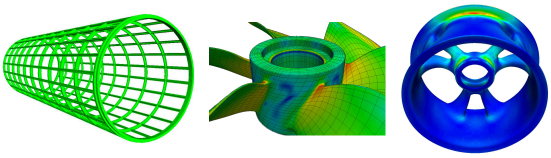
Isogeometric Analysis:
Fundamentals, Applications, and Challenges



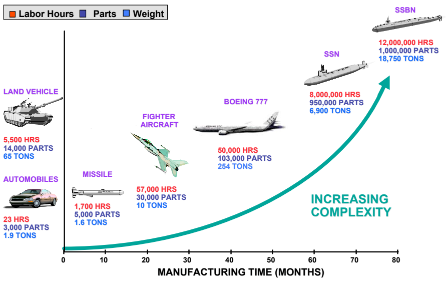
The IGA Crew:

A. Aggrawal, F. Auricchio, I. Babuska, Y. Bazilevs, L. Beirão da Veiga, D. Benson, M. Borden, R. de Borst, V. Calo, E. Cohen, J.A. Cottrell, L. De Lorenzis, T. Elguedj, J. Evans, H. Gomez, I Harari, S. Hossain, M.-C. Hsu, T.J.R. Hughes, D. Kamensky, C. Landis, I. Lee, S. Lipton, J. Liu, S. Morganti, E. Rank, A. Reali, R. Riesenfeld, M. Sacks, G. Sangalli, D. Schillinger, M. Scott, T.Sederberg, N. Sukumar, I. Temizer, B. Urick, C. Verhoosel, Z. Wilson, P. Wriggers, J. Zhang

Engineering *Design* Process

- Engineering designs are encapsulated in CAD systems
- CAD geometry is *"exact"*
- More than 100,000 analyses of CAD designs performed in engineering offices daily

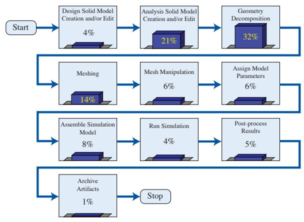
Engineering Designs



Courtesy of General Dynamics / Electric Boat Corporation

Engineering *Analysis* Process

- Finite Element Analysis (FEA) models are created from CAD representations
- Fixing CAD geometry and creating FEA models accounts for more than 80% of overall analysis time and is a major engineering *bottleneck*
- The FEA mesh is also only an *approximate* geometry



(Michael Hardwick and Robert Clay, Sandia National Laboratories)

Engineering *Analysis* Process

- Approximate FEM geometry can create *errors*
- Mesh refinement requires *communication* with CAD
- Adaptive mesh refinement has *not* been widely adopted in industrial practice
- *"Exact" geometry* may be a good idea

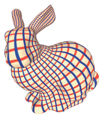
Objectives

- Develop an analysis framework based on functions capable of representing *"exact" geometry*
- *One*, and only one, geometric description
- Vastly *simplify* model development
- Integrate *design and analysis*

