2018

Semester 2

ENGSCI 363 Structures Assignment

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1 Equations and Assumptions.

I made the following assumptions.

- No deflection in the x or z direction.
- No transverse shear of rotation.
- There is only symmetrical loading or geometry.
- Selfweight of the spar can be neglected.

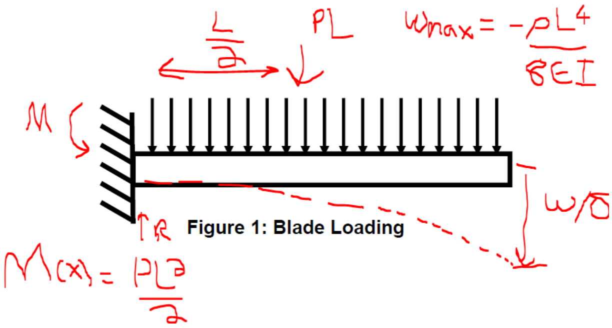


Figure 1: Bending Moment Diagram with relevant equations (Source: Design Brief)

First, the moment at the fixed end is:

$$M = \frac{PL^2}{2} = \frac{500}{2}Nm = 250Nm \tag{1}$$

$$\tag{2}$$

The following equations are used to calculate bending stress and deflection using the 2nd moment of area equation. The 2nd moment of area for a circular spar and a hollow circular spar are $I = \frac{\Pi r^4}{4}$ and $I = \frac{\Pi(r_{Outer}^4 - r_{Inner}^4)}{4}$ respectively. Therefore the formulas for bending stress and deflection are:

$$\sigma_r = \frac{Mr_{Outer}}{I} = \frac{1000}{\pi r^3} \text{ Pa} \tag{3}$$

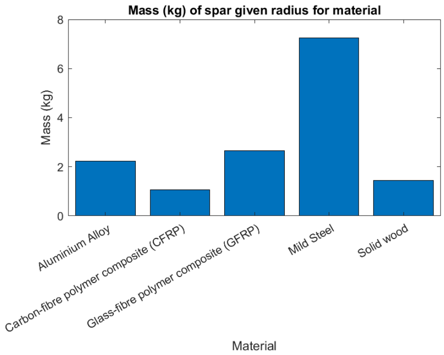
$$\delta = \frac{PL^4}{8EI} = \frac{500}{8EI}\text{m} \tag{4}$$

The volume of a circular solid and hollow rod are $V = \pi r^2 L$ and $V = \pi(r_{Outer}^2 - r_{Inner}^2)L$ respectively. These sets of equations were used in a matlab script to make the relevant calculations for the assignment.

2 Material Comparison

2.1 Method and Calculations

I set the deflection to the maximum deflection to find the radius for each material, then calculated the resulting stress from this radius. If this stress exceeded the allowable stress (Material Strength/FOS) and was smaller than the maximum allowable radius (0.025m), the radius was incremented until it was of a length where both strength and stiffness criteria were met, whilst being less than or equal to the maximum allowable radius. This method was applied to all materials. See the implementation in the appendix code 5 (All figures are to 2dp).



Material	Volume (mm^2)
Mild Steel	9.30×10^3
Aluminium Alloy	8.24×10^3
Solid Wood	24.08×10^3
Glass-Fibre Polymer Composite	14.01×10^3
Carbon - Fibre Polymer Composite	6.27×10^3

Figure 2: Mass(kg) per Material (Left), Material Volumes(Right)

Material	δ (mm)	σ_{Final} (MPa)	σ_b (MPa)	D (mm)	M (kg)	FoS
Mild Steel	4.53	62.49	250	34.41	7.25	4.00
Aluminium Alloy	16.54	74.99	300	32.38	2.22	4.00
Solid wood	11.29	15.00	60	55.37	1.44	4.00
Glass-Fibre Polymer Composite	20	33.79	400	42.24	2.66	11.84
Carbon - Fibre Polymer Composite	20	112.99	800	28.25	1.07	7.08

