2018

Semester 2

ENGSCI 331 Finite Differences

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October 3, 2018

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1 Part 1

1.1 Visualisations

See 1 for the one dimensional comparison and 2 for the two dimensional contour plot.

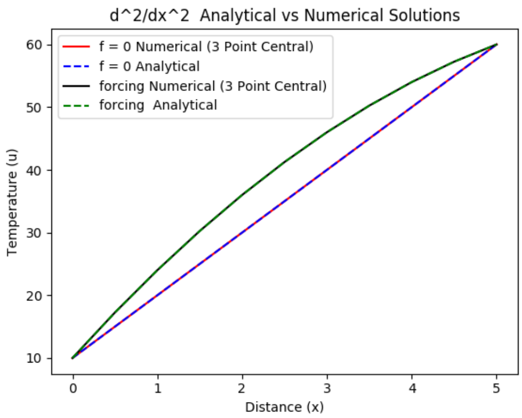

Figure 1: 1D $\frac{d^2u}{dx^2} = 0$: Numerical vs Analytical Comparison

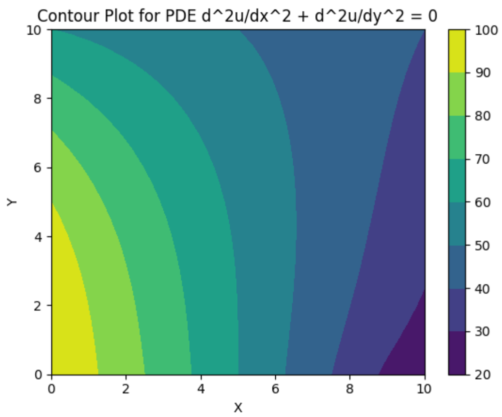

Figure 2: Contour plot for $\frac{d^2u}{dx^2} + \frac{d^2u}{dy^2} = 0$

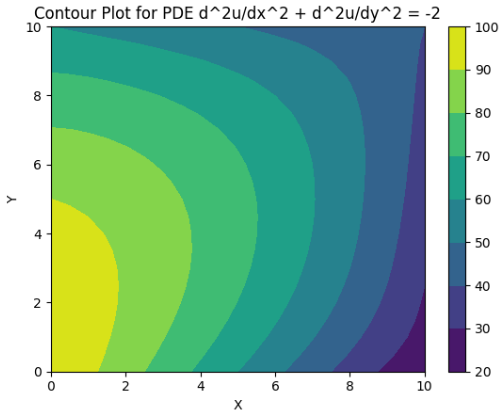

Figure 3: Contour plot for $\frac{d^2u}{dx^2} + \frac{d^2u}{dy^2} = -2$

1.2 Questions

Q1A

The right hand side is an external heat source applied to the system. This is evident as the forcing terms on figure 1 with a higher temperature accross each distance value compared to the non forcing function

Q1B

Increase the number of finite points you use in the stencil, decreasing the truncation error.

Q2

2D interpolation techniques would provide a reasonable approximation. Bilinear intepolation uses linear intepolation in 2D dimensions. Both the finite difference and 2D interpolation techniques use existing points to solve unknown points. Both methods form systems of linear equations to solve unknowns via linear solvers. When considering how the boundary conditions are expressed (as analytic equations), new points are easy to estimate. Both methods may be used.