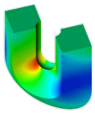
Isogeometric Analysis

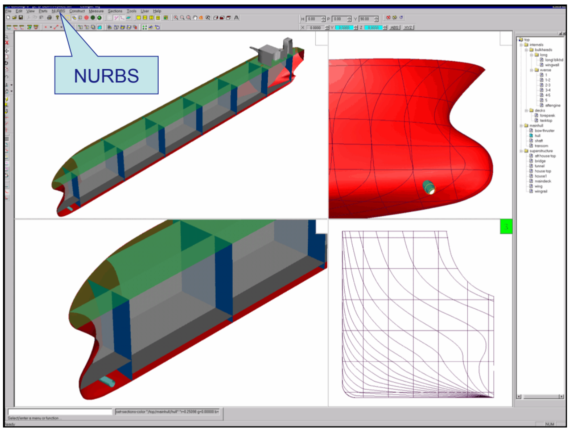
Isogeometric Analysis

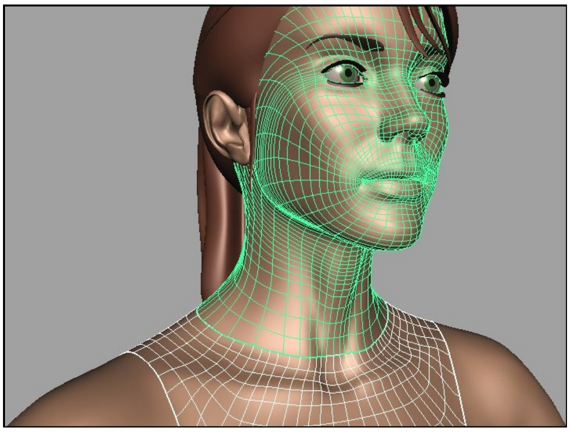
- Based on technologies (e.g., NURBS, T-splines, etc.) from *computational geometry* used in:
  - Design
  - Animation
  - Graphic art
  - Visualization



- Includes standard FEA as a special case, but offers other possibilities:
  - Precise and efficient geometric modeling
  - Simplified mesh refinement
  - Smooth basis functions with compact support
  - Superior approximation properties
  - *Integration* of design and analysis







Isogeometric Analysis
(NURBS, T-Splines, etc.)

FEA

h-, *p*-refinement

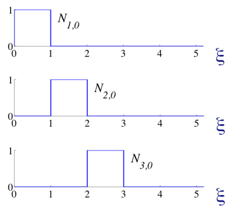
k-refinement

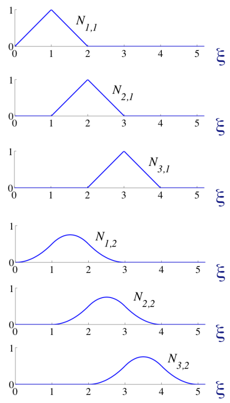
B-Splines

B-spline Basis Functions

$$N_{i,0}(\xi) = \begin{cases} 1 & \text{if } \xi_i \le \xi < \xi_{i+1}, \\ 0 & \text{otherwise} \end{cases}$$

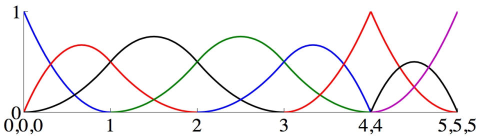
$$N_{i,p}(\xi) = \frac{\xi - \xi_i}{\xi_{i+p} - \xi_i} N_{i,p-1}(\xi) + \frac{\xi_{i+p+1} - \xi}{\xi_{i+p+1} - \xi_{i+1}} N_{i+1,p-1}(\xi)$$





