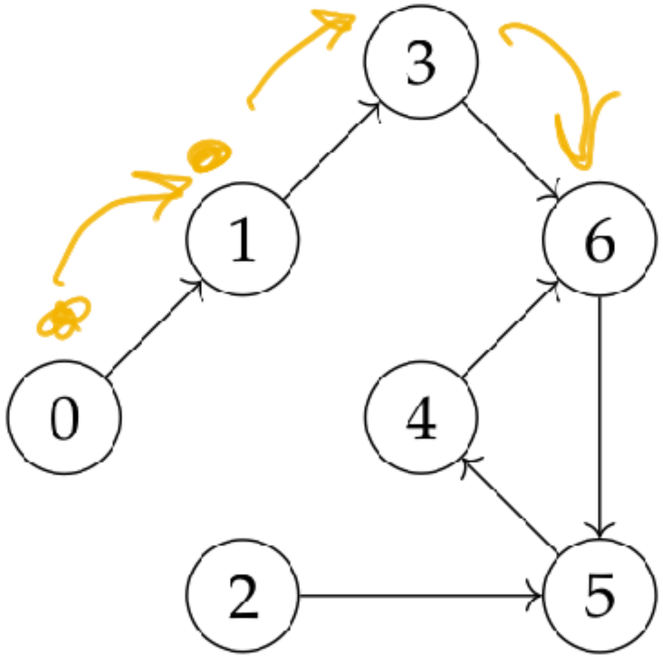
Linear Algebra & Dynamical Systems

Patrik Jansson



Deterministic dyn. sys.

$G = \{0..6\}$

$V = G \to \mathbb{R}$ "stocks"

$next : G \to G$

next 0 = 1; next 1 = 3; ... ..

$e_0 : V$ = "1 item at node 0"

$h : V \to V$ ← spec. of h

$h\,(e\,i) = e\,(next\,i)$

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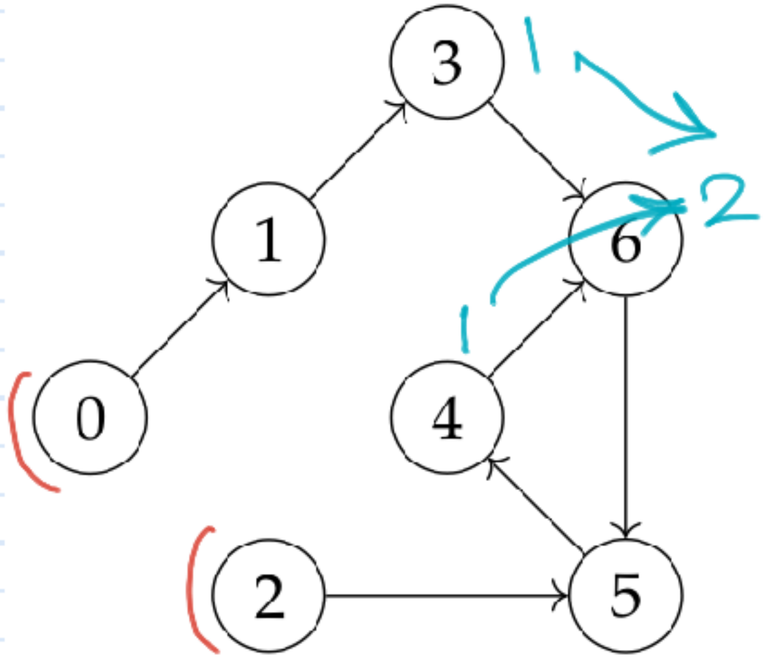

Deterministic dyn. sys.

$$f :: Vector\ \mathbb{R}\ G \to Vector\ \mathbb{R}\ G$$
$$f\ (e\ i) = e\ (next\ i)$$

$$M = \begin{array}{c} \\ r_0 \\ r_1 \\ r_2 \\ r_3 \\ r_4 \\ r_5 \\ r_6 \end{array} \begin{array}{c} \begin{array}{ccccccc} c_0 & c_1 & c_2 & c_3 & c_4 & c_5 & c_6 \end{array} \\ \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 \end{pmatrix} \end{array}$$

