D as a linear transform

Patrik Jansson

$V = \mathbb{R} \to \mathbb{R}$

LinTran(D) ?

$D : V \to V$

D as a linear transform

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$V = \mathbb{R} \to \mathbb{R}$ LinTran(D)!

$D : V \to V$

$H_0(D, zero_F, zero_F) \equiv D(const\ 0) = const\ 0 \equiv True$

$H_2(D, (+_F), (+_F)) \equiv \forall f, g : V.\ D(f +_F g) = Df +_F Dg \equiv True$

$H_1(D, neg, neg) \equiv \forall f : V.\ D(neg\ f) = neg(Df) \equiv True$

D as a linear transform

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$D : V \to V$

Examples: $v_1 = \exp : \mathbb{R} \to \mathbb{R}$ $\quad D \exp = \exp$

let $g_s(t) = \exp(-s \cdot t)$

then $g_s : V$ for every $s : \mathbb{R}$

$D\, g_s = \backslash t \to (-s) \cdot \exp(-s \cdot t) = scale_F\, (-s)\, g_s$

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$D : V \to V$

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$g_s(t) = \exp(-s \cdot t)$

$D \exp = \exp$

$D\, g_s = \text{scale}\ (-s)\ g_s$

$D(f \cdot g) = Df \cdot g + f \cdot Dg$

$\int$ as a linear transform

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$$D : V \to V \qquad D(f \cdot g) = Df \cdot g + f \cdot Dg$$

$$I : V \to V$$

$$I\,f\,x = \int_0^x f = \int_0^x f(t)\,dt$$

$$I\,(Df)\,x = \int_0^x f'(t)\,dt = \Big[\,f(t)\,\Big]_0^x = f\,x - f\,0$$

DSL → δσλ
DSLsofMath

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$\int$ as a linear transform

$D : V \to V \qquad D(f \cdot g) = Df \cdot g + f \cdot Dg$

$I : V \to V$

$I\,f\,x = \int_0^x f = \int_0^x f(t)\,dt$

$I(Df)\;x = f\,x - f\,0$

$I(D(f \cdot g))\,x = (f \cdot g)\,x - (f \cdot g)\,0$

$(f\,x) \cdot (g\,x) - (f\,0) \cdot (g\,0)$

$f(x) \cdot g(x) - f(0) \cdot g(0)$

$\int$ as a linear transform

Patrik Jansson

$$D : V \to V$$

$$I : V \to V$$

$$I\,f\,x = \int_0^x f = \int_0^x f(t)\,dt$$

$$I(Df)\;x = f\,x - f\,0$$

$$D(f\cdot g) = Df\cdot g + f\cdot Dg$$

$$I(D(f\cdot g))\,x = (f\cdot g)\,x - (f\cdot g)\,0 = \overbrace{I(Df\cdot g)}^{\mathbb{R}}\,x + \overbrace{I(f\cdot Dg)}^{\mathbb{R}}\,x$$

Towards Laplace

Patrik Jansson

$$(f \cdot g)\, x - (f \cdot g)\, 0 = I\,(Df \cdot g)\, x + I\,(f \cdot Dg)\, x$$

let $g = g_s = \backslash t \to e^{-st}$; $D\, g_s = scale\,(-s)\, g_s$

Assume $(f \cdot g_s)\, x \to 0$ as $x \to \infty$

$$0 - f\,0 \cdot g_s\,0 = I\,(Df \cdot g_s)\,\infty + I\,(f \cdot scale\,(-s)\, g_s)\,\infty$$

$$-f\,0 \cdot 1 = I\,(Df \cdot g_s)\,\infty + (-s) \cdot \underbrace{I\,(f \cdot g_s)\,\infty}_{\mathbb{R}}$$

Towards Laplace

Patrik Jansson

$(f \cdot g)\, x - (f \cdot g)\, 0 = I\, (Df \cdot g)\, x + I\, (f \cdot Dg)\, x$

let $g = g_s = \backslash t \to e^{-st}$; $D g_s = \text{scale}\, (-s)\, g_s$

Assume $(f \cdot g_s)\, x \to 0$ as $x \to \infty$

$0 - f\,0 \cdot g_s\,0 = I\, (Df \cdot g_s)\, \infty + I\, (f \cdot \text{scale}\, (-s)\, g_s)\, \infty$

$-f\,0 = I\, (Df \cdot g_s) - s \cdot I\, (f \cdot g_s)\, \infty$

DSL $\to$ $\delta\sigma\lambda$

DSLsofMath

Towards Laplace

Patrik Jansson

$$(f\cdot g)\,x - (f\cdot g)\,0 = I\,(Df\cdot g)\,x + I\,(f\cdot Dg)\,x$$

$$\text{let}\quad g = g_s = \backslash t \to e^{-st} \quad ; \; D\,g_s = \text{scale}\,(-s)\,g_s$$

$$\text{Assume}\quad (f\cdot g_s)\,x \to 0 \quad \text{as} \quad x \to \infty$$

$$0 - f\,0 \cdot g_s\,0 = I\,(Df\cdot g_s)\,\infty + I\,(f\cdot \text{scale}\,(-s)\,g_s)\,\infty$$

$$-f\,0 = I\,(Df\cdot g_s)\,\infty - s\cdot I\,(f\cdot g_s)\,\infty$$

$$-f\,0 = L\,(Df)\,s - s\cdot L\,f\,s$$

$$L\,(Df)\,s = -f\,0 + s\cdot L\,f\,s$$

$$\text{let}\; L\,\underbrace{f}_{\mathbb{R}\to\mathbb{R}}\,\underbrace{s}_{\mathbb{C}} = \overbrace{I\,(f\cdot g_s)\,\infty}^{\mathbb{C}}$$

Laplace transform

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$L f s = I (f \cdot g_s) \infty$

$= \int_0^\infty f(t) \cdot e^{-s \cdot t} \, dt$

$g_s t = e^{-st}$

Assume $f x \cdot g_s x \to 0$ as $x \to \infty$

$L (D f) s = - f 0 + s \cdot L f s$

$V \cong \mathbb{R} \to \mathbb{R}$

$W \cong \mathbb{C} \to \mathbb{C}$

$L : V \to W$

$\mathrm{LinTrans}(L, V, W)$ (exercise!)

Continued on Lecture 7.2b
https://jamboard.google.com/d/1s6IX6AGPCG-IgmClUj0sd7cWVmlT2jBlFd6bK0reQAg/edit?usp=sharing