B-spline basis functions of order 0, 1, 2 for a *uniform knot vector:*

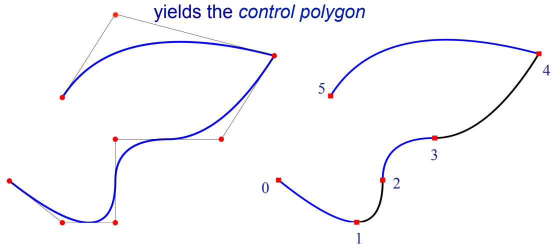
$\Xi = \{0,1,2,3,4,\ldots\}$



Quadratic (p=2) basis functions for an *open, non-uniform knot vector:*

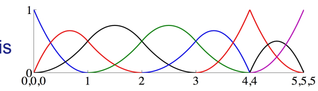
$\Xi = \{0,0,0,1,2,3,4,4,5,5,5\}$

Linear interpolation of control points yields the *control polygon*

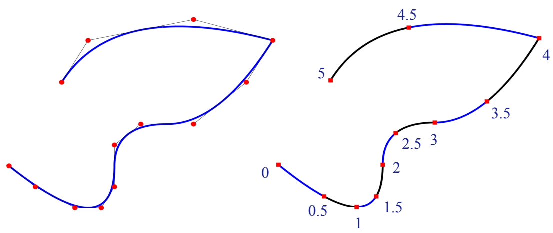


• - control points ■ - knots

Quadratic basis

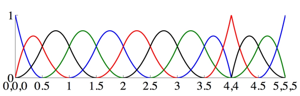


h-refined Curve

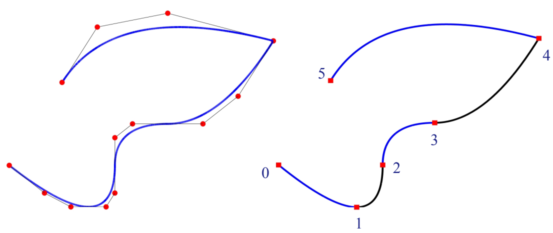


• - control points ■ - knots

Quadratic basis

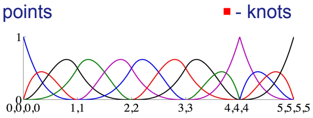


Cubic *p*-refined Curve



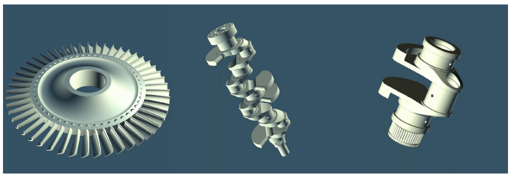
• - control points ■ - knots

Cubic basis

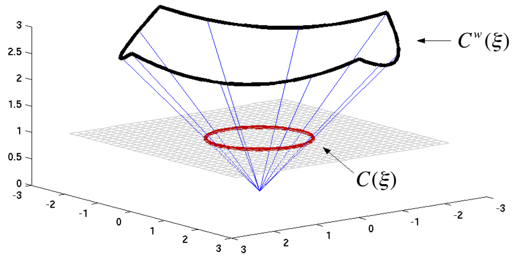


Non-Uniform Rational B-Splines

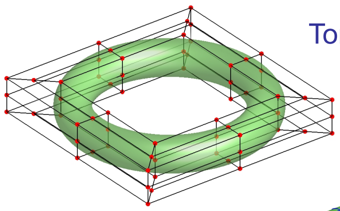
- NURBS are the most commonly used computer aided geometric design (CAGD) technology in engineering



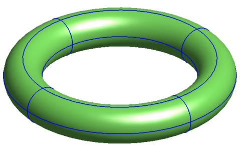
Circle from 3D Piecewise Quadratic Curves



Toroidal Surface

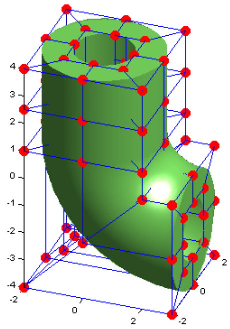


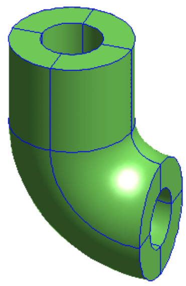
Control net



Mesh

Control Net / Mesh

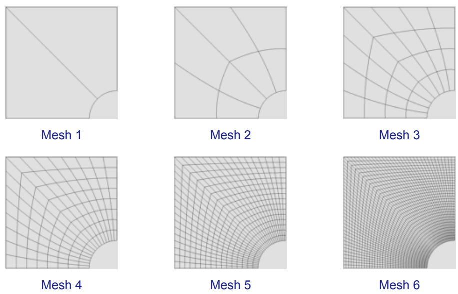




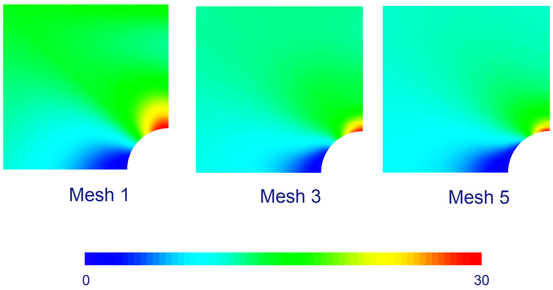
Elastic Plate

- Infinite plate with circular hole under constant stress in *x*-direction
- *k*- and *h*-refinements

