Laplace

Linear transformation

$$L : V \to W$$

$$L(\alpha \triangleleft f + \beta \triangleleft g) = \alpha \triangleleft Lf + \beta \triangleleft Lg$$

$\mathbb{R}$ or $\mathbb{C}$

$(\triangleleft) : \text{Field } a \Rightarrow a \to v \to v$

$V \quad V$

$W \quad W$

$f, g : V$

$Lf, Lg : W$

Laplace

sin & cos from exp

$$f(t) = e^{i \cdot t} = \cos t + i \cdot \sin t$$
$$f(-t) = e^{-i \cdot t} = \cos(-t) + i \cdot \sin(-t)$$
$$= \cos t - i \cdot \sin t$$

$$\frac{f(t) + f(-t)}{2} = \cos t$$

$$\cos t = \frac{e^{i \cdot t} + e^{-i \cdot t}}{2}$$

$$\frac{f(t) - f(-t)}{2} = i \cdot \sin t$$

$$\sin t = \frac{e^{i \cdot t} - e^{-i \cdot t}}{2 \cdot i}$$

Laplace

sin & cos from exp

$$L\cos s = L\left(\backslash t \to \frac{e^{i\cdot t} + e^{-i\cdot t}}{2}\right) s =$$

$$= \frac{1}{2}\left(L\left(\backslash t \to e^{i\cdot t}\right) s + L\left(\backslash t \to e^{-i\cdot t}\right) s\right)$$

$$= \frac{1}{2}\left(\frac{1}{s-i} + \frac{1}{s+i}\right) = \frac{1}{2}\cdot\frac{(s+i)+(s-i)}{(s-i)(s+i)} = \frac{s}{s^2-i^2} = \frac{s}{s^2+1}$$

$$L\left(\backslash t \to e^{a\cdot t}\right) s = \frac{1}{s-a}$$

$$\cos t = \frac{e^{i\cdot t} + e^{-i\cdot t}}{2}$$

$$\sin t = \frac{e^{i\cdot t} - e^{-i\cdot t}}{2\cdot i}$$

Laplace

$$F(s) = \mathcal{L}\{f(t)\}$$

$$\mathcal{L} : FunExp \to FunExp$$

$$\mathcal{L}\,(Exp\,(Ca :*: X)) = Recip\,(X :-: Ca)$$

$$L : (\mathbb{R} \to \mathbb{R}) \to (\mathbb{C} \to \mathbb{C})$$

Math notation

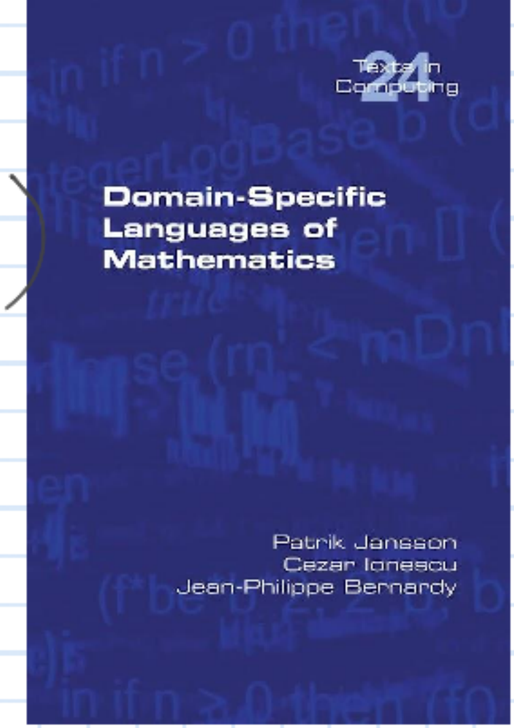
$f(t)$	$F(s)$
e^{at}	$\frac{1}{s-a}$
$\cos t$	$\frac{s}{s^2+1}$
$f'(t)$	$sF(s) - f(0)$
$\vdots$	

DSL

Domain-specific language (in this course)

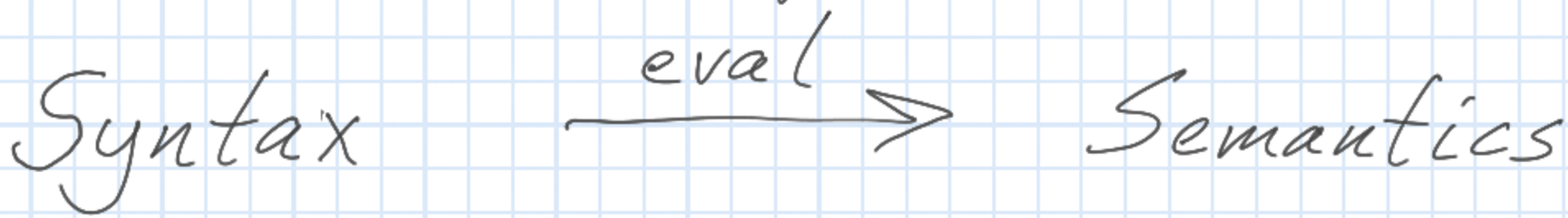
Syntax $\xrightarrow{\text{eval}}$ Semantics

Examples: Ch. 1: Complex, FunExp, Types



DSL

Domain-specific language (in this course)



Examples: Ch.1: Complex, FunExp, Types
Ch2: Prop, FOL, Proof, Set theory

DSL

Domain-specific language (in this course)

Syntax $\xrightarrow{\text{eval}}$ Semantics

Examples: Ch.1: Complex, FunExp, Types
Ch2: Prop, FOL, Proof, Set theory
Ch3: Derivative, typeclass

DSL

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Syntax $\xrightarrow{\text{eval}}$ Semantics

Examples: Ch.1: Complex, FunExp, Types
Ch2: Prop, FOL, Proof, Set theory
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Ch4: Algebraic structure, Homomorphism

DSL

Domain-specific language (in this course)

Syntax $\xrightarrow{\text{eval}}$ Semantics

Examples: Ch.1: Complex, FunExp, Types
Ch2: Prop, FOL, Proof, Set theory
Ch3: Derivative, typeclass
Ch4: Algebraic structure, Homomorphism
Ch5: Polynomial, Power series

DSL

Domain-specific language (in this course)

Syntax $\xrightarrow{\text{eval}}$ Semantics

Examples: Ch.1: Complex, FunExp, Types
Ch2: Prop, FOL, Proof, Set theory
Ch3: Derivative, typeclass
Ch4: Algebraic structure, Homomorphism
Ch5: Polynomial, Power series
Ch6: Taylor, Derivative series, ODE

DSL

Domain-specific language (in this course)

Syntax $\xrightarrow{eval}$ Semantics

Examples: Ch.1: Complex, FunExp, Types
Ch2: Prop, FOL, Proof, Set theory
Ch3: Derivative, typeclass
Ch4: Algebraic structure, Homomorphism
Ch5: Polynomial, Power series
Ch6: Taylor, Derivative series, ODE
Ch7: Vector, Matrix, Linear transformation

DSL

Domain-specific languages of Mathematics

Syntax $\xrightarrow{\text{eval}}$ Semantics

Examples: Ch.1: Complex, FunExp, Types
Ch2: Prop, FOL, Proof, Set theory
Ch3: Derivative, typeclass
Ch4: Algebraic structure, Homomorphism
Ch5: Polynomial, Power series
Ch6: Taylor, Derivative series, ODE
Ch7: Vector, Matrix, Linear transformation
Ch8: Laplace, ODE