

UNIVERSITY OF AUCKLAND
DEPARTMENT OF ACCOUNTING & FINANCE

Finance 788: Research Essay

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requirements for the degree of Bachelor of Commerce
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1 Acknowledgements

Paul Geertsema

2 Abstract

3 Introduction (3 Pages)

4 Research Question(s)

Can neural networks, optimised to maximise financial metrics (e.g., Hedge Portfolio Excess Return, Sharpe Ratio etc.) outperform conventional loss minimisation optimisation strategies, when predicting excess returns in individual equities and equity hedge portfolios?

5 Motivation(s)

6 Literature Review (3 pages)

Overview of literature in asset pricing (761/751), ML application, factor pricing - very brief, 12pt, double spaced,

6.1 Asset Pricing

6.2 Machine Learning

Convexity is an important concept in optimisation Monotonic ranking

6.3 Machine Learning

A couple of recent publications highlight the increased application of machine learning algorithms in financial contexts. Li et al., 2020 Gu et al (2020) explore the comparative use of machine learning in empirical asset pricing.

6.4 Hypothesis

Include examples on the minimisation of sum of the square errors does not contribute to maximising returns

7 Methodology

The required methodology to construct the methods to build, develop, and deploy neural networks with custom objective functions.

7.1 Data

Hou et al., (2020) use an extensive data library to assess 452 anomalies across anomalies literature. Their analysis informs which abnormalities drive the cross section of expected returns. Most abnormalities fail under current standards of empirical finance when using a single hurdle test of absolute t-stat greater or equal to 1.96. Firstly, the paper finds economic fundamentals take precedence over trading frictions in explanatory power, statistical and economic significance. Secondly, micro-caps account for anomalies disproportionately, leading to NYSE breakpoints, value-weighted returns in both portfolio sorts and cross-sectional regressions with weighted least squares. Lastly, arguments in improving anomalies literature credibility follow a closer alignment to economic theory as the field persists to be statistical in nature. Overall, capital market efficiency is higher than expected. Jensen et al., 2021 use the above dataset to explore hierarchical bayesian models of alphas emphasising the joint behaviours of factors, and provide an alternative multiple testing adjustment, more powerful than common methods. Jensen et al., adapt the global dataset to focus only on one-month holding periods for all factors, only include most recent accounting data (quarterly or annually) and add 15 new factors. The authors provide documentation and web-based resources on GitHub to reconstruct an updated dataset from Wharton Research Data Services (WRDS) using SAS Studio. The dataset is exhaustive as illustrated by the both the summary statistics in 15.4 and Global Factor Data Documentation in the author's GitHub repository ¹ Identifier variables (e.g., size group), Accounting variables (e.g., COGS), accounting characteristics (e.g., change in net working capital, solvency ratios etc.), market variables (e.g., share price, excess return), market characteristics (e.g., market equity, 60 month CAPM β), and detailed characteristics (e.g., equity duration, Altman Z-Score) and Foreign Exchange Conversion Rates

¹View summary statistics and in tandem for comprehension

feature in the dataset’s composition. The complete global dataset has 406 characteristics, a superset of the original 153 in Jensen et al., with 2,739,928 firm-year observations, across 524 features, from January 1st 1961 to December 31st 2020. Subsequently, the complete dataset has 1.435 billion data points. **One month lead excess returns** is the designated target variables for prediction as will inform the construction of hedge portfolio to assess relative performance between different optimisation functionalities. The exhaustive nature and accessibility of the global dataset makes it well-suited for exploring optimisation functions, maximising renown financial metrics, in deep neural-networks.

7.2 Infrastructure

The essay necessitates substantial software development, infrastructure, and technological configuration to undertake analysis. Google Cloud Platform Cloud Storage buckets manage large datasets. The deployment of Google Cloud Engine Compute Engine Virtual Machine (VM) Instances, optimised for machine learning, build, train, and evaluate deep neural networks. Cryptographic network protocols, mostly secure shells, establish remote connectivity between local and remote infrastructure to communicate and execute commands. Version control technologies, primarily Git and GitHub, manage and develop methods for analysis with the programming of approximately 40 methods, classes, and functions.

7.3 Data Processing

Analysis requires data preprocessing to form training, validation, and testing subsets from the neural network. The initial training, testing, and validation sets comprise of **1031516**, **706908** , and **1001504** global equity firm-year observations across 524 features, respectively. The division of subsets is chronological with firm-year observations [1961-1990), [1990-2000), [2000-2020] for training, validation, and testing, respectively. However, the computational complexity exceeds resources available at the time of analysis. Subsequently, training sets require further preprocessing in addition to reconfiguring infrastructure. Further preprocessing includes: Reducing the number of factors from

Jensen et al., (2021) to approximately the same number of features (160) Hou et al., (2020) use to explore portfolio replication. The replacement of NaN values in a feature columns with the median value of the respective column to retain observations. The removal of firm-year observations with Micro or Nano size grouping designations, non-overlapping, value weighted by market capitalisation on the New York Stock Exchange (NYSE)). Mega, Large, and Small remain, reflecting equities with market capitalisations greater than the 80th, 50th, and 20th percentile of all NYSE stocks, respectively. Micro equities reside between the 1st and 20th percentiles, with Nano between below the 1st percentile. The rationale for their exclusion is Mega, Large, and Small equities account for the majority of stock market returns (80%). Hypothetically, their omission will not materially influence results. Furthermore, the reduction in numerical feature precision from float64 to float32 effectively halves memory usage. The training, testing, and validation sets comprise of **532218**, **294581**, and **531461** global equity firm-year observations across 160 features after above revisions, respectively. Subsequently, the revised dataset has 217.321,600 data points, suitable to use in analysis. Memory monitoring methods accompany the aforementioned preprocessing adjustments, monitoring CPU and GPU utilisation, reconfiguring GPU's, and configuring application programming interfaces for monitoring modelling performance.

7.4 Feature Encoding

Deep neural networks require tensors as inputs for fitting, training, and evaluating data to make predictions. A tensor is a mathematical object describing the physical properties of an object. In other words, multilinear relationships between sets of algebraic objects related in vector set. Transformation laws govern tensors. Therefore, a tensor is considered an n dimensional arrays in conjunction with associated transformation laws. The dataset must take the form of a tensor for analysis. Essentially, every value for every feature must be encoded to take a numerical form. A feature matrix is correct term for the dataset in this application. Firstly, the labelling of target variable is necessary for training, validating, and testing the model. Secondly, a shuffling of columns and instances

in feature matrix promotes better training as accommodates randomness. This analysis sacrifices shuffling the training set due to memory issues. Lastly, an encoding process encodes both categorical and numerical feature columns. The revised dataset includes eight categorical variables:

1. **size_group**: The aforementioned size grouping in 7.1
2. **permno**: Permanent unique firm identifier from CRSP
3. **permco**: Permanent unique issue identifier from CRSP
4. **crsp_shrcd**: CRSP share code
5. **crsp_exchcd**: Compustat stock exchange code
6. **sic**: Firm SIC industry
7. **ff49**: Classification of stocks in 49 industry groups based on SIC codes and the methodology in F.Fama & K.French 1997 with the addition of the software industry.
8. **adjfct**: Share Adjustment Factor

Categorical encoding transforms the set of instances for a features into a series of binary variables. These may be one-hot encoded, and/or stored in sparse tensors, depending on input and desired application. The remaining variables are numerical where a normalization normalizes each feature to zero mean and unit variance.

7.5 Neural Network Configuration

7.5.1 Multi Layer Perceptron (MLP)

Artificial Neural Nets (ANN) are versatile, powerful, and scalable. They frequently outperform other machine learning algorithms on large and complex problems. Linear threshold units (LTU) compose neural networks. A linear threshold unit (LTU) feeds the weighted sum of input values (??) into a step function (2). A perceptron is a single

layer of LTUs connected to every input. Perceptrons are suitable for both regression and classification tasks. Perceptrons utilize a training algorithm assessing the strength of connections between perceptrons while considering errors. A perceptron is fed one training instance at a time, making predictions for each instance. For every output LTU that produced a wrong prediction, it re-enforces the connection weights using the perceptron learning rule (6) from the inputs that would have contributed to the right prediction. One LTU input layer, multiple LTU hidden layers, and an output LTU layer create an Multi Layer Perceptron (MLP). The step functions in each LTU are replaced by a logistic (??) or ReLU function (4) to enable gradient descent for optimisation. A shared activation function replaces the individual activation functions in the output layer to enable exclusive classification or regression. The linear activation function (5) is most suitable output layer activation function for regression task as predict values between $(-\infty, \infty)$ directly, suitable for evaluating excess returns.

$$z = \mathbf{w}^T \cdot \mathbf{x} \quad (1)$$

$$h_w(\mathbf{x}) = \text{step}(z) \quad (2)$$

$$\sigma(z) = \frac{1}{1 + \exp(-z)} \quad (3)$$

$$\text{ReLU}(z) = \max(0, z) \quad (4)$$

$$\text{Linear}(z) = z \quad (5)$$

$$w_{i,j}^{\text{next step}} = w_{i,j} + \eta(\hat{y}_j - y_j)x_i \quad (6)$$

Where

- $w_{i,j}$ is the connection weights between the i th input neuron and the j th output neuron.
- x_i is the i th input value of the current training instance.
- \hat{y}_j is the output of the j th output neuron for the current training instance.
- y_j is the output of the j th output neuron for the current training instance.

- η is the rate.

The architecture for analysis is a large MLP.

7.5.2 Model Configuration

Figure 1 visualises the topography of a standard neural network.

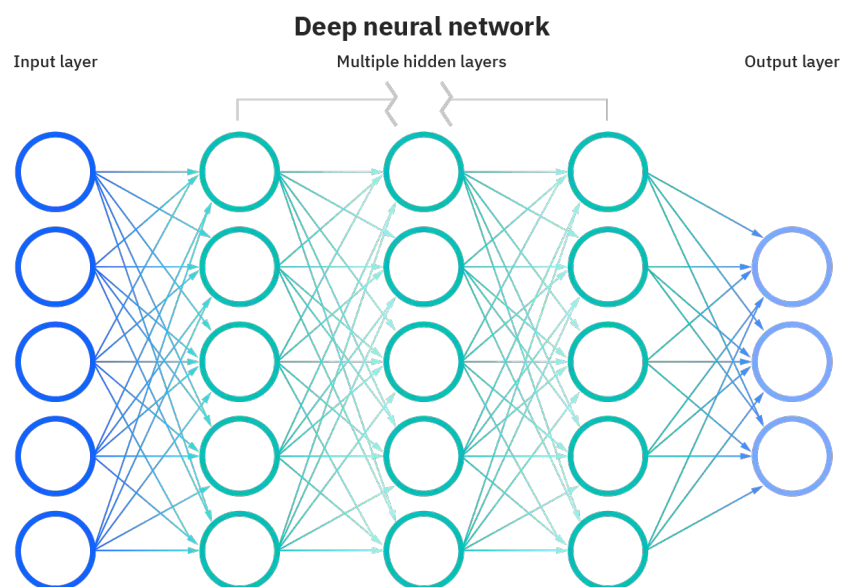


Figure 1: Standard Neural Network Topography (Source: IBM)

The dots and lines represent nodes and connections between nodes, respectively. The architecture of the network is derivative of the model's intended use. The set of

Figure 2 illustrates the configuration for this project (work in progress).

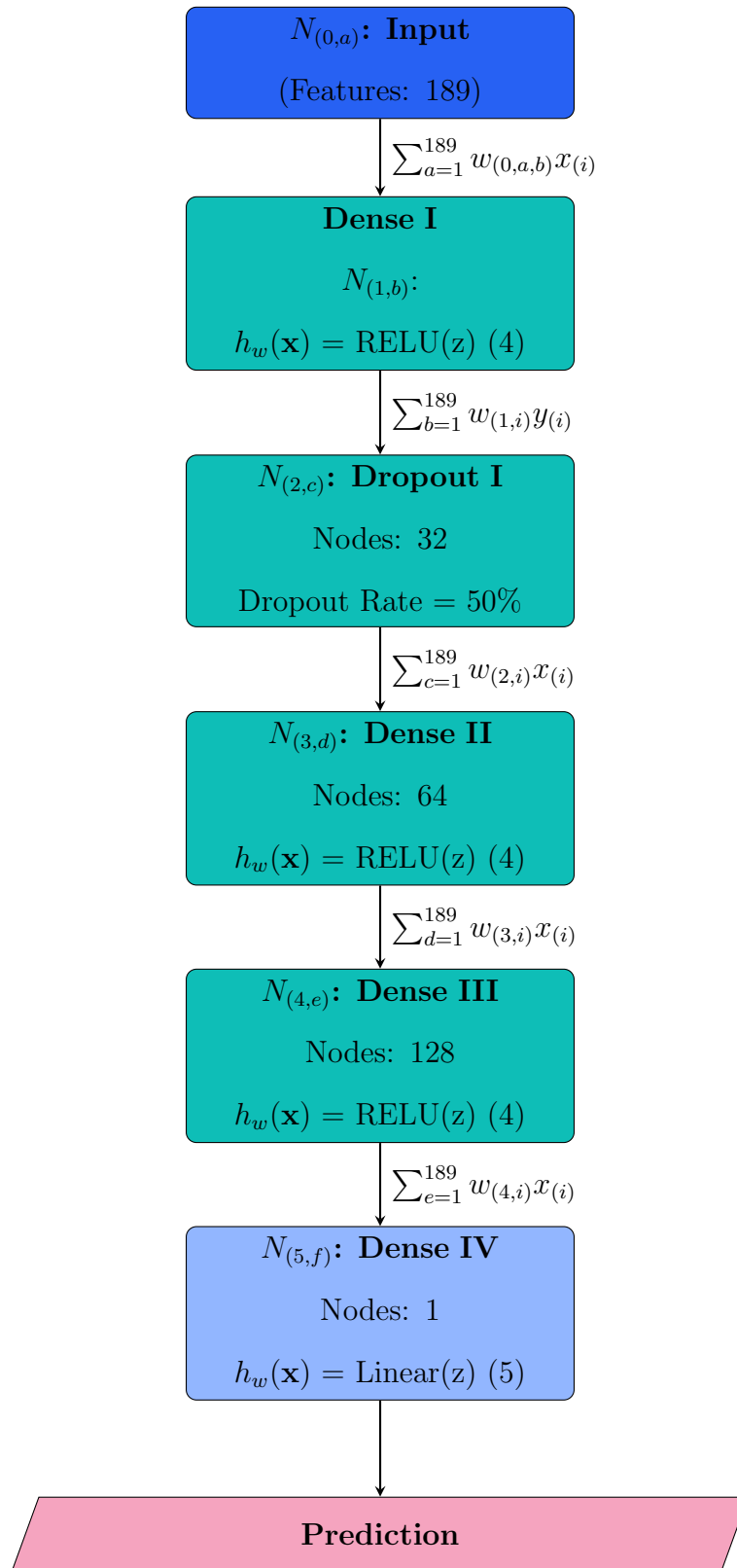


Figure 2: Neural Network Configuration

10 epochs, 207 runs per epochs.

8 Brief Description of Remaining Methods

- Use stochastic gradient descent, train model with 10 epochs (2709 runs per epoch) for each of the six loss functions (tensorflow mse, custom mse, sharpe ratio, sharpe ratio mse, hedge portfolio mse, information ratio). Validate with validation set and evaluate with testign set.
- Make predictions using entire testing set for each model. Merge predictions with fama french factors on month
- Perform newy west regressing hedge portfolio returns on fama factors. Predictions grouped by month. hedge formed by mean of top decile - mean of bottom decile creating hedge return for the month.
- Second set of regressions does a panel regression accounting Heteroskedasticity Robust Covariance as Fama MacBeth did not execute due to rank deficiencies.

9 Work in Progress

9.1 Loss Functions & Performance Metrics

9.1.1 Modelling, Loss, and Optimisation

We summarize the theory surrounding predictive modelling, loss functions, and optimisation algorithms. These functions train models by comparing predictions to realized observations using optimisation algorithms to minimize the loss function. We examine a linear model as our predictive model (7). Mean square error (8) and Gradient Descent

(GD) are basic examples of a loss function and optimisation algorithm, respectively.

$$\hat{y} = mx_i + b \quad (7)$$

$$f(y, (mx_i + b)) = \frac{1}{n} \sum_{i=1}^n (y_i - (mx_i + b))^2 \quad (8)$$

Firstly, gradient descent takes the partial derivatives of the loss function, with the respect to the parameters in our predictive model. In our example, equations 9 and 10 are the partial derivatives for the mean square error loss function.

$$\frac{\partial f(y, (mx_i + b))}{\partial m} = \frac{1}{n} \sum_{i=1}^n -2x_i(y_i - (mx_i + b))^2 \quad (9)$$

$$\frac{\partial f(y, (mx_i + b))}{\partial b} = \frac{1}{n} \sum_{i=1}^n -2(y_i - (mx_i + b))^2 \quad (10)$$

Secondly, the algorithm explores epochs, using a learning rate to update parameters to move in the opposite directions of the partial derivatives until settling in a local minima. This extrema is the optimisation of the loss function, quantifying the accuracy of the predictive model. Ordinary Least Squares (OLS) regressions is an extension of the linear model prevalent in asset pricing. The mathematical rigor and suitability of the OLS estimator section 15.1. The theory underlying OLS informs the common practice in minimising of the sum of least squares when evaluating prediction performance. The mathematical tractability, in accordance with the aforementioned assumption, frame our thinking surrounding the derivation of custom loss functions.

9.1.2 Automatic Differentiation

9.1.3 Loss & Metric Definition

9.2 Hedge Portfolio Construction

9.3 Regression Analysis

10 Limitations

Further research will rely on addressing several limitation.

10.1 Lack of Computational Resources

The lack of resources to explore the entire dataset within reasonable timeframes, and at reasonable costs. The most material inhibitions are the inability to explore all 524 factors for all size groupings, the reducing in level of precision for numerical features, the ability to shuffle training sets at lengths greater than or equal to the input sets

10.2 Loss Functions & Performance Metrics

Table 1 emphasises the separation between training and validation datasets.

Variable	Description	$MSE(y, \hat{y})$	$HP(y, \hat{y})$
θ	Estimation Training	$\hat{\theta}_{MSE}$	$\hat{\theta}_{HP}$
λ	Validation	$\hat{\lambda}_{MSE}$	$\hat{\lambda}_{HP}$

Table 1: Objective (MSE: Mean Square Error, HP: Hedge Portfolio)

10.2.1 Mean Square Error (MSE)

Section 9.1.1 outlines advantages to Ordinary Least Squares. Subsequently, MSE serves as a baseline for loss function and performance metric comparisons. The following function (11) and partial derivative (12) describe Tensorflows's Mean Square Error implementation, both from in-built and custom contexts. Python classes describe equation 11 to enable Tensorflow's automatic differentiation capabilities, approximating the partial derivatives of the loss function (12) with numerical methods. Please note the use of

Hadamard exponentiation ($x^{\circ n}$) as an element-wise operation.

$$f(y, X^T \hat{\theta}) = \frac{\vec{1}}{\vec{1}^T \vec{1}} (y - X^T \hat{\theta})^{\circ 2} \quad (11)$$

$$\frac{\partial f(y, X^T \hat{\theta})}{\partial \hat{\theta}} = \frac{\vec{1}}{\vec{1}^T \vec{1}} (-2(y - X^T \hat{\theta})^{\circ 1}) \quad (12)$$

10.2.2 Hedge Portfolio

Hedge portfolios rely on monotonic ranking functions for optimisation as their monotonic nature preserves or reverses a given ordered set. The analysis cross-section of one-month lead portfolio excess returns using monotonic functions

$$R(y_{i,t}) \quad (13)$$

The ranking function ($R(y_{i,t})$) and thresholds (u, v) form subsets of long and short portfolios. Long (L) or Short (S) sets include excess returns conditioned on the associated monotonic ranking given a threshold, bound by the cardinality of the excess return vector ($|y|$). The subsequent truth sets mathematically express aforementioned time-series hedge portfolios.

$$L = \{y_{i,t} | R(y_{i,t}) \leq u\}$$

$$S = \{y_{i,t} | R(y_{i,t}) \geq v\}$$

$$0 < u \leq |y|$$

$$0 < v \leq |y|$$

$$u < v$$

Equation 14 describes hedge portfolio lead excess returns (H_t) at a given time (t).

$$H_t = \frac{1}{|L|} \sum_{i \in L} y_{i,t} - \frac{1}{|S|} \sum_{i \in S} y_{i,t} \quad (14)$$

Figure 3) illustrates an approximate linear monotonic ranking function with a sample of 100 uniformly distributed excess returns between -10% and 10%. Boundary conditions u and v are set to 20 and 80, respectively. Subsequently, excess returns above (below) the green (blue) dotted line belong to the long (L) (short (S)) set.

