Bezier Extraction

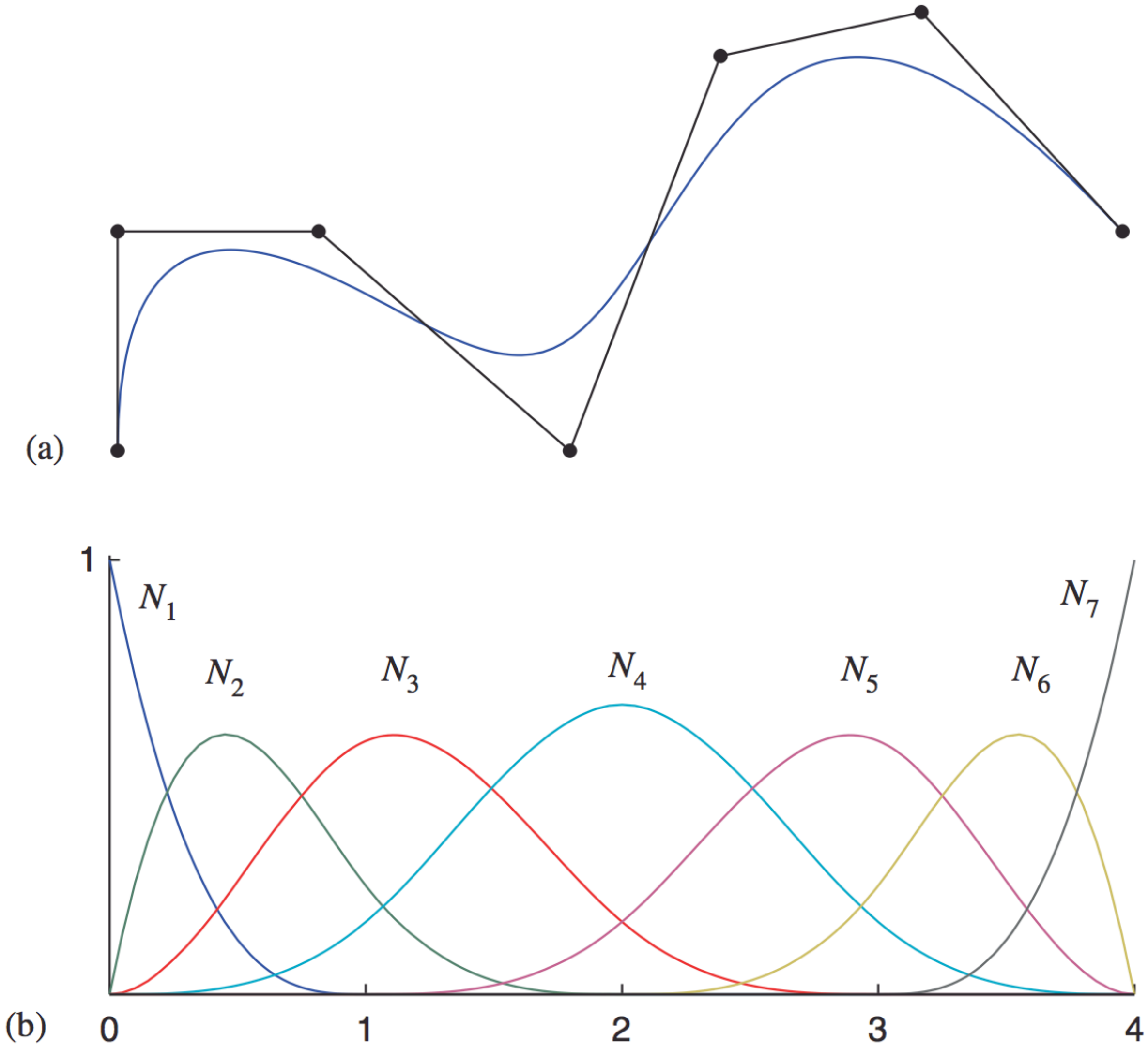


Figure 1. A cubic NURBS curve: (a) the curve and its control net and (b) the basis functions of the curve. The knot vector for the curve is $\{0, 0, 0, 0, 1, 2, 3, 4, 4, 4, 4\}$.

