2-}) / (1+r)$

$$= (0.47 \times (100 - D_2) + 0.53 \times 0) / 1.10$$

$$= \frac{47 - 0.47D_2}{1.10}$$

 $E_0 = (\rho E_{1+} + (1-\rho)E_{1-}) / (1+r)$

$$= (0.47 \times \frac{(241 - D_2)}{1.10} + 0.53 \times \frac{47 - 0.47D_2}{1.10}) / 1.10$$

$$= (102.972727 - 0.427272D_2 + 22.645454 - 0.22644D_2) / 1.10$$

$$= 114.198... - 0.59429...D_2$$

 $F_0 = 114.198347 - 0.594297D_2$
 $D_2 = (114.198347 - F_0) / 0.594297$
 $= 74.370741...$

 $D_0 = \frac{D_2}{(1+r)^2} / \cancel{D_2} = \cancel{D_2} \Rightarrow r_d = \sqrt{\frac{D_2}{D_0}} - 1$
 $r_d = \sqrt{\frac{74.370741...}{35}} - 1 = 0.45769624$
 $= 45.7696\%$

a.

Client:	Date:
Project/Job:	Job No:
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