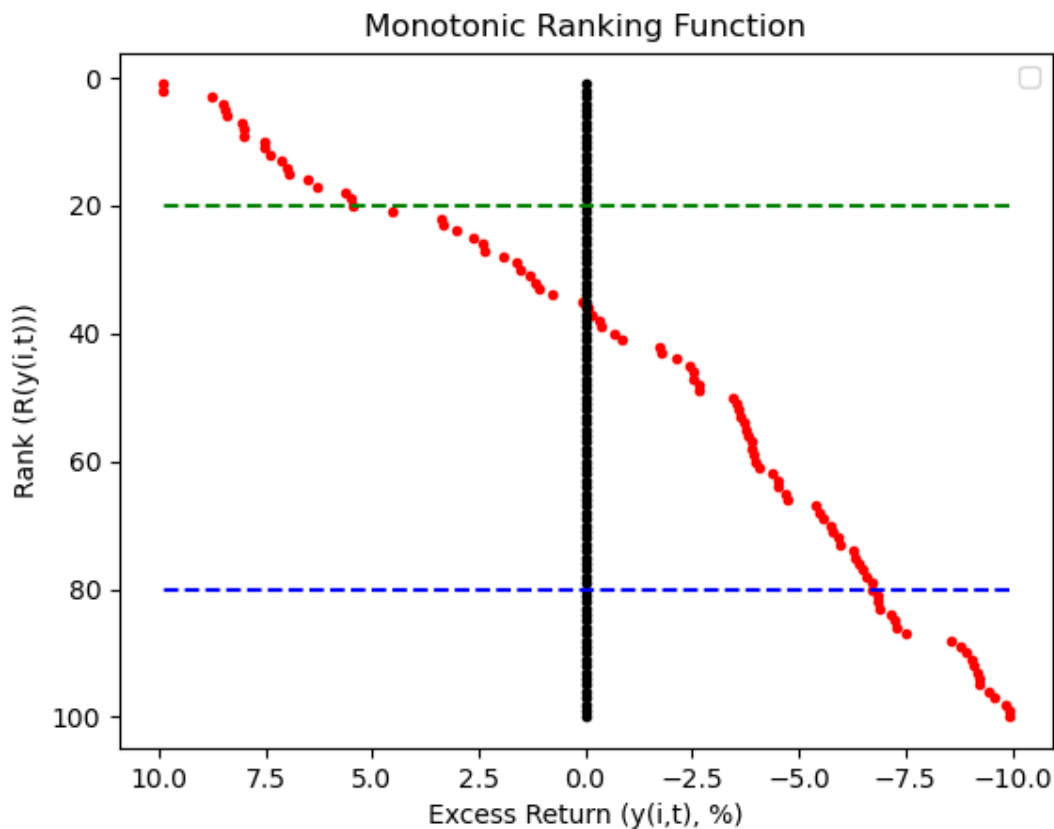


Figure 3: Approximate Linear Monotonic Ranking Function

The permutations in monotonic ranking functions, and subsequent hedge portfolios, are endless. This research essay develops a monotonic ranking function proportionally weighting one month lead excess returns (15). Therefore, equation 16 defines the loss func-

tion.

$$R(\hat{y}) = W \quad (15)$$

$$W := \frac{\hat{y}}{\mathbf{1}\hat{y}}$$

$$\hat{y} = X^T \hat{\theta}$$

$$f_{\hat{\theta}}(X) = \left(\frac{X^T \hat{\theta}}{\mathbf{1} X^T \hat{\theta}} \right)^{\top} X^T \hat{\theta} \quad (16)$$

The above loss function is differentiable using symbolic mathematic as shown in equation 17.

$$\begin{aligned} \frac{\partial f_{\hat{\theta}}(X)}{\partial \hat{\theta}} &= \frac{\partial \left(\left(\frac{X^T \hat{\theta}}{\mathbf{1} X^T \hat{\theta}} \right)^{\top} X^T \hat{\theta} \right)}{\partial \hat{\theta}} \\ \frac{\partial (f_{\hat{\theta}}(X))}{\partial \hat{\theta}} &= \frac{1}{(\hat{\theta}^{\top} X \mathbf{1})} X X^{\top} \hat{\theta} + \frac{1}{\mathbf{1} X^{\top} \hat{\theta}} X X^{\top} \hat{\theta} - \frac{1}{(\hat{\theta}^{\top} X \mathbf{1})^2} \hat{\theta}^{\top} X X^{\top} \hat{\theta} X \mathbf{1} \end{aligned} \quad (17)$$

Our research Subsection 9.1.1 explains the theory supporting loss minimisation. Applying gradient descent methods to the product of the loss function and scalar of -1 transforms the minimisation to maximisation. This transformation leads to finding the argmax of maximisation function with respect to $\hat{\theta}$ (18). The aforementioned transformation is simply and suitable for exploration in the context of the research intent. More sophisticated methods exist for maximisation such as reinforcement learning (??).

$$\operatorname{argmax}_{\hat{\theta}} : \left(\frac{X^T \hat{\theta}}{\mathbf{1} X^T \hat{\theta}} \right)^{\top} X^T \hat{\theta} \quad (18)$$

10.3 Conventional Asset Pricing Models

Conventional asset pricing methodologies persist in academic literature. The use of the Capital Asset Pricing Model (CAPM) persists, regardless of the identifiable shortcomings in market proxies and empirical failings invalidating use (Fama and French, 2004). Nonetheless, this research essay uses the model as a performance metric for comparative

purposes.

$$R_{i,t} - R_{f,t} = \alpha_{i,t} + \beta_{i,t}^1(R_{M,t} - R_{f,t}) \quad (19)$$

E. Fama and K. French (1992) validate the explanatory power of size and value (book-to-market) factors in their ability to capture the cross-sectional variation in average stock returns, in association with market risk, size, leverage, book-to-market, and earnings-price ratios. E. Fama and K. French further their analysis on the common characteristics between stocks and bonds (Fama and French, 2021)¹, and add two additional factors to consider profitability and investment. The main combinations are the Fama French Three (FF3) (20) and Five (FF5) (21) models. E. Fama and K. French consider a momentum factor on international stock returns in subsequent years (Fama and French, 2012). The omission of momentum from the models stand.

$$R_{i,t} - R_{f,t} = \alpha_{i,t} + \beta_{i,t}(R_{M,t} - R_{f,t}) \quad (20)$$

$$R_{i,t} - R_{f,t} = \alpha_{i,t} + \beta_{i,t}(R_{M,t} - R_{f,t}) \quad (21)$$

where

- $R_{i,t} - R_{f,t}$: Portfolio Excess Return on the market for a given portfolio and time, value-weighted using all incorporated US CRSP firms incorporated in the US, listed on the NYSE, AMEX, or NASDAQ.
- $\alpha_{i,t}$: Jensen's alpha indicating mispricing in the asset.
- β_t^1 : Market Risk Factor (co-efficient)
- β_t^1 : Size Factor (co-efficient)
- β_t^2 : Value Factor (co-efficient)
- β_t^2 : Value Factor (co-efficient)

¹Reprinted. Originally published in 1993

- β_t^3 : Profitability Factor (co-efficient)
- β_t^4 : Investment Factor (co-efficient)
- $(R_{M,t} - R_{f,t})$: Market Risk Premium
- SMB_t : Size Premium (small minus big) is the difference in average return between nine small stock and nine large value-weighted portfolios.
- HML_t : Value Premium (high minus low) is the difference in average return between two value and two growth value-weighted portfolios.
- RMW_t : Profitability Premium (robust minus weak) is the difference in average return between two robust operating profitability and two weak operating profitability value-weighted portfolios.
- CMA_t : Investment Premium minus aggressive is the difference in average return on the two conservative and two aggressive investment portfolios

K. French continues to maintain FF3 and FF5 related datasets (French, 2021) E. Fama, with J. MacBeth, developed the Fama-MacBeth regression (Fama and MacBeth, 1973) to estimate factor loadings and prices. The methodology is a two-stage estimation process, similar for estimating factor loadings, and prices, for a given portfolio. The first step requires determining each asset's β exposures by regressing each of n asset returns against m proposed 22. The second step determines the risk premium (factor pricing) for each asset by regressing all asset returns for each of T periods against previously estimated β s

(23).

$$R_{n,t} = \alpha_n + \sum_{f=1}^F \beta_{n,F_f} F_{f,t} + \epsilon_{n,t} \quad (22)$$

$$\forall n \in \{1, \dots, N\}$$

$$R_{i,t} = \gamma_{t,0} + \sum_{f=1}^F \gamma_{t,f} \hat{\beta}_{i,F_f} + \epsilon_{i,t} \quad (23)$$

$$t \in \{1, \dots, N\}$$

Where

- $R_{n,t}$: Return for an asset (n) at a time (t).
- α_n : Jensen's alpha for an asset (n) implying mispricing.
- β_{n,F_f} : An asset's (n) exposure to a factor (f)
- $\hat{\beta}_{i,F_f}$: Estimated factor loading for a factor (f) from regression of asset (i)
- $F_{f,t}$: Risk factor (f) at a given time (t) e.g., SMB, HML etc.,
- $\epsilon_{n,t}$: Residual for an asset at a time (t)
- $\gamma_{t,f}$: Factor pricing for a factor (f)

The residuals of risk-return regressions and co-efficient 'fair game' observations are consistent with the efficient market hypothesis ((Samuelson, 2016), (Fama, 1995), (Fama, 1963), (Fama, 1965) (Fama, 2021)). Subsequently, this research essay uses the excess one month lead returns of the hedge portfolio (??), Capital Asset Pricing Model (19), Fama-French Three Factor Model (20), and Fama-French Five Factor Model (21) as performance metrics for each loss function. Table 2 outlines the proposed performance metrics and loss functions.

Performance Metric	Loss Function
Hedge Portfolio Mean	Hedge Portfolio Excess Returns
Hedge Portfolio FF3	Sharpe Ratio
Hedge Portfolio FF5	Information Ratio
Sharpe Ratio	Validation
Information Ratio	Validation
Mean Square Error	Mean Square Error

Table 2: Proposed Loss Functions and Performance Metrics

Hou et al (2021) provide the required data to derive these performance metric and loss functions.

10.3.1 Sharpe Ratio

Nobel Laurette William F. Sharpe (1994) introduced the Sharpe Ratio (24) as a measure for risk-adjusted returns where $\mathbb{E}[R_a - R_f]$ is the expectation for excess returns and $\sigma(R_a)$ is the standard deviation of excess returns.

$$SR = \mathbb{E} \frac{[R_a - R_f]}{\sigma(R_a)} \quad (24)$$

$$\sigma = \sqrt{\sum_{i=1}^n \frac{(R_a) - \bar{R}_a)^2}{n}}$$

The matrix notation for the Sharpe Ratio loss function () follows:

$$SR = 1\partial SR \quad (25)$$

10.3.2 Information Ratio

The Information Ratio (IR) is another risk-return measure (26), evaluating the performance of a portfolio compared to a selected benchmark e.g., an exchange traded fund.

$\mathbb{E}[R_a - R_b]$ is the excess return on the benchmark. $\sigma(R_a - R_b)$ is the standard deviation of the difference between the portfolio and benchmark known as the tracking error.

$$IR = \mathbb{E} \frac{[R_a - R_b]}{\sigma(R_a - R_b)} \quad (26)$$

Both Sharpe and Information ratios supplement hedge portfolio-related loss functions and performance metrics.

11 Results

11.1 Performance Matrix

Hlavac, 2018 enables the tables in the regression analysis.

11.2 Hedge Portfolio Performance

11.2.1 Mean Square Error (Tensorflow)

<i>One Month Lead Excess Portfolio Return using mean squared error:</i>				
	CAPM	FF3	FF4	FF5
CMA				0.128 (0.096)
HML		-0.118** (0.058)	-0.191*** (0.066)	-0.238*** (0.076)
Mkt-RF	-0.096* (0.049)	-0.087* (0.049)	-0.051 (0.047)	-0.035 (0.048)
RMW			0.199*** (0.063)	0.205*** (0.067)
SMB		-0.039 (0.043)	0.037 (0.039)	0.031 (0.040)
const	-0.018*** (0.004)	-0.018*** (0.004)	-0.019*** (0.004)	-0.019*** (0.004)
Observations	252	252	252	252
R^2	0.029	0.055	0.082	0.088
Adjusted R^2	0.026	0.043	0.067	0.069
Residual Std. Error	0.025(df = 250)	0.025(df = 248)	0.025(df = 247)	0.025(df = 246)
F Statistic	3.816* (df = 1.0; 250.0)	3.080** (df = 3.0; 248.0)	3.644*** (df = 4.0; 247.0)	3.023** (df = 5.0; 246.0)

Note: *p<0.1; **p<0.05; ***p<0.01

11.2.2 Mean Square Error

One Month Lead Excess Portfolio Return using custom mse:				
	CAPM	FF3	FF4	FF5
CMA				0.103 (0.083)
HML		-0.108** (0.044)	-0.157*** (0.053)	-0.195*** (0.063)
Mkt-RF	-0.060* (0.035)	-0.049 (0.034)	-0.025 (0.034)	-0.011 (0.036)
RMW			0.133** (0.056)	0.138** (0.059)
SMB		-0.052 (0.033)	-0.000 (0.033)	-0.006 (0.034)
const	-0.023*** (0.003)	-0.023*** (0.003)	-0.024*** (0.003)	-0.024*** (0.003)
Observations	252	252	252	252
R^2	0.015	0.047	0.063	0.068
Adjusted R^2	0.011	0.035	0.048	0.049
Residual Std. Error	0.022(df = 250)	0.022(df = 248)	0.021(df = 247)	0.021(df = 246)
F Statistic	2.857* (df = 1.0; 250.0)	3.870*** (df = 3.0; 248.0)	3.411*** (df = 4.0; 247.0)	2.990** (df = 5.0; 246.0)

Note: *p<0.1; **p<0.05; ***p<0.01

11.2.3 Hedge Portfolio

<i>One Month Lead Excess Portfolio Return using custom hp:</i>				
	CAPM	FF3	FF4	FF5
CMA				-0.061 (0.038)
HML		-0.064** (0.026)	-0.074*** (0.027)	-0.052** (0.024)
Mkt-RF	0.006 (0.018)	0.010 (0.017)	0.015 (0.017)	0.007 (0.017)
RMW			0.029 (0.030)	0.026 (0.028)
SMB		-0.017 (0.023)	-0.006 (0.023)	-0.003 (0.022)
const	0.012*** (0.001)	0.012*** (0.001)	0.012*** (0.001)	0.012*** (0.001)
Observations	252	252	252	252
R^2	0.001	0.057	0.062	0.072
Adjusted R^2	-0.003	0.046	0.047	0.054
Residual Std. Error	0.009(df = 250)	0.009(df = 248)	0.009(df = 247)	0.009(df = 246)
F Statistic	0.115 (df = 1.0; 250.0)	2.132* (df = 3.0; 248.0)	1.990* (df = 4.0; 247.0)	1.733 (df = 5.0; 246.0)

Note: *p<0.1; **p<0.05; ***p<0.01

11.2.4 Sharpe Ratio

<i>One Month Lead Excess Portfolio Return using custom sharpe:</i>				
	CAPM	FF3	FF4	FF5
CMA				-721.234* (376.766)
HML		-297.974 (236.638)	-322.664 (237.387)	-58.152 (234.498)
Mkt-RF	-335.386 (220.320)	-315.324 (232.523)	-303.168 (218.794)	-394.409* (223.394)
RMW			66.567 (231.154)	33.844 (234.713)
SMB		-93.760 (194.283)	-68.062 (249.031)	-30.708 (243.421)
const	23.034 (16.190)	23.419 (15.903)	23.042 (15.527)	25.094 (15.747)
Observations	252	252	252	252
R^2	0.021	0.030	0.030	0.041
Adjusted R^2	0.017	0.019	0.015	0.022
Residual Std. Error	104.887(df = 250)	104.810(df = 248)	105.012(df = 247)	104.629(df = 246)
F Statistic	2.317 (df = 1.0; 250.0)	1.534 (df = 3.0; 248.0)	1.167 (df = 4.0; 247.0)	1.376 (df = 5.0; 246.0)

Note: *p<0.1; **p<0.05; ***p<0.01

11.2.5 Sharpe Ratio (MSE)

<i>One Month Lead Excess Portfolio Return using custom sharpe mse:</i>				
	CAPM	FF3	FF4	FF5
CMA				996.864** (445.694)
HML		448.559 (297.801)	401.406 (291.798)	35.807 (262.327)
Mkt-RF	-55.441 (178.591)	-51.294 (180.181)	-28.079 (176.186)	98.030 (163.549)
RMW			127.130 (310.092)	172.359 (294.457)
SMB		-29.549 (213.216)	19.531 (244.065)	-32.098 (238.489)
const	-131.292*** (16.512)	-131.559*** (15.873)	-132.280*** (16.058)	-135.116*** (15.706)
Observations	252	252	252	252
R^2	0.001	0.022	0.023	0.046
Adjusted R^2	-0.003	0.010	0.007	0.026
Residual Std. Error	100.581(df = 250)	99.906(df = 248)	100.072(df = 247)	99.075(df = 246)
F Statistic	0.096 (df = 1.0; 250.0)	0.903 (df = 3.0; 248.0)	0.698 (df = 4.0; 247.0)	1.422 (df = 5.0; 246.0)

Note: *p<0.1; **p<0.05; ***p<0.01

11.2.6 Information Ratio

<i>One Month Lead Excess Portfolio Return using custom information:</i>				
	CAPM	FF3	FF4	FF5
CMA				-3.918 (11.851)
HML		-2.394 (5.171)	-7.617 (6.271)	-6.180 (7.500)
Mkt-RF	-6.300 (5.010)	-6.115 (5.471)	-3.543 (5.262)	-4.039 (6.004)
RMW			14.083** (6.487)	13.906** (6.432)
SMB		-0.871 (6.021)	4.566 (7.157)	4.769 (7.231)
const	5.285*** (0.479)	5.288*** (0.479)	5.208*** (0.477)	5.219*** (0.483)
Observations	252	252	252	252
R^2	0.009	0.010	0.020	0.021
Adjusted R^2	0.005	-0.002	0.004	0.001
Residual Std. Error	2.968(df = 250)	2.979(df = 248)	2.970(df = 247)	2.975(df = 246)
F Statistic	1.581 (df = 1.0; 250.0)	0.628 (df = 3.0; 248.0)	2.136* (df = 4.0; 247.0)	1.750 (df = 5.0; 246.0)

Note: *p<0.1; ** p<0.05; ***p<0.01

Fama MacBeth Regressions using individual securities. PooledOLS replaces Fama MacBeth in this instance.

