Isogeometric Methods: Homework #5

Problem 1: Structural Vibrations

In this problem, you will solve the problem of structural vibrations of a solid aluminum NACA 0012 airfoil. Assume that the length of the corresponding aircraft wing is infinitely long, resulting in a plane strain stress state. The resulting problem definition is as depicted in Figure 1. For the problem in consideration, the physical and geometric parameters are as follows:

$$
\begin{aligned}
E &= 69 \text{ GPa} \\
\nu &= 0.35 \\
\rho &= 2700 \text{ kg/m}^3 \\
c &= 1 \text{ m}
\end{aligned}
$$

Compute the first five non-rigid body modes and corresponding natural frequencies, and plot both components of the resulting displacement field and the σ_{xx}, σ_{yy}, and σ_{xy} stress components over the *deformed* physical geometry for each of the computed modes.

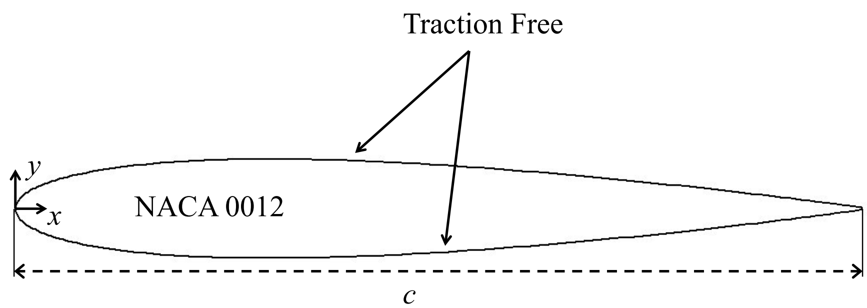

Figure 1: Setup for Problem 1.

Note: The formula for the shape of a NACA 00xx airfoil, with "xx" being replaced by the percentage of thickness to chord, is:

$$y_t = \frac{t}{0.2} c \left[0.2969 \sqrt{\frac{x}{c}} - 0.1260 \left(\frac{x}{c} \right) - 0.3516 \left(\frac{x}{c} \right)^2 + 0.2843 \left(\frac{x}{c} \right)^3 - 0.1036 \left(\frac{x}{c} \right)^4 \right]$$

where c is the chord length, x is the position along the chord from 0 to c, y_t is the half thickness at a given value of x (centerline to surface), and t is the maximum thickness as a fraction of the chord (i.e., $t = \text{xx}/100$).

Problem 2: Wave Propagation

In this problem, you will solve the problem of elastic wave propagation in a thick walled steel cylindrical pressure vessel due to an ultrasonic internal pressure loading of the form:

$$p(t) = \delta p_i \exp i\omega t$$

where ω, the frequency of the loading, is significantly larger than the upper limit of human hearing (greater than 20 MHz). Assume that the ends of the cylinder are capped, resulting in a plane strain stress state. The resulting problem definition is as depicted in Figure 2. For the problem in consideration, the physical and geometric parameters are as follows:

$$\begin{aligned} E &= 200 \text{ GPa} \\ \nu &= 0.3 \\ \rho &= 7850 \text{ kg/m}^3 \\ \delta p_i &= 5 \text{ MPa} \\ r_i &= 75 \text{ mm} \\ t &= 15 \text{ mm} \end{aligned}$$

Compute the resulting amplitude field $\mathbf{u}(\mathbf{x})$ for the two frequency values $\omega = 5$ MHz and $\omega = 30$ MHz. Plot both components of the amplitude field and the two in-plane principal stress amplitudes over the *undeformed* physical geometry for each considered frequency value. Are the resulting elastic waves primary or secondary waves? Estimate the wave-speeds using your computations and compare your results with the theoretical wave-speed values. Discuss any discrepancies you observe.

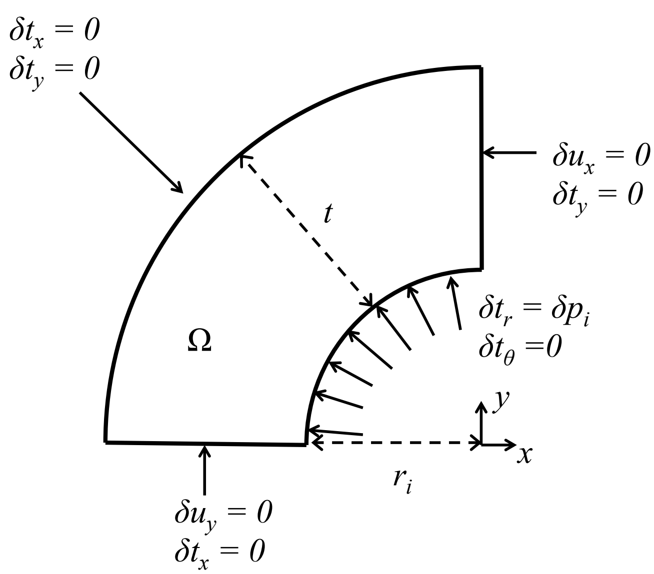

Figure 2: Setup for Problem 2.