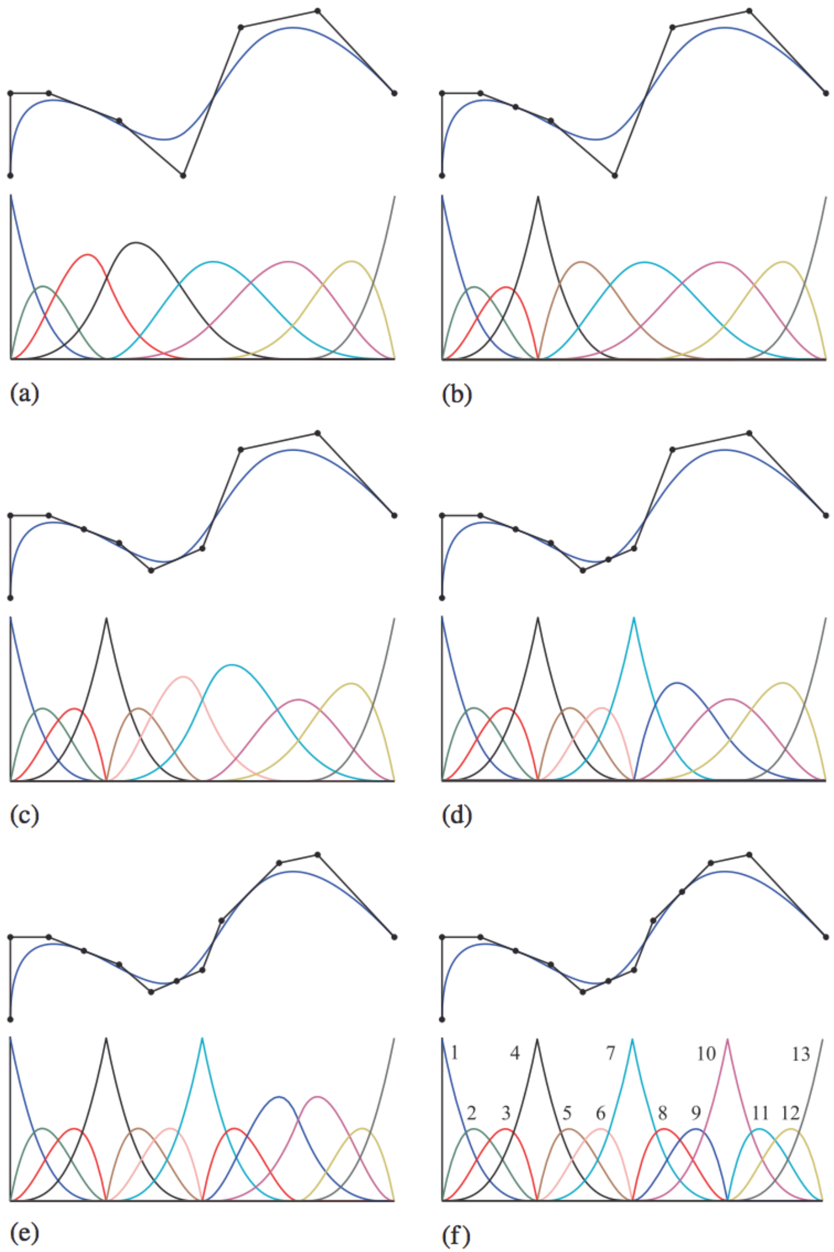


Figure 2. The sequence of basis functions created by inserting the knots $\{1, 1, 2, 2, 3, 3\}$ into the knot vector for the curve in Figure 1. The final set of basis functions in (f) is a collection of piecewise cubic Bézier basis functions. The numbers in (f) denote the numbering scheme of the Bézier basis functions.

$$\begin{Bmatrix} N_1 \\ N_2 \\ N_3 \\ N_4 \\ N_5 \\ N_6 \\ N_7 \end{Bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & \frac{1}{2} & \frac{1}{4} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{2} & \frac{7}{12} & \frac{2}{3} & \frac{1}{3} & \frac{1}{6} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{6} & \frac{1}{3} & \frac{2}{3} & \frac{2}{3} & \frac{2}{3} & \frac{1}{3} & \frac{1}{6} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{6} & \frac{1}{3} & \frac{2}{3} & \frac{7}{12} & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{2} & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{Bmatrix} B_1 \\ B_2 \\ B_3 \\ B_4 \\ B_5 \\ B_6 \\ B_7 \\ B_8 \\ B_9 \\ B_{10} \\ B_{11} \\ B_{12} \\ B_{13} \end{Bmatrix}$$

$$
\begin{Bmatrix} N_1 \\ N_2 \\ N_3 \\ N_4 \\ N_5 \\ N_6 \\ N_7 \end{Bmatrix}
=
\begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & \frac{1}{2} & \frac{1}{4} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \frac{1}{2} & \frac{7}{12} & \frac{2}{3} & \frac{1}{3} & \frac{1}{6} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & \frac{1}{6} & \frac{1}{3} & \frac{2}{3} & \frac{2}{3} & \frac{2}{3} & \frac{1}{3} & \frac{1}{6} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{6} & \frac{1}{3} & \frac{2}{3} & \frac{7}{12} & \frac{1}{2} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{2} & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1
\end{bmatrix}
\begin{Bmatrix} B_1 \\ B_2 \\ B_3 \\ B_4 \\ B_5 \\ B_6 \\ B_7 \\ B_8 \\ B_9 \\ B_{10} \\ B_{11} \\ B_{12} \\ B_{13} \end{Bmatrix}
$$