zero

1 1 2 2

1 1 1 1 1 1 1

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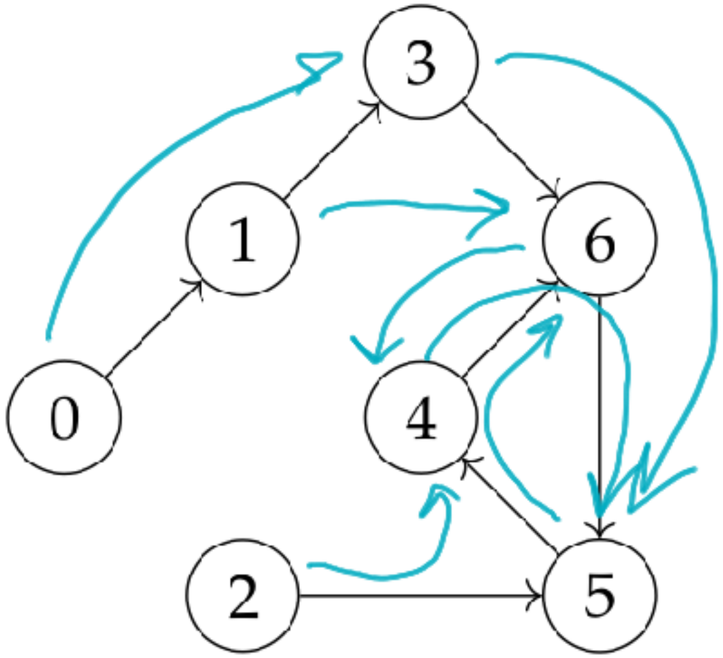

$M^2 = M \cdot M$ = take two steps

$X = 1$
$0 = 0$

$$M^2 = \begin{array}{c} \\ r_0 \\ r_1 \\ r_2 \\ r_3 \\ r_4 \\ r_5 \\ r_6 \end{array} \begin{array}{c} \begin{array}{ccccccc} c_0 & c_1 & c_2 & c_3 & c_4 & c_5 & c_6 \end{array} \\ \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 \end{pmatrix} \end{array}$$

← zero
← zero
← zero

Deterministic dyn. sys.

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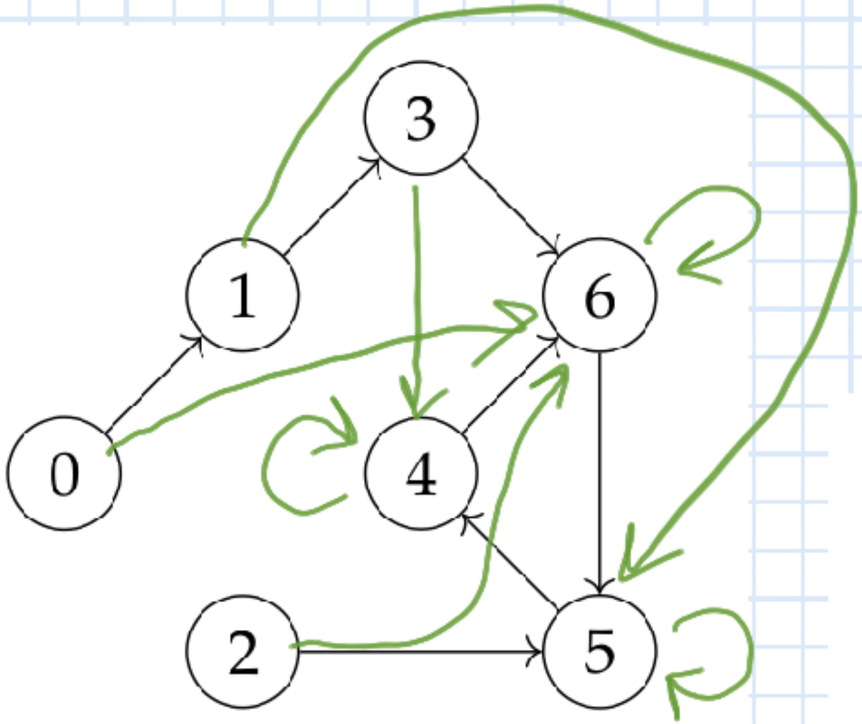

M^3 = take three steps

X = (

0 = 0

Deterministic dyn. sys.

