

# WORK PACKAGE

## Calculation Sheet

Work pack number \_\_\_\_\_ Date \_\_\_\_\_  
 Work pack name \_\_\_\_\_  
 Subject ENGSCI 762 Scheduling  
 By Connor McDowell (cmcd39) Page No 1 of 7

### Question One

Machine shortest processing time.  
 For simplicity,  $i(k)$  is the time taken to process job  $k$ .

We make the assumption  $\frac{w_i(1)}{p_i(1)} \geq \frac{w_i(2)}{p_i(2)} \geq \dots \geq \frac{w_i(n)}{p_i(n)}$ .

Let Job  $i(k+1)$  finish at time  $t$ .

$i(k-1) | i(k) | i(k+1) | i(k+2)$  we assume this ordering is not optimal.

In this sequence, Job  $i(k)$  has a weighted completion time of  $w_i(k) C_i(k) = w_i(k)(t + p_i(k))$ .

Job  $i(k+1)$  has a weighted completion time of  $w_i(k+1) C_i(k+1) = w_i(k+1)(t + p_i(k) + p_i(k+1))$ .

Therefore, the weighted mean completion time is.

$$WC_1 = \sum_{j=1}^n w_{ij} C_{ij}$$

$$\begin{aligned} &= \bar{WC} + (w_i(k) C_i(k)) + (w_i(k+1) C_i(k+1)) \\ &= \bar{WC} + w_i(k)(t + p_i(k)) + w_i(k+1)(t + p_i(k) + p_i(k+1)). \end{aligned}$$

$$\text{where } \bar{WC} = \sum_{j=1}^{k-1} w_{ij} C_{ij} + \sum_{j=k+2}^n w_{ij} C_{ij}.$$

$\bar{WC}$  will remain unchanged when reordering.

Reorder

$i(k-1) | i(k+1) | i(k) | i(k+2)$

Now Weighted Completion Times.

$$WC_2 = \bar{WC} + w_i(k+1)(t + p_i(k+1)) + w_i(k)(t + p_i(k+1) + p_i(k))$$

Change in the weighted completion time.

$$\begin{aligned} WC_2 - WC_1 &= (\bar{WC} + w_i(k+1)(t + p_i(k+1)) + w_i(k)(t + p_i(k+1) + p_i(k))) \\ &\quad - (\bar{WC} + w_i(k)(t + p_i(k)) + w_i(k+1)(t + p_i(k) + p_i(k+1))) \end{aligned}$$

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$$SO WC_2 - WC_1 = w_i(k)(p_i(k+1)) - w_i(k+1)(p_i(k))$$

After eliminating like terms.

We are considering

$$\frac{w_i(1)}{p_i(1)} > \frac{w_i(2)}{p_i(2)} > \frac{w_i(3)}{p_i(3)}$$

This assumes the ratios between weights & processing times are larger than subsequent ratios, minimising the total weighted completion time.

We now consider

$$\frac{w_i(k)}{p_i(k)} < \frac{w_i(k+1)}{p_i(k+1)} \quad ①$$

Take

$$= w_i(k) p_i(k+1) - w_i(k+1) p_i(k)$$

$$= \frac{w_i(k) p_i(k+1)}{p_i(k) p_i(k+1)} - \frac{w_i(k+1) p_i(k)}{p_i(k) p_i(k+1)}$$

$$= \frac{w_i(k)}{p_i(k)} - \frac{w_i(k+1)}{p_i(k+1)}$$

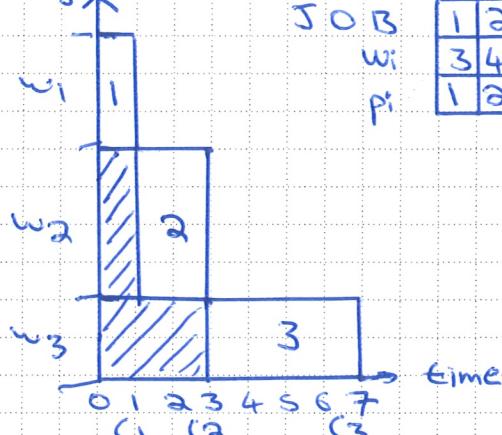
$$= WC_2 - WC_1 = \frac{w_i(k)}{p_i(k)} - \frac{w_i(k+1)}{p_i(k+1)}$$

Therefore due to ①,  $WC_2 < WC_1$ .