B-spline
Basis

$$
\begin{Bmatrix} N_1 \\ N_2 \\ N_3 \\ N_4 \\ N_5 \\ N_6 \\ N_7 \end{Bmatrix}
=
\begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & \frac{1}{2} & \frac{1}{4} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \frac{1}{2} & \frac{7}{12} & \frac{2}{3} & \frac{1}{3} & \frac{1}{6} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & \frac{1}{6} & \frac{1}{3} & \frac{2}{3} & \frac{2}{3} & \frac{2}{3} & \frac{1}{3} & \frac{1}{6} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{6} & \frac{1}{3} & \frac{2}{3} & \frac{7}{12} & \frac{1}{2} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{2} & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1
\end{bmatrix}
\begin{Bmatrix} B_1 \\ B_2 \\ B_3 \\ B_4 \\ B_5 \\ B_6 \\ B_7 \\ B_8 \\ B_9 \\ B_{10} \\ B_{11} \\ B_{12} \\ B_{13} \end{Bmatrix}
$$

Bernstein
Basis

$$\begin{Bmatrix} N_1 \\ N_2 \\ N_3 \\ N_4 \\ N_5 \\ N_6 \\ N_7 \end{Bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & \frac{1}{2} & \frac{1}{4} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{2} & \frac{7}{12} & \frac{2}{3} & \frac{1}{3} & \frac{1}{6} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{6} & \frac{1}{3} & \frac{2}{3} & \frac{2}{3} & \frac{2}{3} & \frac{1}{3} & \frac{1}{6} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{6} & \frac{1}{3} & \frac{2}{3} & \frac{7}{12} & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{2} & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{Bmatrix} B_1 \\ B_2 \\ B_3 \\ B_4 \\ B_5 \\ B_6 \\ B_7 \\ B_8 \\ B_9 \\ B_{10} \\ B_{11} \\ B_{12} \\ B_{13} \end{Bmatrix}$$

Bezier Extraction Operator

Localizing to the first nonempty knot span, or element:

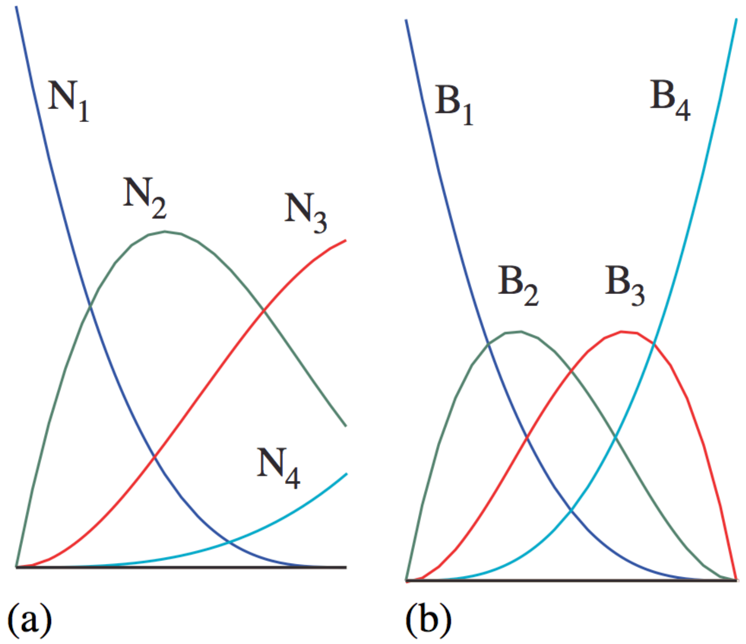


Figure 3. The basis functions over the knot span $[0, 1[$ from (a) the NURBS basis in Figure 1 and (b) the Bernstein basis in Figure 2(f).

