Laplace

$$L : V \to W$$

$$L(\alpha \cdot f + \beta \cdot g) =$$
$$\alpha \cdot Lf + \beta \cdot Lg$$

$$L[f(t)] = F(s)$$

$$L[\sin t] = \frac{1}{s^2+1}$$

$$L \sin s = \frac{1}{s^2+1}$$

$$L \cos s = \frac{s}{s^2+1}$$

$(\cdot) = \text{scale}$

$$f'' + 4 \cdot f' + f = 6 \cdot \cos \quad , \quad f\,0 = 0 \quad , \quad f'\,0 = 0$$

$$L\,f'\,s = -f\,0 + s \cdot L\,f\,s \qquad L\,f\,s = 6 \cdot \frac{s}{(s^2+1)\cdot(s^2+4s+1)}$$

$$L\,(f'' + 4 \cdot f' + f)\,s = L\,(6 \cdot \cos)\,s$$

$$L\,f''\,s + 4 \cdot L\,f'\,s + L\,f\,s = 6 \cdot L\,\cos\,s$$

$$s^2 \cdot L\,f\,s + 4 \cdot s \cdot L\,f\,s + L\,f\,s = 6 \cdot \frac{s}{s^2+1}$$

$$(s^2 + 4s + 1) \cdot L\,f\,s = -{\prime\prime}-$$

$$f'' + 4 \cdot f' + f = 6 \cdot \cos \quad , \quad f\,0 = 0 \quad , \quad f'\,0 = 0$$

$$L\,f'\,s = -f\,0 + s \cdot L\,f\,s$$

$$L\,f\,s = 6 \cdot \frac{s}{(s^2+1)\cdot(s^2+4s+1)} = \frac{3}{2} \cdot \overbrace{\left(\frac{1}{s^2+1} - \frac{1}{s^2+4s+1} \right)}^{4s}$$

$$(s-s_1)\cdot(s-s_2)$$

$$s_1 = -2+\sqrt{3} \quad , \quad s_2 = -2-\sqrt{3}$$

$$\frac{1}{p \cdot q} = \frac{A}{p} + \frac{B}{q}$$

$L \sin s$

$$f'' + 4\cdot f' + f = 6\cdot\cos\ ,\ f\,0 = 0\ ,\ f'\,0 = 0$$

$$L\,f'\,s = -f\,0 + s\cdot L\,f\,s$$

$$\frac{3}{2}\cdot\frac{1}{s^2+4s+1} = \frac{A}{s-s_1} + \frac{B}{s-s_2} \qquad s_1 = -2+\sqrt{3}\ ,\ s_2 = -2-\sqrt{3}$$

$$3 = 2\cdot A\cdot(s-s_2) + 2\cdot B\cdot(s-s_1)$$

$$s=s_1:\quad 3 = 2\cdot A\cdot(s_1-s_2) = 2\cdot A\cdot 2\cdot\sqrt{3} \Rightarrow A = \frac{\sqrt{3}}{4}$$

$$s=s_2:\quad 3 = 2\cdot B\cdot(s_2-s_1) = 2\cdot B\cdot(-2\sqrt{3}) \Rightarrow B = -A$$

$$L\,f\,s = \frac{3}{2}\cdot L\sin s - \frac{\sqrt{3}}{4}\cdot\left(\frac{1}{s-s_1} - \frac{1}{s-s_2}\right)$$

$$f'' + 4\cdot f' + f = 6\cdot\cos\ , \quad f\,0 = 0\ , \quad f'\,0 = 0$$

$$L\,f'\,s = -f\,0 + s\cdot L\,f\,s$$

$$s_1 = -2+\sqrt{3}\ , \quad s_2 = -2-\sqrt{3}$$

$$\approx -0{,}3 \qquad \approx -3{,}7$$

$$L\,f\,s = \frac{3}{2}\cdot L\sin s - \frac{\sqrt{3}}{4}\cdot\left(\frac{1}{s-s_1} - \frac{1}{s-s_2}\right)$$

