The option pricing model and the risk factor of stock

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Paper Overview

The paper presents a methodology to value companies' stock using an options pricing model and the CAPM

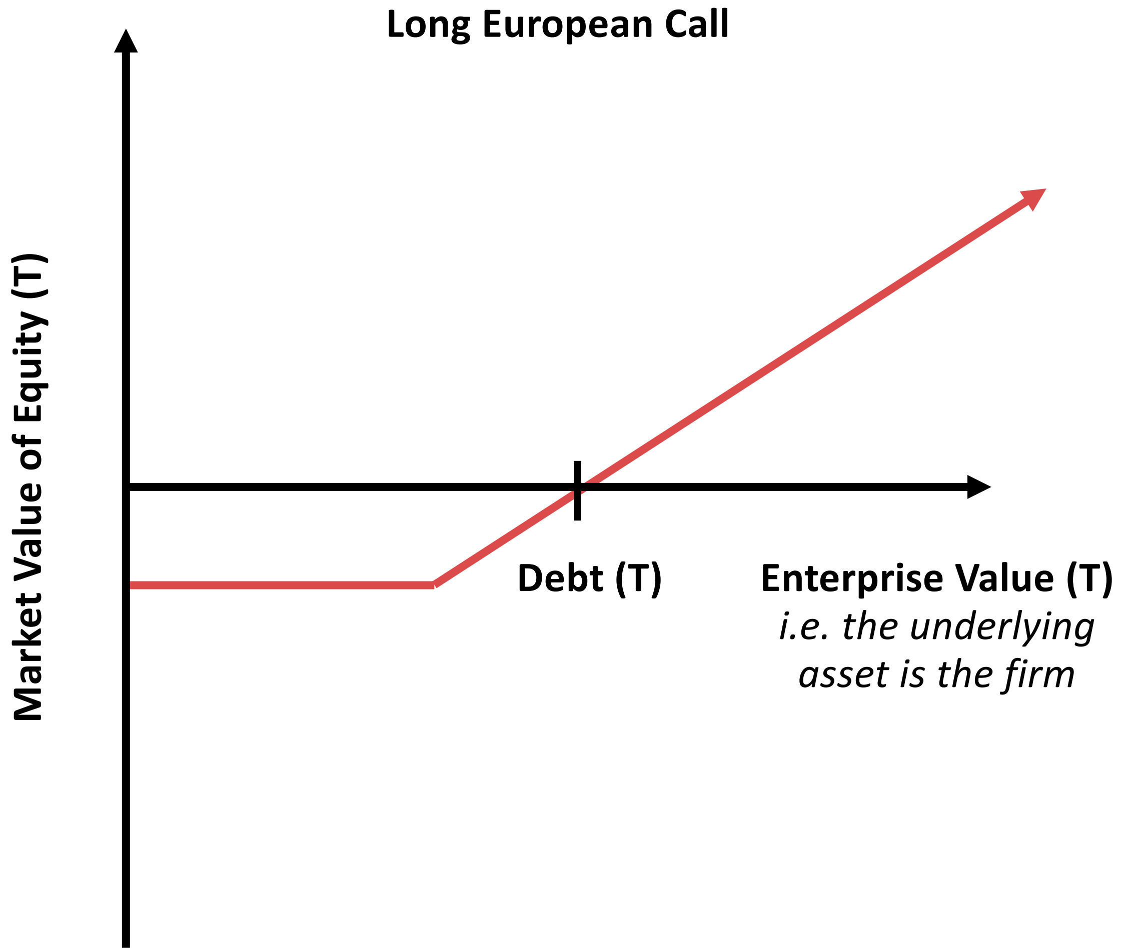
1. Cited nearly 2000 times this paper serves as a widely adopted valuation methodology and risk assessment framework adopted by practitioners and academics alike
2. The paper presents that using an option pricing model (OPM) with the capital asset pricing model (CAPM) yields a theoretically more complete model of corporate security pricing
3. To apply the option pricing model to securities, we first need to recognise that company's equity can be viewed as a European call option on the company
4. Treating a company's equity as a European call allows the option pricing model to be used to value the company's equity
5. In addition to equity valuation this framework can be adopted to assess the risk of a company
6. The paper provides three case studies presenting how this methodology can be used to assess risk and value a company under a range of different scenarios
7. From this presentation you should develop a better understanding of the option pricing model

Understanding the basics

A company's equity can be seen as a European call option with a exercise price equal to the face value of debt

Assumptions

- A firm has one pure-discount bond issue and one common stock issue
- The bond with face value C will mature at T (i.e., T periods from the current period which is denoted by 0) and at that time, 'the firm will liquidate itself
- Up to T, the firm does not experience any net cash flows and pays no dividends to its shareholders
- Under this set of simplifying assumptions, Black-Scholes (1973) observed that common stock can be regarded as a European call option.'
- We view the **stockholders as having an option to buy back the firm** (whose current market value is V) from the bondholders for an exercise price equal to the face value of the firm's debt at time