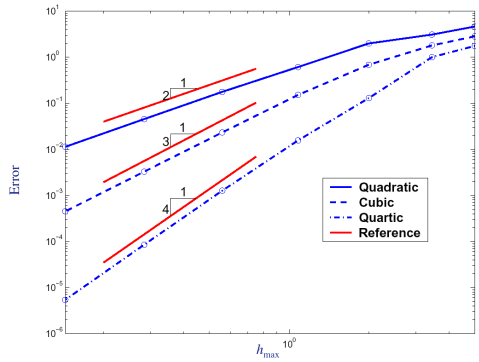
Uniformly Refined NURBS Meshes



Contours of σ_{xx}

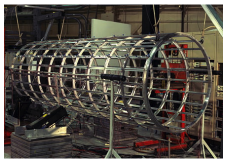


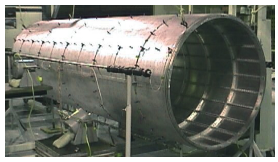
Error in L^2-norm of Stress



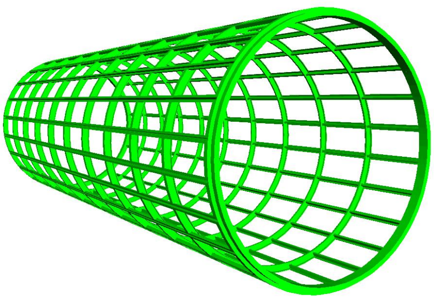
Vibration Analysis

NASA Aluminum Testbed Cylinder (ATC)