2 Part 2

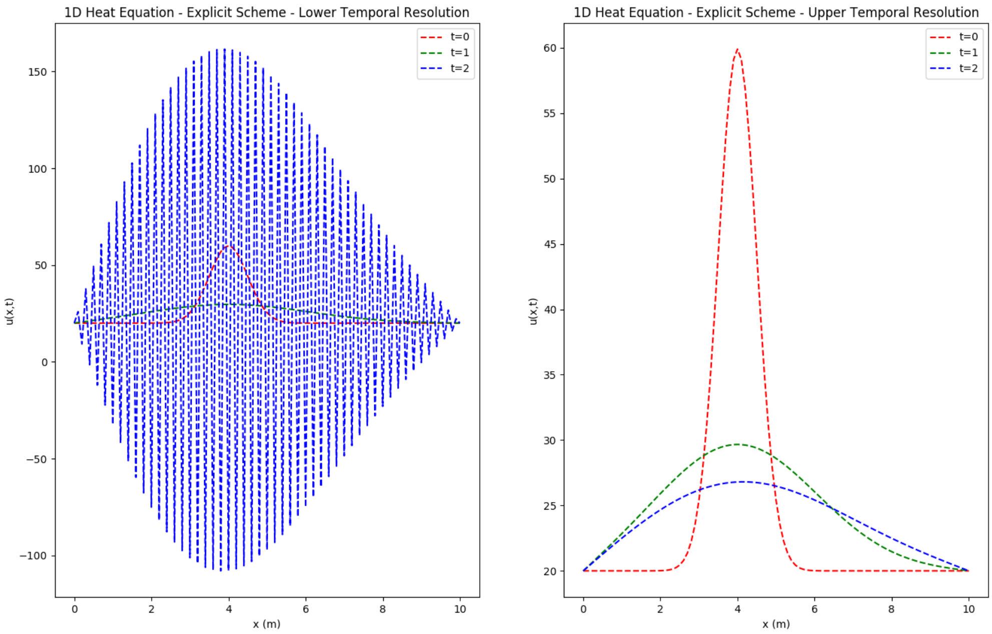

Figure 4: Explicit Scheme Upper and Lower Temporal Resolution Comparison for $\frac{\delta^2 u}{\delta x^2} - \alpha \frac{\delta u}{\delta t} = 0$

Q3

The higher resolution is numerically more accurate. The implicit method with the lower temporal solution oscillates wildly, as seen in 4 on the left. Our numerical model may have exceeded an r value (¡0.5) . The time step would have been too large, resulting in this numerical instability and oscillating behaviour.

2.1 Task 4

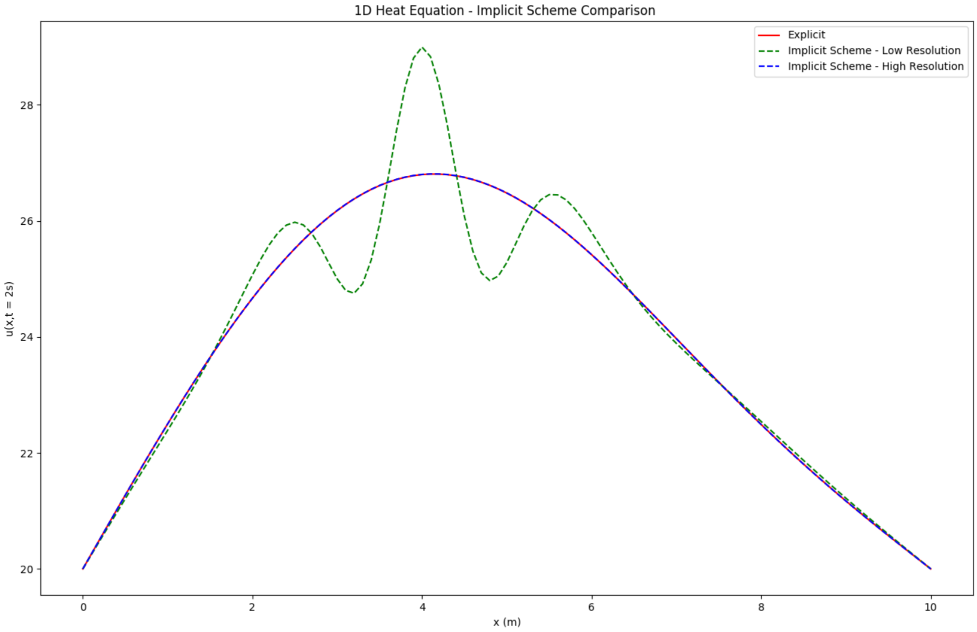

Figure 5: Scheme comparison for lower and upper resolutions for $\frac{\delta^2 u}{\delta x^2} - \alpha \frac{\delta u}{\delta t} = 0$

