2019

Semester 1

Assignment 3: Set Partitioning

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Carts		0	0	0	0	...	0	0	0	0	...	3	4	5	6	...	0	0	0	1	...	8	9	10	11	...	0	0	0	0	...	0	0	0	0	...	0	0	0	0	...	0	0	0	0	...	0	0	1	2		
Jobs	1	1	1	1	1	...	0	0	0	0	...	0	0	0	0	...	0	0	0	0	...	0	0	0	0	...	0	0	0	0	...	0	0	0	0	...	0	0	0	0	...	0	0	0	0	...	0	0	0	0	-	1
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	22	0	0	0	0	...	0	0	0	0	...	1	1	1	1	...	0	0	0	0	...	1	0	0	0	...	0	0	0	0	...	0	0	0	0	...	0	0	0	0	...	0	0	0	1	...	0	1	1	1	<-	1
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	30	0	0	0	0	...	0	0	0	0	...	0	0	0	0	...	0	0	0	0	...	0	0	1	1	...	0	0	0	0	...	0	0	0	0	...	0	0	0	0	...	0	0	0	0	...	0	0	0	0	<-	1
	31	0	0	0	0	...	0	0	0	0	...	0	0	0	0	...	0	0	0	0	...	0	0	0	1	...	0	0	0	0	...	0	0	0	0	...	0	0	0	0	...	0	0	0	0	...	0	0	0	0	<-	1
	32	0	0	0	0	...	0	0	0	0	...	0	0	0	0	...	0	0	0	0	...	0	0	0	0	...	0	0	0	0	...	0	0	0	0	...	0	0	0	0	...	0	0	0	0	...	0	0	0	0	<-	1
	33	0	0	0	0	...	0	0	0	0	...	0	0	0	0	...	0	0	0	0	...	0	0	0	0	...	0	0	0	0	...	0	0	0	0	...	0	0	0	0	...	0	0	0	0	...	0	0	0	0	<-	1
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	37	0	0	0	0	...	0	0	0	0	...	0	0	0	0	...	0	0	0	0	...	0	0	0	0	...	0	0	0	0	...	0	0	0	0	...	0	0	0	0	...	0	0	0	0	...	0	0	0	0	<-	1
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	39	0	0	0	0	...	0	0	0	0	...	0	0	0	0	...	0	0	0	0	...	0	0	0	0	...	0	0	0	0	...	0	0	0	0	...	0	0	0	0	...	0	0	0	0	...	0	0	0	0	<-	1
	40	0	0	0	0	...	0	0	0	0	...	0	0	0	0	...	0	0	0	0	...	0	0	0	0	...	0	0	0	0	...	0	0	0	0	...	0	0	0	0	...	0	0	0	0	...	0	0	0	0	<-	1
	41	0	0	0	0	...	0	0	0	0	...	0	0	0	0	...	0	0	0	0	...	0	0	0	0	...	0	0	0	0	...	0	0	0	0	...	0	0	0	0	...	0	0	0	0	...	0	0	0	0	<-	1
	42	0	0	0	0	...	0	0	0	0	...	0	0	0	0	...	0	0	0	0	...	0	0	0	0	...	0	0	0	0	...	0	0	0	0	...	0	0	0	0	...	0	0	0	0	...	0	0	0	0	<-	1
	43	0	0	0	0	...	0	0	0	0	...	0	0	0	0	...	0	0	0	0	...	0	0	0	0	...	0	0	0	0	...	0	0	0	0	...	0	0	0	0	...	0	0	0	0	...	0	0	0	0	<-	1