Option Pricing Model (OPM) and CAPM assumptions

Viewing the company's equity as a European call option allows us to apply the option pricing model and CAPM to assess the company's equity value and assess the risk profile under a range of scenarios. To apply these two models, we need to make a range of simplifying assumptions

	Under these assumptions both the OPM and CAPM can be derived
1	All individuals have a strictly **concave von Neuman-Morgenstern utility function** and are **expected utility maximisers**
2	There are **homogenous expectations** about the dynamics of firm asset values and of security prices
3	The **capital markets are perfect**: there are **no transaction costs or taxes** and all traders have **free and costless access to all available information**. **Traders are price takers** in the capital markets, i.e., they are atomistic competitors'
4	There are **no costs of voluntary liquidation or bankruptcy**, e.g.. court or reorganization costs, where bankruptcy is defined as the state when the value of the firm's assets is less than the face value of the maturing debt.
5	There is a **known instantaneously riskless interest rate** which is constant through time and is equal for borrowers and lenders
6	Borrowing and short-selling by all investors and free use of all proceeds are allowed
7	The **distribution of firm asset value at the end of any finite time interval is log normal**. The variance of the rate of return on the firm is constant
8	**Trading takes place continuously price changes are continuous**, and assets are infinitely divisible

The Capital Asset Pricing Model (CAPM)

The CAPM discounts a company's cash flows using a discount reflecting the systematic risk of a company

$$\tilde{r}_i = r_F + \beta_i(\tilde{r}_M - r_F)$$

Where:

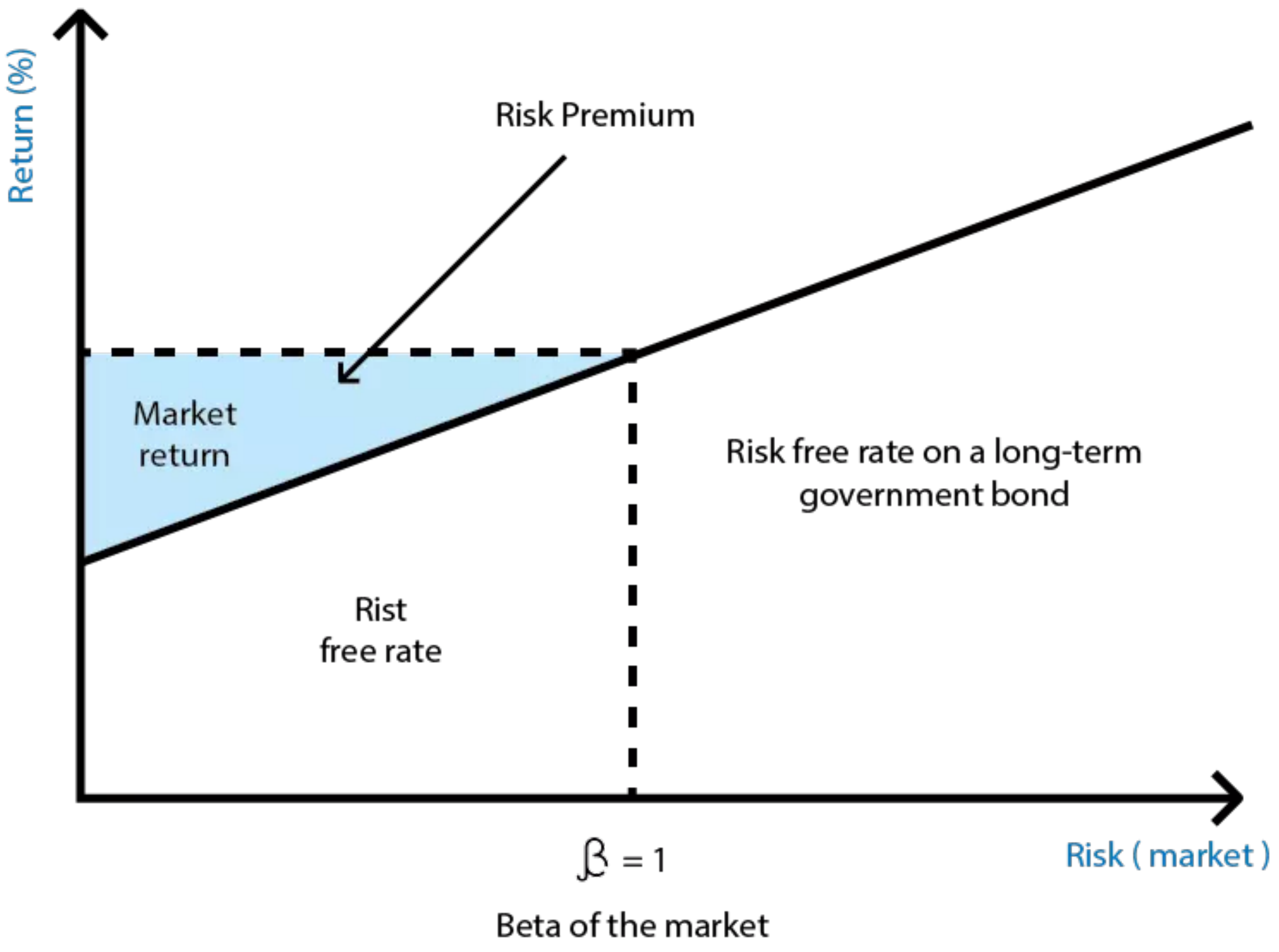
$\tilde{r}_i$ = Expected return company i

