

2D Heat Conduction: Convergence of Isoparametric Finite Element Approximations:

As mentioned earlier, isoparametric finite element approximations exhibit optimal convergence rates in the H^1 -norm for smooth solutions, that is:

$$\|T - T^h\|_{H^1(\Omega^h)} \leq C \left(\frac{h}{L}\right)^k \|u\|_{H^{k+1}(\Omega^h)}$$

where h is a measure of the mesh size and C is a positive constant.

In fact, one can show the L^2 -norm of the error converges at an improved rate of $k+1$ using the so-called Aubin-Nitsche trick. However, the size of the constant C above depends highly on the quality of the finite element mesh. If the mesh contains highly distorted or skewed elements, C becomes very large. To address this issue, appropriate mesh smoothing techniques should be employed based on suitable mesh quality metrics. See, for example, the paper "Mesh Quality Metrics for Isogeometric Bernstein-Bézier Discretizations" by my former PhD student, Luke Engvall.