	44	0	0	0	0	...	0	0	0	0	...	0	0	0	0	...	0	0	0	0	...	0	0	0	0	...	0	0	0	0	...	0	0	0	0	...	0	0	0	0	...	0	0	0	0	...	0	0	0	0	<-	1
	45	0	0	0	0	...	0	0	0	0	...	0	0	0	0	...	0	0	0	0	...	0	0	0	0	...	0	0	0	0	...	0	0	0	0	...	0	0	0	0	...	0	0	0	0	...	0	0	0	0	<-	1
	46	0	0	0	0	...	0	0	0	0	...	0	0	0	0	...	0	0	0	0	...	0	0	0	0	...	0	0	0	0	...	0	0	0	0	...	0	0	0	0	...	0	0	0	0	...	0	0	0	0	<-	1
	47	0	0	0	0	...	0	0	0	0	...	0	0	0	0	...	0	0	0	0	...	0	0	0	0	...	0	0	0	0	...	0	0	0	0	...	0	0	0	0	...	0	0	0	0	...	0	0	0	0	<-	1
	48	0	0	0	0	...	0	0	0	0	...	0	0	0	0	...	0	0	0	0	...	0	0	0	0	...	0	0	0	0	...	0	0	0	0	...	0	0	0	0	...	0	0	0	0	...	0	0	0	0	<-	1
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	50	0	0	0	0	...	0	0	0	0	...	0	0	0	0	...	0	0	0	0	...	0	0	0	0	...	0	0	0	0	...	0	0	0	0	...	0	0	0	0	...	0	0	0	0	...	0	0	0	0	<-	1
	51	0	0	0	0	...	0	0	0	0	...	0	0	0	0	...	0	0	0	0	...	0	0	0	0	...	0	0	0	0	...	0	0	0	0	...	0	0	0	0	...	0	0	0	0	...	0	0	0	0	<-	1
	52	0	0	0	0	...	0	0	0	0	...	0	0	0	0	...	0	0	0	0	...	0	0	0	0	...	0	0	0	0	...	0	0	0	0	...	0	0	0	0	...	0	0	0	0	...	0	0	0	0	<-	1
	53	0	0	0	0	...	0	0	0	0	...	0	0	0	0	...	0	0	0	0	...	0	0	0	0	...	0	0	0	0	...	0	0	0	0	...	0	0	0	0	...	0	0	0	0	...	0	0	0	0	<-	1
	54	0	0	0	0	...	0	0	0	0	...	0	0	0	0	...	0	0	0	0	...	0	0	0	0	...	0	0	0	0	...	0	0	0	0	...	0	0	0	0	...	0	0	0	0	...	0	0	0	0	<-	1
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	56	0	0	0	0	...	0	0	0	0	...	0	0	0	0	...	0	0	0	0	...	0	0	0	0	...	0	0	0	0	...	0	0	0	0	...	0	0	0	0	...	0	0	0	0	...	0	0	0	0	<-	1
	57	0	0	0	0	...	0	0	0	0	...	0	0	0	0	...	0	0	0	0	...	0	0	0	0	...	0	0	0	0	...	0	0	0	0	...	0	0	0	0	...	0	0	0	0	...	0	0	0	0	<-	1
	58	0	0	0	0	...	0	0	0	0	...	0	0	0	0	...	0	0	0	0	...	0	0	0	0	...	0	0	0	0	...	0	0	0	0	...	0	0	0	0	...	0	0	0	0	...	0	0	0	0	<-	1
	59	0	0	0	0	...	0	0	0	0	...	0	0	0	0	...	0	0	0	0	...	0	0	0	0	...	0	0	0	0	...	0	0	0	0	...	0	0	0	0	...	0	0	0	0	...	0	0	0	0	<-	1
	60	0	0	0	0	...	0	0	0	0	...	0	0	0	0	...	0	0	0	0	...	0	0	0	0	...	0	0	0	0	...	0	0	0	0	...	0	0	0	0	...	0	0	0	0	...	0	0	0	0	<-	1