r_F = Risk free rate

$$\beta_i \equiv \text{cov}\,(\tilde{r}_i, \tilde{r}_M)/\sigma^2(\tilde{r}_M)$$

β_i = Companies systematic risk

- Investors differentiate assets only according to the assets' expected rates of return and their contribution to the variance of investors' efficient portfolios
- Non systematic risk is diversified away



The Option Pricing Model (OPM)

The OPM can be applied to a company under the assumptions that equity can be viewed as a European call on the company

Black-Scholes Overview

- The option pricing model as derived by Black-Scholes (1973) applies to European-type options
- They create a perfect hedge, at each instant of time, composed of one unit long (short) of the underlying security and a short (long) position on a number of options
- The return on a completely hedged position should be equal to the riskless return on the investment in order to eliminate arbitrage opportunities

European call formula

$$S = VN(d_1) - C\,e^{-r_F T} N(d_2),$$

Where:

- V is the current value of the underlying asset
- σ^2 is the instantaneous variance of percentage returns on V
- C is the exercise price of the option
- T is the time to maturity
- rF is the riskless interest rate
- N(e) is the standardized normal cumulative probability density function, and

$$d_1 \equiv \frac{\ln(V/C) + (r_F + \frac{1}{2}\sigma^2)T}{\sigma\sqrt{T}}$$

$$d_2 \equiv d_1 - \sigma\sqrt{T}.$$

Application to equity

The value of the stock is an increasing function of:

- The value of equity (S)
- the value of the firm (V)
- the riskless interest rate (rF)
- the variance of the percentage return of the firm (σ^2) and
- the time to liquidation (T); and

It is a decreasing function of:

- the face value of the debt (C)

The risk of equity

The paper uses math proofs to present the factors impacting the relationship between the risk and return of equity relative to the firm

Relationship between β_s *and* β_V

- The paper presents that if the systematic risk of the firm is constant over time, the instantaneous risk of the equity will not necessarily be stable or known with certainty for the time period in question
- The analysis indicates that the relationship between the systematic risk of the firm and of its equity is not only a positive function of the firm's leverage V/S as shown by Hamada (1972)
- But that it is a positive function of the face value of debt C and a negative function of the value of the firm V, the riskless interest rate rF, the variance of the firm's percentage returns and the time to maturity of the firm's debt T

$$\frac{\partial \beta_s}{\partial V} < 0, \quad \frac{\partial \beta_s}{\partial C} > 0, \quad \frac{\partial \beta_s}{\partial r_F} < 0, \quad \frac{\partial \beta_s}{\partial \sigma^2} < 0, \quad \frac{\partial \beta_s}{\partial T} < 0.$$

Relationship between r_s *and* r_V

$$\partial \bar{r}_s = (\bar{r}_V - r_F)\frac{C e^{-r_F T} N(d_2)}{S}$$

- From this expression we can see explicitly the terms that contribute to the higher required expected rate of return by stockholders due to leverage

$$\frac{\partial \bar{r}_s}{\partial V} < 0, \quad \frac{\partial \bar{r}_s}{\partial C} > 0, \quad \frac{\partial \bar{r}_s}{\partial r_F} \gtreqless 0, \quad \frac{\partial \bar{r}_s}{\partial \sigma^2} < 0, \quad \frac{\partial \bar{r}_s}{\partial T} < 0,$$

Case Studies

Presenters:

Toby Riley
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Case Studies

Setting

Comparative static approach

- Comparison between two firms A and B:
- Initially identical before changing one of firm B's relevant characteristics.
- Firm B represents Firm A at another point in time

- Each case study looks at the changes to a firm's systematic risk, expected rate of return and market value of the firm's debt and equity.
- Purpose is to examine the redistribution potential from one security to the other when perfect me first rules do not exist

- What are perfect me first rules?