Swapping  $i(k+1)$  &  $i(k)$  produced a better solution so the original pairing order cannot be an optimal solution so you continue swaps, therefore we assume if this is the last swap, this is the optimal solution.

Geometric Representation

Weight



$$\begin{aligned} \sum w_i c_i &= \boxed{1} + \boxed{2} + \boxed{3} + \boxed{4} \\ &= 3 + 8 + 8 + 10 \\ &= 29 \end{aligned}$$

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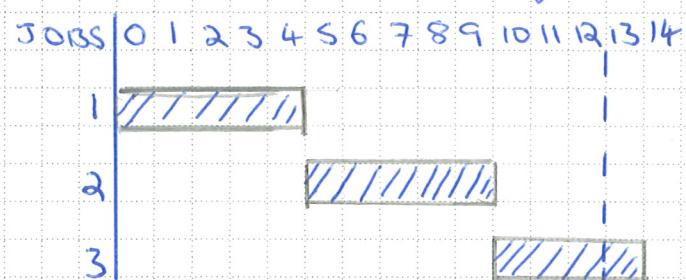
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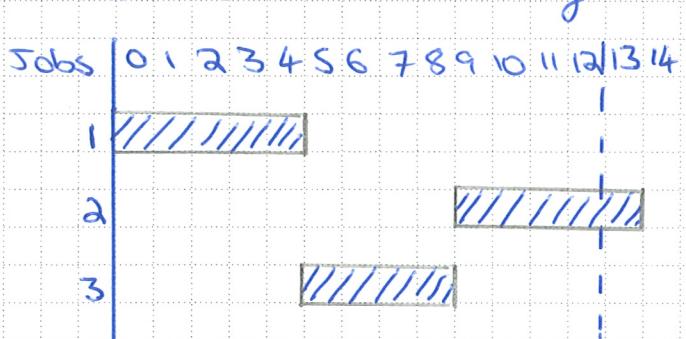
Question 2

JOBS	1	2	3
$r_i$	0	2	5
$d_i$	13	13	13
$C_{in}$	5	10	14
$L_{in}$	-8	-3	1
$p_i$	5	5	4
$C_{(a)}$	5	14	9
$L_{(a)}$	-8	1	-4

(1).  $r_i$  is in Non decreasing order:



$1/r_i, d_i = l_{max}$



Both  $l_{max} = 10$

b).  $1/r_i / C_{max}$ .

If due dates are ~~∞~~ <sup>∞</sup> (as in don't exist)  
 Then  $C_{max} = l_{max}$ .

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## Calculation Sheet

### Question Three

$i: r_i = 0, d_i = d, \text{prec} / L_{\max}$ .

1: $P = 4$
$r = 7$
$r' = 7$

2: $P = 7$
$r = 13$
$r' = 13$

$p = \text{process time}$   
 $r = \text{current release date}$   
 $r' = \text{updated release date}$

3: $P = 7$
$r = 20$
$r' = 20$

4: $P = 12$
$r = 30$
$r' = 30$

5: $P = 2$
$r = 15$
$r' = 37$

6: $P = 6$
$r = 14$
$r' = 14$

7: $P = 8$
$r = 29$
$r' = 29$

9: $P = 4$
$r = 21$
$r' = 37$

8: $P = 2$
$r = 19$
$r' = 20$

10: $P = 6$
$r = 24$
$r' = 24$

WORKING: Update the release dates to reflect the maximum value between the (precedence release date + the precedence process time) & the current nodes release date.