11.3 Individual Panel Regressions

11.3.1 Realised Excess Returns

Dep. Variable:	ret_exc_lead1m	R-squared:	0.0015			
Estimator:	PooledOLS	R-squared (Between):	-0.0742			
No. Observations:	531461	R-squared (Within):	0.0011			
Date:	Tue, Jan 11 2022	R-squared (Overall):	0.0015			
Time:	11:47:07	Log-likelihood	3.019e+05			
Cov. Estimator:	Robust					
		F-statistic:	813.86			
Entities:	7282	P-value	0.0000			
Avg Obs:	72.983	Distribution:	F(1,531459)			
Min Obs:	1.0000					
Max Obs:	252.00	F-statistic (robust):	521.20			
		P-value	0.0000			
Time periods:	252	Distribution:	F(1,531459)			
Avg Obs:	2109.0					
Min Obs:	1869.0					
Max Obs:	3400.0					
Parameter		Std. Err.	T-stat	P-value	Lower CI	Upper CI
const	0.0067	0.0002	34.227	0.0000	0.0063	0.0071
Mkt-RF	0.0012	5.072e-05	22.830	0.0000	0.0011	0.0013

Dep. Variable:	ret_exc_lead1m	R-squared:	0.0022
Estimator:	PooledOLS	R-squared (Between):	-0.0683
No. Observations:	531461	R-squared (Within):	0.0013
Date:	Tue, Jan 11 2022	R-squared (Overall):	0.0022
Time:	11:47:08	Log-likelihood	3.02e+05
Cov. Estimator:	Robust		
		F-statistic:	387.46
Entities:	7282	P-value	0.0000
Avg Obs:	72.983	Distribution:	F(3,531457)
Min Obs:	1.0000		
Max Obs:	252.00	F-statistic (robust):	238.06
		P-value	0.0000
Time periods:	252	Distribution:	F(3,531457)
Avg Obs:	2109.0		
Min Obs:	1869.0		
Max Obs:	3400.0		

	Parameter	Std. Err.	T-stat	P-value	Lower CI	Upper CI
const	0.0070	0.0002	35.904	0.0000	0.0066	0.0073
Mkt-RF	0.0012	5.393e-05	23.130	0.0000	0.0011	0.0014
SMB	-0.0006	7.995e-05	-7.5937	0.0000	-0.0008	-0.0005
HML	-0.0009	7.383e-05	-11.770	0.0000	-0.0010	-0.0007

Dep. Variable:	ret_exc_lead1m	R-squared:	0.0057
Estimator:	PooledOLS	R-squared (Between):	-0.0922
No. Observations:	531461	R-squared (Within):	0.0053
Date:	Tue, Jan 11 2022	R-squared (Overall):	0.0057
Time:	11:47:09	Log-likelihood	3.03e+05
Cov. Estimator:	Robust		
		F-statistic:	757.73
Entities:	7282	P-value	0.0000
Avg Obs:	72.983	Distribution:	F(4,531456)
Min Obs:	1.0000		
Max Obs:	252.00	F-statistic (robust):	526.03
		P-value	0.0000
Time periods:	252	Distribution:	F(4,531456)
Avg Obs:	2109.0		
Min Obs:	1869.0		
Max Obs:	3400.0		

	Parameter	Std. Err.	T-stat	P-value	Lower CI	Upper CI
const	0.0091	0.0002	44.309	0.0000	0.0087	0.0095
Mkt-RF	0.0006	5.855e-05	9.9063	0.0000	0.0005	0.0007
SMB	-0.0022	8.173e-05	-27.137	0.0000	-0.0024	-0.0021
HML	0.0007	7.583e-05	9.1125	0.0000	0.0005	0.0008
RMW	-0.0037	0.0001	-36.210	0.0000	-0.0039	-0.0035

Dep. Variable:	ret_exc_lead1m	R-squared:	0.0063
Estimator:	PooledOLS	R-squared (Between):	-0.0995
No. Observations:	531461	R-squared (Within):	0.0063
Date:	Tue, Jan 11 2022	R-squared (Overall):	0.0063
Time:	11:47:10	Log-likelihood	3.031e+05
Cov. Estimator:	Robust		
		F-statistic:	675.07
Entities:	7282	P-value	0.0000
Avg Obs:	72.983	Distribution:	F(5,531455)
Min Obs:	1.0000		
Max Obs:	252.00	F-statistic (robust):	478.68
		P-value	0.0000
Time periods:	252	Distribution:	F(5,531455)
Avg Obs:	2109.0		
Min Obs:	1869.0		
Max Obs:	3400.0		

	Parameter	Std. Err.	T-stat	P-value	Lower CI	Upper CI
const	0.0084	0.0002	41.037	0.0000	0.0080	0.0088
Mkt-RF	0.0009	5.779e-05	15.519	0.0000	0.0008	0.0010
SMB	-0.0024	8.158e-05	-28.972	0.0000	-0.0025	-0.0022
HML	-0.0001	9.671e-05	-1.4864	0.1372	-0.0003	4.58e-05
RMW	-0.0036	0.0001	-35.222	0.0000	-0.0038	-0.0034
CMA	0.0022	0.0002	14.439	0.0000	0.0019	0.0025

11.3.2 Mean Square Error (Tensorflow)

Dep. Variable:	predict	R-squared:	4.125e-05
Estimator:	PooledOLS	R-squared (Between):	-0.0132
No. Observations:	531461	R-squared (Within):	-0.0002
Date:	Wed, Jan 12 2022	R-squared (Overall):	4.125e-05
Time:	00:12:16	Log-likelihood	1.669e+05
Cov. Estimator:	Clustered		
		F-statistic:	21.921
Entities:	7282	P-value	0.0000
Avg Obs:	72.983	Distribution:	F(1,531459)
Min Obs:	1.0000		
Max Obs:	252.00	F-statistic (robust):	0.2592
		P-value	0.6107
Time periods:	252	Distribution:	F(1,531459)
Avg Obs:	2109.0		
Min Obs:	1869.0		
Max Obs:	3400.0		

	Parameter	Std. Err.	T-stat	P-value	Lower CI	Upper CI
const	0.0297	0.0026	11.270	0.0000	0.0246	0.0349
Mkt-RF	0.0245	0.0481	0.5091	0.6107	-0.0698	0.1188

Dep. Variable:	predict	R-squared:	0.0025
Estimator:	PooledOLS	R-squared (Between):	-0.0054
No. Observations:	531461	R-squared (Within):	0.0012
Date:	Wed, Jan 12 2022	R-squared (Overall):	0.0025
Time:	00:12:22	Log-likelihood	1.676e+05
Cov. Estimator:	Clustered		
		F-statistic:	446.89
Entities:	7282	P-value	0.0000
Avg Obs:	72.983	Distribution:	F(3,531457)
Min Obs:	1.0000		
Max Obs:	252.00	F-statistic (robust):	9.6855
		P-value	0.0000
Time periods:	252	Distribution:	F(3,531457)
Avg Obs:	2109.0		
Min Obs:	1869.0		
Max Obs:	3400.0		

	Parameter	Std. Err.	T-stat	P-value	Lower CI	Upper CI
const	0.0304	0.0025	11.999	0.0000	0.0254	0.0353
Mkt-RF	0.0418	0.0429	0.9749	0.3296	-0.0423	0.1259
SMB	-0.1284	0.0668	-1.9225	0.0545	-0.2593	0.0025
HML	-0.2302	0.0537	-4.2891	0.0000	-0.3354	-0.1250

Dep. Variable:	predict	R-squared:	0.0026
Estimator:	PooledOLS	R-squared (Between):	-0.0049
No. Observations:	531461	R-squared (Within):	0.0014
Date:	Wed, Jan 12 2022	R-squared (Overall):	0.0026
Time:	00:12:27	Log-likelihood	1.676e+05
Cov. Estimator:	Clustered		
		F-statistic:	342.47
Entities:	7282	P-value	0.0000
Avg Obs:	72.983	Distribution:	F(4,531456)
Min Obs:	1.0000		
Max Obs:	252.00	F-statistic (robust):	7.1605
		P-value	0.0000
Time periods:	252	Distribution:	F(4,531456)
Avg Obs:	2109.0		
Min Obs:	1869.0		
Max Obs:	3400.0		

	Parameter	Std. Err.	T-stat	P-value	Lower CI	Upper CI
const	0.0300	0.0026	11.679	0.0000	0.0250	0.0351
Mkt-RF	0.0526	0.0467	1.1261	0.2601	-0.0389	0.1441
SMB	-0.1024	0.0688	-1.4884	0.1366	-0.2372	0.0324
HML	-0.2553	0.0619	-4.1277	0.0000	-0.3766	-0.1341
RMW	0.0599	0.0814	0.7362	0.4616	-0.0996	0.2195

Dep. Variable:	predict	R-squared:	0.0027
Estimator:	PooledOLS	R-squared (Between):	-0.0036
No. Observations:	531461	R-squared (Within):	0.0013
Date:	Wed, Jan 12 2022	R-squared (Overall):	0.0027
Time:	00:12:33	Log-likelihood	1.676e+05
Cov. Estimator:	Clustered		
		F-statistic:	285.26
Entities:	7282	P-value	0.0000
Avg Obs:	72.983	Distribution:	F(5,531455)
Min Obs:	1.0000		
Max Obs:	252.00	F-statistic (robust):	6.0751
		P-value	0.0000
Time periods:	252	Distribution:	F(5,531455)
Avg Obs:	2109.0		
Min Obs:	1869.0		
Max Obs:	3400.0		

	Parameter	Std. Err.	T-stat	P-value	Lower CI	Upper CI
const	0.0304	0.0026	11.662	0.0000	0.0253	0.0355
Mkt-RF	0.0360	0.0503	0.7153	0.4744	-0.0627	0.1347
SMB	-0.0948	0.0709	-1.3370	0.1812	-0.2337	0.0442
HML	-0.2117	0.0764	-2.7691	0.0056	-0.3615	-0.0618
RMW	0.0540	0.0795	0.6796	0.4967	-0.1018	0.2098
CMA	-0.1154	0.1137	-1.0148	0.3102	-0.3382	0.1075

11.3.3 Mean Square Error

Dep. Variable:	predict	R-squared:	6.43e-05			
Estimator:	PooledOLS	R-squared (Between):	-0.0089			
No. Observations:	531461	R-squared (Within):	-0.0001			
Date:	Wed, Jan 12 2022	R-squared (Overall):	6.43e-05			
Time:	00:12:41	Log-likelihood	2.055e+05			
Cov. Estimator:	Clustered					
		F-statistic:	34.175			
Entities:	7282	P-value	0.0000			
Avg Obs:	72.983	Distribution:	F(1,531459)			
Min Obs:	1.0000					
Max Obs:	252.00	F-statistic (robust):	0.5528			
		P-value	0.4572			
Time periods:	252	Distribution:	F(1,531459)			
Avg Obs:	2109.0					
Min Obs:	1869.0					
Max Obs:	3400.0					
	Parameter	Std. Err.	T-stat	P-value	Lower CI	Upper CI
const	0.0214	0.0024	8.9293	0.0000	0.0167	0.0261
Mkt-RF	0.0284	0.0383	0.7435	0.4572	-0.0465	0.1034

Dep. Variable:	predict	R-squared:	0.0017
Estimator:	PooledOLS	R-squared (Between):	-0.0045
No. Observations:	531461	R-squared (Within):	0.0011
Date:	Wed, Jan 12 2022	R-squared (Overall):	0.0017
Time:	00:12:46	Log-likelihood	2.06e+05
Cov. Estimator:	Clustered		
		F-statistic:	308.29
Entities:	7282	P-value	0.0000
Avg Obs:	72.983	Distribution:	F(3,531457)
Min Obs:	1.0000		
Max Obs:	252.00	F-statistic (robust):	9.0802
		P-value	0.0000
Time periods:	252	Distribution:	F(3,531457)
Avg Obs:	2109.0		
Min Obs:	1869.0		
Max Obs:	3400.0		

	Parameter	Std. Err.	T-stat	P-value	Lower CI	Upper CI
const	0.0219	0.0023	9.4144	0.0000	0.0174	0.0265
Mkt-RF	0.0416	0.0338	1.2299	0.2187	-0.0247	0.1079
SMB	-0.0977	0.0497	-1.9678	0.0491	-0.1951	-0.0004
HML	-0.1762	0.0413	-4.2623	0.0000	-0.2572	-0.0952

Dep. Variable:	predict	R-squared:	0.0018
Estimator:	PooledOLS	R-squared (Between):	-0.0043
No. Observations:	531461	R-squared (Within):	0.0012
Date:	Wed, Jan 12 2022	R-squared (Overall):	0.0018
Time:	00:12:52	Log-likelihood	2.06e+05
Cov. Estimator:	Clustered		
		F-statistic:	234.78
Entities:	7282	P-value	0.0000
Avg Obs:	72.983	Distribution:	F(4,531456)
Min Obs:	1.0000		
Max Obs:	252.00	F-statistic (robust):	6.7420
		P-value	0.0000
Time periods:	252	Distribution:	F(4,531456)
Avg Obs:	2109.0		
Min Obs:	1869.0		
Max Obs:	3400.0		

	Parameter	Std. Err.	T-stat	P-value	Lower CI	Upper CI
const	0.0217	0.0023	9.2469	0.0000	0.0171	0.0263
Mkt-RF	0.0486	0.0368	1.3196	0.1870	-0.0236	0.1208
SMB	-0.0808	0.0524	-1.5431	0.1228	-0.1835	0.0218
HML	-0.1926	0.0475	-4.0579	0.0000	-0.2856	-0.0995
RMW	0.0390	0.0620	0.6283	0.5298	-0.0826	0.1606

Dep. Variable:	predict	R-squared:	0.0018
Estimator:	PooledOLS	R-squared (Between):	-0.0037
No. Observations:	531461	R-squared (Within):	0.0011
Date:	Wed, Jan 12 2022	R-squared (Overall):	0.0018
Time:	00:12:58	Log-likelihood	2.06e+05
Cov. Estimator:	Clustered		
		F-statistic:	192.28
Entities:	7282	P-value	0.0000
Avg Obs:	72.983	Distribution:	F(5,531455)
Min Obs:	1.0000		
Max Obs:	252.00	F-statistic (robust):	5.7039
		P-value	0.0000
Time periods:	252	Distribution:	F(5,531455)
Avg Obs:	2109.0		
Min Obs:	1869.0		
Max Obs:	3400.0		

	Parameter	Std. Err.	T-stat	P-value	Lower CI	Upper CI
const	0.0219	0.0024	9.2120	0.0000	0.0173	0.0266
Mkt-RF	0.0389	0.0395	0.9851	0.3246	-0.0385	0.1163
SMB	-0.0764	0.0538	-1.4202	0.1555	-0.1818	0.0290
HML	-0.1670	0.0601	-2.7777	0.0055	-0.2849	-0.0492
RMW	0.0355	0.0613	0.5796	0.5622	-0.0846	0.1557
CMA	-0.0674	0.0884	-0.7631	0.4454	-0.2406	0.1058

11.3.4 Hedge Portfolio

Dep. Variable:	predict	R-squared:	0.0001			
Estimator:	PooledOLS	R-squared (Between):	-0.0012			
No. Observations:	531461	R-squared (Within):	-8.467e-05			
Date:	Wed, Jan 12 2022	R-squared (Overall):	0.0001			
Time:	00:14:18	Log-likelihood	8.196e+05			
Cov. Estimator:	Clustered					
		F-statistic:	58.421			
Entities:	7282	P-value	0.0000			
Avg Obs:	72.983	Distribution:	F(1,531459)			
Min Obs:	1.0000					
Max Obs:	252.00	F-statistic (robust):	1.5779			
		P-value	0.2091			
Time periods:	252	Distribution:	F(1,531459)			
Avg Obs:	2109.0					
Min Obs:	1869.0					
Max Obs:	3400.0					
	Parameter	Std. Err.	T-stat	P-value	Lower CI	Upper CI
const	-0.0001	0.0006	-0.1636	0.8700	-0.0014	0.0011
Mkt-RF	0.0117	0.0093	1.2561	0.2091	-0.0066	0.0300

Dep. Variable:	predict	R-squared:	0.0010
Estimator:	PooledOLS	R-squared (Between):	0.0004
No. Observations:	531461	R-squared (Within):	0.0001
Date:	Wed, Jan 12 2022	R-squared (Overall):	0.0010
Time:	00:14:24	Log-likelihood	8.198e+05
Cov. Estimator:	Clustered		
		F-statistic:	176.39
Entities:	7282	P-value	0.0000
Avg Obs:	72.983	Distribution:	F(3,531457)
Min Obs:	1.0000		
Max Obs:	252.00	F-statistic (robust):	6.0298
		P-value	0.0004
Time periods:	252	Distribution:	F(3,531457)
Avg Obs:	2109.0		
Min Obs:	1869.0		
Max Obs:	3400.0		

	Parameter	Std. Err.	T-stat	P-value	Lower CI	Upper CI
const	9.651e-06	0.0006	0.0154	0.9877	-0.0012	0.0012
Mkt-RF	0.0147	0.0089	1.6520	0.0985	-0.0027	0.0322
SMB	-0.0224	0.0150	-1.4946	0.1350	-0.0518	0.0070
HML	-0.0403	0.0114	-3.5497	0.0004	-0.0626	-0.0181

Dep. Variable:	predict	R-squared:	0.0010
Estimator:	PooledOLS	R-squared (Between):	0.0006
No. Observations:	531461	R-squared (Within):	0.0001
Date:	Wed, Jan 12 2022	R-squared (Overall):	0.0010
Time:	00:14:29	Log-likelihood	8.198e+05
Cov. Estimator:	Clustered		
		F-statistic:	133.80
Entities:	7282	P-value	0.0000
Avg Obs:	72.983	Distribution:	F(4,531456)
Min Obs:	1.0000		
Max Obs:	252.00	F-statistic (robust):	4.5136
		P-value	0.0012
Time periods:	252	Distribution:	F(4,531456)
Avg Obs:	2109.0		
Min Obs:	1869.0		
Max Obs:	3400.0		

	Parameter	Std. Err.	T-stat	P-value	Lower CI	Upper CI
const	-3.686e-05	0.0006	-0.0577	0.9540	-0.0013	0.0012
Mkt-RF	0.0162	0.0102	1.5819	0.1137	-0.0039	0.0362
SMB	-0.0189	0.0152	-1.2493	0.2116	-0.0487	0.0108
HML	-0.0437	0.0137	-3.1976	0.0014	-0.0705	-0.0169
RMW	0.0080	0.0178	0.4487	0.6537	-0.0269	0.0429

Dep. Variable:	predict	R-squared:	0.0011			
Estimator:	PooledOLS	R-squared (Between):	0.0007			
No. Observations:	531461	R-squared (Within):	0.0002			
Date:	Wed, Jan 12 2022	R-squared (Overall):	0.0011			
Time:	00:14:35	Log-likelihood	8.199e+05			
Cov. Estimator:	Clustered					
		F-statistic:	113.57			
Entities:	7282	P-value	0.0000			
Avg Obs:	72.983	Distribution:	F(5,531455)			
Min Obs:	1.0000					
Max Obs:	252.00	F-statistic (robust):	4.5345			
		P-value	0.0004			
Time periods:	252	Distribution:	F(5,531455)			
Avg Obs:	2109.0					
Min Obs:	1869.0					
Max Obs:	3400.0					
	Parameter	Std. Err.	T-stat	P-value	Lower CI	Upper CI
const	4.601e-05	0.0007	0.0705	0.9438	-0.0012	0.0013
Mkt-RF	0.0125	0.0115	1.0816	0.2794	-0.0101	0.0351
SMB	-0.0172	0.0156	-1.1022	0.2704	-0.0479	0.0134
HML	-0.0339	0.0186	-1.8263	0.0678	-0.0704	0.0025
RMW	0.0067	0.0176	0.3798	0.7041	-0.0277	0.0411
CMA	-0.0257	0.0260	-0.9892	0.3226	-0.0767	0.0253

11.3.5 Sharpe Ratio

Dep. Variable:	predict	R-squared:	0.0005			
Estimator:	PooledOLS	R-squared (Between):	-0.0182			
No. Observations:	531461	R-squared (Within):	0.0022			
Date:	Wed, Jan 12 2022	R-squared (Overall):	0.0005			
Time:	00:13:05	Log-likelihood	-4.309e+06			
Cov. Estimator:	Clustered					
		F-statistic:	251.19			
Entities:	7282	P-value	0.0000			
Avg Obs:	72.983	Distribution:	F(1,531459)			
Min Obs:	1.0000					
Max Obs:	252.00	F-statistic (robust):	5.9721			
		P-value	0.0145			
Time periods:	252	Distribution:	F(1,531459)			
Avg Obs:	2109.0					
Min Obs:	1869.0					
Max Obs:	3400.0					
	Parameter	Std. Err.	T-stat	P-value	Lower CI	Upper CI
const	515.46	11.658	44.213	0.0000	492.61	538.31
Mkt-RF	377.10	154.31	2.4438	0.0145	74.660	679.55

Dep. Variable:	predict	R-squared:	0.0009
Estimator:	PooledOLS	R-squared (Between):	-0.0157
No. Observations:	531461	R-squared (Within):	0.0026
Date:	Wed, Jan 12 2022	R-squared (Overall):	0.0009
Time:	00:13:10	Log-likelihood	-4.309e+06
Cov. Estimator:	Clustered		
		F-statistic:	165.26
Entities:	7282	P-value	0.0000
Avg Obs:	72.983	Distribution:	F(3,531457)
Min Obs:	1.0000		
Max Obs:	252.00	F-statistic (robust):	3.0642
		P-value	0.0268
Time periods:	252	Distribution:	F(3,531457)
Avg Obs:	2109.0		
Min Obs:	1869.0		
Max Obs:	3400.0		