Figure 1: Condensed A matrix Representation

The entire $\min c^T X$, $Ax \leq b$ formulation can be found in cmcd398 762 Assignment 3 Worksheet.xlxs. The time intervals index the end of the period. For example t = 1 is the end of the first interval.

1.1.2 Formulation: Matlab Implementation

The following script was used to formulate the problem and solve both the LP relaxation and IP.

Listing 1: LP Relaxation and IP Implementation

```
% Connor McDowall, cmcd398, 530913386
% This script conducts Question One for the problem
% load in the problem data

jobi = [1,2,3,4,5,6,7,8,9,10];
pi = [3,5,7,2,7,7,8,2,8,4];
di = [12,16,20,20,20,20,29,19,25,22];
ri = [0,7,16,16,21,8,4,12,11,17];

% Initialise the A matrix as an array for 10 jobs and 60 times intervals
A = [];
c = [];
A_add = zeros(70,1);
count = 1;
rollcol = 0;


% Use many for loops for the function
for i = 1:length(jobi)
    % Populate the A matrix with all possible time intervals
    for j = 1:(60 - pi(i)-ri(i)+1)
        % Add a new column to the A matrix
        A = [A,A_add];
        A(length(jobi) + count + (ri(i)):length(jobi)+ count +(ri(i)) + ...
            pi(i)-1,end) = 1;
        % Determine the lateness, therefore the tardiness for the cost
        % function
        c = [c,max(0, count +(ri(i)) + pi(i)-1-di(i))];
        count = count + 1;
     end
    % Add all the ones at the top of the Matrix for this job
    A(jobi(i),rollcol +1:rollcol +(60 - pi(i)-ri(i)+1)) = 1;
    % Reset the count and increment
    count = 1;
    % Increment the rolling number of columns to help add new columns
    rollcol = rollcol + (60 - pi(i)-ri(i)+1);
end

% Create the b matrix
b = ones(70,1);

% Save A matrix
% save A;

% Write the A matrix and Cost matrix into an excel file to check the
% Correct structure and show in assignment.
xlswrite('Amatrix.xlsx',A)
xlswrite('cmatrix.xlsx',c)
```

```

% Set up the bounds properly to get the correct mix of equality and
% inequality constraints
Aeq =A(1:10,1:end);
beq =b(1:10,1);
Aineq = A(11:end,1:end);
bineq = b(11:end,1);
lb = zeros(445,1);
ub = ones(445,1);
intcon = ones(445,1);

% Uses linprog to calculate the solution to the linear relaxation
X = linprog(c,Aineq,bineq,Aeq,beq,lb,ub);
% Use the sumproduct to work out what the minimum cost is
obj_LP_Relaxation = c*X;

% Uses intlinprog to calculate the solution to the integer programme
Xint = intlinprog(c,intcon,Aineq,bineq,Aeq,beq,lb,ub);

% Calculate the integer solutions objective value
obj_Int = c*Xint;

% Find out the start time, end time and tardiness of each job for the
% integer solution(Write to an excel file and work out manually).
xlswrite('Xint.xlsx',transpose(Xint))
```

1.2 Part 2

Job i	1	2	3	4	5	6	7	8	9	10
pi	3	5	7	2	7	7	8	2	8	4
di	12	16	20	20	20	20	29	19	25	22
ri	0	7	16	16	21	8	4	12	11	17
Start time	1	12	32	19	39	25	4	17	46	21
End time	4	17	39	21	46	32	12	19	54	25
Ti	0	1	19	1	26	12	0	0	29	3

The objective value function ($\min c^T x$) for both LP Relaxation and IP is 91 as the formulation creates naturally integer problem, therefore a naturally integer solution.

2 Question Two

2.1 Original LP Relaxation

The solution is z = 10.333.