NODE 1:  $r = r' = 7$

NODE 2:  $\text{Max}(7+4, 13) = 13$

NODE 3:  $\text{Max}(7+4, 20) = 20$

NODE 4:  $\text{Max}(7+20, 30) = 30$

NODE 5:  $\text{Max}(8+29, 20+7, 15) = 37$

NODE 6:  $r = r' = 14$

NODE 7:  $\text{Max}(14+6, 29) = 29$

NODE 8:  $\text{Max}(6+14, 29) = 29$

NODE 9:  $\text{Max}(8+29, 21) = 37$

NODE 10:  $\text{Max}(20+2, 24) = 24$

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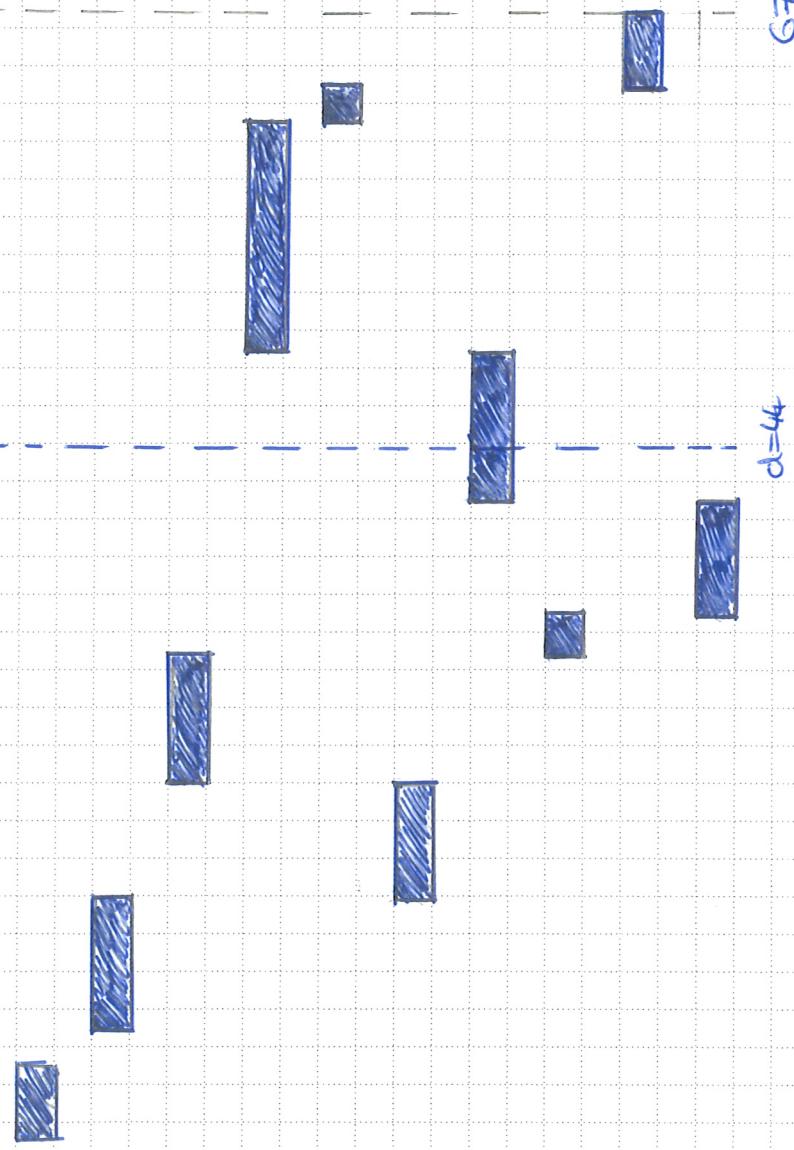
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Question 3b Gantt chart.

Line

02 468 0194 1618 02638 00324 28505 03455 00548 00684 00682

1 2 3 4 5 6 7 8 9 10



d=4

G7



Start

Each box represents 2 time units.  
 Length of tasks written the numbers in the box

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Question 4: IP Model

$1 \leq d_i, r_i \leq L_{\max}$

Let  $t_j$  be the time the job starts  
 $x_{ij}$  be 1 if Job  $i$  precedes Job  $j$ .

Minimise  $L_{\max}$

s/t

$$x_{ji} + x_{ij} = 1 \quad \forall i, j \neq j$$

$$t_j \geq t_i + p_i - M x_{ji} \quad \forall i, j \neq j$$

$$L_{\max} \geq t_i + p_i - d_i \quad \forall i$$

$$t_i \geq r_i \quad \forall i$$

$$x_{ji} = 1 \quad j \in P_i \quad \forall i$$

where

$$i \in \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$$

$$j \in \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$$

$$P_i \in \{[7, [1], [1], [3], [3, 7], [1], [6], [6], [7], [8]\}$$

$$p_i \in \{3, 5, 7, 2, 7, 7, 8, 2, 8, 4\}$$

$$d_i \in \{12, 16, 20, 20, 29, 19, 25, 22\}$$

$$r_i \in \{0, 7, 16, 16, 21, 8, 4, 22, 11, 17\}$$

$M > 0$  is a 'sufficiently large' number.