Table 1

Variables of the firm	Firm A	Firm B	General
Current market value of firm	V_0^A	V_0^B	V_0
Terminal market value of firm	V_T^A	V_T^B	V_T
Current market value of shares	S_0^A	S_0^B	S_0
Current market value of debt	D_0^A	D_0^B	D_0
Systematic risk of firm	β_V^A	β_V^B	β_V
Systematic risk of shares	β_S^A	β_S^B	β_S
Variance of rate of return of the firm	σ_A^2	σ_B^2	σ^2
Rate of return of the firm	r_V^A	r_V^B	r_V
Rate of return of the shares	r_S^A	r_S^B	r_S
Face value of debt maturing at T	C_A	C_B	C

Case Studies

Case study I – Rate of return variability and changes due acquisitions and divestitures

Under these assumptions

- Using equation 3 the market value of the two firms is equal, and they are in the same risk class with the same systematic risk
- The only difference is their total variability of returns in (d).
- Under these assumptions it can be proven that firm B will have higher value of debt and firm A will have higher value of equity
- Equity can be regarded as call option as shown before and options value is an increasing function of variability of returns.
- As a result, firm A has a lower debt to equity ratio than firm B

Assumptions

(a) $C_A = C_B,$

(b) $\bar{V}_T^A = \bar{V}_T^B,$

(c) $\operatorname{cov}(\tilde{V}_t^A, \tilde{V}_t^M) = \operatorname{cov}(\tilde{V}_t^B, \tilde{V}_t^M), \qquad 0 \leqq t \leqq T.$

$\tilde{V}_t^M$ is defined at the end of section 3, and

(d) $\sigma_A^2 > \sigma_B^2.$

$$V_0^J = \left[\bar{V}_T^J - \frac{\lambda \operatorname{cov}(\tilde{V}_T^J, \tilde{V}_T^M)}{\sigma(\tilde{V}_T^M)}\right] \Big/ (1+R_F). \qquad (3)$$

$$S = VN(d_1) - C\,e^{-r_F T} N(d_2), \qquad (4)$$

Case Studies

Case study I – Rate of return variability and changes due acquisitions and divestitures

Numerical Example

- Calculate d1 and d2

$$d_1 = \frac{\ln \frac{1000}{500} + \left(0.08 + \frac{1}{2}(0.1)\right)5}{0.1\sqrt{5}} = 6.006$$

$$d_2 = 6.006 - 0.1\sqrt{5} = 5.7831$$

- Then get values of standard normal table and sub into equation (4)
- Differences in the total return variation thus causes differences in the market value of the firm’s debt and equity.
- If the derivative of the systematic equity risk w.r.t to total variation of return is negative, what will happen to expected return on equity?

$$S = VN(d_1) - C\,e^{-r_F T}N(d_2), \tag{4}$$

$$V_0^A = V_0^B = \$1{,}000,$$

$$C_A = C_B = \$500,$$

$$\sigma_A^2 = 0.10\ (10\%), \qquad \sigma_B^2 = 0.05\ (5\%),$$

$$R_F = 0.08\ (8\%).$$

Then, for $T = 5$ (e.g., five years) we find, using eq. (4), that

$$S_0^A = \$675, \qquad S_0^B = \$666,$$

$$D_0^A = \$325, \qquad D_0^B = \$334.$$

If we raise the variance of the rate of return of firm A to $\sigma_A^2 = 0.15$, then

$$S_0^A = \$688, \qquad D_0^A = \$312.$$

Alternatively, if we lowered the face value of the debt of both firms to \$400, we obtain (for $\sigma_A^2 = 0.10$ and $\sigma_B^2 = 0.05$)

$$S_0^A = \$736, \qquad S_0^B = \$732,$$

and

$$D_0^A = \$264, \qquad D_0^B = \$268.$$

Case Studies

Case study II – Changes with the scale of a firm and the problem of dilution

Firm rescale

Assume that

(a) $\tilde{V}_t^A = \alpha \tilde{V}_t^B, \quad 0 \leqq t \leqq T.$

This implies

$$\tilde{V}_T^A = \alpha \tilde{V}_T^B,$$

and

$$\operatorname{cov}(\tilde{V}_t^A, \tilde{V}_t^M) = \alpha \operatorname{cov}(\tilde{V}_t^B, \tilde{V}_t^M), \quad 0 \leqq t \leqq T,$$

which together with the valuation eq. (3) yields

(b) $V_0^A = \alpha V_0^B.$

Assumption (a) also implies that the two firms' rates of return have perfect positive correlation and therefore

(c) $\sigma_A^2 = \sigma_B^2,$

(d) $\beta_V^A = \beta_V^B.$

If we further assume that

(e) $C_A = \alpha C_B,$

then from eq. (4) we see that $d_1^A = d_1^B$ and $d_2^A = d_2^B$, so

$$S_0^A = (\alpha V_0^B) N(d_1^B) - (\alpha C_B)\, e^{-r_F T} N(d_2^B) = \alpha S_0^B.$$

$$\beta_S = N(d_1) \frac{V}{S} \beta_V \equiv \eta_S \beta_V. \qquad \text{Equation 8}$$

Using equation 8 and substituting in the relationship above yields

$$\beta_S^A = \frac{\alpha V_0^B}{\alpha S_0^B} N(d_1^B) \beta_V^B = \beta_S^B.$$

