

## Plane Strain Elastostatics : The Patch Test:

To conclude our discussion of plane strain elastostatics, we briefly introduce the patch test. The patch test was introduced by Irons in the 1960s as a simple test indicating whether sequences of finite element approximations utilizing a particular element type converge. The patch test is passed if constant strain states are exactly reproduced by all finite element approximations using the particular element type, that is, by arbitrary patches of elements using the particular element type. In constructing an arbitrary patch, it is important to use an irregular mesh, because some element types pass the patch test in certain special configurations but not others.

All isoparametric finite elements pass the patch test if the stiffness matrix is fully integrated as isoparametric finite element approximations are complete. If all the terms in the stiffness matrix are underintegrated, however, the patch test is failed as the stiffness matrix loses rank. The patch test has proven especially useful in assessing the viability of non-conforming finite element approximations for which  $\{J_h\}^h \notin C^0(\Omega^h)$ .

While the patch test has long been used by engineers to assess the quality of a particular finite element type, passing the patch test is neither necessary nor sufficient for convergence of finite element approximations using the particular element type. Nonetheless, as reproducing constant strain states is a desirable property to have, the patch test is still quite useful.