Table 1: Material Comparison

3 Geometry Comparison

3.1 Methodology, Failure Modes and Calculations

I started with the maximum thickess for each material. I calculated the 2nd moment of area, stress and stiffness using the inner and outer radii. If both stiffness and stength criteria were met and the volume was lower than previous iterations (initial volume set at infinity), the inner radius, volume, stress, displacement and mass were saved. The inner radius was slowly incremented until just below the outer radius. The outer radius was set at the maximum (0.025m). With a larger outer radius, the second moment of area increases more proportionally than the increase in the vertical distance from the centroid. The second moment of area also increases with a a larger inner radius. Subsequently, the bending stress and deflection decrease. Therefore, the thickness can be as small as possible, minimising volume and therefore cost. See the code in the appendix 5.

The following failure modes have not been considered in our analysis

- Wave buckling
- Local buckling
- Smooth kinking
- Torsion
- Shearing
- Puncture

Material	δ (mm)	σ_{Final} (MPa)	D (Inner,(mm)	t (mm)	FoS
Aluminium Alloy	10.38	74.98	46.19	1.91	4.00
Glass - Fibre Polymer Composite	19.59	39.99	41.85	4.08	11.84
Carbon - Fibre Polymer Composite	16.98	199.85	48.67	0.66	7.08

Table 2: Geometry Comparison (2dp, r_{Outer} of 25mm)

Material	Volume (mm^3)	Mass (kg)
Aluminium Alloy	2.88×10^3	0.77
Glass Fibre Polymer Composite	5.88×10^3	1.11
Carbon Fibre Polymer Composite	1.03×10^3	0.17

Table 3: Geometry Comparison Volumes (2dp, r_{Outer} of 25mm)

4 Design Recommendation

A hollow carbon fibre polymer composite rod meets both the bending and stiffness criteria whilst delivering the smallest mass of 0.17kg.

4.1 Durability

The carbon fibre polymer composite is the most durable of the five materials. Carbon fibre doesn't deteriorate over time unlike wood (rots unless sealed/treated) and mild steel (corrodes unless treated). Carbon fibre does not thermally expand or shrink as much as the other materials. Therefore, carbon fibre can function in a wide range of operating environments. Carbon fibre has a higher strength(1st) and stiffness(2nd) thresholds than the other materials, therefore the hollow tube can have a a lot thinner radius than the other materials. Carbon fibre is light weight yet strong.

4.2 Cost

Carbon fibre is the most expensive of the five materials. The other materials are listed in descending order by cost: glass fibre polymer composite, aluminium alloy, mild steel, and solid wood. The hollow materials are cheaper as use less material, however are more complicated to manufacture resulting in other manufacturing costs. The hollow carbon fibre polymer composite rod would be cheaper than the solid variant.

4.3 Durability

Carbon fibre polymer composite is the most difficult to manufacture. A precursor is prepared, drawm into long strands/fibres. The fibres are stabilised first then carbonised, drawing non carbon atoms out of the working material by heating them at high temperatures. The surface is treated thereafter through slight oxidation. The fibres are sized to protect them during wrapping. The fibre sheets are them wound into shapes (solid circular rods) or wrapped in molds to be heated and pressed. Fibre glass is manufactured through melting raw materials and forming fibres from the pot. Solid wood is the easiest to manufacter (carve), followed by steel (melt and set in molds) then aluminium alloys (melt, set in molds and shaped). Manufacturing hollow rods adds complexity to the process.

4.4 Conclusion

A hollow carbon fibre rod meets the strength and stiffness requirements, minimises mass, volume and maximises durability. However, It is one of the most costly and difficult to manufacture.