Localizing to the first element:

$$\begin{Bmatrix} N_1 \\ N_2 \\ N_3 \\ N_4 \\ N_5 \\ N_6 \\ N_7 \end{Bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & \frac{1}{2} & \frac{1}{4} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{2} & \frac{7}{12} & \frac{2}{3} & \frac{1}{3} & \frac{1}{6} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{6} & \frac{1}{3} & \frac{2}{3} & \frac{2}{3} & \frac{2}{3} & \frac{1}{3} & \frac{1}{6} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{6} & \frac{1}{3} & \frac{2}{3} & \frac{7}{12} & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{2} & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{Bmatrix} B_1 \\ B_2 \\ B_3 \\ B_4 \\ B_5 \\ B_6 \\ B_7 \\ B_8 \\ B_9 \\ B_{10} \\ B_{11} \\ B_{12} \\ B_{13} \end{Bmatrix}$$

Localizing to the first element:

Local ***Element*** Extraction Operator

$$\begin{Bmatrix} N_1 \\ N_2 \\ N_3 \\ N_4 \\ N_5 \\ N_6 \\ N_7 \end{Bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & \frac{1}{2} & \frac{1}{4} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{2} & \frac{7}{12} & \frac{2}{3} & \frac{1}{3} & \frac{1}{6} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{6} & \frac{1}{3} & \frac{2}{3} & \frac{2}{3} & \frac{2}{3} & \frac{1}{3} & \frac{1}{6} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{6} & \frac{1}{3} & \frac{2}{3} & \frac{7}{12} & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{2} & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{Bmatrix} B_1 \\ B_2 \\ B_3 \\ B_4 \\ B_5 \\ B_6 \\ B_7 \\ B_8 \\ B_9 \\ B_{10} \\ B_{11} \\ B_{12} \\ B_{13} \end{Bmatrix}$$