Case Studies

Case study II – Changes with the scale of a firm and the problem of dilution

Result

- The financing policy is devoid of redistribution effects
- For proportional upscale of the firm's operations firm can issue debt until the proportional increase in the face value of the debt is equal to the proportional increase in value of the firm with the remaining financing being done with equity.
- This equivalent to issuing new debt and equity proportionately with the rise in the firm's scale.
- Any other combination of debt and equity will result in watering down or diluting of either debt or equity

From the option pricing model can derive:

$$D = D_V V + D_C C,$$

$$S = S_V V + S_C C,$$

$$\frac{\partial \beta_S}{\partial V} V + \frac{\partial \beta_S}{\partial C} C = 0,$$

and

$$\frac{\partial \beta_D}{\partial V} V + \frac{\partial \beta_D}{\partial C} C = 0.$$

Case Studies

Case study III – Conglomerate merger

Setting

- Three firms, Firm G, Firm A and Firm B
- Conglomerate merger occurs where Firm A and Firm B merge together to become the new Firm G
- Firm G has the exact same assets as held by Firm A and Firm B
- What is a conglomerate merger?

Assumptions

(a) $$\tilde{V}_t^G = \tilde{V}_t^A + \tilde{V}_t^B, \qquad 0 \leqq t \leqq T,$$

This implies

(a′) $$V_0^G = V_0^A + V_0^B,$$

and

(a″) $$\beta_V^G = \gamma\beta_V^A + (1-\gamma)\beta_V^B, \quad \text{where} \quad \gamma = V_0^A / V_0^G,$$

(b) $$C_G = C_A + C_B,$$

(c) $$\rho(\tilde{r}_V^A, \tilde{r}_V^B) < 1,$$

Case Studies

Case study III – Conglomerate merger

Assumptions continued

(d) $\sigma_A^2 = \sigma_B^2,$

(e) $V_0^A/C_A = V_0^B/C_B.$

Assumptions (c) and (d) imply that[39]

(f) $\sigma_G^2 < \sigma_A^2 = \sigma_B^2,$

while assumptions (a′), (b) and (e) yield[40]

(g) $V_0^G/C_G = V_0^A/C_A = V_0^B/C_B.$

Result

From the results (f) and (g) combined with the analysis of Case I, we see that eq. (4) implies

$$S_0^G < S_0^A + S_0^B \quad \text{and} \quad D_0^G > D_0^A + D_0^B.$$

So, who is better off in this scenario, debt holder or equity holders?

Case Studies

Case study III – Conglomerate merger

Implications

- The total return deviation or risk of default for firm G is less than what it is for either firm A or firm B
- Using case I, this results in the market equity of firm G being less than firm A and B combined and the market value of debt for firm G being greater than the combined value of firm A and B
- In this conglomerate merger scenario debt holders come out better off than equity holders.

How to correct this?

- In this example stockholder are not protected against a financial policy that can change their wealth.
- In order to make equity holders no worse off as a result of the merger, more debt can be issued with the same seniority and then a certain portion of the merged firm's equity is retired
- When will this process stop?

Case Studies

Case study III – Conglomerate merger

Numerical Example

$$V_0^A = V_0^B = \$1000,$$

$$S_0^A = S_0^B,$$

$$\sigma_A^2 = \sigma_B^2 = \sigma^2,$$

$$C_A = C_B = \$500,$$

$$T = 5 \quad \text{(e.g., 5 years)},$$

$$r = 0.08.$$

If $\sigma^2 = 0.10$, then

$$S_0^A = S_0^B = \$675.2 \quad \text{and} \quad S_0^A + S_0^B = \$1350.4.$$

f the correlation between the percentage return on A and B is $\rho = 0$, then for the
ıerged firm ($C_G = C_A + C_B = \$1000$),

$$S_0^G = \$1332.5 \quad \text{and} \quad D_0^G = \$667.5,$$

nd hence

$$L_S = \$1350.4 - \$1332.5 = \$17.9.$$

And hence

$$L_S = \$1350.4 - \$1332.5 = \$17.9.$$

If we issue additional debt with face value of \$560 and with the proceeds retire part of the equity[66] we obtain

$$S_0^{G'} = \$1013.27 \quad \text{and} \quad D_0^{G'} = \$986.7.$$

The market value of the old bonds is \$986.7 (1000/1560) = \$649.6, exactly like their combined value before the merger (i.e., $D_0^A + D_0^B = 2 \times 324.8 = \$649.6$).

Wealth of equity holders is now equal to the current market value of equity (\$1013.3) plus the cash payment equal to the market value of new debt (\$337.1) which together equals 1350.4 which is the same as the combined equity value before the merger.

Case IV: Spin-offs

What is a spin-off? Can you list any recent examples?