	Parameter	Std. Err.	T-stat	P-value	Lower CI	Upper CI
const	513.97	11.583	44.372	0.0000	491.26	536.67
Mkt-RF	299.33	156.47	1.9131	0.0557	-7.3368	606.00
SMB	448.45	202.84	2.2109	0.0270	50.891	846.00
HML	287.99	219.41	1.3126	0.1893	-142.05	718.03

Dep. Variable:	predict	R-squared:	0.0011
Estimator:	PooledOLS	R-squared (Between):	-0.0164
No. Observations:	531461	R-squared (Within):	0.0030
Date:	Wed, Jan 12 2022	R-squared (Overall):	0.0011
Time:	00:13:16	Log-likelihood	-4.309e+06
Cov. Estimator:	Clustered		
		F-statistic:	140.23
Entities:	7282	P-value	0.0000
Avg Obs:	72.983	Distribution:	F(4,531456)
Min Obs:	1.0000		
Max Obs:	252.00	F-statistic (robust):	2.6567
		P-value	0.0311
Time periods:	252	Distribution:	F(4,531456)
Avg Obs:	2109.0		
Min Obs:	1869.0		
Max Obs:	3400.0		

	Parameter	Std. Err.	T-stat	P-value	Lower CI	Upper CI
const	511.59	11.602	44.097	0.0000	488.85	534.33
Mkt-RF	372.53	166.58	2.2364	0.0253	46.042	699.03
SMB	625.16	240.74	2.5968	0.0094	153.31	1097.0
HML	116.87	242.71	0.4815	0.6301	-358.83	592.57
RMW	407.57	267.94	1.5211	0.1282	-117.59	932.73

Dep. Variable:	predict	R-squared:	0.0014
Estimator:	PooledOLS	R-squared (Between):	-0.0147
No. Observations:	531461	R-squared (Within):	0.0036
Date:	Wed, Jan 12 2022	R-squared (Overall):	0.0014
Time:	00:13:22	Log-likelihood	-4.309e+06
Cov. Estimator:	Clustered		
		F-statistic:	151.56
Entities:	7282	P-value	0.0000
Avg Obs:	72.983	Distribution:	F(5,531455)
Min Obs:	1.0000		
Max Obs:	252.00	F-statistic (robust):	4.7366
		P-value	0.0002
Time periods:	252	Distribution:	F(5,531455)
Avg Obs:	2109.0		
Min Obs:	1869.0		
Max Obs:	3400.0		

	Parameter	Std. Err.	T-stat	P-value	Lower CI	Upper CI
const	508.43	11.738	43.317	0.0000	485.43	531.44
Mkt-RF	513.58	162.35	3.1634	0.0016	195.38	831.78
SMB	560.35	247.29	2.2660	0.0235	75.674	1045.0
HML	-254.64	301.76	-0.8439	0.3988	-846.08	336.80
RMW	457.76	254.35	1.7997	0.0719	-40.768	956.28
CMA	981.43	381.11	2.5752	0.0100	234.46	1728.4

11.3.6 Sharpe Ratio (MSE)

Dep. Variable:	predict	R-squared:	0.0012			
Estimator:	PooledOLS	R-squared (Between):	-3.227e-05			
No. Observations:	531461	R-squared (Within):	0.0036			
Date:	Wed, Jan 12 2022	R-squared (Overall):	0.0012			
Time:	00:13:29	Log-likelihood	-4.012e+06			
Cov. Estimator:	Clustered					
		F-statistic:	615.62			
Entities:	7282	P-value	0.0000			
Avg Obs:	72.983	Distribution:	F(1,531459)			
Min Obs:	1.0000					
Max Obs:	252.00	F-statistic (robust):	11.803			
		P-value	0.0006			
Time periods:	252	Distribution:	F(1,531459)			
Avg Obs:	2109.0					
Min Obs:	1869.0					
Max Obs:	3400.0					
	Parameter	Std. Err.	T-stat	P-value	Lower CI	Upper CI
const	-2.0195	7.2544	-0.2784	0.7807	-16.238	12.199
Mkt-RF	337.61	98.269	3.4355	0.0006	145.00	530.21

Dep. Variable:	predict	R-squared:	0.0021
Estimator:	PooledOLS	R-squared (Between):	-0.0008
No. Observations:	531461	R-squared (Within):	0.0062
Date:	Wed, Jan 12 2022	R-squared (Overall):	0.0021
Time:	00:13:35	Log-likelihood	-4.012e+06
Cov. Estimator:	Clustered		
		F-statistic:	371.06
Entities:	7282	P-value	0.0000
Avg Obs:	72.983	Distribution:	F(3,531457)
Min Obs:	1.0000		
Max Obs:	252.00	F-statistic (robust):	7.5525
		P-value	0.0000
Time periods:	252	Distribution:	F(3,531457)
Avg Obs:	2109.0		
Min Obs:	1869.0		
Max Obs:	3400.0		

	Parameter	Std. Err.	T-stat	P-value	Lower CI	Upper CI
const	-3.2212	7.1863	-0.4482	0.6540	-17.306	10.864
Mkt-RF	265.80	99.851	2.6619	0.0078	70.091	461.50
SMB	398.64	125.68	3.1719	0.0015	152.31	644.96
HML	175.73	127.91	1.3739	0.1695	-74.971	426.43

Dep. Variable:	predict	R-squared:	0.0021
Estimator:	PooledOLS	R-squared (Between):	-0.0009
No. Observations:	531461	R-squared (Within):	0.0063
Date:	Wed, Jan 12 2022	R-squared (Overall):	0.0021
Time:	00:13:41	Log-likelihood	-4.012e+06
Cov. Estimator:	Clustered		
		F-statistic:	281.52
Entities:	7282	P-value	0.0000
Avg Obs:	72.983	Distribution:	F(4,531456)
Min Obs:	1.0000		
Max Obs:	252.00	F-statistic (robust):	5.5799
		P-value	0.0002
Time periods:	252	Distribution:	F(4,531456)
Avg Obs:	2109.0		
Min Obs:	1869.0		
Max Obs:	3400.0		

	Parameter	Std. Err.	T-stat	P-value	Lower CI	Upper CI
const	-2.6172	7.2031	-0.3633	0.7163	-16.735	11.501
Mkt-RF	247.17	96.079	2.5726	0.0101	58.859	435.48
SMB	353.68	169.79	2.0830	0.0372	20.894	686.46
HML	219.27	167.82	1.3066	0.1914	-109.66	548.20
RMW	-103.70	232.82	-0.4454	0.6560	-560.02	352.62

Dep. Variable:	predict	R-squared:	0.0024
Estimator:	PooledOLS	R-squared (Between):	-0.0015
No. Observations:	531461	R-squared (Within):	0.0070
Date:	Wed, Jan 12 2022	R-squared (Overall):	0.0024
Time:	00:13:47	Log-likelihood	-4.012e+06
Cov. Estimator:	Clustered		
		F-statistic:	251.06
Entities:	7282	P-value	0.0000
Avg Obs:	72.983	Distribution:	F(5,531455)
Min Obs:	1.0000		
Max Obs:	252.00	F-statistic (robust):	4.6489
		P-value	0.0003
Time periods:	252	Distribution:	F(5,531455)
Avg Obs:	2109.0		
Min Obs:	1869.0		
Max Obs:	3400.0		

	Parameter	Std. Err.	T-stat	P-value	Lower CI	Upper CI
const	-4.0803	7.2899	-0.5597	0.5757	-18.368	10.208
Mkt-RF	312.48	111.24	2.8092	0.0050	94.460	530.50
SMB	323.67	173.60	1.8645	0.0623	-16.579	663.91
HML	47.253	189.32	0.2496	0.8029	-323.81	418.31
RMW	-80.461	226.84	-0.3547	0.7228	-525.05	364.13
CMA	454.42	286.99	1.5834	0.1133	-108.07	1016.9

11.3.7 Information Ratio

Dep. Variable:	predict	R-squared:	0.0003
Estimator:	PooledOLS	R-squared (Between):	-0.0262
No. Observations:	531461	R-squared (Within):	0.0003
Date:	Wed, Jan 12 2022	R-squared (Overall):	0.0003
Time:	00:13:54	Log-likelihood	-2.199e+06
Cov. Estimator:	Clustered		
		F-statistic:	143.01
Entities:	7282	P-value	0.0000
Avg Obs:	72.983	Distribution:	F(1,531459)
Min Obs:	1.0000		
Max Obs:	252.00	F-statistic (robust):	1.0833
		P-value	0.2980
Time periods:	252	Distribution:	F(1,531459)
Avg Obs:	2109.0		
Min Obs:	1869.0		
Max Obs:	3400.0		

	Parameter	Std. Err.	T-stat	P-value	Lower CI	Upper CI
const	12.010	0.2909	41.291	0.0000	11.440	12.580
Mkt-RF	5.3702	5.1597	1.0408	0.2980	-4.7426	15.483

Dep. Variable:	predict	R-squared:	0.0029
Estimator:	PooledOLS	R-squared (Between):	-0.0261
No. Observations:	531461	R-squared (Within):	0.0065
Date:	Wed, Jan 12 2022	R-squared (Overall):	0.0029
Time:	00:13:59	Log-likelihood	-2.199e+06
Cov. Estimator:	Clustered		
		F-statistic:	520.75
Entities:	7282	P-value	0.0000
Avg Obs:	72.983	Distribution:	F(3,531457)
Min Obs:	1.0000		
Max Obs:	252.00	F-statistic (robust):	7.6605
		P-value	0.0000
Time periods:	252	Distribution:	F(3,531457)
Avg Obs:	2109.0		
Min Obs:	1869.0		
Max Obs:	3400.0		

	Parameter	Std. Err.	T-stat	P-value	Lower CI	Upper CI
const	12.069	0.2872	42.030	0.0000	11.507	12.632
Mkt-RF	7.1435	4.4295	1.6127	0.1068	-1.5383	15.825
SMB	-12.519	4.1587	-3.0104	0.0026	-20.670	-4.3684
HML	-19.960	5.1500	-3.8758	0.0001	-30.054	-9.8665

Dep. Variable:	predict	R-squared:	0.0030
Estimator:	PooledOLS	R-squared (Between):	-0.0257
No. Observations:	531461	R-squared (Within):	0.0068
Date:	Wed, Jan 12 2022	R-squared (Overall):	0.0030
Time:	00:14:05	Log-likelihood	-2.199e+06
Cov. Estimator:	Clustered		
		F-statistic:	400.77
Entities:	7282	P-value	0.0000
Avg Obs:	72.983	Distribution:	F(4,531456)
Min Obs:	1.0000		
Max Obs:	252.00	F-statistic (robust):	5.8743
		P-value	0.0001
Time periods:	252	Distribution:	F(4,531456)
Avg Obs:	2109.0		
Min Obs:	1869.0		
Max Obs:	3400.0		

	Parameter	Std. Err.	T-stat	P-value	Lower CI	Upper CI
const	12.034	0.2895	41.574	0.0000	11.467	12.601
Mkt-RF	8.2350	4.7134	1.7472	0.0806	-1.0031	17.473
SMB	-9.8842	5.7640	-1.7148	0.0864	-21.181	1.4130
HML	-22.512	6.1096	-3.6847	0.0002	-34.487	-10.537
RMW	6.0775	7.4930	0.8111	0.4173	-8.6086	20.764

Dep. Variable:	predict	R-squared:	0.0031
Estimator:	PooledOLS	R-squared (Between):	-0.0253
No. Observations:	531461	R-squared (Within):	0.0069
Date:	Wed, Jan 12 2022	R-squared (Overall):	0.0031
Time:	00:14:11	Log-likelihood	-2.199e+06
Cov. Estimator:	Clustered		
		F-statistic:	327.62
Entities:	7282	P-value	0.0000
Avg Obs:	72.983	Distribution:	F(5,531455)
Min Obs:	1.0000		
Max Obs:	252.00	F-statistic (robust):	5.9797
		P-value	0.0000
Time periods:	252	Distribution:	F(5,531455)
Avg Obs:	2109.0		
Min Obs:	1869.0		
Max Obs:	3400.0		

	Parameter	Std. Err.	T-stat	P-value	Lower CI	Upper CI
const	12.059	0.2881	41.859	0.0000	11.494	12.624
Mkt-RF	7.1143	4.8513	1.4665	0.1425	-2.3941	16.623
SMB	-9.3693	6.1042	-1.5349	0.1248	-21.333	2.5948
HML	-19.560	8.4300	-2.3203	0.0203	-36.083	-3.0376
RMW	5.6787	7.3114	0.7767	0.4373	-8.6515	20.009
CMA	-7.7979	11.363	-0.6863	0.4925	-30.068	14.473

12 Discussion

13 Contributions

14 Conclusion

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15 Appendix

15.1 Ordinary Least Squares (OLS)

The OLS regression is the most prominent statistical model in asset pricing theory. Rosenfeld (2021) summarises OLS. The composition of the true OLS (27) model includes four components. Firstly, \mathbf{X} , an $n \times k$ matrix of k independent variables for n observations. Secondly, \mathbf{y} , an $n \times 1$ vector of observation on the dependent variable. Thirdly, ϵ , an $n \times 1$ vector of unexplained error. Lastly, θ , a $k \times 1$ vector of parameters to be estimated.

$$y = X\theta + \epsilon \quad (27)$$

15.1.1 Estimation Criteria

The criteria to obtain the parameter estimate ($\hat{\theta}$) relies on the minimisation of the sum of squared residuals (28). We highlight the observed residuals (e) are distinct from unexplained disturbances (ϵ). Equation 29 derives residuals by taking the difference between observations based on parameter estimates.

$$\sum e_i^2 \quad (28)$$

$$e = y - X\hat{\theta} \quad (29)$$

Expanding the quadratic $e^T e$ after substituting in equation 29 leads to the alternative expression of the sum of squared residuals in equation 30. Minimizing the sum of square residuals requires taking the partial derivative of equation 30 with respect to the estimated parameters (equation) using matrix differentiation (31). It is imperative X has full rank where all vectors in the matrix are linearly independent, validating both the presence of

a positive definite matrix and minimum.

$$e^T e = y^T y - 2\hat{\theta}^T X^T y + \hat{\theta}^T X^T \hat{\theta} X \quad (30)$$

$$\frac{\partial e^T e}{\partial \hat{\theta}} = -2X^T y + 2X^T X \hat{\theta} = 0 \quad (31)$$

We find the expression for the Ordinary Least Squares (OLS) estimator (32) after rearranging equation 31 to normal form, utilizing inverse matrices to form identity matrices, and simplifying.

$$\begin{aligned} 2X^T X \hat{\theta} &= 2X^T y \\ (X^T X)^{-1}(X^T X) \hat{\theta} &= (X^T X)^{-1} X^T y \\ I \hat{\theta} &= (X^T X)^{-1} X^T y \\ \hat{\theta} &= (X^T X)^{-1} (X^T y) \end{aligned} \quad (32)$$

Therefore, we can use the OLS estimator to make predictions with OLS (33).

$$\hat{y} = X^T \hat{\theta} \quad (33)$$

15.1.2 Properties of OLS Estimators

There are six key properties in addition to the satisfaction in minimizing the summation of squared residuals.

1. The residuals are uncorrelated with the observed values of X i.e., $X^T e = 0$.
2. The sum of the residuals is zero i.e., $\sum e_i = 0$.
3. The sample mean of the residuals is zero i.e., $\bar{e} = \frac{\sum e_i}{n} = 0$.
4. The regression hyperplane passes through the means of observed values i.e., $\frac{e}{n} \frac{y - X\theta}{n} = 0$. Since $\bar{e} = 0$ assumed, it is implied $\bar{y} = \bar{x}\bar{\theta}$.

5. The residuals are uncorrelated with the predicted y i.e., $\hat{y} = X\hat{\theta}$, $\hat{y}^T e = (X\hat{\beta})^T e = b^T X^T e = 0$
6. The mean of \hat{y} for the sample will equal the mean of the y .

15.1.3 The Gauss-Markov Theorem

However, OLS makes Gauss-Markov assumptions about the true model to make inferences regarding β from $\hat{\beta}$. The intention of the Gauss-Markov Theorem, conditional on the below assumptions, states the OLS estimator is the best linear, unbiased, and efficient estimator:

$$y = x\beta + \epsilon$$

$$E[\epsilon|X] = 0 \tag{34}$$

$$E(\epsilon\epsilon^T|X) = \Omega = \sigma^2 I \tag{35}$$

$$\epsilon|X \sim N[0, \sigma^2 I] \text{ (hypothesis testing)}$$

- X is an $n \times k$ matrix of full rank
- X must be generated randomly, or fixed, by a mechanism uncorrelated to disturbances.

Equation 34 implies $E(y) = X\beta$ as no observations of the independent variables convey any information about the expected values of the disturbances. Equation 35 captures homoskedasticity and no autocorrelation assumptions.

15.2 Tensorflow Model Performance

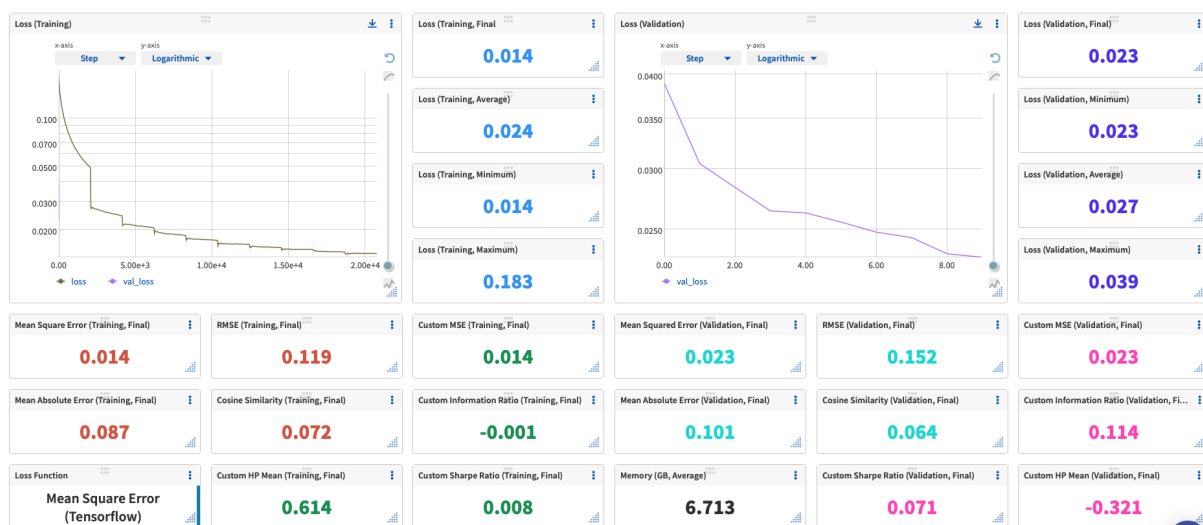


Figure 4: Mean Square Error Model (Tensorflow)