$$f\,t = \frac{3}{2}\cdot\sin t - \frac{\sqrt{3}}{4}\cdot\left(\exp(s_1\cdot t) - \exp(s_2\cdot t)\right)$$

$$f'\,t = \frac{3}{2}\cdot\cos t - \frac{\sqrt{3}}{4}\cdot\left(s_1\cdot \qquad - s_2\cdot \qquad\right)$$

$$f''\,t = -\frac{3}{2}\sin t - \frac{\sqrt{3}}{4}\left(s_1^2\cdot \qquad - s_2^2\cdot \qquad\right)$$

$$f'' + 4\cdot f' + f = 6\cdot\cos\ , \quad f\,0 = 0\ , \quad f'\,0 = 0$$

$$s_1 = -2+\sqrt{3}\ , \quad s_2 = -2-\sqrt{3}$$

$$f\,t = \frac{3}{2}\cdot\sin t - \frac{\sqrt{3}}{4}\cdot\left(\exp(s_1\cdot t) - \exp(s_2\cdot t)\right)$$

$$f'\,t = \frac{3}{2}\cdot\cos t - \frac{\sqrt{3}}{4}\cdot\left(s_1\cdot\exp(s_1\cdot t) - s_2\cdot\exp(s_2\cdot t)\right)$$

$$f''\,t = -\frac{3}{2}\cdot\sin t - \frac{\sqrt{3}}{4}\cdot\left(s_1^2\cdot\exp(s_1\cdot t) - s_2^2\cdot\exp(s_2\cdot t)\right)$$

$$\left.\begin{array}{l}
\operatorname{dot}(\ )\ (1,4,1) = 4\cdot\frac{3}{2}\cdot\cos t = 6\cdot\cos t = RHS \\
\operatorname{dot}\ (1,4,1) = (1+4\cdot s_1 + s_1^2)\cdot\exp(s_1\cdot t) = 0 \\
\operatorname{dot}\ (1,4,1) = (1+4 s_2 + s_2^2)\cdot\exp(s_2\cdot t) = 0
\end{array}\right\}\ \text{OK!}$$

$$f'' + 4 \cdot f' + f = 6 \cdot \cos\ , \quad f\,0 = 0\ , \quad f'0 = 0$$

$$L\,f'\,s = -f\,0 + s \cdot L\,f\,s \qquad s_1 = -2 + \sqrt{3}\ , \ s_2 = -2 - \sqrt{3}$$

$$L\,f\,s = \frac{3}{2} \cdot L\sin s - \frac{\sqrt{3}}{4} \cdot \left(\frac{1}{s - s_1} - \frac{1}{s - s_2} \right)$$

$$f\,t = \frac{3}{2} \cdot \sin t - \frac{\sqrt{3}}{4} \cdot \left(\exp(s_1 \cdot t) - \exp(s_2 \cdot t) \right)$$

$$f'\,t = \frac{3}{2} \cdot \cos t - \frac{\sqrt{3}}{4} \cdot \left(s_1 \cdot \exp(s_1 \cdot t) - s_2 \cdot \exp(s_2 \cdot t) \right)$$

$$f''\,t = -\frac{3}{2} \cdot \sin t - \frac{\sqrt{3}}{4} \cdot \left(s_1^2 \cdot \exp(s_1 \cdot t) - s_2^2 \cdot \exp(s_2 \cdot t) \right)$$

$$\left.\begin{aligned} f\,0 &= 0 - \frac{\sqrt{3}}{4} \cdot (1 - 1) = 0 \\ f'\,0 &= \frac{3}{2} \cdot 1 - \frac{\sqrt{3}}{4} \cdot (s_1 \cdot 1 - s_2 \cdot 1) = \frac{3}{2} - \frac{\sqrt{3}}{4} \cdot 2 \cdot \sqrt{3} = \frac{3}{2} - \frac{3}{2} = 0 \end{aligned}\right\} \text{OK}$$