$$M^3 = \begin{array}{c} \\ r_0 \\ r_1 \\ r_2 \\ r_3 \\ r_4 \\ r_5 \\ r_6 \end{array} \begin{array}{c} \begin{array}{ccccccc} c_0 & c_1 & c_2 & c_3 & c_4 & c_5 & c_6 \end{array} \\ \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 \end{pmatrix} \end{array}$$

zero

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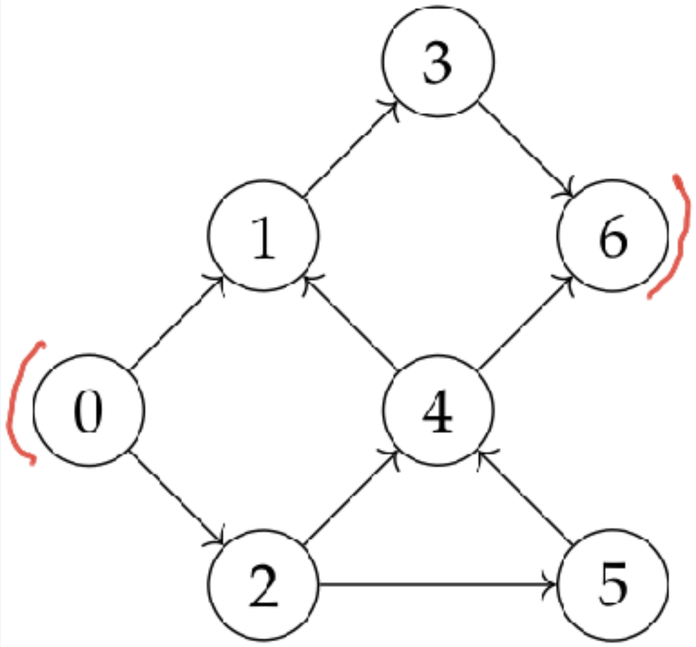

Nondeterministic dyn. sys.

next : G → [G]

next 0 = [1, 2]

⋮

next 6 = []

zero

$$M = \begin{array}{c} \\ r_0 \\ r_1 \\ r_2 \\ r_3 \\ r_4 \\ r_5 \\ r_6 \end{array} \begin{array}{c} \begin{array}{ccccccc} c_0 & c_1 & c_2 & c_3 & c_4 & c_5 & c_6 \end{array} \\ \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 \end{pmatrix} \end{array}$$

← zero

DSL → δσλ

DSLsofMath

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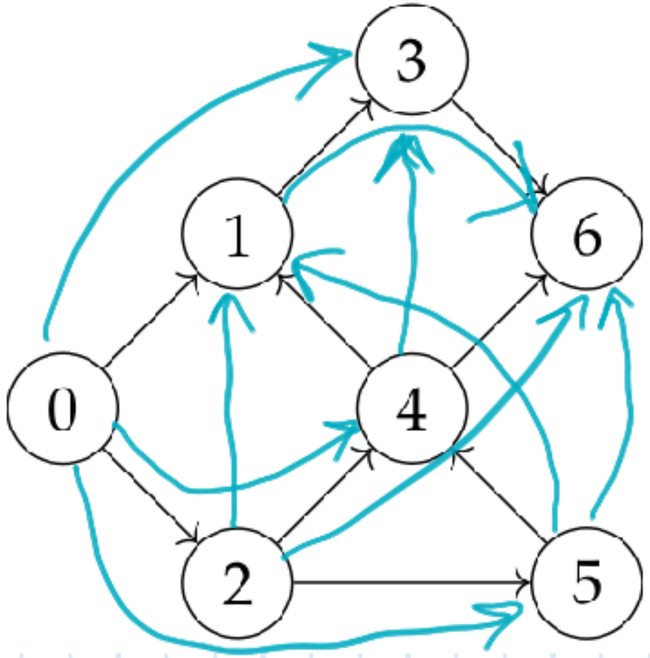

Nondeterministic dyn. sys.

