Types & equations

Patrik Jansson

$x + 2 = 3$

$x + 3 = 2$

$x : \mathbb{N}$

$x : \mathbb{N}$?

$x : \mathbb{Z}$?

$\boxed{x = 1}$

No solution ↯

$x = -1$

DSL → δσλ

DSLsOfMath

Types & equations

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$x + 3 = 2$ $\quad x : \mathbb{Z}$

$x * 2 = 3$ $\quad x : \mathbb{Z}$

— ! ! — $\quad x : \mathbb{Q}$

No!

$x = 3/2 = 1.5$

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$x + 3 = 2$ $\quad x : \mathbb{Z}$

$x * 2 = 3$ $\quad x : \mathbb{Q}$

$x \hat{} 2 = 3$ $\quad x : \mathbb{Q}$?

$x : \mathbb{R}$ $\quad x = \sqrt{3}$

No! ∄ no rational solving $x^2 = 3$

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$x + 3 = 2$ $\quad x : \mathbb{Z}$

$x * 2 = 3$ $\quad x : \mathbb{Q}$

$x \hat{} 2 = 3$ $\quad x : \mathbb{R}$

$x \hat{} 2 = -1$ $\quad x : \mathbb{R}$? (No! because $x^2 \geq 0$ forall $x : \mathbb{R}$)

$x : \mathbb{C}$ $\quad x = \sqrt{-1}$

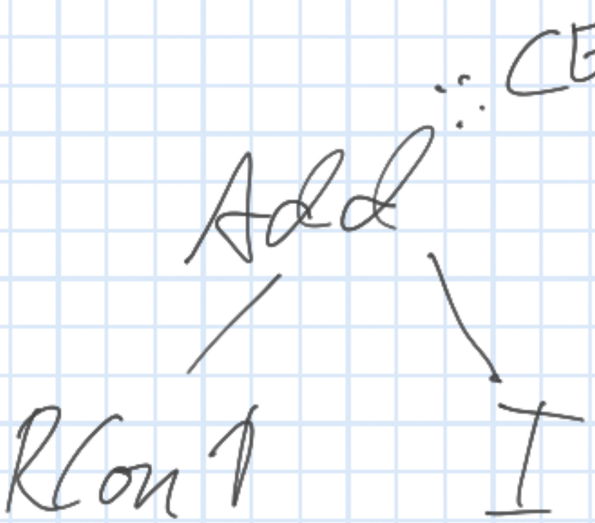
$x = i$

Patrik Jansson

Complex

Syntax

```
Add :: CE -> CE -> CE
```



eval

Semantics

```
addCC :: CC -> CC -> CC
addCC (PlusI a b)
      (PlusI x y) =
  PlusI (a+x) (b+y)
```

eval

```
PlusI 1 1
```

See book section 1.5

Complex multiplication

$$(a+ib)\cdot(x+iy) =$$

treat this as a polynomial in i & $i^2=-1$

$$a\cdot x + a\cdot iy + ib\cdot x + ib\cdot iy =$$

$$ax + i\cdot(ay+bx) + i^2 by =$$

$$\underbrace{(ax-by)}_{\text{real}} + i\cdot\underbrace{(ay+bx)}_{\text{imag}}$$