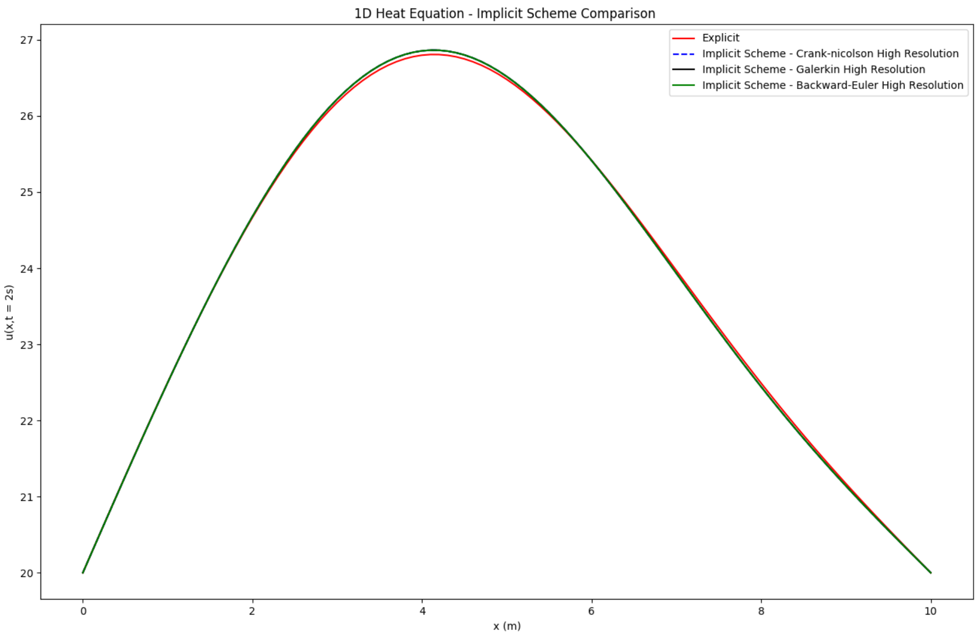

Figure 6: Scheme comparison for upper resolutions frt $\frac{\delta^2 u}{\delta x^2} - \alpha \frac{\delta u}{\delta t} = 0$

Q4a

Implicit methods are more numerically stable and are convergent but are computationally more intensive numerically than the explicit methods, therefore less efficient. The methods can use relatively large time steps and still converge. Implicit methods are computationally strenous as a system of simulataneous equations must be solved at each time step.

Q4b

It is reasonable. As seen on 5, the r value exceeds the stability threshold for the lower temporal resolution. With the higher temporal resolution, the numerical stability threshold is met accross all implicit methods (6). It is reasonable to assume, as shown with the explicit and imlpicit methods being apporximately the same. The implicit methods should be more accurate than the explicit. If they are about the same, they are reasonable.