M^2 = take two steps

M^6 = all zeroes

$$M = \begin{array}{c} \\ r_0 \\ r_1 \\ r_2 \\ r_3 \\ r_4 \\ r_5 \\ r_6 \end{array} \begin{array}{c} \begin{array}{ccccccc} c_0 & c_1 & c_2 & c_3 & c_4 & c_5 & c_6 \end{array} \\ \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 \end{pmatrix} \end{array}$$

zero

DSL → δσλ
DSLsofMath

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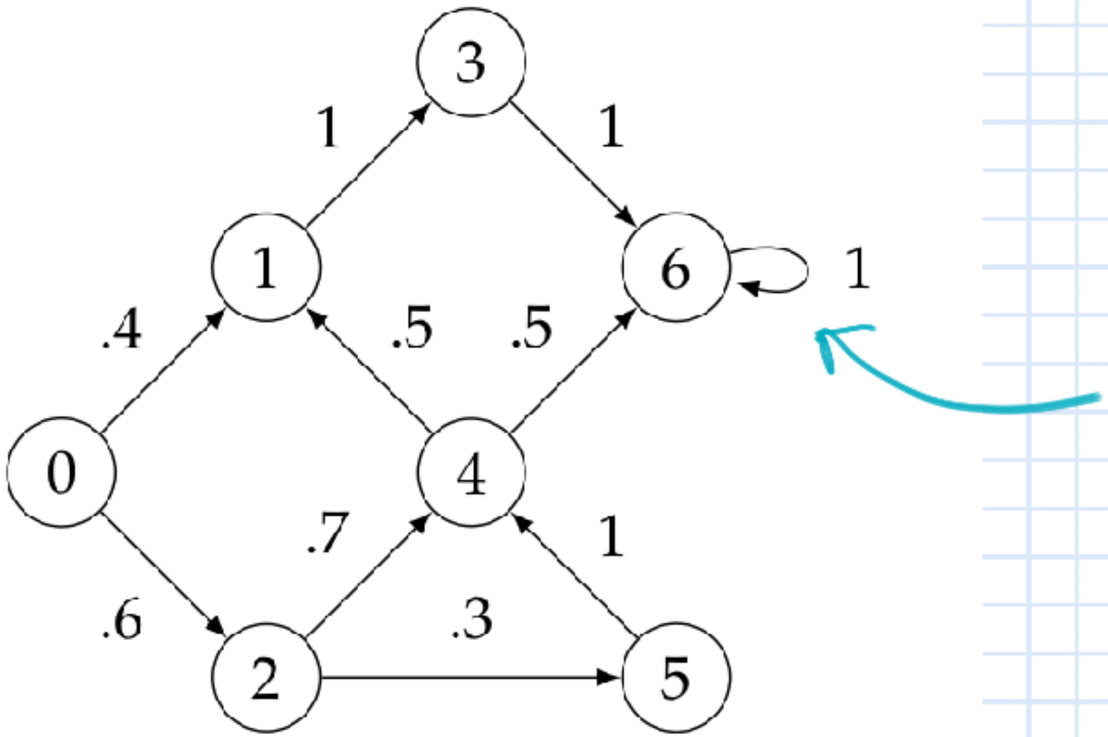

$$V = \{ p : G \to \mathrm{Prob} \mid \sum_c p_c = 1 \}$$

Probabilistic / stochastic dyn. sys.

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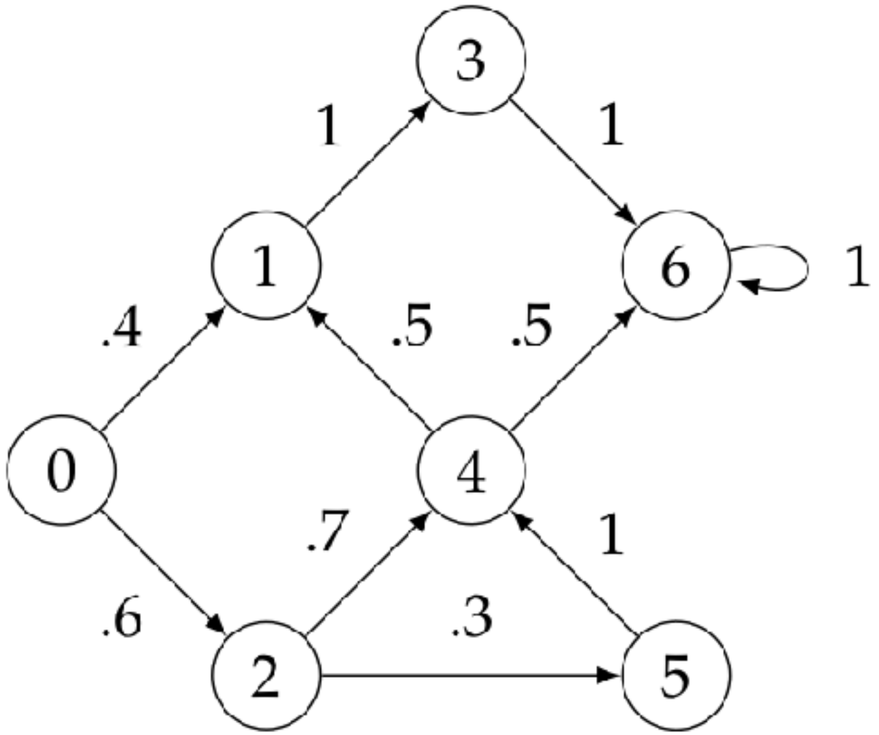