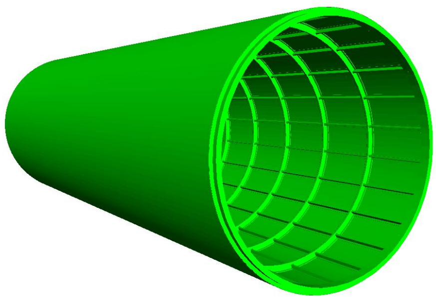


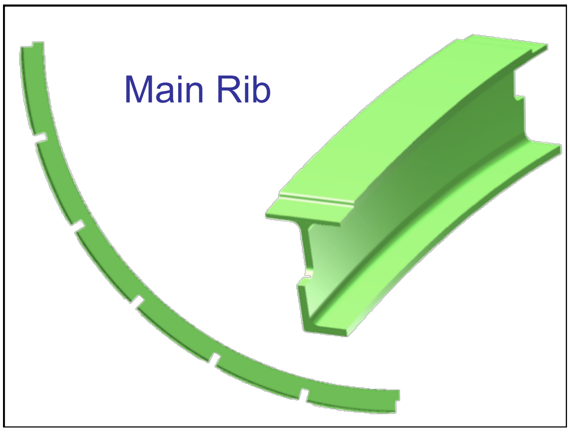


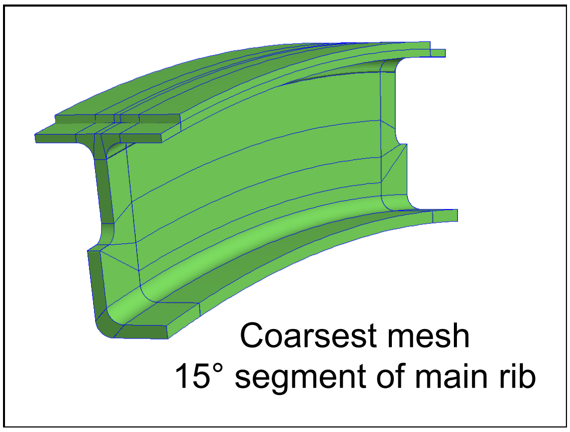
NASA ATC Frame

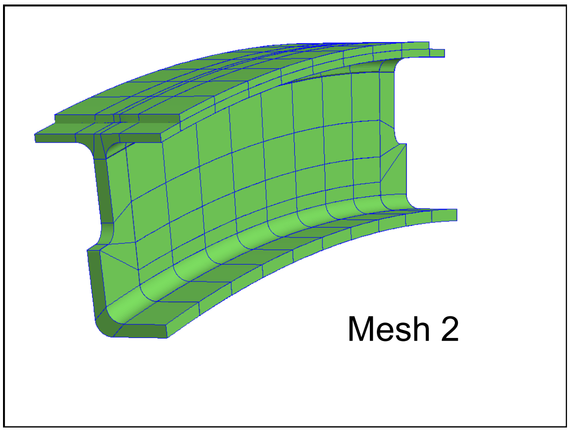


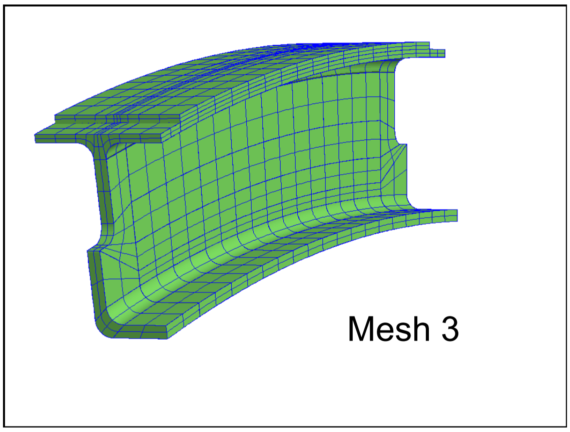
NASA ATC Frame and Skin

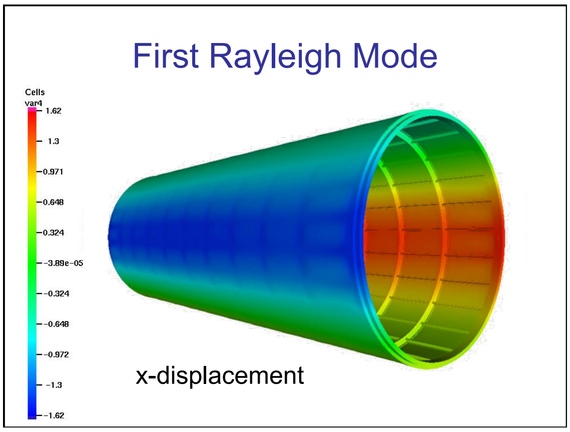


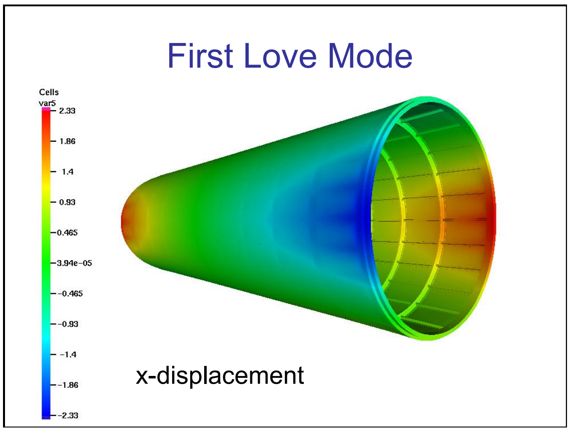

Main Rib


Coarsest mesh
15° segment of main rib

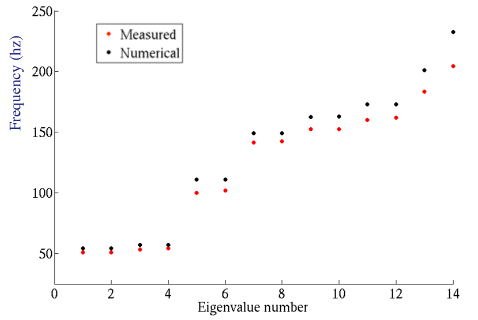

Mesh 2


Mesh 3


First Rayleigh Mode
x-displacement

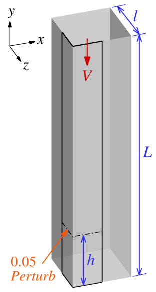

First Love Mode
x-displacement