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### Question 5

The scoring equation used was: (Dispatch Rule).

$$I_j(t) = \frac{w_j / (p_j + s_{ij})}{e^{\max(d_j - (p_j + s_{ij}) - t, 0)} / K\bar{p}}$$

Where  $K$  is the scaling parameter  $K=1.6$ .

$w_j$  is the weight for job  $j$

$p_j$  is the processing time for job  $j$ .

$s_{ij}$  is the setup time for the colours from job  $i$  to  $j$ .

$d_j$  is the due date of job  $j$ .

$t$  is the time

$\bar{p}$  is the average processing time of the remaining jobs including job  $j$  itself.

The test score achieved was 650.700

Test Score: 650.700.

K	Jobs	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	From\To	1	2	3	k	2	Range								
1.6	p_j	11	12	7	2	11	14	8	16	14	10	7	10	15	3	2	10	13	6		1	0	5	7										
650.7000	r_j	4	8	7	47	31	11	13	55	45	19	71	28	9	11	22	14	28	16		2	7	0	9										
	d_j	66	123	36	166	114	70	189	122	120	58	152	182	213	70	199	34	86	171		3	2	5	0										
	Colour	1	2	3	1	2	3	1	2	3	1	2	3	1	2	3	1	2	3															
	wj	9	7	1	3	4	7	5	9	4	2	8	6	6	4	10	1	10	1															
Time=	0 Col= 1	Score=	0.021947726	0.000385493	0.02	3.1E-05	0	0.01	0	0	0	0.01	0	0	0	0.01	0	0.02	0.01	0	BestScore=	0.02 Best Job=	1 pj= 11	rj= 4	wj= 9	dj= 66	col= 1	Setup=	0	Start= 4	Cj= 15	Tj=	0	Fj= 11
Time=	15 Col= 1	Score=	0.05744538	0.000977735	0.04	7.6E-05	0	0.03	0	0	0	0.02	0	0	0	0.02	0	0.06	0.02	0	BestScore=	0.06 Best Job=	16 pj= 10	rj= 14	wj= 1	dj= 34	col= 1	Setup=	0	Start= 15	Cj= 25	Tj=	0	Fj= 11
Time=	25 Col= 1	Score=	0.110728868	0.001859769	0.07	0.00014	0	0.07	0	0	0	0.04	0	0	0	0.04	0	0.1	0.03	0	BestScore=	0.07 Best Job=	3 pj= 7	rj= 7	wj= 1	dj= 36	col= 3	Setup=	7	Start= 32	Cj= 39	Tj=	3	Fj= 32
Time=	39 Col= 3	Score=	0.276496547	0.005093522	0.14	0.00024	0.01	0.16	0	0.01	0	0.11	0	0	0	0.11	0	0.08	0.08	0	BestScore=	0.16 Best Job=	6 pj= 14	rj= 11	wj= 7	dj= 70	col= 3	Setup=	0	Start= 39	Cj= 53	Tj=	0	Fj= 42
Time=	53 Col= 3	Score=	0.692307692	0.011307778	0.14	0.00046	0.01	0.41	0	0.02	0.01	0.17	0	0	0	0.27	0	0.08	0.2	0	BestScore=	0.27 Best Job=	14 pj= 3	rj= 11	wj= 4	dj= 70	col= 2	Setup=	5	Start= 58	Cj= 61	Tj=	0	Fj= 50
Time=	61 Col= 2	Score=	0.5	0.023209285	0.06	0.00068	0.02	0.3	0	0.03	0.02	0.12	0.01	0	0	0.91	0	0.06	0.35	0	BestScore=	0.35 Best Job=	17 pj= 13	rj= 28	wj= ##	dj= 86	col= 2	Setup=	0	Start= 61	Cj= 74	Tj=	0	Fj= 46
Time=	74 Col= 2	Score=	0.5	0.050049202	0.06	0.00135	0.05	0.3	0	0.07	0.04	0.12	0.01	0	0	1.33	0	0.06	0.77	0	BestScore=	0.12 Best Job=	10 pj= 10	rj= 19	wj= 2	dj= 58	col= 1	Setup=	7	Start= 81	Cj= 91	Tj=	33	Fj= 72