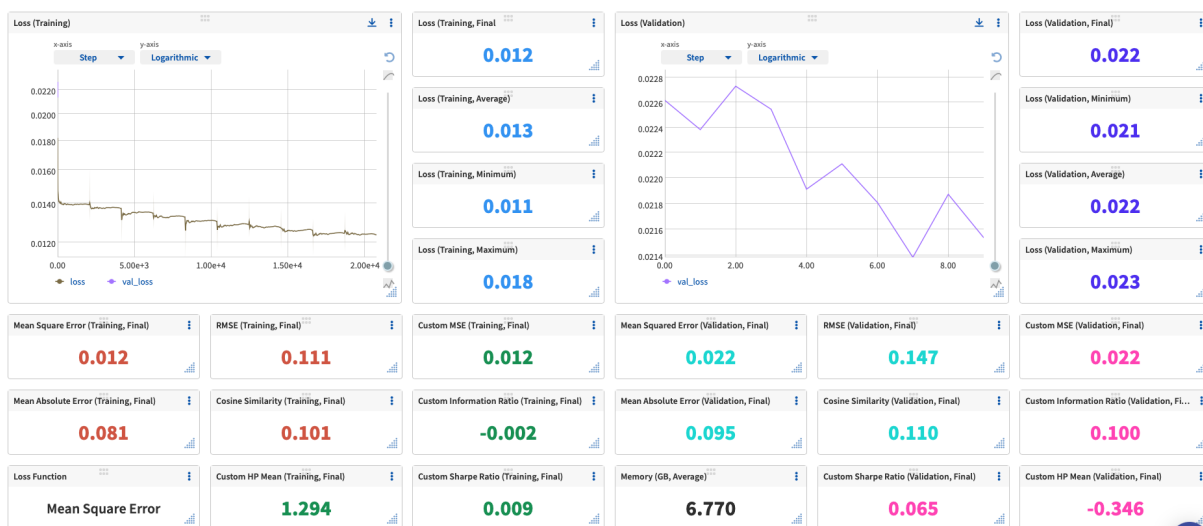


Figure 5: Mean Square Error Loss Function

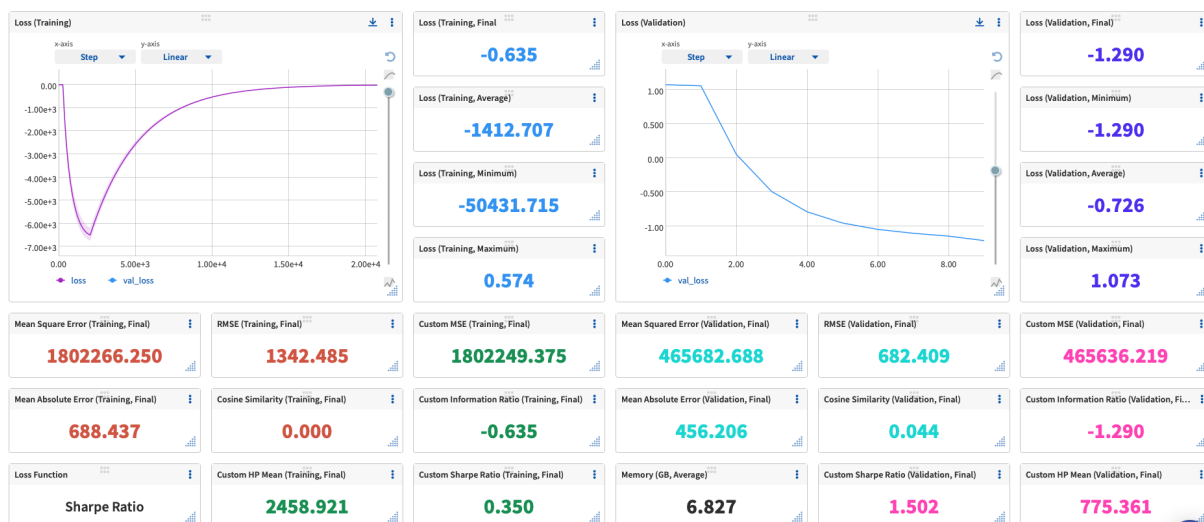


Figure 6: Sharpe Ratio Loss Function

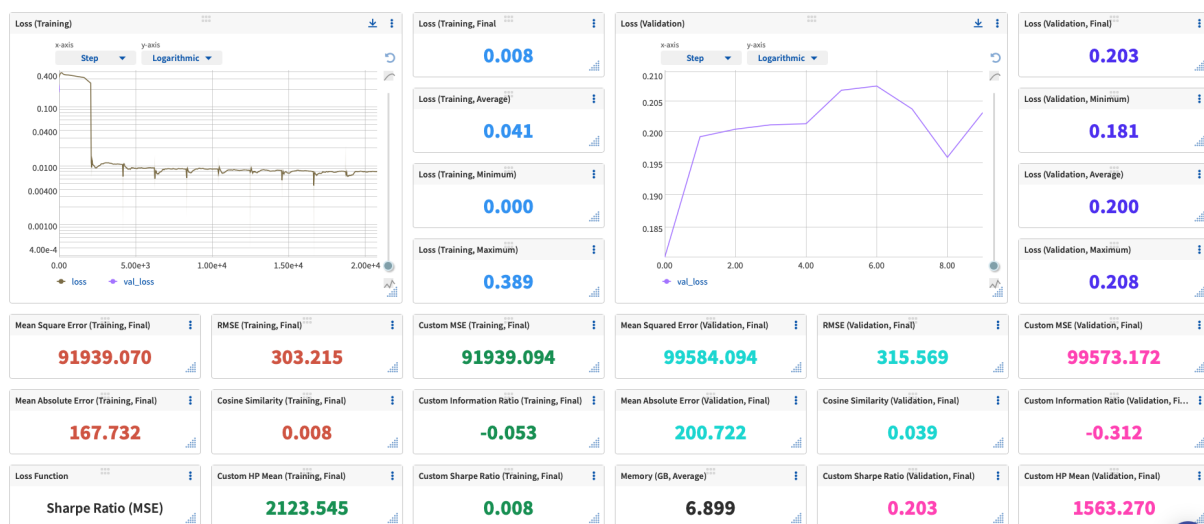


Figure 7: Sharpe Ratio (MSE) Loss Function

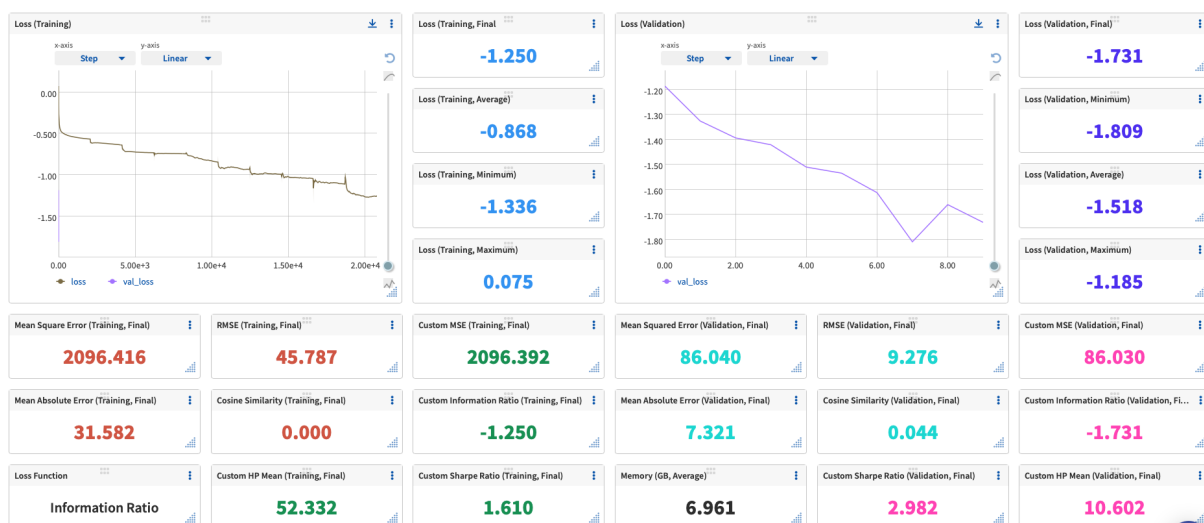


Figure 8: Information Ratio Loss Function

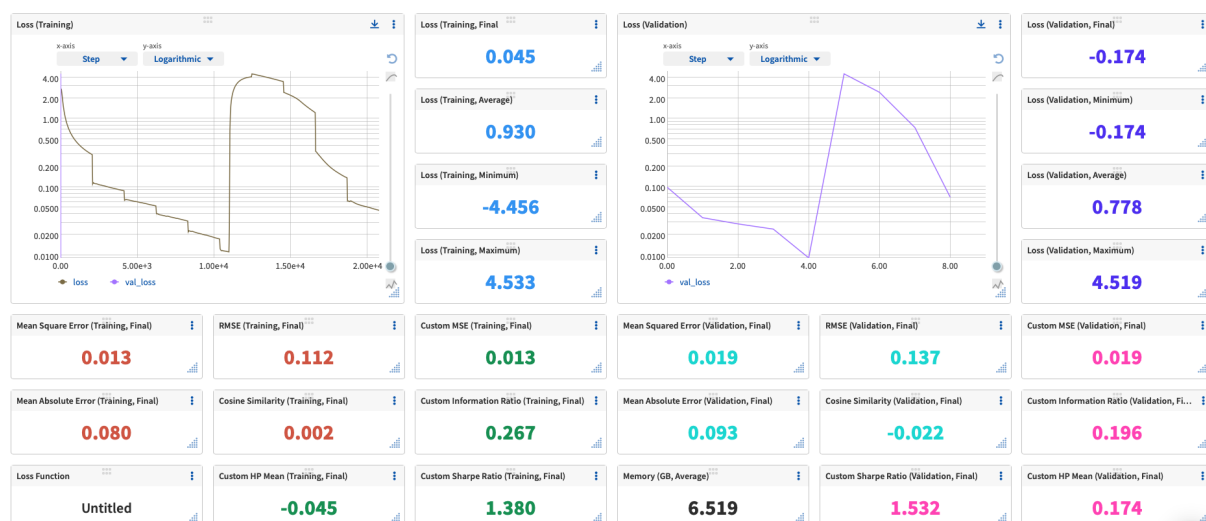


Figure 9: Hedge Portfolio Loss Function

15.3 Technical Details

15.3.1 Organisation

This research essay uses data science best practise (Wilson et al., 2016). Data and results saved regularly and reproducible. Data retention in all forms receives high levels of attention. Project files synchronises continuously to Google Drive (Google LLC, 2020). Git (Linus Torvalds, 2020) manages version control protocols for source code, data, documents, and results. Git stores a complete history of versions using Git hashes. These hashes are strings unique to each state of the publicly available finance-honours repository¹. Git hashes enable discretisation of finance-honours development, enabling the accessibility and recollection of all previous states given a unique git hash. This functionality enables reproducibility, error correction, and the ability to revert to previous models.

15.3.2 Version Control

Git, hosted by GitHub, provides a comprehensive set of version control technologies and range of benefits. Firstly, Git enables collaborative functionalities. The master version of a project is accessible for all who have access to the repository. Each contributor can create custom copies of branches through pull requests on the master branch. Contributors can commit changes to custom branches and push these changes to the master branch through push requests. Product managers can review push requests, approving valid requests for integrating changes to the master branch. Collaborative efforts are possible with commit messages describing contributions from each contributor. This research essay has only one contributor, rendering collaborative functionalities redundant in this instance. Git ensures the storage of code, work, and author histories. The descriptive nature of commit logs ensures journal accuracy.

¹<https://github.com/CMCD1996/finance-honours>

15.3.3 Directories

This research essay follows directory structure recommendations from Wilson et al (2016). Organisation is crucial as the modelling of artificial neural networks involves integrating a range of optimisation models, data files and documents. Directory management is most efficient and comprehensive. **finance-honours** is the root directory containing the following sub directories: bin, data, doc, src, and results. The **bin** sub directory contains external scripts and compiled programmes. The **data** sub directory contains all raw data associated with the project. The **doc** sub directory stores user guides, academic resources, research reports and project deliverables. The **results** sub directory contains the outputs from project analysis. The **src** sub directory stores the source code for preparing datasets, partitioning sets of geographies with varying granularities. All files were continuously backed up using Google Drive and Git.

15.3.4 Python

Python 3.9.7 is the primary programming language for this research essay. The language is omnipresent, widespread in software development. Python's language design makes the language highly productive and simple to use. Python can hand off computationally straining tasks to C/C++ using supporting first-class integration capabilities. The language also has a very active and supportive community. Python is the most popular coding language on the planet defined by the PYPL PopularitY of Programming Language Index. As at December 2021, Python has 30.21% of all language tutorial search instances on Google (PYPL, 2021). Python's dynamic, low cost, and open source nature makes programming quick.

15.3.5 Package Management

The Anaconda package management platform for Python (Anaconda, Inc., 2020) is the chosen coding environment. Anaconda is a well defined, free platform, with known versions of python packages such as matplotlib, numpy, and pip. The use of this environment ensures reproducibility and consistency across infrastructure. Pip is the default package

manager for Python, included in the Anaconda package. Pip manages package installation and updates.

15.3.6 Code Style

The PEP8 style for Python Code is formatting style for development code Guido Van Rossum and Coghlan, 2001. Yapf, a formatter maintained by Google, manages formatting. Standardised formatting is important as makes supports readability, optimisation, and consistency. Docstrings and rigorous commenting are important in documentation. A docstring is a Python inline comment describing function use, inputs, and outputs. An unique docstring belongs to each Python class and function. The Google style docstring is most appropriate because of it's readability, writing ease, and consistency with Google's Style Guide. The parsing of yapf docstrings enables automated documentation generators to create docstring documents describing functions and classes.

15.3.7 Infrastructure

This research essay deploys variations in artificial neural networks of changing size and complexity. Analysis either took place locally, or remotely, depending on the computational requirements for the particular analysis. An Apple MacBook Pro 13 Inch 2019 with 8 GB 2133 MHz LPDDR3 memory and 1.4 GHz Quad-Core Intel Core i5 processor handles simple tasks locally. A Virtual Machine Instance on the Google Cloud Platform **Insert specification before submission** handles complex tasks remotely.

15.3.8 Documentation

The research essay documentation keeps an accurate record of key design decisions. Commit histories (15.3.2) is the most important form of documentation. Application of auxiliary documentation methods are supplementary.

15.4 Dataset Summary Statistics

	count	mean	std	min	25%	50%	75%	max
permno	2739928.0	5.405281e+04	2.782267e+04	10000.0000	2.651800e+04	5.715400e+04	8.018600e+04	9.343600e+04
permco	2739928.0	1.843974e+04	1.402881e+04	3.0000	7.702000e+03	1.640850e+04	2.321000e+04	5.766700e+04
crsp_shred	2739928.0	1.089520e+01	4.571000e-01	10.0000	1.100000e+01	1.100000e+01	1.100000e+01	1.200000e+01
crsp_exched	2739928.0	2.127400e+00	9.343000e-01	1.0000	1.000000e+00	3.000000e+00	3.000000e+00	3.000000e+00
sic	2692217.0	4.605936e+03	1.921398e+03	100.0000	3.271000e+03	4.011000e+03	6.036000e+03	9.999000e+03
ff49	2674304.0	3.037380e+01	1.341740e+01	1.0000	1.800000e+01	3.400000e+01	4.300000e+01	4.900000e+01
adjfct	2739928.0	2.838700e+00	1.267170e+01	0.0000	1.000000e+00	1.000000e+00	2.000000e+00	1.215000e+03
shares	2739928.0	6.078630e+01	2.852566e+02	0.0830	4.399000e+00	1.251900e+01	3.808200e+01	2.920640e+04
me	2739928.0	2.241254e+03	1.473073e+04	1.1708	4.367020e+01	1.565628e+02	7.167608e+02	2.255969e+06
me_company	2739928.0	2.283180e+03	1.527340e+04	1.1708	4.387450e+01	1.574086e+02	7.211363e+02	2.255969e+06
prc	2739928.0	2.876220e+01	6.488772e+02	0.0078	7.875000e+00	1.612500e+01	2.912500e+01	1.416000e+05
prc_local	2739928.0	2.876220e+01	6.488772e+02	0.0078	7.875000e+00	1.612500e+01	2.912500e+01	1.416000e+05
dolvol	2580622.0	3.282292e+08	2.520900e+09	0.0000	1.070786e+06	7.165154e+06	7.076108e+07	8.441730e+11
ret	2719460.0	1.640000e-02	1.672000e-01	-1.0000	-5.880000e-02	4.100000e-03	7.410000e-02	2.400000e+01
ret_local	2719460.0	1.640000e-02	1.672000e-01	-1.0000	-5.880000e-02	4.100000e-03	7.410000e-02	2.400000e+01
ret_exc	2719460.0	1.270000e-02	1.673000e-01	-1.0068	-6.250000e-02	7.000000e-04	7.060000e-02	2.399690e+01
ret_lag_dif	2739928.0	1.000000e+00	0.000000e+00	1.0000	1.000000e+00	1.000000e+00	1.000000e+00	1.000000e+00
ret_exc_lead1m	2732542.0	6.400000e-03	1.559000e-01	-1.0113	-6.560000e-02	-1.800000e-03	6.710000e-02	1.988170e+01
market_equity_rank_x	2739928.0	5.982920e+01	2.380660e+01	1.0000	4.000000e+01	6.000000e+01	8.000000e+01	9.950000e+01
enterprise_value_rank_x	2480615.0	5.845440e+01	2.501660e+01	1.0000	3.800000e+01	5.900000e+01	8.000000e+01	9.950000e+01
book_equity_rank_x	2452453.0	5.800700e+01	2.593820e+01	1.0000	3.800000e+01	5.900000e+01	8.000000e+01	9.950000e+01
assets_rank_x	2522907.0	5.751850e+01	2.635510e+01	1.0000	3.700000e+01	5.900000e+01	8.000000e+01	9.950000e+01
sales_rank_x	2509790.0	5.691950e+01	2.717080e+01	1.0000	3.600000e+01	5.900000e+01	8.000000e+01	9.950000e+01
net_income_rank_x	2517298.0	5.581200e+01	2.878360e+01	1.0000	3.300000e+01	6.000000e+01	8.000000e+01	9.950000e+01
bidask_x	2739928.0	1.289000e-01	3.351000e-01	0.0000	0.000000e+00	0.000000e+00	0.000000e+00	1.000000e+00
prc_high_x	2355383.0	2.540480e+01	2.608370e+01	0.1790	9.250000e+00	1.850000e+01	3.300000e+01	4.617600e+02
prc_low_x	2365005.0	2.211970e+01	2.325750e+01	0.0818	7.640000e+00	1.600000e+01	2.880000e+01	4.175300e+02

	count	mean	std	min	25%	50%	75%	max
tvol_x	2580622.0	8.316484e+06	2.941295e+07	0.0000	9.875000e+04	5.510000e+05	3.923700e+06	6.485186e+08
div1m_me_x	2718102.0	1.300000e-03	3.700000e-03	0.0000	0.000000e+00	0.000000e+00	0.000000e+00	9.010000e-02
div3m_me_x	2718121.0	4.000000e-03	6.000000e-03	0.0000	0.000000e+00	0.000000e+00	6.700000e-03	1.164000e-01
div6m_me_x	2660395.0	8.100000e-03	1.170000e-02	0.0000	0.000000e+00	0.000000e+00	1.360000e-02	1.472000e-01
div12m_me_x	2548844.0	1.670000e-02	2.350000e-02	0.0000	0.000000e+00	3.800000e-03	2.780000e-02	4.015000e-01
chcsho_1m_x	2720001.0	3.200000e-03	2.550000e-02	-0.1168	0.000000e+00	0.000000e+00	0.000000e+00	1.096800e+00
chcsho_3m_x	2681179.0	1.240000e-02	6.180000e-02	-0.1424	0.000000e+00	0.000000e+00	3.300000e-03	1.686700e+00
chcsho_6m_x	2624125.0	2.810000e-02	1.189000e-01	-0.1880	0.000000e+00	9.000000e-04	1.070000e-02	3.832600e+00
chcsho_12m_x	2514147.0	6.190000e-02	2.297000e-01	-0.2696	0.000000e+00	4.700000e-03	3.390000e-02	8.477000e+00
eqnpo_1m_x	2718435.0	-1.500000e-03	2.310000e-02	-0.6801	-0.000000e+00	0.000000e+00	0.000000e+00	1.263000e-01
eqnpo_3m_x	2677912.0	-6.200000e-03	5.200000e-02	-0.9973	-1.800000e-03	0.000000e+00	8.000000e-03	1.696000e-01
eqnpo_6m_x	2618619.0	-1.350000e-02	8.900000e-02	-1.5754	-7.400000e-03	0.000000e+00	1.640000e-02	2.788000e-01
eqnpo_12m_x	2504936.0	-2.670000e-02	1.474000e-01	-2.2489	-2.450000e-02	0.000000e+00	3.340000e-02	4.743000e-01
ret_1_0_x	2541516.0	1.490000e-02	1.481000e-01	-0.7242	-6.120000e-02	7.900000e-03	7.690000e-02	2.176500e+00
ret_2_0_x	2521767.0	2.960000e-02	2.125000e-01	-0.8327	-8.110000e-02	1.480000e-02	1.176000e-01	3.342500e+00
ret_3_0_x	2503682.0	4.400000e-02	2.649000e-01	-0.8864	-9.610000e-02	2.270000e-02	1.506000e-01	5.000000e+00
ret_3_1_x	2502019.0	2.870000e-02	2.108000e-01	-0.8310	-8.140000e-02	1.440000e-02	1.167000e-01	3.342500e+00
ret_6_0_x	2447794.0	8.830000e-02	3.970000e-01	-0.9396	-1.267000e-01	4.500000e-02	2.336000e-01	8.555600e+00
ret_6_1_x	2446030.0	7.230000e-02	3.553000e-01	-0.9171	-1.184000e-01	3.700000e-02	2.059000e-01	8.411800e+00
ret_9_0_x	2393988.0	1.336000e-01	5.093000e-01	-0.9721	-1.466000e-01	6.750000e-02	3.069000e-01	9.857100e+00
ret_9_1_x	2392087.0	1.168000e-01	4.700000e-01	-0.9555	-1.414000e-01	5.930000e-02	2.812000e-01	9.273700e+00
ret_12_0_x	2341375.0	1.813000e-01	6.179000e-01	-0.9783	-1.593000e-01	9.080000e-02	3.773000e-01	1.301590e+01
ret_12_1_x	2339380.0	1.635000e-01	5.789000e-01	-0.9728	-1.558000e-01	8.200000e-02	3.514000e-01	1.223080e+01
ret_12_7_x	2337747.0	7.050000e-02	3.478000e-01	-0.9055	-1.163000e-01	3.610000e-02	2.015000e-01	8.509400e+00
ret_18_1_x	2239551.0	2.625000e-01	7.812000e-01	-0.9850	-1.710000e-01	1.321000e-01	4.926000e-01	2.048480e+01
ret_24_1_x	2145964.0	3.596000e-01	9.260000e-01	-0.9890	-1.717000e-01	1.837000e-01	6.267000e-01	1.484620e+01
ret_24_12_x	2142652.0	1.821000e-01	6.037000e-01	-0.9678	-1.493000e-01	9.260000e-02	3.714000e-01	1.345160e+01
ret_36_1_x	1976435.0	5.673000e-01	1.234400e+00	-0.9935	-1.548000e-01	2.964000e-01	8.916000e-01	1.914000e+01
ret_36_12_x	1972590.0	3.838000e-01	9.482000e-01	-0.9864	-1.546000e-01	2.006000e-01	6.490000e-01	1.702520e+01