Spin-offs are also known as divestures

- A division of a single firm into two separate corporate entities
  - Take a portion of a firm's assets and create a legally independent firm with these assets, usually unrelated to the remaining operations of the firm
- Spin-offs depend on distributing the shares of the new equity solely to shareholders of the parent company
- Subsequently, shareholder appropriate a portion of bondholder's collateral as no longer have any claim to the asset's of the new firm
  - **Why would this be an issue to bondholders?**

Mathematical Illustrations of a Spin-off

How are shareholders and bondholders different? What does this create?

- (1) implies there is no economic dependence between divisions A & B, implying linear combinations of value in (2)
- The composition of Firm G is two economically independent divisions A & B. At time 0, Firm G unexpectedly spins off B, therefore (3)
- Subsequently, Debtholders of A find that their position deteriorates as less asset service as collateral for debt. Furthermore, leverage increase from loss of asset leading to (4)
- However, the paper assumes variance remains constant, observing (5) which combined with (2) yields
- (5) informs the lack of protection against investment and financial decisions of the firm by class of security holders may deteriorate positions
- Anticipation of firm decisions prevents redistribution effects. Over-anticipation may reverse effects

Case IV: Spin-offs

$$\tilde{V}_t^G = \tilde{V}_t^A + \tilde{V}_t^B, \ 0 \le t \le T \quad (1)$$

$$\tilde{V}_0^G = \tilde{V}_0^A + \tilde{V}_0^B \quad (2)$$

$$C_G = C_A \quad (3)$$

$$\uparrow \frac{V}{C} := \beta_S^A > \beta_S^G, \beta_D^A > \beta_D^G, \sigma_A^2 \neq \sigma_G^2 \quad (4)$$

$$D_0^A \le D_0^G \quad (5)$$

$$S_0^A + S_0^B > S_0^G \quad (6)$$

Table 1

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Current market value of shares	S_0^A	S_0^B	S_0
Current market value of debt	D_0^A	D_0^B	D_0
Systematic risk of firm	β_V^A	β_V^B	β_V
Systematic risk of shares	β_S^A	β_S^B	β_S
Variance of rate of return of the firm	σ_A^2	σ_B^2	σ^2
Rate of return of the firm	r_V^A	r_V^B	r_V
Rate of return of the shares	r_S^A	r_S^B	r_S
Face value of debt maturing at T	C_A	C_B	C

Application to Corporate Investment Decisions

What is a spin-off? Can you list any recent examples?

Case studies do not consider corporate decision making

- Authors continue to assume no side payments, perfect ‘me first’ rules, or transaction costs
- Jensen-Long (1972) and Merton-Sabrahmanyam (1974) proved that an unlevered firm in a perfectly competitive environment under uncertainty acts to maximise its current value
- Assumed held for levered firms. However, not the case in a world stockholders control investment decisions
- For example, Stockholders will select an investment opportunity with higher variance of percentage returns between two mutually exclusive investment opportunities with equal opportunity in terms of expected net cash flow (discounted for systematic risk)

$$\frac{\partial V}{\partial I} \geq 0 \tag{7}$$

$$\frac{\partial S}{\partial I} = \frac{\partial S}{\partial V} \times \frac{\partial V}{\partial I} + \frac{\partial S}{\partial \sigma^2} \times \frac{\partial \sigma^2}{\partial I} \geq 0 \tag{8}$$