ATC Frame and Skin



Nonlinear Solids

Square Tube Buckling

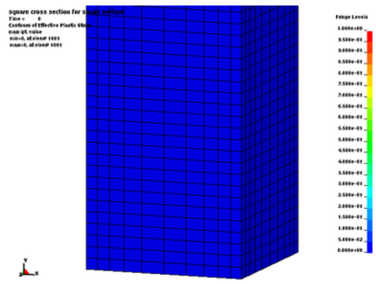


- Standard benchmark for automobile crashworthiness
- Quarter symmetry
- Perturbation to initiate buckling mode
- J_2 plasticity with linear isotropic hardening

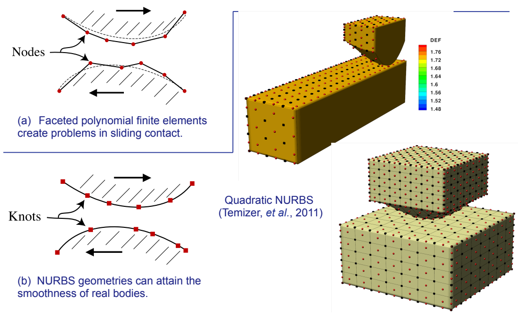
(D. Benson *et al.*)

Smooth Functions are Robust

C^3 quartics

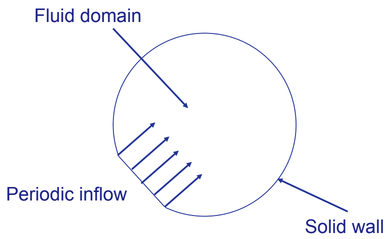


Sliding Contact



Fluid-Structure Interaction

Balloon Containing an Incompressible Fluid

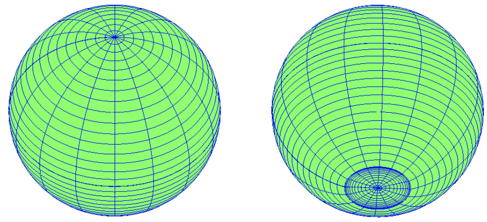


(Re = 4 x 10^5)