	count	mean	std	min	25%	50%	75%	max
ret_48_12_x	1821582.0	5.938000e-01	1.256400e+00	-0.9918	-1.358000e-01	3.161000e-01	9.172000e-01	1.811810e+01
ret_48_1_x	1826053.0	7.976000e-01	1.577300e+00	-0.9965	-1.285000e-01	4.175000e-01	1.176300e+00	1.772000e+01
ret_60_1_x	1691563.0	1.064400e+00	2.014800e+00	-0.9985	-9.170000e-02	5.486000e-01	1.492300e+00	2.754720e+01
ret_60_12_x	1686573.0	8.258000e-01	1.611700e+00	-0.9960	-1.096000e-01	4.364000e-01	1.200000e+00	2.063640e+01
ret_60_36_x	1680619.0	3.857000e-01	9.340000e-01	-0.9860	-1.429000e-01	2.072000e-01	6.479000e-01	1.808570e+01
seas_1_1an_x	2426517.0	1.420000e-02	1.421000e-01	-0.6705	-6.040000e-02	7.600000e-03	7.560000e-02	1.823500e+00
seas_1_1na_x	1870192.0	1.490000e-02	4.360000e-02	-0.2355	-7.800000e-03	1.280000e-02	3.460000e-02	3.871000e-01
seas_2_5an_x	1599992.0	1.520000e-02	6.790000e-02	-0.2970	-2.260000e-02	1.180000e-02	4.810000e-02	6.337000e-01
at_gr1_x	2426455.0	2.641000e-01	9.239000e-01	-0.7398	4.800000e-03	9.050000e-02	2.391000e-01	3.163840e+01
ca_gr1_x	2184566.0	3.206000e-01	1.336600e+00	-0.8313	-3.830000e-02	9.400000e-02	2.815000e-01	4.636900e+01
nca_gr1_x	2183067.0	3.950000e-01	1.682300e+00	-0.8737	-1.530000e-02	8.250000e-02	2.844000e-01	5.781320e+01
lt_gr1_x	2408077.0	3.042000e-01	9.791000e-01	-0.8021	-2.990000e-02	8.560000e-02	2.894000e-01	1.783760e+01
cl_gr1_x	2190296.0	2.996000e-01	8.898000e-01	-0.8494	-6.490000e-02	1.114000e-01	3.701000e-01	1.634630e+01
ncl_gr1_x	2075342.0	9.926000e-01	5.509500e+00	-1.0000	-1.023000e-01	3.970000e-02	3.376000e-01	1.990000e+02
be_gr1_x	2311345.0	3.178000e-01	1.301000e+00	-0.9166	5.900000e-03	9.660000e-02	2.271000e-01	3.373330e+01
debt_gr1_x	2158693.0	7.838000e-01	4.707200e+00	-1.0000	-1.456000e-01	1.900000e-02	3.292000e-01	1.090000e+02
sale_gr1_x	2362404.0	2.228000e-01	6.711000e-01	-0.9960	5.000000e-03	1.032000e-01	2.478000e-01	1.370570e+01
cogs_gr1_x	2358805.0	2.142000e-01	6.122000e-01	-0.9619	-4.700000e-03	1.032000e-01	2.613000e-01	1.190030e+01
sga_gr1_x	1997437.0	1.844000e-01	3.963000e-01	-1.0000	1.340000e-02	1.044000e-01	2.389000e-01	6.765800e+00
opex_gr1_x	2387208.0	1.949000e-01	4.470000e-01	-0.7668	7.900000e-03	1.058000e-01	2.505000e-01	7.187400e+00
capx_gr1_x	2147147.0	6.016000e-01	2.183000e+00	-1.3370	-2.236000e-01	1.144000e-01	6.251000e-01	3.425000e+01
inv_gr1_x	1910333.0	2.595000e-01	9.931000e-01	-1.0000	-6.850000e-02	8.260000e-02	2.909000e-01	1.698080e+01
at_gr3_x	2114339.0	9.104000e-01	2.670800e+00	-0.8797	8.870000e-02	3.426000e-01	8.167000e-01	6.899070e+01
ca_gr3_x	1898998.0	9.832000e-01	3.187300e+00	-0.9099	2.890000e-02	3.230000e-01	8.289000e-01	7.748590e+01
nca_gr3_x	1897746.0	1.592100e+00	6.786800e+00	-0.9628	4.280000e-02	3.455000e-01	1.005000e+00	1.792615e+02
lt_gr3_x	2091277.0	1.135900e+00	3.376000e+00	-0.8936	3.580000e-02	3.474000e-01	9.457000e-01	5.633890e+01
cl_gr3_x	1906078.0	9.845000e-01	2.656400e+00	-0.9194	9.000000e-03	3.652000e-01	9.754000e-01	4.535460e+01
ncl_gr3_x	1803330.0	4.168200e+00	2.242620e+01	-1.0000	-1.231000e-01	2.914000e-01	1.285200e+00	8.323333e+02
be_gr3_x	1998122.0	1.009400e+00	3.275200e+00	-0.9384	7.210000e-02	3.326000e-01	7.902000e-01	6.699660e+01

	count	mean	std	min	25%	50%	75%	max
debt_gr3_x	1882647.0	3.622500e+00	2.086590e+01	-1.0000	-2.165000e-01	2.251000e-01	1.145100e+00	4.310000e+02
sale_gr3_x	2063618.0	8.605000e-01	2.814400e+00	-1.0000	7.210000e-02	3.286000e-01	7.527000e-01	8.620390e+01
cogs_gr3_x	2052669.0	7.935000e-01	2.179500e+00	-1.0000	4.870000e-02	3.267000e-01	7.894000e-01	4.537560e+01
sga_gr3_x	1713690.0	6.540000e-01	1.324200e+00	-1.0000	9.470000e-02	3.366000e-01	7.294000e-01	2.400000e+01
opex_gr3_x	2073541.0	7.171000e-01	1.625000e+00	-0.8979	7.650000e-02	3.367000e-01	7.689000e-01	2.833740e+01
capx_gr3_x	1846897.0	1.692700e+00	5.902400e+00	-1.2088	-2.368000e-01	3.214000e-01	1.355700e+00	1.128462e+02
cash_gr1a_x	2396920.0	1.480000e-02	1.380000e-01	-1.1898	-1.600000e-02	2.800000e-03	3.520000e-02	8.303000e-01
inv_gr1a_x	2351255.0	1.250000e-02	5.090000e-02	-0.3723	-7.000000e-04	7.000000e-04	2.250000e-02	2.978000e-01
rec_gr1a_x	2363716.0	2.190000e-02	6.430000e-02	-0.4405	-2.700000e-03	1.190000e-02	4.270000e-02	3.340000e-01
ppeg_gr1a_x	2178200.0	5.240000e-02	1.039000e-01	-0.8431	8.900000e-03	3.670000e-02	8.330000e-02	5.756000e-01
lti_gr1a_x	2205853.0	5.400000e-03	4.060000e-02	-0.4964	0.000000e+00	0.000000e+00	1.100000e-03	3.478000e-01
intan_gr1a_x	2110874.0	1.080000e-02	6.690000e-02	-0.9608	-7.000000e-04	0.000000e+00	1.700000e-03	5.336000e-01
debtst_gr1a_x	2395084.0	3.900000e-03	6.220000e-02	-0.5236	-5.000000e-03	0.000000e+00	1.320000e-02	4.847000e-01
ap_gr1a_x	2267822.0	1.460000e-02	4.890000e-02	-0.2766	-3.900000e-03	6.100000e-03	2.540000e-02	2.945000e-01
txp_gr1a_x	2057276.0	9.000000e-04	1.130000e-02	-0.0902	-9.000000e-04	0.000000e+00	2.200000e-03	9.250000e-02
debtlt_gr1a_x	2411829.0	1.770000e-02	9.970000e-02	-0.6085	-1.080000e-02	0.000000e+00	3.540000e-02	5.760000e-01
txdltc_gr1a_x	2135161.0	2.300000e-03	1.280000e-02	-0.1302	0.000000e+00	0.000000e+00	4.800000e-03	8.330000e-02
coa_gr1a_x	2167569.0	3.450000e-02	1.005000e-01	-0.7908	-4.200000e-03	2.200000e-02	7.140000e-02	4.923000e-01
col_gr1a_x	2191221.0	1.980000e-02	6.480000e-02	-0.4855	-5.500000e-03	1.350000e-02	4.240000e-02	3.834000e-01
cowc_gr1a_x	2146736.0	1.440000e-02	8.680000e-02	-0.6052	-1.810000e-02	9.000000e-03	4.750000e-02	4.185000e-01
ncoa_gr1a_x	2185140.0	4.890000e-02	1.438000e-01	-1.8841	-5.500000e-03	2.970000e-02	9.040000e-02	7.494000e-01
ncol_gr1a_x	2174709.0	6.300000e-03	3.310000e-02	-0.3605	-1.100000e-03	1.900000e-03	1.180000e-02	3.338000e-01
nncoa_gr1a_x	2147813.0	4.270000e-02	1.424000e-01	-1.8841	-9.700000e-03	2.500000e-02	8.290000e-02	7.692000e-01
oa_gr1a_x	2167557.0	8.310000e-02	2.025000e-01	-2.5884	-3.400000e-03	6.800000e-02	1.668000e-01	8.176000e-01
ol_gr1a_x	2174709.0	2.620000e-02	8.090000e-02	-0.6433	-4.900000e-03	2.070000e-02	5.460000e-02	5.422000e-01
fna_gr1a_x	2497393.0	5.700000e-03	6.030000e-02	-0.7055	0.000000e+00	0.000000e+00	0.000000e+00	6.896000e-01
fnl_gr1a_x	2418391.0	2.150000e-02	1.353000e-01	-1.2296	-1.620000e-02	1.000000e-04	5.400000e-02	1.130300e+00
nfna_gr1a_x	2418391.0	-1.580000e-02	1.552000e-01	-1.1078	-5.900000e-02	-9.000000e-04	2.760000e-02	1.384100e+00
gp_gr1a_x	2387365.0	3.580000e-02	1.161000e-01	-0.8663	-2.200000e-03	2.080000e-02	7.290000e-02	1.372100e+00

	count	mean	std	min	25%	50%	75%	max
ebitda_gr1a_x	2390711.0	9.700000e-03	9.740000e-02	-0.8685	-1.050000e-02	9.300000e-03	3.840000e-02	1.237100e+00
ebit_gr1a_x	2392217.0	5.200000e-03	9.760000e-02	-0.8536	-1.310000e-02	6.700000e-03	3.280000e-02	1.345400e+00
ope_gr1a_x	2056758.0	9.400000e-03	1.005000e-01	-0.9869	-1.390000e-02	1.090000e-02	3.950000e-02	1.233300e+00
ni_gr1a_x	2402691.0	8.000000e-04	1.303000e-01	-1.6889	-1.340000e-02	3.900000e-03	2.430000e-02	2.739400e+00
nix_gr1a_x	2402691.0	6.000000e-04	1.422000e-01	-1.8549	-1.540000e-02	3.800000e-03	2.570000e-02	2.791300e+00
dp_gr1a_x	2309627.0	3.900000e-03	1.560000e-02	-0.3935	0.000000e+00	2.500000e-03	7.500000e-03	1.932000e-01
fincf_gr1a_x	2053075.0	1.220000e-02	2.465000e-01	-2.0255	-5.480000e-02	2.700000e-03	7.330000e-02	1.485100e+00
ocf_gr1a_x	2334713.0	1.000000e-04	1.397000e-01	-0.9941	-4.190000e-02	2.900000e-03	4.640000e-02	1.151200e+00
fcf_gr1a_x	2181931.0	-7.300000e-03	1.637000e-01	-1.1368	-6.050000e-02	-4.000000e-04	5.020000e-02	1.202900e+00
nwc_gr1a_x	2164316.0	2.640000e-02	1.763000e-01	-1.4272	-2.650000e-02	1.650000e-02	7.240000e-02	9.090000e-01
eqnetis_gr1a_x	2052797.0	1.170000e-02	2.127000e-01	-1.9975	-1.000000e-02	0.000000e+00	1.380000e-02	1.207600e+00
dltnetis_gr1a_x	2373431.0	-3.100000e-03	1.313000e-01	-0.7874	-2.580000e-02	0.000000e+00	2.250000e-02	7.003000e-01
dstnetis_gr1a_x	2290818.0	7.000000e-04	8.970000e-02	-0.8063	-1.090000e-02	0.000000e+00	1.870000e-02	7.197000e-01
dbnetis_gr1a_x	2374474.0	-2.600000e-03	1.670000e-01	-1.0269	-4.130000e-02	0.000000e+00	4.330000e-02	1.017900e+00
netis_gr1a_x	2052412.0	8.700000e-03	2.717000e-01	-2.0764	-6.040000e-02	1.700000e-03	7.550000e-02	1.539900e+00
eqnpo_gr1a_x	2047069.0	-1.040000e-02	2.148000e-01	-1.1821	-1.480000e-02	0.000000e+00	1.310000e-02	1.940900e+00
tax_gr1a_x	2398103.0	3.100000e-03	2.840000e-02	-0.2157	-3.800000e-03	1.000000e-03	1.140000e-02	2.047000e-01
eqbb_gr1a_x	1893504.0	1.700000e-03	3.370000e-02	-0.3806	0.000000e+00	0.000000e+00	3.000000e-04	2.809000e-01
eqis_gr1a_x	2000469.0	1.360000e-02	2.117000e-01	-2.0255	-2.500000e-03	0.000000e+00	5.700000e-03	1.226200e+00
div_gr1a_x	2382722.0	1.100000e-03	1.270000e-02	-0.2183	0.000000e+00	0.000000e+00	1.200000e-03	2.439000e-01
eqpo_gr1a_x	1891334.0	2.900000e-03	4.380000e-02	-0.4620	-1.000000e-04	0.000000e+00	4.100000e-03	3.915000e-01
capx_gr1a_x	2184434.0	7.400000e-03	5.440000e-02	-0.4868	-7.300000e-03	2.300000e-03	1.940000e-02	4.471000e-01
be_gr1a_x	2311289.0	4.620000e-02	1.699000e-01	-2.0718	1.600000e-03	3.510000e-02	8.970000e-02	8.561000e-01
cash_gr3a_x	2081646.0	2.960000e-02	1.755000e-01	-2.5781	-1.260000e-02	9.500000e-03	6.320000e-02	9.052000e-01
inv_gr3a_x	2033267.0	2.900000e-02	8.700000e-02	-0.6971	0.000000e+00	6.800000e-03	5.550000e-02	4.115000e-01
rec_gr3a_x	2047864.0	4.970000e-02	1.082000e-01	-0.7795	1.400000e-03	3.280000e-02	8.960000e-02	4.887000e-01
ppeg_gr3a_x	1890568.0	1.277000e-01	2.118000e-01	-2.1282	3.190000e-02	1.080000e-01	2.163000e-01	9.231000e-01
lti_gr3a_x	1864897.0	1.290000e-02	7.040000e-02	-0.6566	0.000000e+00	0.000000e+00	8.800000e-03	4.683000e-01
intan_gr3a_x	1784074.0	2.520000e-02	1.171000e-01	-1.7938	-0.000000e+00	0.000000e+00	2.360000e-02	6.632000e-01

	count	mean	std	min	25%	50%	75%	max
debtst_gr3a_x	2078323.0	8.500000e-03	7.970000e-02	-0.8315	-6.500000e-03	3.000000e-04	2.440000e-02	5.514000e-01
ap_gr3a_x	1936459.0	3.440000e-02	8.510000e-02	-0.4973	-3.000000e-04	1.600000e-02	4.880000e-02	4.801000e-01
txp_gr3a_x	1751204.0	1.900000e-03	1.400000e-02	-0.0976	-1.200000e-03	0.000000e+00	4.400000e-03	1.079000e-01
debtlt_gr3a_x	2098723.0	4.090000e-02	1.579000e-01	-1.1700	-1.120000e-02	1.060000e-02	1.011000e-01	7.496000e-01
txdltc_gr3a_x	1843283.0	6.200000e-03	2.480000e-02	-0.2172	0.000000e+00	0.000000e+00	1.330000e-02	1.273000e-01
coa_gr3a_x	1880953.0	7.660000e-02	1.701000e-01	-1.4412	6.100000e-03	6.190000e-02	1.549000e-01	6.791000e-01
col_gr3a_x	1907173.0	4.420000e-02	9.650000e-02	-0.9653	4.300000e-03	3.750000e-02	8.380000e-02	4.559000e-01
cowc_gr3a_x	1861920.0	3.210000e-02	1.338000e-01	-1.0405	-2.130000e-02	2.260000e-02	9.140000e-02	5.604000e-01
ncoa_gr3a_x	1899708.0	1.091000e-01	2.575000e-01	-4.5815	1.230000e-02	1.026000e-01	2.250000e-01	8.112000e-01
ncol_gr3a_x	1887939.0	1.640000e-02	5.970000e-02	-0.5782	-0.000000e+00	9.000000e-03	3.080000e-02	4.104000e-01
nncoa_gr3a_x	1861492.0	9.300000e-02	2.474000e-01	-3.9391	1.200000e-03	8.690000e-02	2.030000e-01	8.094000e-01
oa_gr3a_x	1880920.0	1.840000e-01	3.641000e-01	-5.1474	4.560000e-02	2.082000e-01	3.829000e-01	9.247000e-01
ol_gr3a_x	1887939.0	6.020000e-02	1.295000e-01	-1.1795	1.270000e-02	5.900000e-02	1.138000e-01	6.233000e-01
fna_gr3a_x	2302373.0	1.560000e-02	8.920000e-02	-1.1421	0.000000e+00	0.000000e+00	0.000000e+00	7.162000e-01
fnl_gr3a_x	2105333.0	4.560000e-02	2.040000e-01	-1.8999	-1.910000e-02	2.600000e-02	1.304000e-01	8.753000e-01
nfna_gr3a_x	2105333.0	-3.150000e-02	2.282000e-01	-1.3255	-1.318000e-01	-2.310000e-02	4.440000e-02	2.048000e+00
gp_gr3a_x	2074121.0	7.850000e-02	1.870000e-01	-1.2858	4.200000e-03	5.550000e-02	1.554000e-01	1.274100e+00
ebitda_gr3a_x	2079592.0	2.410000e-02	1.330000e-01	-1.0362	-8.600000e-03	2.410000e-02	7.360000e-02	1.478800e+00
ebit_gr3a_x	2081034.0	1.490000e-02	1.346000e-01	-1.1637	-1.460000e-02	1.620000e-02	6.010000e-02	1.985300e+00
ope_gr3a_x	1772515.0	2.290000e-02	1.350000e-01	-1.1140	-1.410000e-02	2.540000e-02	7.260000e-02	1.382600e+00
ni_gr3a_x	2095331.0	5.500000e-03	1.607000e-01	-2.0040	-1.480000e-02	8.900000e-03	4.110000e-02	3.365400e+00
nix_gr3a_x	2095331.0	5.200000e-03	1.722000e-01	-2.2144	-1.670000e-02	8.800000e-03	4.270000e-02	3.330500e+00
dp_gr3a_x	1998657.0	9.200000e-03	2.780000e-02	-0.6566	5.000000e-04	7.400000e-03	1.760000e-02	3.627000e-01
ocf_gr3a_x	2026157.0	1.030000e-02	1.536000e-01	-0.9623	-3.950000e-02	1.100000e-02	6.680000e-02	1.459300e+00
fcf_gr3a_x	1875380.0	-2.300000e-03	1.806000e-01	-0.9594	-6.520000e-02	3.500000e-03	6.430000e-02	1.668700e+00
nwc_gr3a_x	1880705.0	5.470000e-02	2.333000e-01	-3.1433	-2.400000e-02	4.470000e-02	1.438000e-01	9.475000e-01
dltnetis_gr3a_x	2057295.0	-7.000000e-03	1.381000e-01	-0.9437	-3.150000e-02	0.000000e+00	2.360000e-02	8.602000e-01
dstnetis_gr3a_x	1975805.0	-1.000000e-04	7.960000e-02	-0.7776	-1.420000e-02	0.000000e+00	1.680000e-02	6.541000e-01
dbnetis_gr3a_x	2058325.0	-7.400000e-03	1.681000e-01	-1.2437	-4.610000e-02	0.000000e+00	4.140000e-02	1.075700e+00