5 Appendices: Code

Listing 1: Calculations Script

```
% This script calculates the values required for the assignment
% Connor McDowall cmcd398 530913386

% Place in Inputs
del = 500; %N
moment = 250;
len = 1; %m
deflect = 0.02; %m
maxdiameter = 0.05; %m
fos = 4; % Factor of safety
names = {'Mild Steel','Aluminium Alloy','Solid wood','Glass-fibre polymer composite (GFRP)','Carbon-fibre polymer composite (CFRP)'};
density = [7800,2700,600,1900,1700]; %kg/m^3
YoungsModulus = [200,70,12,20,100]; %GPa
Strength = [250,300,60,400,800]; %MPa

% Convert the units
YoungsModulus = YoungsModulus.*1e9; %Convert to Pa
Strength = Strength.*1e6; %Convert to Pa

% Calculate 2nd Moment of Area for maximum deflection
SMA = ((del.*len.^4)./(8.*YoungsModulus.*deflect));

% Calculate radius
r = ((4.*SMA)./pi()).^(1/4);

% Calculate from resulting material using deflection radius
stress = ((4.*moment)./(pi().*(r.^3)));
maximumStress = (Strength./fos);

% Set condition to increment radius if maximum stress exceeds strength
% until is doesn't. Both the deflection and stress will decrease

for i = 1:5
    if (stress(i)> maximumStress(i) && r(i) < maxdiameter/2)
        while stress(i)> maximumStress(i)
            r(i) = r(i) + 0.00001; %increment radius by 0.01mm
            stress(i) = (1000./(pi().*r(i).^3));
        end
    end
end

% Calculate ending stress and deflection
FinalDiameter = r.*2.*1000;
FinalStress = stress;
FinalDeflection = (del*4./((r.^4)*8.*YoungsModulus.*pi()))*1000;
```

```
FinalFoS = Strength./FinalStress;
FinalStress = FinalStress./1e6; %MPa
FinalStrength = Strength./1e6; %MPa

% Plot a bar graph

% Calculate the volume
volume = pi().*r.^2.*len;
% Calculate the mass
mass = density.*volume;

% Plot the bar graph
c =categorical(names);
bar(c,mass);
xlabel('Material');
ylabel('Mass (kg)');
title('Mass (kg) of spar given radius for material')
saveas(gcf,'bar.png');

% Part Two
% Determine suitable wall dimensions
% Create new input vectors
namesGeo = {'Aluminium Alloy','Glass-fibre polymer composite (
    GFRP)','Carbon-fibre polymer composite (CFRP)'};
densityGeo = [density(2),density(4),density(5)];
YoungsModulusGeo = [70,20,100]*1e9; %GPa
StrengthGeo = [300,400,800]*1e6; %MPa

% Set thickess to be 0.1mm
t = [0.001,0.001,0.001];

% Initialise volume, make as large as possible.
VolFinal = [inf,inf,inf];
StressGeoFinal = [0,0,0];
StiffGeoFinal = [0,0,0];
MassGeoFinal = [0,0,0];
rin = [0,0,0];
FosGeoFinal = [0,0,0];
rnom = 0.025;

for i = 1:3
    % Can't have them equal eachother otherwise not a hollow tube
        .
    while rin(i) < rnom
        % Calculate the 2nd moment of area
        SMAGeo(i) = (pi()./4).*((rnom.^4)-(rin(i).^4));
        % Calculate stress and stiffness (Displacement)
        StressGeo(i) = moment.*rnom./SMAGeo(i);
        StiffGeo(i) = ((del.*len.^4)./(8.*YoungsModulusGeo(i).*
            SMAGeo(i)));
        % Calculate volume
```

```
        VolGeo(i) = pi().*len.*(rnom^2 - rin(i)^2);
        % Check, if criteria met, save the minimum volume.
        if (StressGeo(i)<= (StrengthGeo(i)/4)) && (StiffGeo(i) <=
            deflect) && (VolGeo(i)<= VolFinal(i))
            VolFinal(i) = VolGeo(i)*1e7;
            StressGeoFinal(i) = (StressGeo(i)/1e6);
            FosGeoFinal(i) = (StrengthGeo(i)/1e6)/StressGeoFinal
               (i);
            StiffGeoFinal(i) = StiffGeo(i)*1000;
            MassGeoFinal(i) = VolGeo(i)*densityGeo(i);
            Tfinal(i) = (rnom -rin(i))*1000;
            dinFinal(i) = rin(i)*2*1000;
        end
        rin(i) = rin(i) + 0.000001;
    end
end
```