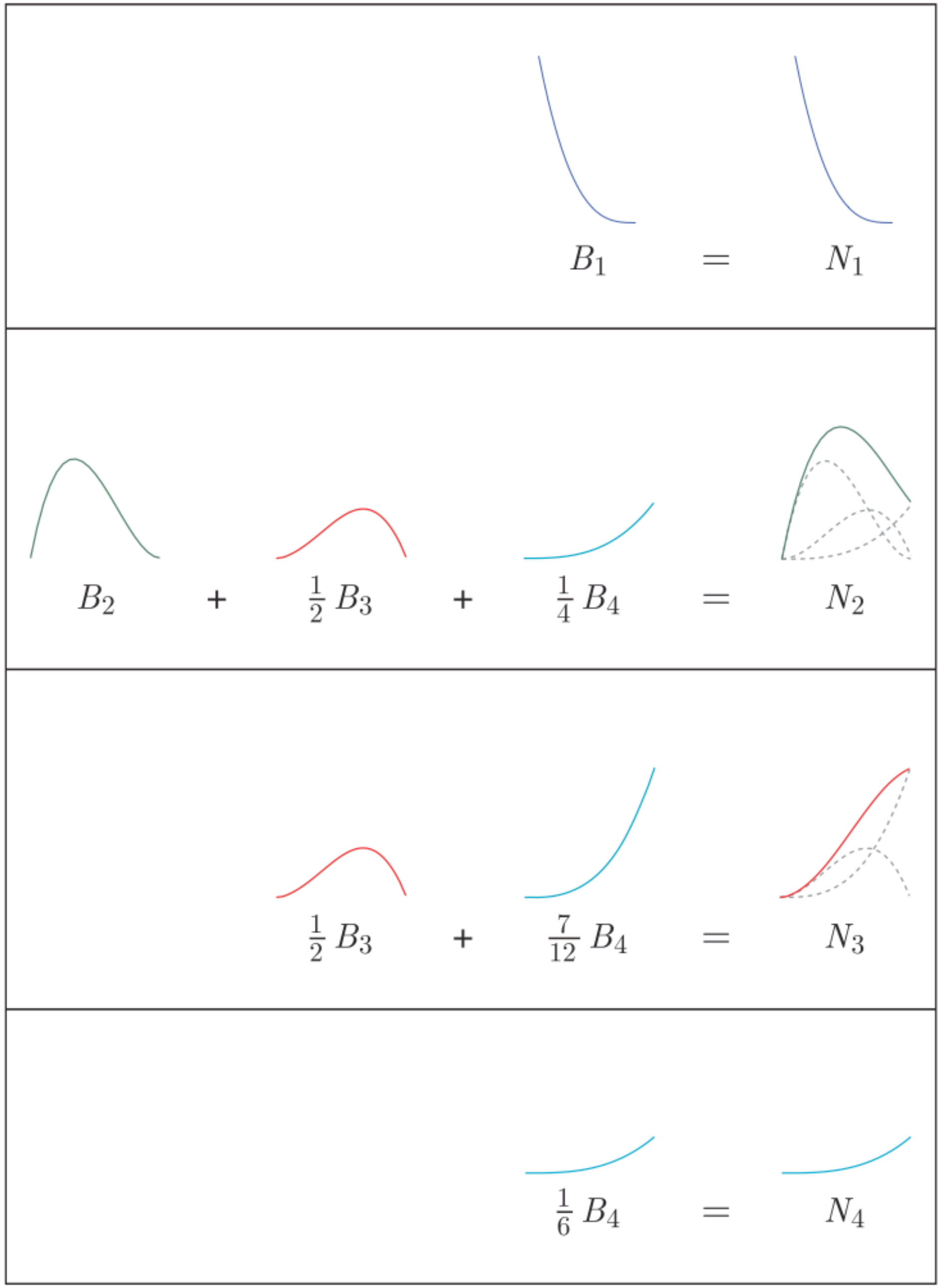
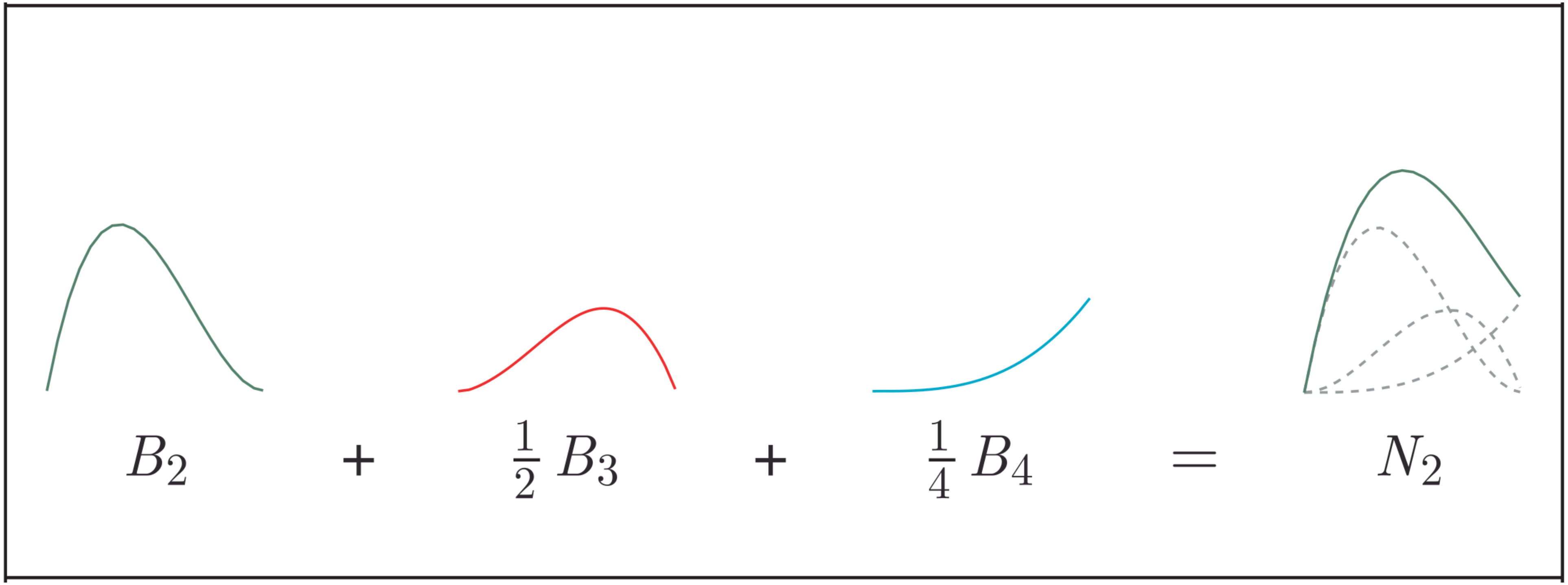


Figure 4. After knot insertion the original basis functions can be written as a linear combination of the basis functions for the Bézier elements.


$B_2 + \frac{1}{2} B_3 + \frac{1}{4} B_4 = N_2$

Localizing to the second element:

$$\begin{Bmatrix} N_1 \\ N_2 \\ N_3 \\ N_4 \\ N_5 \\ N_6 \\ N_7 \end{Bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & \frac{1}{2} & \frac{1}{4} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{2} & \frac{7}{12} & \frac{2}{3} & \frac{1}{3} & \frac{1}{6} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{6} & \frac{1}{3} & \frac{2}{3} & \frac{2}{3} & \frac{2}{3} & \frac{1}{3} & \frac{1}{6} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{6} & \frac{1}{3} & \frac{2}{3} & \frac{7}{12} & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{2} & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{Bmatrix} B_1 \\ B_2 \\ B_3 \\ B_4 \\ B_5 \\ B_6 \\ B_7 \\ B_8 \\ B_9 \\ B_{10} \\ B_{11} \\ B_{12} \\ B_{13} \end{Bmatrix}$$