	1	2	3	4	5	6
Variables:	0	0	0	0	0.333333	0.666667
Index:	1	2	3	4	5	6
Cost:	7	5	12	11	1	2
A	1	1	1	1	1	1
B	0	0	0	0	0	0
C	0	0	0	0	0	0
Shift 1	0	1	0	1	1	0
Shift 2	0	1	1	1	0	1
Shift 3	0	0	1	0	0	0
Shift 4	1	0	0	0	1	1
Shift 5	0	0	1	1	0	0
Shift 6	1	0	0	0	0	1
Shift 7	0	0	1	0	0	1

	7	8	9	10	11	12
Variables:	0	0	0.833333333	0	0	0.166667
Index:	7	8	9	10	11	12
Cost:	14	12	2	13	6	9
A	0	0	0	0	0	0
B	1	1	1	1	1	1
C	0	0	0	0	0	0
Shift 1	0	0	0	0	0	1
Shift 2	0	1	0	1	1	0
Shift 3	1	0	1	0	0	0
Shift 4	0	0	0	0	1	1
Shift 5	1	0	1	0	1	0
Shift 6	0	1	0	0	0	1
Shift 7	1	0	0	0	1	1

	13	14	15	16	17			
Variables:	0.166667	0.5	0	0	0.333333			
Index:	13	14	15	16	17			
Cost:	9	2	13	6	9	10.33333	Objective Value	
A	0	0	0	0	0	1	=	1
B	0	0	0	0	0	1	=	1
C	1	1	1	1	1	1	=	1
Shift 1	1	0	0	0	1	1	>=	1
Shift 2	0	0	1	0	1	1	>=	1
Shift 3	1	0	0	1	0	1	>=	1
Shift 4	1	0	0	0	0	1.333333	>=	1
Shift 5	1	1	0	0	0	1.5	>=	1
Shift 6	1	0	0	1	0	1	>=	1
Shift 7	1	0	0	1	0	1	>=	1

Staff- Shift Row Coverage	1	2	3	4	5	6	7
A	0.333333	0.666667	0	1	0	0.666667	0.666667
B	0.166667	0	0.833333	0.166667	0.833333	0.166667	0.166667
C	0.5	0.333333	0.166667	0.166667	0.666667	0.166667	0.166667

Shift-Shift Row Coverage	1	2	3	4	5	6	7
1		0.333333	0.166667	0.666667	0.166667	0.333333	0.333333
2			0	0.666667	0	0.666667	0.666667
3				0.166667	1	0.166667	0.166667
4					0.166667	1	1
5						0.166667	0.166667
6							1
7							

	UB:	Row Coverage	Type
Next Branch	B3	0.833333333	Staff - Shift
	13	0.166666667	Shift - Shift

Figure 2: The solution to the LP Relaxation with z = 10.333

2.2 Staff-Shift Constraint Branching

2.2.1 Depth = 1, Y_{B3}

Variables:	-1.5E-08	0	2.98E-08	-1E-07	0.333333	0.666667	-4.5E-08	0	1.000000066	0	0	0	0.333333	0.333333	0	0	0.333333			
Index:	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17			
Cost:	7	5	12	11	1	2	14	12	2	13	6	9	9	2	13	6	9	10.33333	Objective Value	
A	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	1	=	1
B	0	0	0	0	0	0	1	1	1	1	1	1	0	0	0	0	0	1	=	1
C	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	=	1
Shift 1	0	1	0	1	1	0	0	0	0	0	0	1	1	0	0	0	1	1	>=	1
Shift 2	0	1	1	1	0	1	0	1	0	1	1	0	0	0	1	0	1	1	>=	1
Shift 3	0	0	1	0	0	0	1	0	1	0	0	0	1	0	0	1	0	1.333333	>=	1
Shift 4	1	0	0	0	1	1	0	0	0	0	1	1	1	0	0	0	0	1.333333	>=	1
Shift 5	0	0	1	1	0	0	1	0	1	0	1	0	1	1	0	0	0	1.666667	>=	1
Shift 6	1	0	0	0	0	1	0	1	0	0	0	1	1	0	0	1	0	1	>=	1
Shift 7	0	0	1	0	0	1	1	0	0	0	1	1	1	0	0	1	0	1	>=	1
UB: B3	1	1	1	1	1	1	1	0	1	0	0	0	1	1	1	1	1			