3 Part 3

3.1 Task 5

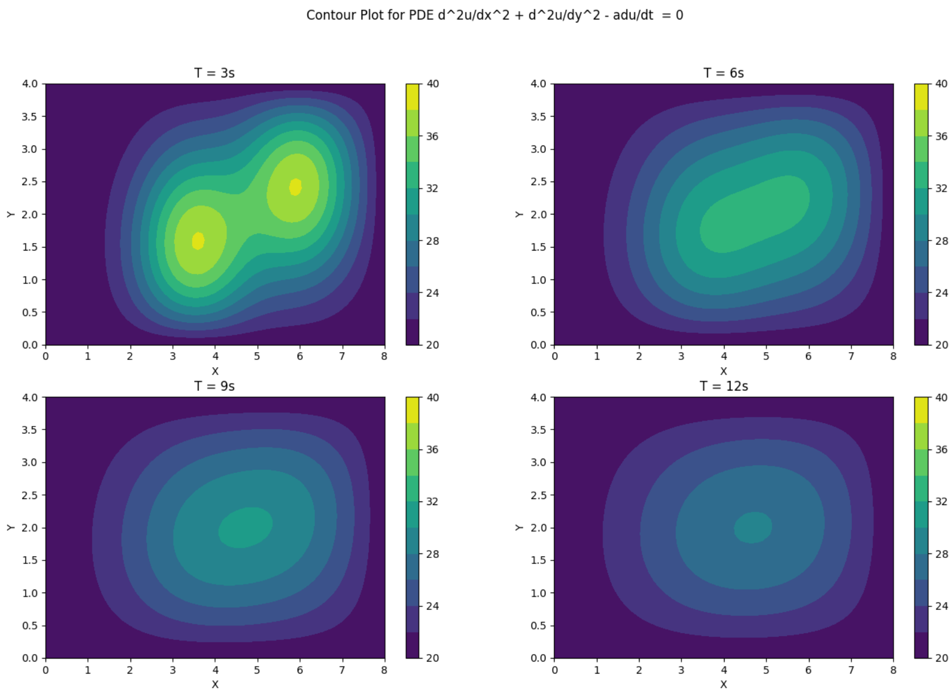

Figure 7: PDE for $\frac{\delta^2 u}{\delta x^2} + \frac{\delta^2 u}{\delta y^2} - \alpha \frac{\delta u}{\delta t} = 0$ where $\alpha = 10$

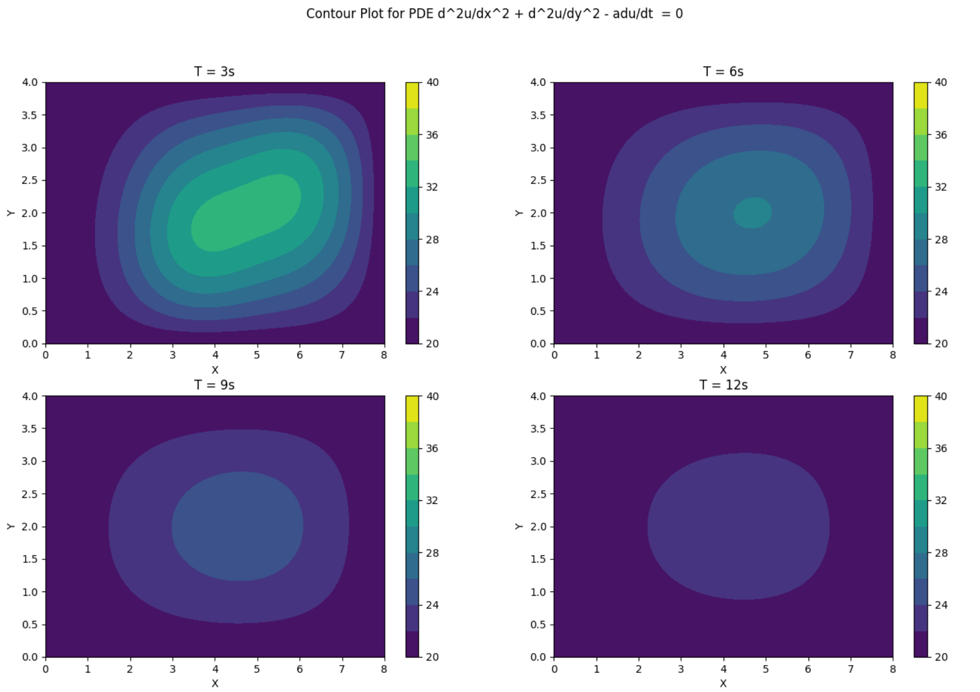

Figure 8: PDE for $\frac{\delta^2 u}{\delta x^2} + \frac{\delta^2 u}{\delta y^2} - \alpha\frac{\delta u}{\delta t} = 0$ where $\alpha = 5$

3.1.1 Q5

The temperature distribution diminishes over time. Initial, there are large temperarture discrepancies. As time progresses, the discrepancies decreases and the peak temperatures are lower. Contour bands are larger and cooler as time progresses.

As seen in 8, the peak temperatures are much lower with the overall surface cooler. A lower α (thermal diffusivity) decreases the rate heat is transferred from hot parts of the plate to cooler parts. For this reason, the contours are wider with less intense heat as the plate cools rather than transferring heat. This behaviour decreases the heat flow efficiency through the plate.