Localizing to the third element:

$$\begin{Bmatrix} N_1 \\ N_2 \\ N_3 \\ N_4 \\ N_5 \\ N_6 \\ N_7 \end{Bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & \frac{1}{2} & \frac{1}{4} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{2} & \frac{7}{12} & \frac{2}{3} & \frac{1}{3} & \frac{1}{6} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{6} & \frac{1}{3} & \frac{2}{3} & \frac{2}{3} & \frac{2}{3} & \frac{1}{3} & \frac{1}{6} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{6} & \frac{1}{3} & \frac{2}{3} & \frac{7}{12} & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{2} & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{Bmatrix} B_1 \\ B_2 \\ B_3 \\ B_4 \\ B_5 \\ B_6 \\ B_7 \\ B_8 \\ B_9 \\ B_{10} \\ B_{11} \\ B_{12} \\ B_{13} \end{Bmatrix}$$

Localizing to the fourth element:

$$
\begin{Bmatrix} N_1 \\ N_2 \\ N_3 \\ N_4 \\ N_5 \\ N_6 \\ N_7 \end{Bmatrix}
=
\begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & \frac{1}{2} & \frac{1}{4} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \frac{1}{2} & \frac{7}{12} & \frac{2}{3} & \frac{1}{3} & \frac{1}{6} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & \frac{1}{6} & \frac{1}{3} & \frac{2}{3} & \frac{2}{3} & \frac{2}{3} & \frac{1}{3} & \frac{1}{6} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{6} & \frac{1}{3} & \frac{2}{3} & \frac{7}{12} & \frac{1}{2} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{4} & \frac{1}{2} & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1
\end{bmatrix}
\begin{Bmatrix} B_1 \\ B_2 \\ B_3 \\ B_4 \\ B_5 \\ B_6 \\ B_7 \\ B_8 \\ B_9 \\ B_{10} \\ B_{11} \\ B_{12} \\ B_{13} \end{Bmatrix}
$$

$$\begin{Bmatrix} N_1^1 \\ N_2^1 \\ N_3^1 \\ N_4^1 \end{Bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & \frac{1}{2} & \frac{1}{4} \\ 0 & 0 & \frac{1}{2} & \frac{7}{12} \\ 0 & 0 & 0 & \frac{1}{6} \end{bmatrix} \begin{Bmatrix} B_1^1 \\ B_2^1 \\ B_3^1 \\ B_4^1 \end{Bmatrix} \qquad \begin{Bmatrix} N_1^2 \\ N_2^2 \\ N_3^2 \\ N_4^2 \end{Bmatrix} = \begin{bmatrix} \frac{1}{4} & 0 & 0 & 0 \\ \frac{7}{12} & \frac{2}{3} & \frac{1}{3} & \frac{1}{6} \\ \frac{1}{6} & \frac{1}{3} & \frac{2}{3} & \frac{2}{3} \\ 0 & 0 & 0 & \frac{1}{6} \end{bmatrix} \begin{Bmatrix} B_1^2 \\ B_2^2 \\ B_3^2 \\ B_4^2 \end{Bmatrix}$$

$$\begin{Bmatrix} N_1^3 \\ N_2^3 \\ N_3^3 \\ N_4^3 \end{Bmatrix} = \begin{bmatrix} \frac{1}{6} & 0 & 0 & 0 \\ \frac{2}{3} & \frac{2}{3} & \frac{1}{3} & \frac{1}{6} \\ \frac{1}{6} & \frac{1}{3} & \frac{2}{3} & \frac{7}{12} \\ 0 & 0 & 0 & \frac{1}{4} \end{bmatrix} \begin{Bmatrix} B_1^3 \\ B_2^3 \\ B_3^3 \\ B_4^3 \end{Bmatrix} \qquad \begin{Bmatrix} N_1^4 \\ N_2^4 \\ N_3^4 \\ N_4^4 \end{Bmatrix} = \begin{bmatrix} \frac{1}{6} & 0 & 0 & 0 \\ \frac{7}{12} & \frac{1}{2} & 0 & 0 \\ \frac{1}{4} & \frac{1}{2} & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{Bmatrix} B_1^4 \\ B_2^4 \\ B_3^4 \\ B_4^4 \end{Bmatrix}$$

$$
\begin{Bmatrix} N_1^1 \\ N_2^1 \\ N_3^1 \\ N_4^1 \end{Bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & \frac{1}{2} & \frac{1}{4} \\ 0 & 0 & \frac{1}{2} & \frac{7}{12} \\ 0 & 0 & 0 & \frac{1}{6} \end{bmatrix} \begin{Bmatrix} B_1^1 \\ B_2^1 \\ B_3^1 \\ B_4^1 \end{Bmatrix} \qquad \begin{Bmatrix} N_1^2 \\ N_2^2 \\ N_3^2 \\ N_4^2 \end{Bmatrix} = \begin{bmatrix} \frac{1}{4} & 0 & 0 & 0 \\ \frac{7}{12} & \frac{2}{3} & \frac{1}{3} & \frac{1}{6} \\ \frac{1}{6} & \frac{1}{3} & \frac{2}{3} & \frac{2}{3} \\ 0 & 0 & 0 & \frac{1}{6} \end{bmatrix} \begin{Bmatrix} B_1^2 \\ B_2^2 \\ B_3^2 \\ B_4^2 \end{Bmatrix}
$$