Staff- Shift Row Coverage	1	2	3	4	5	6	7
A	0.333333	0.666667	2.98E-08	1	-7.2E-08	0.666667	0.666667
B	0	0	1	0	1	0	-4.5E-08
C	0.666667	0.333333	0.333333	0.333333	0.666667	0.333333	0.333333

Shift-Shift Row Coverage	1	2	3	4	5	6	7
1		0.333333	0.333333	0.666667	0.333333	0.333333	0.333333
2			2.98E-08	0.666667	-7.2E-08	0.666667	0.666667
3				0.333333	1.333333	0.333333	0.333333
4					0.333333	1	1
5						0.333333	0.333333
6							1
7							

	UB:	Row Coverage
Next Branch	A2	0.666666605
Key	1	0
UB:	Keep Column	Remove Column

Figure 3: The solution to the LP Relaxation with a $Y_{B3} = 1$ contraint branch and z = 10.333

2.2.2 Depth = 2, Y_{A2}

Variables:	0	0.5	2.98E-08	0	0	0.5	0	0	1	0	0	0	0.5	0.5	0	-9.9E-09	-3.3E-07			
Index:	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17			
Cost:	7	5	12	11	1	2	14	12	2	13	6	9	9	2	13	6	9	11	Objective Value	
A	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	1	=	1
B	0	0	0	0	0	0	1	1	1	1	1	1	0	0	0	0	0	1	=	1
C	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	=	1
Shift 1	0	1	0	1	1	0	0	0	0	0	0	1	1	0	0	0	1	1	>=	1
Shift 2	0	1	1	1	0	1	0	1	0	1	1	0	0	0	1	0	1	1	>=	1
Shift 3	0	0	1	0	0	0	1	0	1	0	0	0	1	0	0	1	0	1.5	>=	1
Shift 4	1	0	0	0	1	1	0	0	0	0	1	1	1	0	0	0	0	1	>=	1
Shift 5	0	0	1	1	0	0	1	0	1	0	1	0	1	1	0	0	0	2	>=	1
Shift 6	1	0	0	0	0	1	0	1	0	0	0	1	1	0	0	1	0	1	>=	1
Shift 7	0	0	1	0	0	1	1	0	0	0	1	1	1	0	0	1	0	1	>=	1
UB: B3	1	1	1	1	1	1	1	0	1	0	0	0	1	1	1	1	1			
UB:A2	0	1	1	1	0	1	1	0	1	0	0	0	1	1	1	1	1			

Staff- Shift Row Coverage	1	2	3	4	5	6	7
A	0.5	1	2.98E-08	0.5	2.98E-08	0.5	0.5
B	0	0	1	0	1	0	0
C	0.5	-3.3E-07	0.5	0.5	1	0.5	0.5

Shift-Shift Row Coverage	1	2	3	4	5	6	7
1		0.5	0.5	0.5	0.5	0.5	0.5
2			2.98E-08	0.5	2.98E-08	0.5	0.5
3				0.5	1.5	0.5	0.5
4					0.5	1	1
5						0.5	0.5
6							1
7							

	UB:	Row Coverage
Next Branch	A1	0.500000358
Key	1	0
UB:	Keep Column	Remove Column

Figure 4: The solution to the LP Relaxation with a $Y_{A2} = 1$ contraint branch and z = 11

