Concluding Remarks:

With our discussion on plane strain elastodynamics complete, we have now concluded our introduction to finite element methods. Note that we have barely scratched the surface of the world of finite element methods in this class, and there are many directions one could pursue in further study of this subject. For instance, one could continue to study finite element analysis of structural mechanics. This pursuit could involve, for instance, an examination of finite element methods for the analysis of three-dimensional elastic structures, slender structures such as rods and beams, and thin structures such as plates and shells. Locking phenomena occur in each of these applications and require special treatment. This pursuit could also involve study of finite element analysis of structural vibrations and wave propagation and even nonlinear phenomena such as plasticity, contact, buckling, and fracture. Nonlinear finite element analysis in particular requires specialized algorithms.

Alternatively, one could study finite element analysis of fluid mechanics. There are a plethora of finite element methods for advective-diffusive systems such as those governed by the incompressible and compressible Navier-Stokes equations. Both continuous and discontinuous Galerkin methods are popular in practice. Most finite element methods for fluid mechanics applications include stabilization mechanisms to combat instabilities that arise from the application of Galerkin's method to advective-dominated problems. Many finite element methods for fluid mechanics applications also include discontinuity or shock capturing operators that provide highly localized numerical dissipation in the vicinity of sharp layers and shocks. A theoretical basis for stabilized methods is provided by the recently introduced framework of variational multiscale analysis.

There are also specialized finite element methods for other applications of interest. For instance, the simulation of electromagnetic phenomena requires the use of so-called curl-conforming finite element functions. Coupled problems, such as those arising in fluid-structure interaction, also require specialized finite element treatments.

Finally, one could study advanced finite element methods. Adaptive finite element methods employ a postriori error estimation and local mesh adaptivity to arrive at accurate predictions of quantities of interest at minimal cost. Immersed finite element methods enable the use of non-body fitted meshes in finite element analysis, bypassing the need for generating a high-quality body-fitted finite element mesh. Isogeometric finite element methods enable the simulation of physical phenomena directly on Computer Aided Design (CAD) geometries. The interested reader is encouraged to dive into the research literature to learn more about these advanced methodologies.