	count	mean	std	min	25%	50%	75%	max
tax_gr3a_x	2090131.0	6.500000e-03	3.600000e-02	-0.2190	-4.800000e-03	2.700000e-03	1.970000e-02	2.106000e-01
div_gr3a_x	2069485.0	2.200000e-03	1.420000e-02	-0.2110	0.000000e+00	0.000000e+00	4.200000e-03	2.609000e-01
capx_gr3a_x	1877910.0	1.340000e-02	6.720000e-02	-0.6838	-6.700000e-03	6.500000e-03	3.240000e-02	3.679000e-01
capx_at_x	2305667.0	6.630000e-02	7.300000e-02	-0.0305	1.920000e-02	4.470000e-02	8.570000e-02	6.092000e-01
spi_at_x	2376699.0	-1.010000e-02	4.960000e-02	-1.3123	-2.700000e-03	0.000000e+00	0.000000e+00	1.961000e-01
xido_at_x	2513016.0	-5.000000e-04	1.800000e-02	-0.4152	0.000000e+00	0.000000e+00	0.000000e+00	1.762000e-01
nri_at_x	2375825.0	-1.080000e-02	6.070000e-02	-1.5759	-4.600000e-03	0.000000e+00	0.000000e+00	2.675000e-01
gp_sale_x	2468341.0	8.440000e-02	3.062100e+00	-124.7476	2.080000e-01	3.345000e-01	5.045000e-01	9.763000e-01
ebitda_sale_x	2470375.0	-3.073000e-01	4.409900e+00	-171.6176	5.970000e-02	1.272000e-01	2.277000e-01	7.373000e-01
ebit_sale_x	2470818.0	-3.840000e-01	4.578500e+00	-185.0447	3.170000e-02	8.990000e-02	1.721000e-01	6.154000e-01
pi_sale_x	2473639.0	-4.469000e-01	4.876400e+00	-184.2990	1.190000e-02	7.260000e-02	1.445000e-01	7.101000e-01
ni_sale_x	2474362.0	-4.693000e-01	4.796100e+00	-184.2990	7.200000e-03	4.550000e-02	9.440000e-02	5.566000e-01
nix_sale_x	2472905.0	-4.745000e-01	4.848700e+00	-184.2990	6.200000e-03	4.620000e-02	9.640000e-02	6.508000e-01
ocf_sale_x	2414346.0	-3.439000e-01	3.755000e+00	-140.2577	-1.520000e-02	5.800000e-02	1.448000e-01	1.412300e+00
fcf_sale_x	2267091.0	-5.418000e-01	4.134400e+00	-125.9694	-1.053000e-01	-1.100000e-03	6.670000e-02	1.210500e+00
gp_at_x	2503159.0	3.011000e-01	2.895000e-01	-1.2660	1.023000e-01	2.659000e-01	4.563000e-01	1.412300e+00
ebitda_at_x	2505194.0	7.710000e-02	1.992000e-01	-2.1076	2.950000e-02	1.080000e-01	1.699000e-01	5.122000e-01
ebit_at_x	2506116.0	4.100000e-02	1.986000e-01	-2.1142	1.820000e-02	7.130000e-02	1.269000e-01	4.730000e-01
fi_at_x	2185678.0	1.660000e-02	2.114000e-01	-2.6041	2.010000e-02	6.410000e-02	9.800000e-02	3.716000e-01
cop_at_x	2259456.0	1.333000e-01	1.925000e-01	-1.1882	3.940000e-02	1.365000e-01	2.302000e-01	1.940400e+00
ni_at_x	2514966.0	-5.000000e-03	2.045000e-01	-2.8828	3.400000e-03	3.510000e-02	7.410000e-02	3.332000e-01
ope_be_x	2108352.0	1.569000e-01	5.427000e-01	-8.8149	9.490000e-02	2.136000e-01	3.261000e-01	3.725100e+00
ni_be_x	2444347.0	-1.1990000e-02	5.962000e-01	-10.7541	1.720000e-02	9.500000e-02	1.504000e-01	1.450500e+00
nix_be_x	2444347.0	-2.270000e-02	6.187000e-01	-11.9515	1.490000e-02	9.590000e-02	1.526000e-01	1.558300e+00
ocf_be_x	2375509.0	4.150000e-02	5.350000e-01	-7.2459	-3.990000e-02	1.089000e-01	2.199000e-01	4.068700e+00
fcf_be_x	2219533.0	-1.352000e-01	6.520000e-01	-9.8959	-2.117000e-01	-4.000000e-03	1.206000e-01	2.895100e+00
gp_beve_x	2404319.0	6.940000e-01	1.236500e+00	-11.0645	2.172000e-01	4.625000e-01	8.366000e-01	1.753110e+01
ebitda_beve_x	2406313.0	5.730000e-02	1.310800e+00	-38.6063	9.750000e-02	1.837000e-01	2.972000e-01	3.290900e+00
ebit_beve_x	2406990.0	-2.510000e-02	1.386000e+00	-41.0563	5.220000e-02	1.282000e-01	2.282000e-01	2.800000e+00

	count	mean	std	min	25%	50%	75%	max
fi_bev_x	2116451.0	-8.600000e-02	1.345800e+00	-38.5103	4.190000e-02	9.910000e-02	1.608000e-01	2.274200e+00
cop_bev_x	2188818.0	3.139000e-01	8.344000e-01	-8.9448	8.920000e-02	2.259000e-01	4.111000e-01	1.607970e+01
gp_ppen_x	2466653.0	2.766900e+00	6.510900e+00	-130.5385	4.559000e-01	1.518900e+00	3.353000e+00	1.035052e+02
ebitda_ppen_x	2468488.0	-1.134000e-01	1.280070e+01	-558.0000	1.689000e-01	4.726000e-01	1.116300e+00	3.389320e+01
fcf_ppen_x	2270795.0	-8.658000e-01	1.104610e+01	-423.4211	-3.778000e-01	-1.180000e-02	3.338000e-01	3.272670e+01
fincf_at_x	2181057.0	6.050000e-02	2.270000e-01	-0.9085	-4.100000e-02	1.800000e-03	8.120000e-02	1.643700e+00
netis_at_x	2180970.0	2.900000e-02	2.576000e-01	-1.3681	-4.860000e-02	0.000000e+00	5.940000e-02	1.592800e+00
eqnetis_at_x	2181226.0	5.680000e-02	1.918000e-01	-0.3507	-8.000000e-04	6.000000e-04	1.520000e-02	1.488800e+00
eqis_at_x	2142004.0	7.050000e-02	1.912000e-01	-0.1034	0.000000e+00	3.200000e-03	2.280000e-02	1.535600e+00
dbnetis_at_x	2487875.0	-2.120000e-02	1.573000e-01	-1.3624	-3.980000e-02	-8.000000e-04	2.270000e-02	6.456000e-01
dltnetis_at_x	2487184.0	-2.430000e-02	1.364000e-01	-1.2268	-3.180000e-02	-2.200000e-03	1.200000e-03	5.184000e-01
dstnetis_at_x	2428021.0	3.500000e-03	6.050000e-02	-0.4789	-5.100000e-03	0.000000e+00	1.130000e-02	4.836000e-01
eqnpo_at_x	2177364.0	-4.470000e-02	1.949000e-01	-1.4673	-1.110000e-02	8.000000e-04	2.020000e-02	4.462000e-01
eqbb_at_x	2059717.0	1.250000e-02	3.500000e-02	-0.0026	0.000000e+00	0.000000e+00	5.300000e-03	4.018000e-01
div_at_x	2500964.0	1.160000e-02	2.170000e-02	0.0000	0.000000e+00	1.900000e-03	1.660000e-02	3.183000e-01
oaccruals_at_x	2261617.0	-1.580000e-02	1.522000e-01	-2.2637	-7.200000e-02	-1.830000e-02	4.760000e-02	6.719000e-01
oaccruals_ni_x	2260635.0	-5.853000e-01	6.180500e+00	-71.4418	-1.208700e+00	-2.712000e-01	6.967000e-01	8.515790e+01
taccruals_at_x	2240180.0	-3.100000e-02	2.045000e-01	-2.4802	-9.100000e-02	-1.180000e-02	4.930000e-02	1.294200e+00
taccruals_ni_x	2238904.0	-1.448100e+00	8.683400e+00	-131.5096	-1.516600e+00	-1.946000e-01	7.622000e-01	6.728570e+01
noa_at_x	2142866.0	6.816000e-01	4.649000e-01	-1.1515	4.896000e-01	6.884000e-01	8.418000e-01	1.038840e+01
be_bev_x	2368048.0	1.343100e+00	2.666700e+00	0.0326	5.543000e-01	8.086000e-01	1.190400e+00	6.053070e+01
debt_bev_x	2416506.0	4.732000e-01	6.162000e-01	0.0000	1.399000e-01	3.804000e-01	6.012000e-01	1.276120e+01
cash_bev_x	2397575.0	8.357000e-01	3.110100e+00	0.0000	3.800000e-02	1.245000e-01	4.276000e-01	8.007360e+01
pstk_bev_x	2418755.0	2.720000e-02	1.704000e-01	0.0000	0.000000e+00	0.000000e+00	0.000000e+00	7.089400e+00
debtlt_bev_x	2412477.0	3.446000e-01	4.482000e-01	0.0000	5.390000e-02	2.671000e-01	4.815000e-01	9.026500e+00
debtst_bev_x	2403343.0	1.233000e-01	2.903000e-01	0.0000	3.200000e-03	3.390000e-02	1.172000e-01	5.633000e+00
int_debt_x	1959042.0	1.258000e-01	3.153000e-01	0.0000	5.310000e-02	7.610000e-02	1.063000e-01	7.750000e+00
int_debtlt_x	1874541.0	3.393000e-01	1.552500e+00	0.0000	6.360000e-02	9.400000e-02	1.485000e-01	4.145000e+01
ebitda_debt_x	2242375.0	2.161600e+00	2.312980e+01	-362.2105	1.666000e-01	3.959000e-01	9.501000e-01	5.562212e+02

	count	mean	std	min	25%	50%	75%	max
profit_cl_x	2270271.0	4.298000e-01	1.566600e+00	-11.9038	2.114000e-01	5.648000e-01	1.016300e+00	6.155300e+00
ocf_cl_x	2269486.0	5.390000e-02	1.456200e+00	-14.9568	-1.363000e-01	2.183000e-01	5.993000e-01	5.976400e+00
ocf_debt_x	2189764.0	1.253200e+00	1.968000e+01	-264.1167	-7.590000e-02	1.564000e-01	5.185000e-01	4.307215e+02
cash_lt_x	2487462.0	7.781000e-01	2.113200e+00	0.0000	4.150000e-02	1.312000e-01	5.084000e-01	2.990910e+01
inv_act_x	2124755.0	2.719000e-01	2.276000e-01	0.0000	4.860000e-02	2.538000e-01	4.448000e-01	9.113000e-01
rec_act_x	2130411.0	3.499000e-01	2.071000e-01	0.0000	1.990000e-01	3.479000e-01	4.754000e-01	9.455000e-01
debtst_debt_x	2235158.0	2.916000e-01	3.181000e-01	0.0000	3.900000e-02	1.578000e-01	4.582000e-01	1.000000e+00
cl_lt_x	2271050.0	5.408000e-01	2.822000e-01	0.0172	3.033000e-01	5.188000e-01	7.861000e-01	1.000000e+00
debtlt_debt_x	2251637.0	7.215000e-01	3.158000e-01	0.0000	5.637000e-01	8.571000e-01	9.724000e-01	1.000000e+00
lt_ppen_x	2467297.0	1.413180e+01	4.095230e+01	0.0809	1.032300e+00	2.019600e+00	5.768200e+00	7.630447e+02
debtlt_be_x	2439883.0	7.140000e-01	1.464700e+00	0.0000	3.360000e-02	3.025000e-01	7.618000e-01	2.225160e+01
opex_at_x	2503218.0	9.413000e-01	8.196000e-01	0.0029	3.295000e-01	7.872000e-01	1.304500e+00	7.158500e+00
nwc_at_x	2253296.0	2.724000e-01	2.457000e-01	-0.7924	8.520000e-02	2.536000e-01	4.349000e-01	9.547000e-01
debt_at_x	2514980.0	2.331000e-01	2.095000e-01	0.0000	5.090000e-02	1.957000e-01	3.591000e-01	1.428700e+00
debt_be_x	2444508.0	9.825000e-01	1.972300e+00	0.0000	9.520000e-02	4.426000e-01	1.023800e+00	3.440000e+01
ebit_int_x	2038745.0	1.266250e+01	1.784445e+02	-3702.0000	1.253300e+00	4.003000e+00	1.124330e+01	3.302250e+03
inv_days_x	2394275.0	8.869850e+01	1.683021e+02	0.0000	9.009300e+00	5.392190e+01	1.091676e+02	3.574195e+03
rec_days_x	2403668.0	3.602296e+02	9.967740e+02	0.0000	3.863530e+01	5.827670e+01	8.822010e+01	7.354934e+03
ap_days_x	2314657.0	1.459695e+03	7.489965e+03	0.7812	2.587680e+01	4.209780e+01	7.865320e+01	1.412089e+05
cash_conversion_x	1836443.0	1.256743e+02	2.122532e+02	0.0000	4.172550e+01	8.193360e+01	1.398610e+02	3.521431e+03
cash_cl_x	2262167.0	1.419800e+00	3.231200e+00	0.0000	1.124000e-01	3.726000e-01	1.177400e+00	3.650000e+01
caliq_cl_x	2241081.0	2.487700e+00	3.827100e+00	0.0581	9.004000e-01	1.378900e+00	2.376600e+00	4.066670e+01
ca_cl_x	2252774.0	3.162200e+00	3.912700e+00	0.0824	1.372500e+00	2.102000e+00	3.307100e+00	4.119530e+01
inv_turnover_x	1990611.0	1.861590e+01	4.951140e+01	0.0438	2.956600e+00	5.130900e+00	1.205000e+01	7.307939e+02
at_turnover_x	2482416.0	1.084900e+00	9.318000e-01	0.0000	3.768000e-01	9.269000e-01	1.525100e+00	9.298300e+00
rec_turnover_x	2400338.0	1.234110e+01	2.636800e+01	0.0000	4.039600e+00	6.187900e+00	9.236800e+00	2.787135e+02
ap_turnover_x	2229997.0	1.163840e+01	1.238900e+01	-0.1258	4.826800e+00	8.918500e+00	1.434510e+01	1.336129e+02
sale_bev_x	2408388.0	2.269200e+00	2.923100e+00	0.0000	7.623000e-01	1.580300e+00	2.598800e+00	3.887110e+01
sale_be_x	2437063.0	2.732600e+00	3.718300e+00	0.0000	9.001000e-01	1.758000e+00	3.096000e+00	5.438940e+01

	count	mean	std	min	25%	50%	75%	max
div_ni_x	1963756.0	3.126000e-01	5.775000e-01	0.0000	0.000000e+00	1.650000e-01	4.135000e-01	1.293670e+01
sale_nwc_x	2017664.0	9.746900e+00	2.267620e+01	0.0000	2.066900e+00	3.971600e+00	7.750900e+00	3.110241e+02
tax_pi_x	1999061.0	3.279000e-01	3.117000e-01	-7.2981	2.705000e-01	3.654000e-01	4.329000e-01	5.548900e+00
cash_at_x	2496082.0	1.581000e-01	2.035000e-01	0.0000	2.580000e-02	7.260000e-02	2.026000e-01	9.799000e-01
ni_emp_x	2332173.0	-1.044570e+01	1.898294e+02	-3810.3810	4.055000e-01	4.200600e+00	1.703640e+01	1.438498e+03
sale_emp_x	2328826.0	2.691786e+02	5.003031e+02	0.0000	6.301400e+01	1.411000e+02	2.763478e+02	7.782523e+03
sale_emp_gr1_x	2120715.0	1.123000e-01	4.553000e-01	-0.9563	-3.330000e-02	5.300000e-02	1.513000e-01	7.027000e+00
emp_gr1_x	2048454.0	7.670000e-02	2.504000e-01	-1.3333	-3.060000e-02	4.520000e-02	1.538000e-01	1.483100e+00
ni_inc8q_x	1837805.0	3.116800e+00	3.262400e+00	0.0000	0.000000e+00	2.000000e+00	7.000000e+00	8.000000e+00
noa_gr1a_x	2130139.0	1.277000e-01	4.002000e-01	-0.7366	-1.750000e-02	4.940000e-02	1.574000e-01	1.075230e+01
ppeinv_gr1a_x	2130674.0	1.104000e-01	2.282000e-01	-0.5663	9.400000e-03	5.870000e-02	1.436000e-01	3.078700e+00
lnoa_gr1a_x	2042945.0	3.180000e-02	9.170000e-02	-0.5778	-3.800000e-03	1.370000e-02	4.740000e-02	7.544000e-01
capx_gr2_x	1996106.0	1.219100e+00	4.305300e+00	-1.4277	-2.477000e-01	2.272000e-01	1.043000e+00	7.697220e+01
saleq_gr1_x	2256822.0	2.428000e-01	8.315000e-01	-1.0000	-1.270000e-02	9.890000e-02	2.606000e-01	1.574840e+01
niq_be_x	2153966.0	5.000000e-04	1.393000e-01	-2.0216	1.600000e-03	2.420000e-02	4.290000e-02	6.993000e-01
niq_at_x	2218680.0	-2.200000e-03	6.080000e-02	-0.6672	0.000000e+00	8.200000e-03	2.060000e-02	1.818000e-01
niq_be_chg1_x	1961181.0	-7.700000e-03	1.339000e-01	-2.0038	-1.650000e-02	-6.000000e-04	1.090000e-02	1.227600e+00
niq_at_chg1_x	2044996.0	3.000000e-04	5.400000e-02	-0.4547	-7.100000e-03	-0.000000e+00	5.600000e-03	8.413000e-01
dsale_dinv_x	1796036.0	-4.380000e-02	8.780000e-01	-19.4778	-1.460000e-01	2.150000e-02	1.949000e-01	5.598300e+00
dsale_drec_x	2136436.0	-3.080000e-02	6.202000e-01	-7.3996	-1.418000e-01	1.500000e-03	1.418000e-01	7.637700e+00
dgp_dsale_x	2120443.0	2.720000e-02	5.405000e-01	-5.9700	-7.530000e-02	2.300000e-03	8.380000e-02	1.201120e+01
dsale_dsga_x	1827645.0	2.310000e-02	3.643000e-01	-2.2251	-8.920000e-02	-1.000000e-04	9.360000e-02	6.963700e+00
saleq_su_x	1944544.0	1.618000e-01	1.699500e+00	-16.0960	-8.666000e-01	1.532000e-01	1.125000e+00	3.358810e+01
niq_su_x	1972831.0	-1.123000e-01	1.940400e+00	-50.8463	-7.565000e-01	5.100000e-03	7.529000e-01	2.019490e+01
capex_abn_x	1806456.0	1.173000e-01	9.626000e-01	-1.1469	-3.685000e-01	-6.920000e-02	2.932000e-01	1.196350e+01
op_atl1_x	2415570.0	1.320000e-01	2.472000e-01	-6.9463	4.860000e-02	1.355000e-01	2.227000e-01	1.125400e+00
gp_atl1_x	2413733.0	3.639000e-01	3.763000e-01	-1.9036	1.162000e-01	3.032000e-01	5.356000e-01	2.788000e+00
ope_bell_x	2010286.0	2.202000e-01	6.575000e-01	-13.6285	1.063000e-01	2.425000e-01	3.880000e-01	4.617600e+00
cop_atl1_x	2237311.0	1.409000e-01	2.863000e-01	-3.8344	4.500000e-02	1.505000e-01	2.563000e-01	1.923400e+00