2.2.3 Depth = 3, Y_{A1}

Variables:	0	1	0	0	0	0	-7.3E-07	0	1.000000732	0	0	0	1	-5E-07	-7.4E-07	1.58E-06	0			
Index:	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17			
Cost:	7	5	12	11	1	2	14	12	2	13	6	9	9	2	13	6	9	15.99999	Objective Value	
A	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	1	=	1
B	0	0	0	0	0	0	1	1	1	1	1	1	0	0	0	0	0	1	=	1
C	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	=	1
Shift 1	0	1	0	1	1	0	0	0	0	0	0	1	1	0	0	0	1	2	>=	1
Shift 2	0	1	1	1	0	1	0	1	0	1	1	0	0	0	1	0	1	0.999999	>=	1
Shift 3	0	0	1	0	0	0	1	0	1	0	0	0	1	0	0	1	0	2.000002	>=	1
Shift 4	1	0	0	0	1	1	0	0	0	0	1	1	1	0	0	0	0	1	>=	1
Shift 5	0	0	1	1	0	0	1	0	1	0	1	0	1	1	0	0	0	1.999999	>=	1
Shift 6	1	0	0	0	0	1	0	1	0	0	0	1	1	0	0	1	0	1.000002	>=	1
Shift 7	0	0	1	0	0	1	1	0	0	0	1	1	1	0	0	1	0	1.000001	>=	1
UB: B3	1	1	1	1	1	1	1	0	1	0	0	0	1	1	1	1	1			
UB:A2	0	1	1	1	0	1	1	0	1	0	0	0	1	1	1	1	1			
UB:A1	0	1	0	1	1	0	1	0	1	0	0	0	1	1	1	1	1			

Staff- Shift Row Coverage	1	2	3	4	5	6	7
A	1	1	0	0	0	0	0
B	0	0	1	0	1	0	-7.3E-07
C	1	-7.4E-07	1.000002	1	0.999999	1.000002	1.000002

Shift-Shift Row Coverage	1	2	3	4	5	6	7
1		1	1	1	1	1	1
2			0	0	0	0	0
3				1	2	1.000002	1.000001
4					1	1	1
5						1	0.999999
6							1.000002
7							

	UB:	Row Coverage
Next Branch		Max Depth Reached
Key	1	0
UB:	Keep Column	Remove Column

Figure 5: The solution to the LP Relaxation with a $Y_{A1} = 1$ contraint branch and z = 16