Localized Extraction Operators

$$
\begin{Bmatrix} N_1^3 \\ N_2^3 \\ N_3^3 \\ N_4^3 \end{Bmatrix} = \begin{bmatrix} \frac{1}{6} & 0 & 0 & 0 \\ \frac{2}{3} & \frac{2}{3} & \frac{1}{3} & \frac{1}{6} \\ \frac{1}{6} & \frac{1}{3} & \frac{2}{3} & \frac{7}{12} \\ 0 & 0 & 0 & \frac{1}{4} \end{bmatrix} \begin{Bmatrix} B_1^3 \\ B_2^3 \\ B_3^3 \\ B_4^3 \end{Bmatrix} \qquad \begin{Bmatrix} N_1^4 \\ N_2^4 \\ N_3^4 \\ N_4^4 \end{Bmatrix} = \begin{bmatrix} \frac{1}{6} & 0 & 0 & 0 \\ \frac{7}{12} & \frac{1}{2} & 0 & 0 \\ \frac{1}{4} & \frac{1}{2} & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{Bmatrix} B_1^4 \\ B_2^4 \\ B_3^4 \\ B_4^4 \end{Bmatrix}
$$

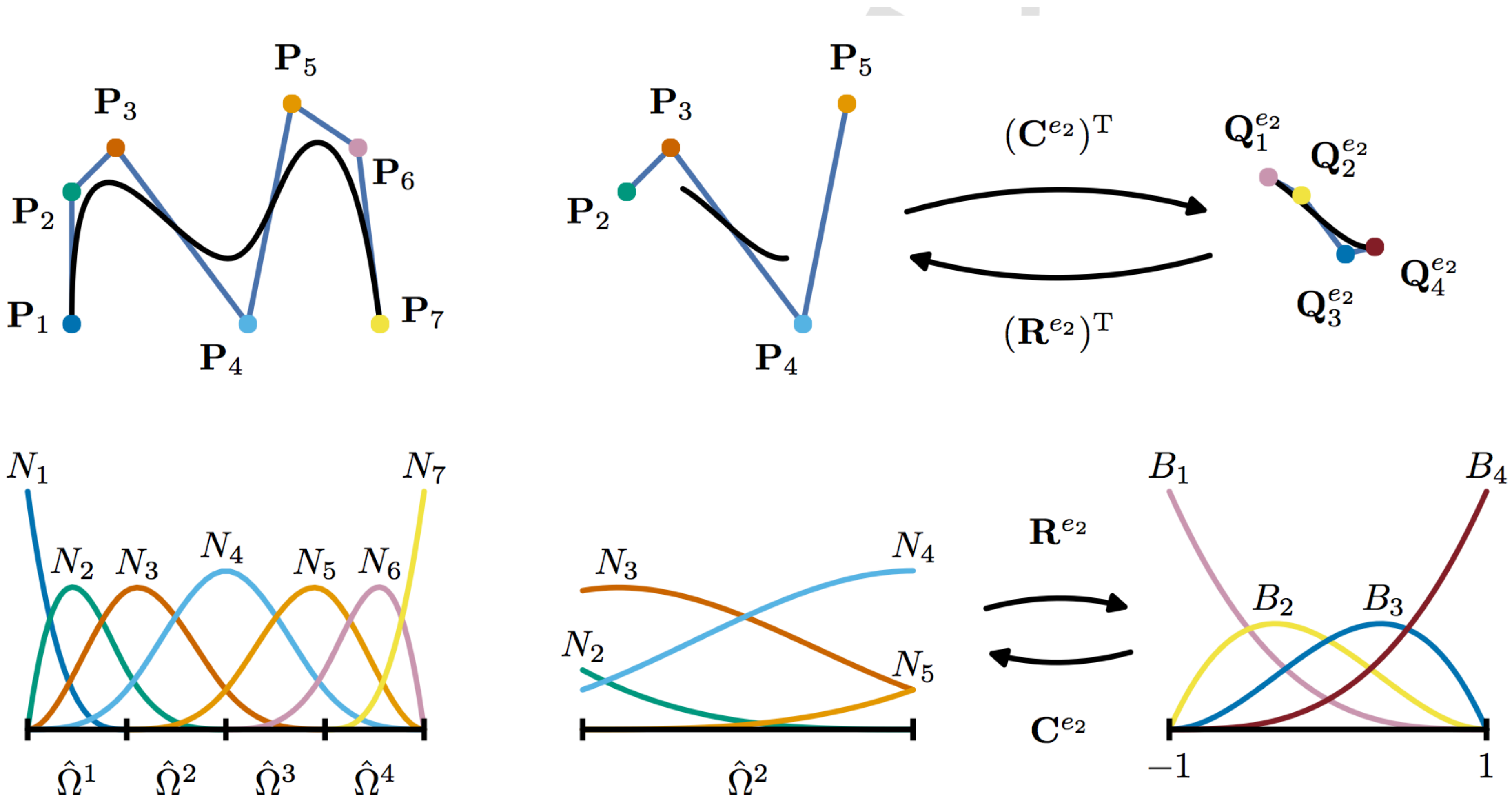
$$\begin{Bmatrix} N_1^1 \\ N_2^1 \\ N_3^1 \\ N_4^1 \end{Bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & \frac{1}{2} & \frac{1}{4} \\ 0 & 0 & \frac{1}{2} & \frac{7}{12} \\ 0 & 0 & 0 & \frac{1}{6} \end{bmatrix} \begin{Bmatrix} B_1^1 \\ B_2^1 \\ B_3^1 \\ B_4^1 \end{Bmatrix}$$