$$V = \{ p : G \to \mathrm{Prob} \mid \sum_c p_c = 1 \}$$

↓ ↓ ↓ ↓ ... columns sum to 1 = 100%

$$M = \begin{array}{c} \\ r_0 \\ r_1 \\ r_2 \\ r_3 \\ r_4 \\ r_5 \\ r_6 \end{array} \begin{array}{c} \begin{array}{ccccccc} c_0 & c_1 & c_2 & c_3 & c_4 & c_5 & c_6 \end{array} \\ \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ .4 & 0 & 0 & 0 & .5 & 0 & 0 \\ .6 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & .7 & 0 & 0 & 1 & 0 \\ 0 & 0 & .3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & .5 & 0 & 1 \end{pmatrix} \end{array}$$

Probabilistic / stochastic dyn. sys.

[For more, see Markov chains.]

DSL → δσλ
DSLsOfMath

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Deterministic
Non-deterministic
Stochastic / Probabilistic
⋮

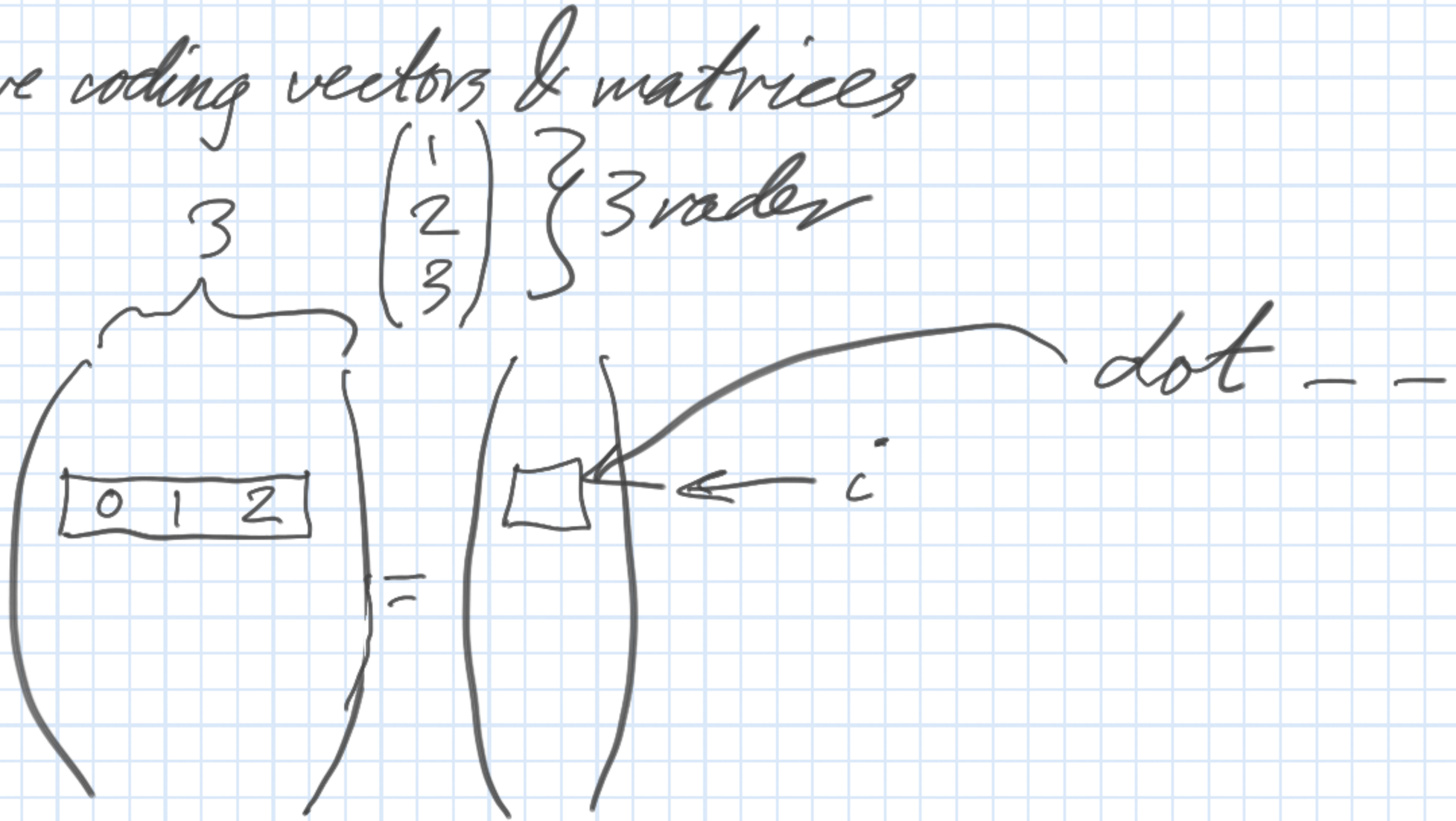
} transition matrices

[More: Monadic Dynamic Systems]

DSL → δσλ
DSLsOfMath

Next:

- Live coding vectors & matrices



Laplace

$$L : V \to W$$

$$L(\alpha \cdot f + \beta \cdot g) =$$

$$\alpha \cdot Lf + \beta \cdot Lg$$

$$L(f + 2 \cdot f' + 3 \cdot f'') =$$

$$Lf + 2 \cdot Lf' + 3 \cdot Lf''$$

$$V \cong \mathbb{R} \to \mathbb{R}$$

$$W \cong \mathbb{C} \to \mathbb{C}$$

$$\alpha, \beta : \mathbb{R}$$

$$f, g : V = \mathbb{R} \to \mathbb{R}$$