2.2.4 Branch and Bound Tree

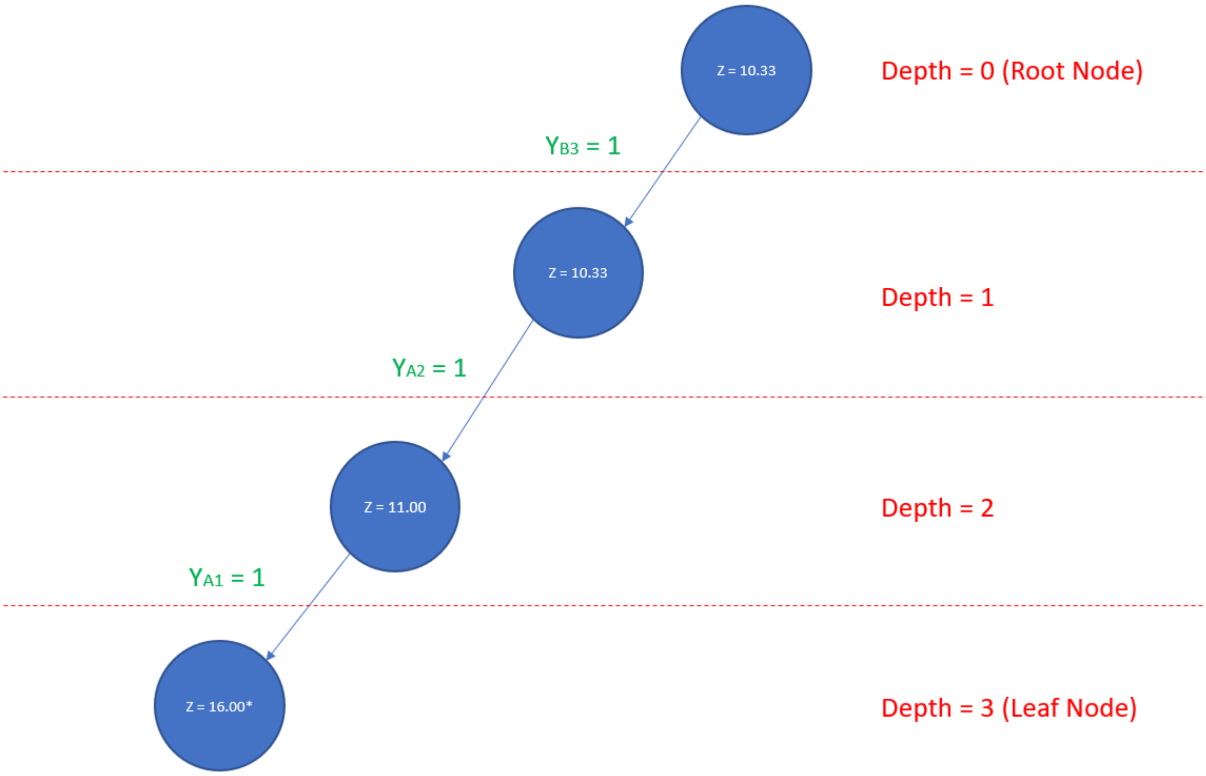

Figure 6: The constraint branch and bound tree with max depth of 3

2.3 Shift-Shift Constraint Branching

The row coverage table suggests you create contraints branches on the Y_{13} variable as per the conditions in the handout. This is the closest to integer branch with the smallest first row and smallest second row index.

2.3.1 Y_{13}

The problem wasn't resolved (objective function) as the question didn't ask for a new solution or to resolve the problem, hence z = 10.333 (it's the LP Relaxation)

Variables:	0	0	0	0	0.333333	0.666667	0	0	0.833333333	0	0	0.166667	0.166667	0.5	0	0	0.333333			
Index:	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17			
Cost:	7	5	12	11	1	2	14	12	2	13	6	9	9	2	13	6	9	10.333333	Objective Value	
A	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	1	=	1
B	0	0	0	0	0	0	1	1	1	1	1	1	0	0	0	0	0	1	=	1
C	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	=	1
Shift 1	0	1	0	1	1	0	0	0	0	0	0	1	1	0	0	0	1	1	>=	1
Shift 2	0	1	1	1	0	1	0	1	0	1	1	0	0	0	1	0	1	1	>=	1
Shift 3	0	0	1	0	0	0	1	0	1	0	0	0	1	0	0	1	0	1	>=	1
Shift 4	1	0	0	0	1	1	0	0	0	0	1	1	1	0	0	0	0	1.3333333	>=	1
Shift 5	0	0	1	1	0	0	1	0	1	0	1	0	1	1	0	0	0	1.5	>=	1
Shift 6	1	0	0	0	0	1	0	1	0	0	0	1	1	0	0	1	0	1	>=	1
Shift 7	0	0	1	0	0	1	1	0	0	0	1	1	1	0	0	1	0	1	>=	1
13 (0) Branch	1	1	1	1	1	1	1	1	1	1	1	1	0	1	1	1	1			
13 (1) Branch	1	0	0	0	0	1	0	1	0	1	1	0	1	1	1	0	0			

Staff- Shift Row Coverage	1	2	3	4	5	6	7
A	0.333333	0.666667	0	1	0	0.666667	0.666667
B	0.166667	0	0.833333	0.166667	0.833333	0.166667	0.166667
C	0.5	0.333333	0.166667	0.166667	0.666667	0.166667	0.166667

Shift-Shift Row Coverage	1	2	3	4	5	6	7
1		0.333333	0.166667	0.666667	0.166667	0.333333	0.333333
2			0	0.666667	0	0.666667	0.666667
3				0.166667	1	0.166667	0.166667
4					0.166667	1	1
5						0.166667	0.166667
6							1
7							

	UB:	Row Coverage
Proposed Branch	13	0.166666667
Key	1	0
(X) Branch	Keep Column	Remove Column

Figure 7: The solution to the LP Relaxation with a Y_{13} contraint branches (0 and 1) and the original LP relaxation.