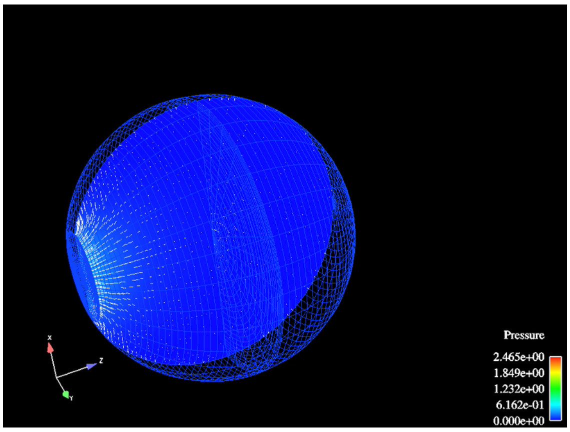
From Wall ' 06, Tezduyar ' 07

Balloon Containing an Incompressible Fluid

- Quadratic NURBS for both solid and fluid
- Boundary layer meshing

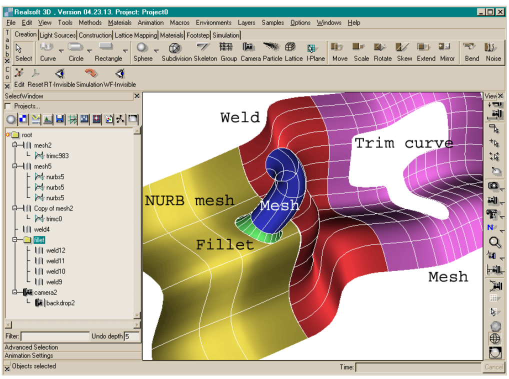


(a) Top view (b) Bottom view

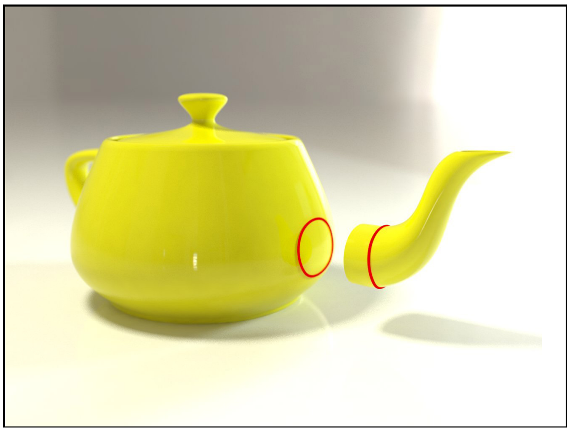


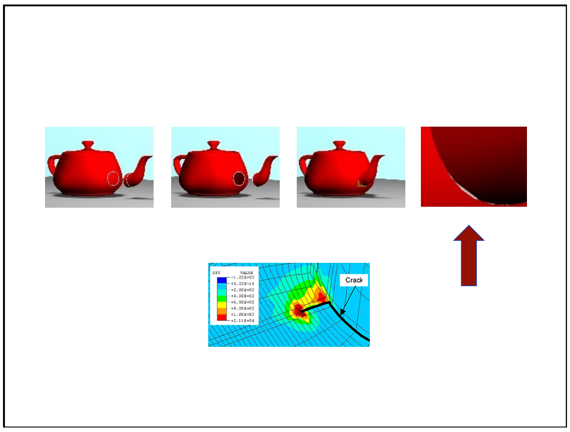
Problems with NURBS-based Engineering Design

- Water-tight merging of patches
- Trimmed surfaces



Courtesy of Juan Santocono



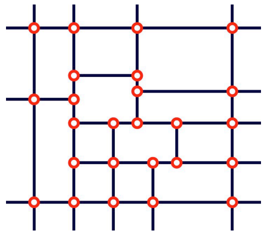

Crack

Analysis Suitable IGA Technologies

NURBS

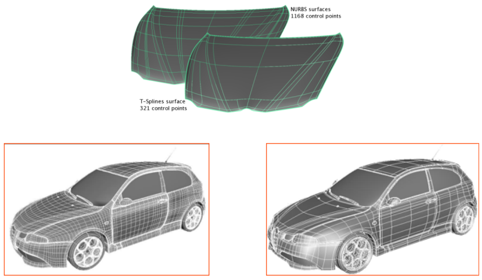
- Water tight (i.e., no gaps, no overlaps, no holes, etc.) No
- Linearly independent Yes
- Partition of unity property Yes
- Affine covariance Yes
- Pass standard patch tests Yes
- Locally refinable No
- Accommodate extraordinary points (i.e., star points) No
- Trimless option No
- Simply implemented in finite element analysis codes Yes
- Designers must use them and, ideally, like them Yes/No

T-splines (T. Sederberg)