$$(\cdot) = scale : \mathbb{R} \to V \to V$$

Laplace

$$L : V \to W$$

$$L(\alpha \cdot f + \beta \cdot g) =$$

$$\alpha \cdot Lf + \beta \cdot Lg$$

$$L(f + 2 \cdot f' + 3 \cdot f'') =$$

$$Lf + 2 \cdot Lf' + 3 \cdot Lf''$$

$$L[f(t)] = F(s)$$

$$L\begin{bmatrix} \sin t \\ \sin a \cdot t \end{bmatrix} = \frac{1}{s^2+1}$$

$$L(\backslash t \to \sin t) = \backslash s \to \frac{1}{s^2+1}$$

$$L \sin s = \frac{1}{s^2+1}$$

$$L \cos s = \frac{s}{s^2+1}$$

$(\cdot) = \text{scale}$

$$f'' + 4 \cdot f' + f = 6 \cdot \cos \quad , \quad f0 = 0 \quad , \quad f'0 = 0$$

$$L f' s = -f0 + s \cdot L f s \rightarrow L f s = 6 \cdot \frac{s}{(s^2+1) \cdot (s^2+4s+1)}$$

$$L (f'' + 4 \cdot f' + f) s = L (6 \cdot \cos) s$$

$$L f'' s + 4 \cdot L f' s + L f s = 6 \cdot L \cos s$$

$$s^2 \cdot L f s + 4 \cdot s \cdot L f s + L f s = 6 \cdot \frac{s}{s^2+1}$$

$$(s^2 + 4s + 1) \cdot L f s = -11 - \frac{s}{s^2+1}$$

$(\cdot) = \text{scale}$

$$f'' + 4 \cdot f' + f = 6 \cdot \cos \,, \quad f0 = 0 \,, \quad f'0 = 0$$

$$L f' s = -f0 + s \cdot L f s$$

$$L f s = 6 \cdot \frac{s}{(s^2+1) \cdot (s^2+4s+1)} = \frac{A}{s - r_1} + \frac{B}{s - r_2} + \frac{C}{s - r_3} + \frac{D}{s - r_4}$$

$$r_1 = i \,, \; r_2 = -i \,,$$