Isogeometric Methods: Homework #2

Problem 1:

For a given Bézier element $\hat{\Omega}^e = [\xi_l, \xi_{l+1})$, determine an *explicit analytical expression* for the univariate element extraction operator $\mathbf{C}^e$ in terms of the knots $\xi_{l-p}, \ldots, \xi_{l+p+1}$ for polynomial degrees $p = 1, 2, 3$. ***Hint:*** *The element extraction operator may not depend on all of the knots.*

Problem 2:

Write a MATLAB function which constructs the element extraction operators $\mathbf{C}^e$ and the corresponding IEN array for a two-dimensional B-spline basis. Your function should take the form:

```
function [n_el,C_operators,IEN] = Extract_Basis(p_1,p_2,n_1,n_2,Xi_1,Xi_2)
```

where `n_el` is the number of Bézier elements n_{el}, `C_operators` is an array storing the element extraction operators $\{\mathbf{C}^e\}_{e=1}^{n_{el}}$, `IEN` is an array mapping local basis function/element numbers to global basis function numbers, `p_1` and `p_2` are the polynomial degrees in directions ξ_1 and ξ_2 respectively, `n_1` and `n_2` are the number of basis functions in directions ξ_1 and ξ_2, and `Xi_1` and `Xi_2` are the univariate knot vectors.

Problem 3:

Write a MATLAB function which computes the Bézier control points $\mathbf{P}^{b,e}$ and weights $\mathbf{w}^{b,e}$ corresponding to a NURBS surface given the control points $\mathbf{P}$ and weights $\mathbf{w}$, the element extraction operators $\mathbf{C}^e$, and the corresponding IEN array. Your function should take the form:

```
function [P_b,w_b] = Extract_Geometry(d,p_1,p_2,n_el,C_operators,IEN,P,w)
```

where `P_b` is an array storing the Bézier control points for the NURBS surface, `w_b` is an array storing the Bézier weights for the NURBS surface, `d` is the spatial dimension of physical space, `p_1` and `p_2` are the polynomial degrees, `n_el` is the number of Bézier elements, `C_operators` is an array storing the element extraction operators, `IEN` is an array mapping local basis function/element numbers to global basis function numbers, `P` is an array storing the control points for the NURBS surface, and `w` is an array storing the weights for the NURBS surface.

Problem 3:

Part 1: Use the MATLAB function `NURBS_Pipe_Elbow` to compute the NURBS parametrization of a pipe with an elbow bend with $h = 4$, $w = 3$, $r = 0.5$, and $R = 3$.

Part 2: Use the MATLAB function `NURBS_Surface_Refine` to perform (uniform) knot insertion for the NURBS surface obtained in Part 1.

Part 3: Use the MATLAB functions `Extract_Basis` and `Extract_Geometry` to compute the Bézier control points $\mathbf{P}^{b,e}$ and weights $\mathbf{w}^{b,e}$ for the refined NURBS surface obtained in Part 2. Plot the refined NURBS surface and the corresponding Bézier control net, and print the element extraction operators $\{\mathbf{C}^e\}_{e=1}^{n_{el}}$ and corresponding IEN array.

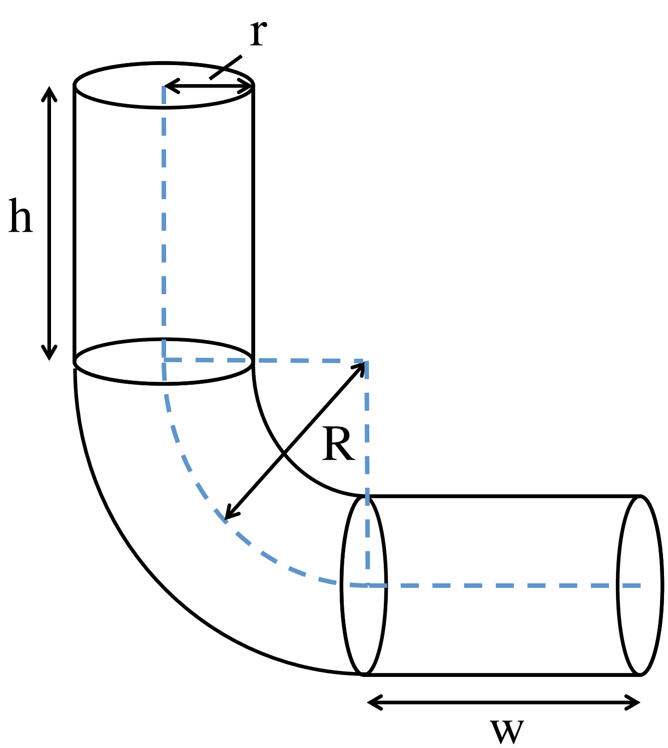

Figure 1: Pipe with an elbow bend.