	count	mean	std	min	25%	50%	75%	max
pi_nix_x	1959639.0	1.615000e+00	6.861000e-01	0.1059	1.340700e+00	1.572900e+00	1.777900e+00	1.989360e+01
ocf_at_x	2449158.0	1.150000e-02	1.872000e-01	-1.8184	-2.140000e-02	4.090000e-02	1.033000e-01	5.979000e-01
op_at_x	2505194.0	1.113000e-01	1.575000e-01	-1.2330	4.200000e-02	1.205000e-01	1.892000e-01	5.662000e-01
ocf_at_chg1_x	2333855.0	2.300000e-03	1.627000e-01	-1.0782	-4.770000e-02	-1.000000e-04	4.630000e-02	1.390100e+00
at_be_x	2452393.0	3.714900e+00	4.779700e+00	1.0000	1.469000e+00	2.029600e+00	3.240900e+00	5.963100e+01
niq_saleq_std_x	1902197.0	1.360600e+00	1.149800e+01	0.0008	1.930000e-02	4.260000e-02	1.236000e-01	3.177766e+02
roe_be_std_x	1799259.0	1.611000e-01	4.732000e-01	0.0021	2.230000e-02	4.760000e-02	1.133000e-01	9.225400e+00
tangibility_x	2201788.0	6.502000e-01	1.916000e-01	0.0025	5.540000e-01	6.638000e-01	7.614000e-01	1.684700e+00
earnings_variability_x	1752776.0	8.639000e-01	1.037400e+00	0.0243	2.577000e-01	5.765000e-01	1.052900e+00	1.145280e+01
aliq_at_x	2174808.0	8.263000e-01	8.005000e-01	0.1044	5.792000e-01	6.946000e-01	8.423000e-01	2.803980e+01
f_score_x	1978727.0	4.911500e+00	1.728500e+00	0.0000	4.000000e+00	5.000000e+00	6.000000e+00	9.000000e+00
o_score_x	2127585.0	-1.902100e+00	3.035200e+00	-9.3872	-3.598500e+00	-2.309000e+00	-8.857000e-01	2.287030e+01
z_score_x	2126989.0	5.526800e+00	9.357000e+00	-37.3359	1.992200e+00	3.446700e+00	5.637300e+00	1.744239e+02
intrinsic_value_x	1899809.0	1.317903e+03	5.258077e+03	0.0982	3.489040e+01	1.295681e+02	5.706605e+02	1.130984e+05
kz_index_x	2167838.0	-1.126290e+01	5.190800e+01	-1723.5716	-6.936600e+00	-1.467200e+00	5.962000e-01	8.903350e+01
gpoa_ch5_x	1799428.0	-5.000000e-03	1.939000e-01	-1.1201	-7.080000e-02	-2.900000e-03	5.560000e-02	1.669700e+00
roe_ch5_x	1718355.0	-1.400000e-02	5.543000e-01	-7.5143	-7.570000e-02	-6.100000e-03	5.400000e-02	7.791500e+00
roa_ch5_x	1824336.0	4.900000e-03	1.917000e-01	-1.6595	-3.640000e-02	-1.800000e-03	2.650000e-02	3.283900e+00
cfoa_ch5_x	1759171.0	1.520000e-02	1.825000e-01	-0.9610	-5.570000e-02	2.500000e-03	7.100000e-02	2.175100e+00
gmar_ch5_x	1777826.0	4.200000e-02	9.593000e-01	-24.3597	-4.330000e-02	2.700000e-03	5.140000e-02	3.059480e+01
ni_ar1_x	1798398.0	2.127000e-01	6.110000e-01	-3.9640	-1.463000e-01	1.674000e-01	5.078000e-01	9.144200e+00
ni_ivol_x	1798398.0	5.090000e-02	1.054000e-01	0.0003	7.900000e-03	1.910000e-02	4.640000e-02	1.756800e+00
at_me_x	2522907.0	2.710400e+00	4.953100e+00	0.0086	5.788000e-01	1.230200e+00	2.687400e+00	1.923122e+02
be_me_x	2452453.0	7.411000e-01	7.141000e-01	0.0050	3.072000e-01	5.729000e-01	9.557000e-01	2.516310e+01
debt_me_x	2515141.0	7.136000e-01	1.647800e+00	0.0000	3.610000e-02	2.333000e-01	7.145000e-01	6.550580e+01
netdebt_me_x	2515141.0	4.707000e-01	1.480500e+00	-3.4965	-6.240000e-02	1.146000e-01	5.472000e-01	5.866260e+01
cash_me_x	2496218.0	2.459000e-01	5.843000e-01	0.0000	3.340000e-02	9.500000e-02	2.301000e-01	1.478940e+01
sale_me_x	2509790.0	1.848400e+00	3.088100e+00	0.0000	3.854000e-01	9.080000e-01	2.049100e+00	7.507530e+01
gp_me_x	2504145.0	4.729000e-01	6.845000e-01	-5.3506	1.503000e-01	2.955000e-01	5.587000e-01	1.896990e+01

	count	mean	std	min	25%	50%	75%	max
ebitda_me_x	2506237.0	1.594000e-01	2.707000e-01	-5.8474	5.650000e-02	1.331000e-01	2.363000e-01	5.597900e+00
ebit_me_x	2507305.0	9.600000e-02	2.455000e-01	-7.4186	3.000000e-02	9.500000e-02	1.716000e-01	3.506600e+00
ope_me_x	2183835.0	1.085000e-01	2.516000e-01	-8.0248	3.920000e-02	1.084000e-01	1.911000e-01	3.793500e+00
ni_me_x	2517298.0	1.200000e-03	3.459000e-01	-18.9294	5.500000e-03	4.900000e-02	8.530000e-02	9.917000e-01
nix_me_x	2517298.0	-1.200000e-03	3.693000e-01	-20.3694	4.100000e-03	4.920000e-02	8.650000e-02	1.036200e+00
cop_me_x	2259562.0	2.183000e-01	5.014000e-01	-3.5452	4.550000e-02	1.406000e-01	2.768000e-01	2.124680e+01
ocf_me_x	2450553.0	4.280000e-02	2.747000e-01	-5.6691	-1.830000e-02	5.360000e-02	1.205000e-01	5.711200e+00
fcf_me_x	2303306.0	-7.030000e-02	3.536000e-01	-8.5448	-1.065000e-01	-2.600000e-03	5.530000e-02	4.202300e+00
div_me_x	2501593.0	1.780000e-02	2.950000e-02	0.0000	0.000000e+00	3.900000e-03	2.660000e-02	1.049700e+00
eqbb_me_x	2059868.0	1.380000e-02	3.780000e-02	-0.0037	0.000000e+00	0.000000e+00	7.800000e-03	8.704000e-01
eqis_me_x	2142182.0	4.550000e-02	1.388000e-01	-0.1339	1.000000e-04	3.500000e-03	1.830000e-02	5.839400e+00
eqpo_me_x	2058263.0	3.150000e-02	5.660000e-02	-0.0013	0.000000e+00	1.120000e-02	4.150000e-02	1.725500e+00
equpo_me_x	2177501.0	-1.430000e-02	1.450000e-01	-6.1142	-8.100000e-03	1.200000e-03	3.130000e-02	1.442900e+00
eqnetis_me_x	2181408.0	3.130000e-02	1.401000e-01	-0.6866	-1.400000e-03	7.000000e-04	1.260000e-02	5.679700e+00
at_me_v_x	2480516.0	1.759600e+00	3.280300e+00	0.0085	5.638000e-01	1.008000e+00	1.587100e+00	6.916660e+01
bev_me_v_x	2404633.0	6.919000e-01	5.487000e-01	0.0009	3.194000e-01	6.308000e-01	9.482000e-01	1.692550e+01
ppen_me_v_x	2459710.0	3.322000e-01	3.872000e-01	0.0000	5.950000e-02	1.893000e-01	4.753000e-01	6.654400e+00
be_me_v_x	2410201.0	6.153000e-01	8.336000e-01	0.0050	2.513000e-01	4.357000e-01	7.057000e-01	2.914710e+01
cash_me_v_x	2460357.0	2.333000e-01	6.611000e-01	0.0000	2.350000e-02	6.940000e-02	1.825000e-01	1.486960e+01
sale_me_v_x	2472091.0	1.265200e+00	1.765600e+00	0.0000	3.146000e-01	7.343000e-01	1.550900e+00	3.775600e+01
gp_me_v_x	2467238.0	3.453000e-01	4.647000e-01	-2.4081	1.209000e-01	2.305000e-01	4.284000e-01	1.314000e+01
ebitda_me_v_x	2469299.0	1.012000e-01	2.101000e-01	-5.5869	5.090000e-02	1.060000e-01	1.669000e-01	2.711700e+00
ebit_me_v_x	2470075.0	6.010000e-02	2.226000e-01	-6.8743	2.670000e-02	7.470000e-02	1.222000e-01	2.601300e+00
cop_me_v_x	2243652.0	1.516000e-01	2.798000e-01	-2.3844	4.200000e-02	1.203000e-01	2.126000e-01	8.747500e+00
ocf_me_v_x	2431339.0	3.150000e-02	1.968000e-01	-4.7377	-1.650000e-02	4.340000e-02	9.350000e-02	2.334400e+00
fcf_me_v_x	2286863.0	-3.800000e-02	2.261000e-01	-6.0410	-8.630000e-02	-2.300000e-03	4.670000e-02	1.728000e+00
debt_me_v_x	2480615.0	3.008000e-01	3.502000e-01	0.0000	4.020000e-02	2.106000e-01	4.607000e-01	7.224300e+00
pstk_me_v_x	2479267.0	1.480000e-02	5.860000e-02	0.0000	0.000000e+00	0.000000e+00	0.000000e+00	1.220500e+00
debtlt_me_v_x	2476104.0	2.224000e-01	2.443000e-01	0.0000	1.400000e-02	1.446000e-01	3.542000e-01	2.411300e+00

	count	mean	std	min	25%	50%	75%	max
debtst_mev_x	2461067.0	8.090000e-02	2.125000e-01	0.0000	9.000000e-04	1.690000e-02	7.010000e-02	5.292900e+00
dltnetis_mev_x	2453443.0	-3.150000e-02	1.853000e-01	-3.5613	-3.440000e-02	-1.900000e-03	1.200000e-03	6.324000e-01
dstnetis_mev_x	2393968.0	4.100000e-03	9.390000e-02	-1.0163	-4.800000e-03	0.000000e+00	1.110000e-02	1.122900e+00
dbnetis_mev_x	2454176.0	-2.880000e-02	2.223000e-01	-4.4848	-4.210000e-02	-6.000000e-04	2.280000e-02	1.188800e+00
netis_mev_x	2164671.0	-8.300000e-03	2.729000e-01	-4.6395	-5.040000e-02	0.000000e+00	5.030000e-02	5.358400e+00
fincf_mev_x	2164802.0	3.700000e-02	2.405000e-01	-2.3006	-4.040000e-02	1.300000e-03	7.090000e-02	6.822000e+00
aliq_mat_x	2036506.0	5.016000e-01	2.661000e-01	0.0270	3.052000e-01	4.793000e-01	6.504000e-01	3.973200e+00
eq_dur_x	2193667.0	1.598720e+01	5.630900e+00	0.2861	1.413720e+01	1.612420e+01	1.764670e+01	3.430355e+02
beta_60m_x	2090801.0	1.153800e+00	6.856000e-01	-1.7467	6.897000e-01	1.081600e+00	1.528500e+00	4.912400e+00
ivol_capm_60m_x	2090801.0	1.172000e-01	6.560000e-02	0.0288	7.050000e-02	1.002000e-01	1.454000e-01	5.392000e-01
resff3_12_1_x	2274040.0	-2.210000e-02	2.736000e-01	-1.1550	-1.908000e-01	-8.900000e-03	1.610000e-01	7.899000e-01
resff3_6_1_x	2273172.0	-5.420000e-02	5.396000e-01	-2.9537	-3.435000e-01	-2.040000e-02	2.734000e-01	1.925800e+00
mispricing_mgmt_x	2414716.0	4.896000e-01	1.856000e-01	0.0147	3.610000e-01	5.047000e-01	6.284000e-01	9.427000e-01
mispricing_perf_x	2649116.0	5.208000e-01	2.065000e-01	0.0099	3.773000e-01	5.270000e-01	6.749000e-01	9.881000e-01
zero_trades_21d_x	2568596.0	9.102000e-01	2.670500e+00	0.0000	1.800000e-03	3.700000e-03	7.200000e-03	2.100980e+01
dolvol_126d_x	2527407.0	1.272436e+07	5.041472e+07	36.1000	6.023594e+04	3.756701e+05	3.493927e+06	1.038495e+09
dolvol_var_126d_x	2527340.0	1.275800e+00	7.751000e-01	0.2622	7.587000e-01	1.088500e+00	1.545100e+00	8.289100e+00
turnover_126d_x	2527415.0	4.300000e-03	6.800000e-03	0.0000	9.000000e-04	2.200000e-03	5.300000e-03	2.857000e-01
turnover_var_126d_x	2527348.0	1.251900e+00	7.609000e-01	0.2796	7.459000e-01	1.058700e+00	1.509700e+00	7.678300e+00
zero_trades_126d_x	2527415.0	9.170000e-01	2.511100e+00	0.0000	1.900000e-03	4.000000e-03	1.771000e-01	1.949730e+01
zero_trades_252d_x	2472485.0	9.236000e-01	2.470600e+00	0.0001	2.000000e-03	4.300000e-03	2.625000e-01	1.910030e+01
bidaskhl_21d_x	2474735.0	1.470000e-02	1.810000e-02	0.0011	5.600000e-03	9.400000e-03	1.710000e-02	5.318000e-01
rvolhl_21d_x	2474735.0	2.130000e-02	1.570000e-02	0.0000	1.100000e-02	1.720000e-02	2.680000e-02	1.854000e-01
beta_21d_x	2469080.0	8.736000e-01	1.205700e+00	-11.1429	2.238000e-01	8.042000e-01	1.458000e+00	1.276490e+01
ivol_capm_21d_x	2469080.0	2.710000e-02	1.960000e-02	0.0018	1.400000e-02	2.160000e-02	3.380000e-02	2.415000e-01
iskew_capm_21d_x	2469046.0	2.407000e-01	8.745000e-01	-3.5665	-2.542000e-01	2.053000e-01	7.097000e-01	3.715300e+00
coskew_21d_x	2469074.0	-1.530000e-02	3.111000e-01	-1.4678	-2.232000e-01	-2.070000e-02	1.886000e-01	1.347500e+00
beta_dimson_21d_x	2469080.0	9.503000e-01	1.950600e+00	-19.3713	4.290000e-02	8.515000e-01	1.798400e+00	2.341690e+01
ivol_ff3_21d_x	2469080.0	2.640000e-02	1.930000e-02	0.0018	1.360000e-02	2.100000e-02	3.300000e-02	2.340000e-01

	count	mean	std	min	25%	50%	75%	max
iskew_ff3_21d_x	2469068.0	1.990000e-01	7.943000e-01	-3.1203	-2.632000e-01	1.696000e-01	6.344000e-01	3.455800e+00
ivol_hxz4_21d_x	2332649.0	2.680000e-02	1.960000e-02	0.0018	1.370000e-02	2.130000e-02	3.350000e-02	2.397000e-01
iskew_hxz4_21d_x	2332643.0	1.777000e-01	7.585000e-01	-3.0805	-2.681000e-01	1.513000e-01	6.005000e-01	3.275600e+00
rmax5_21d_x	2469033.0	3.860000e-02	2.910000e-02	0.0022	1.960000e-02	3.050000e-02	4.810000e-02	3.544000e-01
rmax1_21d_x	2469033.0	6.730000e-02	5.830000e-02	0.0035	3.110000e-02	5.000000e-02	8.280000e-02	8.996000e-01
rvol_21d_x	2469080.0	2.970000e-02	2.060000e-02	0.0018	1.590000e-02	2.400000e-02	3.690000e-02	2.515000e-01
rskew_21d_x	2469038.0	2.439000e-01	8.740000e-01	-3.5810	-2.529000e-01	2.077000e-01	7.136000e-01	3.808400e+00
ami_126d_x	2427976.0	2.294900e+00	1.277990e+01	0.0000	6.300000e-03	8.310000e-02	7.621000e-01	7.242321e+02
beta_252d_x	2434576.0	8.972000e-01	6.011000e-01	-1.8325	4.682000e-01	8.481000e-01	1.259200e+00	4.013900e+00
ivol_capm_252d_x	2434576.0	2.910000e-02	1.710000e-02	0.0050	1.700000e-02	2.480000e-02	3.650000e-02	1.684000e-01
betadown_252d_x	2406390.0	1.001300e+00	7.817000e-01	-3.9821	5.127000e-01	9.352000e-01	1.414000e+00	5.699200e+00
prec_highprec_252d_x	2434268.0	7.724000e-01	1.997000e-01	0.0167	6.610000e-01	8.272000e-01	9.318000e-01	1.000000e+00
rvol_252d_x	2434576.0	3.110000e-02	1.740000e-02	0.0052	1.870000e-02	2.680000e-02	3.870000e-02	1.690000e-01
corr_1260d_x	1904407.0	3.603000e-01	1.650000e-01	-0.0374	2.362000e-01	3.573000e-01	4.786000e-01	8.219000e-01
betabab_1260d_x	1893789.0	1.075600e+00	5.871000e-01	-0.3259	6.475000e-01	1.000500e+00	1.410600e+00	4.274900e+00
rmax5_rvol_21d_x	2343331.0	1.232900e+00	5.049000e-01	0.1125	8.720000e-01	1.159900e+00	1.512600e+00	4.328700e+00
age_x	2739928.0	2.184690e+02	1.888040e+02	1.0000	7.900000e+01	1.590000e+02	3.000000e+02	1.115000e+03
qmj_x	1825615.0	8.990000e-02	9.763000e-01	-1.7027	-7.318000e-01	1.204000e-01	9.350000e-01	1.701100e+00
qmj_prof_x	2502382.0	9.110000e-02	9.846000e-01	-1.7036	-7.339000e-01	1.300000e-01	9.456000e-01	1.698800e+00
qmj_growth_x	1825622.0	3.610000e-02	9.739000e-01	-1.7018	-7.911000e-01	4.900000e-02	8.716000e-01	1.702100e+00
qmj_safety_x	2579701.0	8.730000e-02	9.713000e-01	-1.7012	-7.189000e-01	1.215000e-01	9.239000e-01	1.708800e+00

Table 3: Summary Statistics

15.5 Code

All files, resources, and code is available for download from Github. The document listing function and class docstring is available for download [here](#). Furthermore, the coding listings for this research essay follow. Try update.