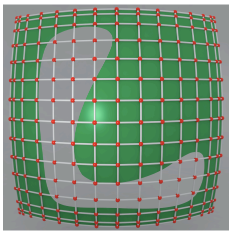


Unstructured NURBS Mesh

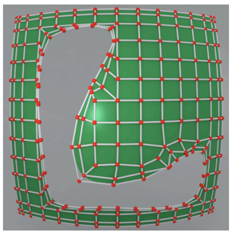
Reduced Number of Control Points



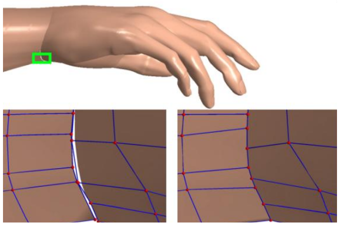
Trimmed NURBS



Untrimmed T-spline



Water-tight merging of patches



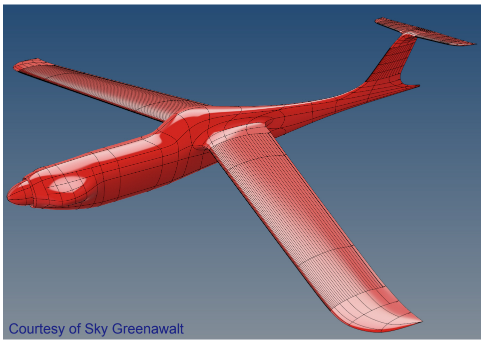
Water-tight untrimmed T-spline

Analysis Suitable IGA Technologies

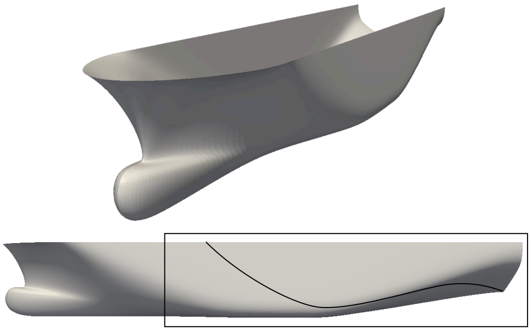
T-splines (M. Scott *et al.*)

- Water tight (i.e., no gaps, no overlaps, no holes, etc.) Yes
- Linearly independent Yes
- Partition of unity property Yes
- Affine covariance Yes
- Pass standard patch tests Yes
- Locally refinable Yes
- Accommodate extraordinary points (i.e., star points) Yes
- Trimless option Yes
- Simply implemented in finite element analysis codes Yes
- Designers must use them and, ideally, like them Yes

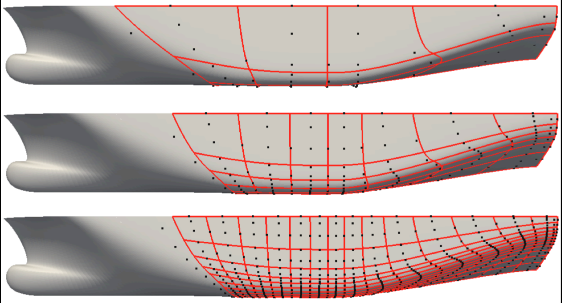
T-splines of Arbitrary Topology



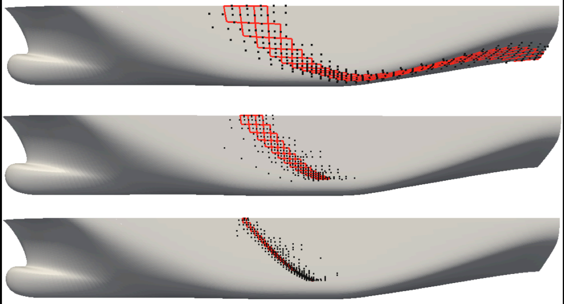
Local Refinement with T-splines



Refinement of Bezier Elements and Additional Control Points

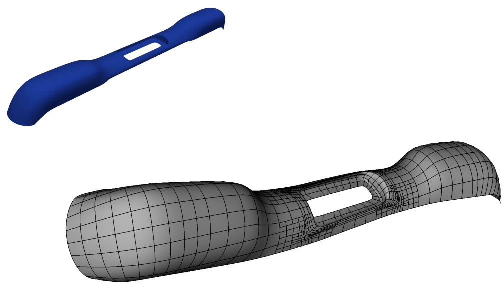


Refinement of Bezier Elements and Additional Control Points



Design-through-analysis: Surfaces

- Idea:
  - Create surface geometry with *commercial* CAD software and use it directly in *commercial* FEA software
- Goal:
  - Eliminate geometry repair, feature removal, and mesh generation
- Test case:
  - Import T-spline from Rhino (with T-Spline, Inc. plug-in) directly into LS-DYNA for shell analysis



Mode 7