$$\begin{Bmatrix} N_1^2 \\ N_2^2 \\ N_3^2 \\ N_4^2 \end{Bmatrix} = \begin{bmatrix} \frac{1}{4} & 0 & 0 & 0 \\ \frac{7}{12} & \frac{2}{3} & \frac{1}{3} & \frac{1}{6} \\ \frac{1}{6} & \frac{1}{3} & \frac{2}{3} & \frac{2}{3} \\ 0 & 0 & 0 & \frac{1}{6} \end{bmatrix} \begin{Bmatrix} B_1^2 \\ B_2^2 \\ B_3^2 \\ B_4^2 \end{Bmatrix}$$

$$\mathbf{N}^e = \mathbf{C}^e \mathbf{B}^e$$

$$\begin{Bmatrix} N_1^3 \\ N_2^3 \\ N_3^3 \\ N_4^3 \end{Bmatrix} = \begin{bmatrix} \frac{1}{6} & 0 & 0 & 0 \\ \frac{2}{3} & \frac{2}{3} & \frac{1}{3} & \frac{1}{6} \\ \frac{1}{6} & \frac{1}{3} & \frac{2}{3} & \frac{7}{12} \\ 0 & 0 & 0 & \frac{1}{4} \end{bmatrix} \begin{Bmatrix} B_1^3 \\ B_2^3 \\ B_3^3 \\ B_4^3 \end{Bmatrix}$$

$$\begin{Bmatrix} N_1^4 \\ N_2^4 \\ N_3^4 \\ N_4^4 \end{Bmatrix} = \begin{bmatrix} \frac{1}{6} & 0 & 0 & 0 \\ \frac{7}{12} & \frac{1}{2} & 0 & 0 \\ \frac{1}{4} & \frac{1}{2} & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{Bmatrix} B_1^4 \\ B_2^4 \\ B_3^4 \\ B_4^4 \end{Bmatrix}$$



$\mathbf{P}_1$
$\mathbf{P}_2$
$\mathbf{P}_3$
$\mathbf{P}_4$
$\mathbf{P}_5$
$\mathbf{P}_6$
$\mathbf{P}_7$
$\mathbf{P}_2$
$\mathbf{P}_3$
$\mathbf{P}_4$
$\mathbf{P}_5$
$(\mathbf{C}^{e_2})^{\mathrm{T}}$
$(\mathbf{R}^{e_2})^{\mathrm{T}}$
$\mathbf{Q}_1^{e_2}$
$\mathbf{Q}_2^{e_2}$
$\mathbf{Q}_3^{e_2}$
$\mathbf{Q}_4^{e_2}$
N_1
N_2
N_3
N_4
N_5
N_6
N_7
$\hat{\Omega}^1$
$\hat{\Omega}^2$
$\hat{\Omega}^3$
$\hat{\Omega}^4$
N_2
N_3
N_4
N_5
$\hat{\Omega}^2$
$\mathbf{R}^{e_2}$
$\mathbf{C}^{e_2}$
B_1
B_2
B_3
B_4
-1
1