Problem 3: Elastodynamics

In this problem, you will solve the problem of time-dependent elastic deformation of a thick walled steel cylindrical pressure vessel due to a dynamic internal pressure loading of the form:

$$p(t) = p_i \left(\frac{t}{t_{peak}} \right) F \left(\frac{t_{peak} - t}{t_{decay}} \right)$$

where $F(\eta) = 0.5\,(1 + \tanh(\eta))$, t_{peak} corresponds to the time of peak loading, and t_{decay} is the period of decay. The resulting non-dimensional pressure field $\tilde{p}(t) := p(t)/p_i$ is illustrated in Figure 3 for $t_{peak} = 0.05$ ms and $t_{decay} = 0.15$ ms. This problem is motivated by deformation due to detonation of a gas inside of a steel pipe. Note that such a problem is three-dimensional in nature, however, with resulting pressure waves traversing in both radial and axial directions. Assume that the ends of the cylinder are capped, resulting in a plane strain stress state. The resulting problem definition is as depicted in Figure 4. For the problem in consideration, the physical and geometric parameters are as follows:

$$\begin{aligned} E &= 200 \text{ GPa} \\ \nu &= 0.3 \\ \rho &= 7850 \text{ kg/m}^3 \\ p_i &= 30 \text{ MPa} \\ t_{peak} &= 0.05 \text{ ms} \\ t_{decay} &= 0.15 \text{ ms} \\ r_i &= 75 \text{ mm} \\ t_{pipe} &= 15 \text{ mm} \end{aligned}$$

Assuming that there is no viscous damping and that the vessel is initially at rest, compute the resulting time-dependent displacement field $\mathbf{u}(\mathbf{x}, t)$ up to a final time of five milliseconds. Plot the two in-plane principal stress amplitudes over the *undeformed* physical geometry at several representative time instances, and plot the inner radius (i.e., $r = r_i$) hoop stress $\sigma_{\theta\theta}$ versus time. Assess the accuracy of your results, and explain your methodology for determining your results are sufficiently accurate.

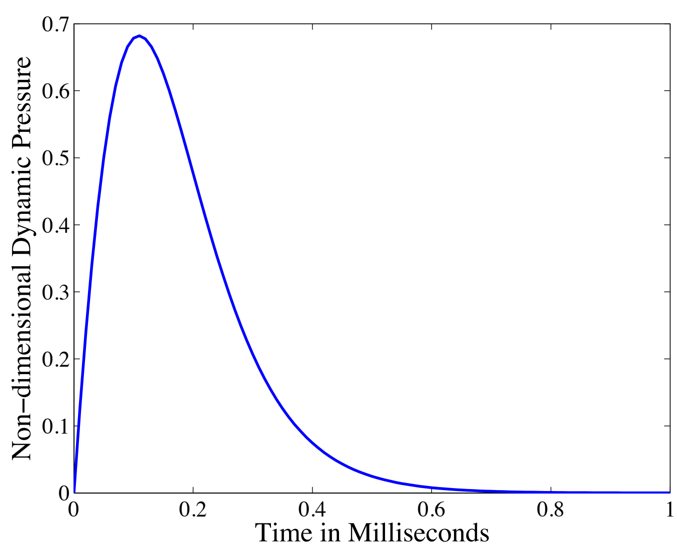

Figure 3: Non-dimensional dynamic pressure field for Problem 3.

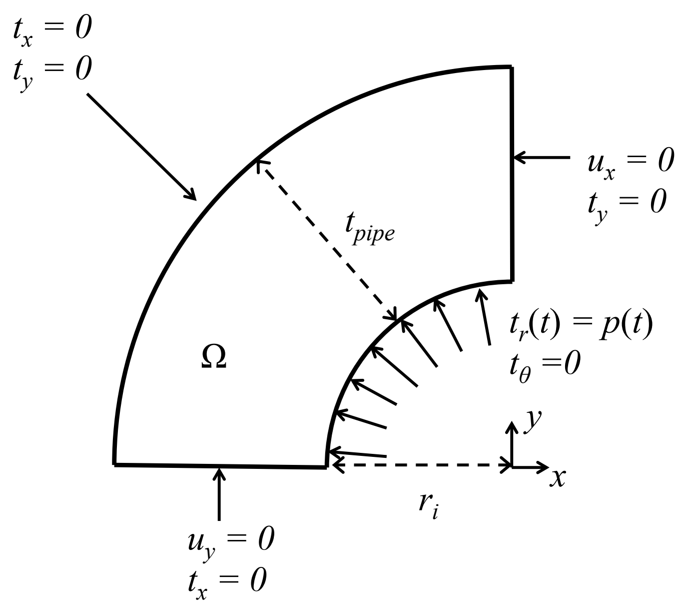

Figure 4: Setup for Problem 3.