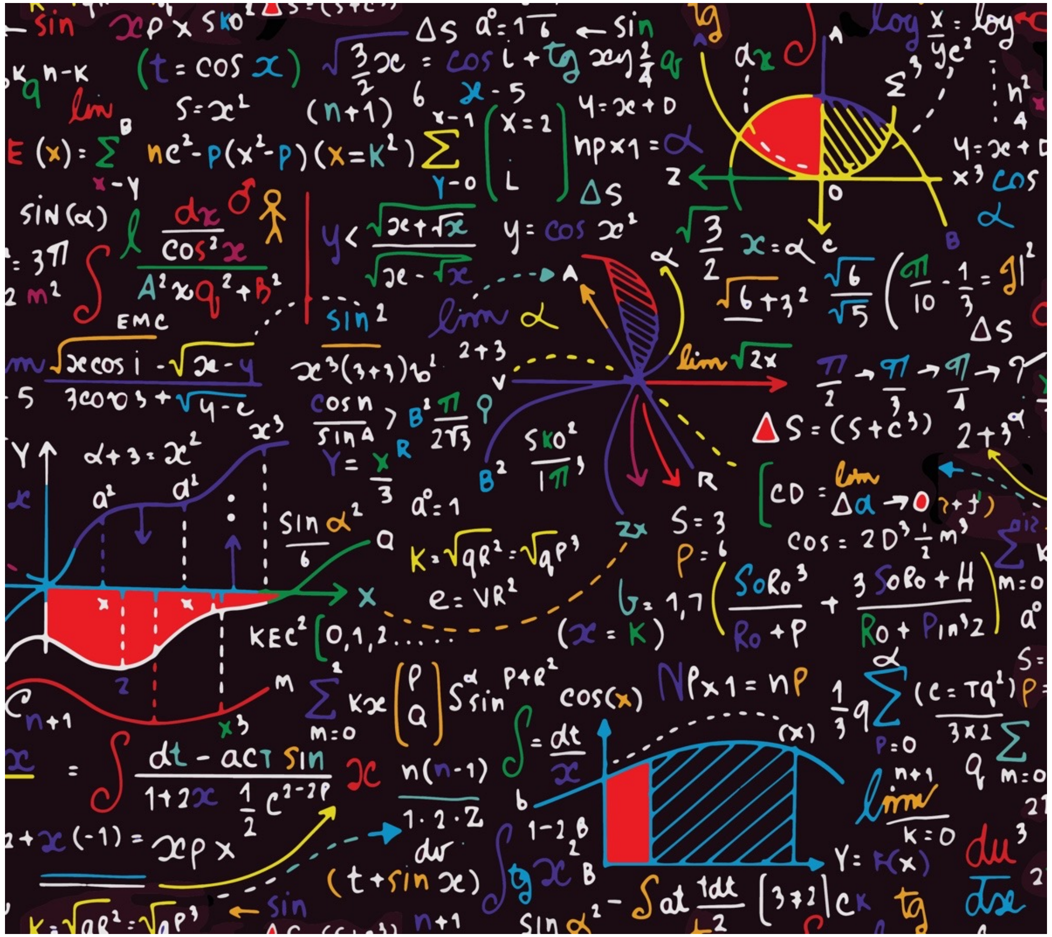
- A more profitable investment may be rejected in favour of an opportunity with higher variance of percentage returns
- A pure-equity firms will invest under (7) while a levered firm will only accept a project under (8)
- One interpretation is CoC used in investment decision making is a negative function of the change in the firm’s rate of return variance if investment is accepted
- Levered firms do not maximise the market value of the firm due to the externality affecting shareholders. **What does this mean?**

Implications for Empirical Studies

What implications can we hypothesize?

The authors derive several implications

- Most based on the result systematic risk and rate of return variance of levered equity and risky debt are non-stationary, in general
- Therefore, the rate of return distributions of this debt and equity will generally also be non-stationary, presenting a number of statistical difficulties in measuring security risk, testing the efficiency of the capital markets, or the validity of the Capital Asset Pricing Model
- Prior research informs individual common stock sensitivities are non-stationary but little empirical work utilizes information concerning changes in the firm's asset and capital structure, to predict changes in securities' risk, except for Hamada (1972), finding changes in leverage inform non-stationary nature of common stock systematic risk



Implications for Empirical Studies

What implications are there for capital markets?

The authors derive several implications

- Efficient capital markets immediately compound new information concerning asset values into security prices
- There is an expectations to find empirical relationships between changes in security prices, or their systematic risk, and the appearance of new information in the market concerning:
    - Variance of the firm's rate of return
    - Riskless interest rate
    - Time to maturity of debt
    - Face value of debt to asset ratios
- Changes to financial or asset structure should affect these variables, therefore prices and systematic risk

Implications for Empirical Studies

Is it possible to eliminate information asymmetries?

The authors derive several implications

- Subsequently, expect on average that the realized rate of return on securities will be affected not only by changes in the expected terminal value but also changes in systematic risk, but compounds problems measuring and interpreting the excess realized rates of returns due to information effects.
- The paper's analysis builds on Fama-Fisher-Jensen-Roll (1969) discovering positive dividends following stock splits had positive excess realized rates of returns, proposing this effect is in-part from the redistribution effect of the unanticipated dividend rise.
- The paper would predict an adverse effect upon the value of the firm's debt. The information hypothesis would predict the inverse

Implications for Empirical Studies

What is a random walk?