Time=	91 Col= 1	Score=	0.818181818	0.151296047	0.07	0.01148	0.16	0.33	0	0.22	0.11	0.2	0.03	0	0	0.5	0	0.1	0.56	0	BestScore=	0.22 Best Job=	8 pj= 16	rj= 55	wj= 9	dj= 122	col= 2	Setup=	5	Start= 96	Cj= 112	Tj=	0	Fj= 57
Time=	112 Col= 2	Score=	0.5	0.583333333	0.06	0.01315	0.36	0.3	0	0.56	0.17	0.12	0.11	0.01	0	1.33	0	0.06	0.77	0	BestScore=	0.58 Best Job=	2 pj= 12	rj= 8	wj= 7	dj= 123	col= 2	Setup=	0	Start= 112	Cj= 124	Tj=	1	Fj= 116
Time=	124 Col= 2	Score=	0.5	0.583333333	0.06	0.02805	0.36	0.3	0.01	0.56	0.17	0.12	0.24	0.02	0	1.33	0.01	0.06	0.77	0.01	BestScore=	0.36 Best Job=	5 pj= 11	rj= 31	wj= 4	dj= 114	col= 2	Setup=	0	Start= 124	Cj= 135	Tj=	21	Fj= 104
Time=	135 Col= 2	Score=	0.5	0.583333333	0.06	0.05976	0.36	0.3	0.02	0.56	0.17	0.12	0.52	0.04	0	1.33	0.01	0.06	0.77	0.01	BestScore=	0.52 Best Job=	11 pj= 7	rj= 71	wj= 8	dj= 152	col= 2	Setup=	0	Start= 135	Cj= 142	Tj=	0	Fj= 71
Time=	142 Col= 2	Score=	0.5	0.583333333	0.06	0.10541	0.36	0.3	0.03	0.56	0.17	0.12	0.91	0.06	0.01	1.33	0.03	0.06	0.77	0.02	BestScore=	0.17 Best Job=	9 pj= 14	rj= 45	wj= 4	dj= 120	col= 3	Setup=	9	Start= 151	Cj= 165	Tj=	45	Fj= 120
Time=	165 Col= 3	Score=	0.692307692	0.411764706	0.14	0.75	0.25	0.5	0.15	0.43	0.29	0.17	0.67	0.33	0.02	0.5	0.31	0.08	0.56	0.17	BestScore=	0.75 Best Job=	4 pj= 2	rj= 47	wj= 3	dj= 166	col= 1	Setup=	2	Start= 167	Cj= 169	Tj=	3	Fj= 122
Time=	169 Col= 1	Score=	0.818181818	0.411764706	0.07	1.5	0.25	0.33	0.25	0.43	0.19	0.2	0.67	0.35	0.04	0.5	0.22	0.1	0.56	0.08	BestScore=	0.35 Best Job=	12 pj= 10	rj= 28	wj= 6	dj= 182	col= 3	Setup=	7	Start= 176	Cj= 186	Tj=	4	Fj= 158
Time=	186 Col= 3	Score=	0.692307692	0.411764706	0.14	0.75	0.25	0.5	0.5	0.43	0.29	0.17	0.67	0.6	0.16	0.5	2.06	0.08	0.56	0.17	BestScore=	2.06 Best Job=	15 pj= 2	rj= 22	wj= ##	dj= 199	col= 3	Setup=	0	Start= 186	Cj= 188	Tj=	0	Fj= 166
Time=	188 Col= 3	Score=	0.692307692	0.411764706	0.14	0.75	0.25	0.5	0.5	0.43	0.29	0.17	0.67	0.6	0.21	0.5	2.79	0.08	0.56	0.17	BestScore=	0.5 Best Job=	7 pj= 8	rj= 13	wj= 5	dj= 189	col= 1	Setup=	2	Start= 190	Cj= 198	Tj=	9	Fj= 185
Time=	198 Col= 1	Score=	0.818181818	0.411764706	0.07	1.5	0.25	0.33	0.63	0.43	0.19	0.2	0.67	0.35	0.4	0.5	1.11	0.1	0.56	0.08	BestScore=	0.4 Best Job=	13 pj= 15	rj= 9	wj= 6	dj= 213	col= 1	Setup=	0	Start= 198	Cj= 213	Tj=	0	Fj= 204
Time=	213 Col= 1	Score=	0.818181818	0.411764706	0.07	1.5	0.25	0.33	0.63	0.43	0.19	0.2	0.67	0.35	0.4	0.5	1.11	0.1	0.56	0.08	BestScore=	0.08 Best Job=	18 pj= 6	rj= 16	wj= 1	dj= 171	col= 3	Setup=	7	Start= 220	Cj= 226	Tj=	55	Fj= 210

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