The authors derive several implications

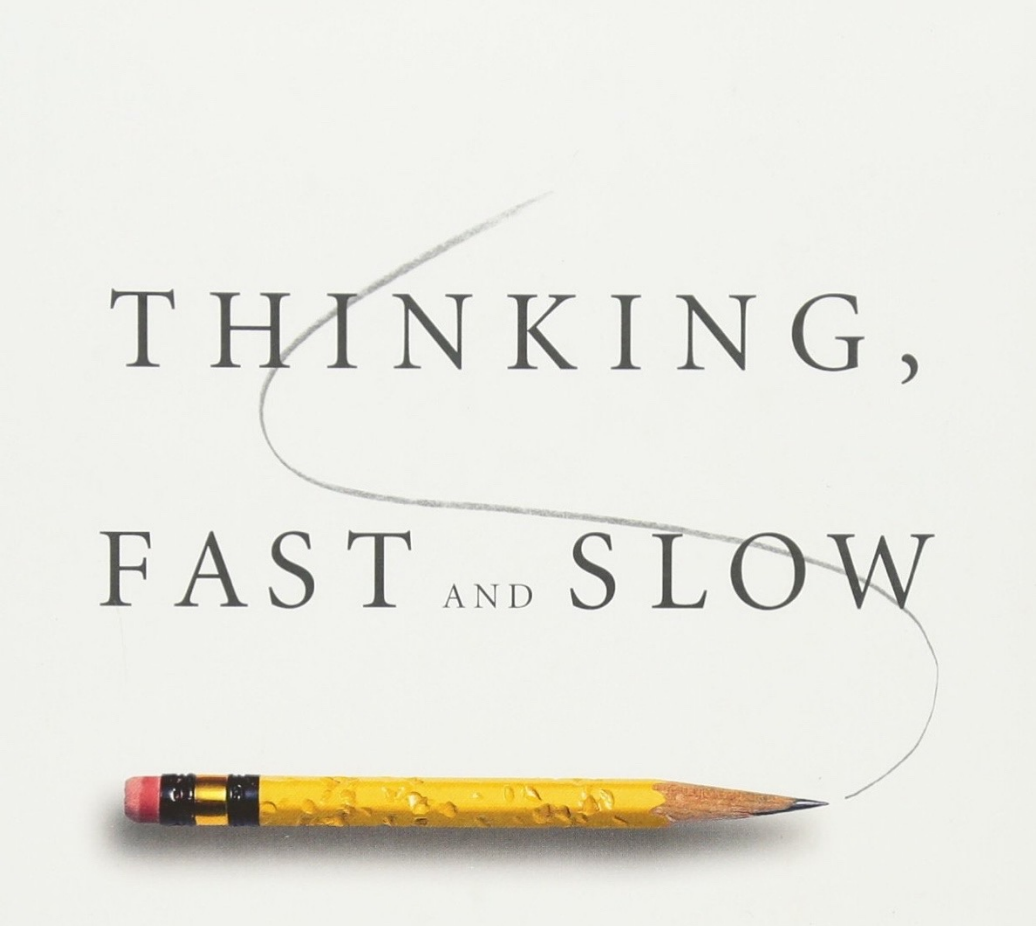
- Test to the efficient capital assumption assumes the distribution of common stock returns behaves as a random walk, relying on the stationarity of the return’s distribution
- However, this is not possible as the rate of return of common stock in a levered firm is a non-stationary functions of the rate of return of the assets of the firm
- The random walk assumptions is at best a first approximation, not correct for all classes of firm

Implications for Empirical Studies

What is a good proxy for the market?

The authors derive several implications

- The authors model has important implications for testing the validity of the CAPM using returns data of leverage equity
- Merton (1970) offers warnings about equity returns in empirical studies as regressions introduce systematics biases from complexities
- Black-Jensen-Scholes (1972) and Fama-MacBeth (1973) develop techniques to avoid selection bias from regression phenomena by introducing lagged CAPM testing and forming portfolios comprising of both non-stationary and stationary betas
- It is suggestive proxies for the market index should not consist entirely of equity



Conclusion

Proposition of new models with implications

- Combines Capital Asset Pricing Model and Option Pricing Model and applies to the derivation of equity's value and systematic risk
- Develops two models and presents newly found properties of the options pricing model.
- Considers the effects of the properties on share and bondholders, showing how anticipated changes in firm capital and asset structure can differently affect the debt and equity of a firm
- Considers several theoretical and empirical implications of the model including investment policy, and non-stationarity effects to the systematic levered equity and risky debt

Strengths & Weaknesses

What are additional strengths and weaknesses?

Paper has several strengths and weaknesses

- Strengths
  - Thorough analytic derivations of model(s)
  - Support from case studies involving divestures, acquisition, spin-offs, conglomerate mergers, dilution and scaling
  - Extensive support from prior literature
- Weaknesses
  - Heavy reliance on simplifying assumptions
  - Lack of empirical analysis/simulation to validate models

Literature Review & Future Research

Extensive preceding literature informs formulation, application, & implication

1. **Option Pricing Model**
   Black–Scholes, 1973
2. **Value Maximisation**
   Jensen-Long, 1972; Merton-Subrahmanyam, 1974
3. **Non-Stationarity of Common Stock Beta**
   Blume, 1968/1971, Gonedes, 1973; Hamada, 1972
4. **Capital Asset Pricing Model**
   Black-Jensen-Scholes, 1972; Fama-Macbeth, 1973
5. **Information Effects and Realised Rates of Return**
   Fama-Fisher-Jensen-Roll, 1969
6. Options Arbitrage in Imperfect Markets,
   Figlewski, 1989,

Further research

1. Test the implications proposed
   E.g., Non-stationarity, Efficiencies, Proxies
2. Apply simulations to vary model parameters to find numerical bounds to variables
3. Incorporate market frictions into analysis

Thank you