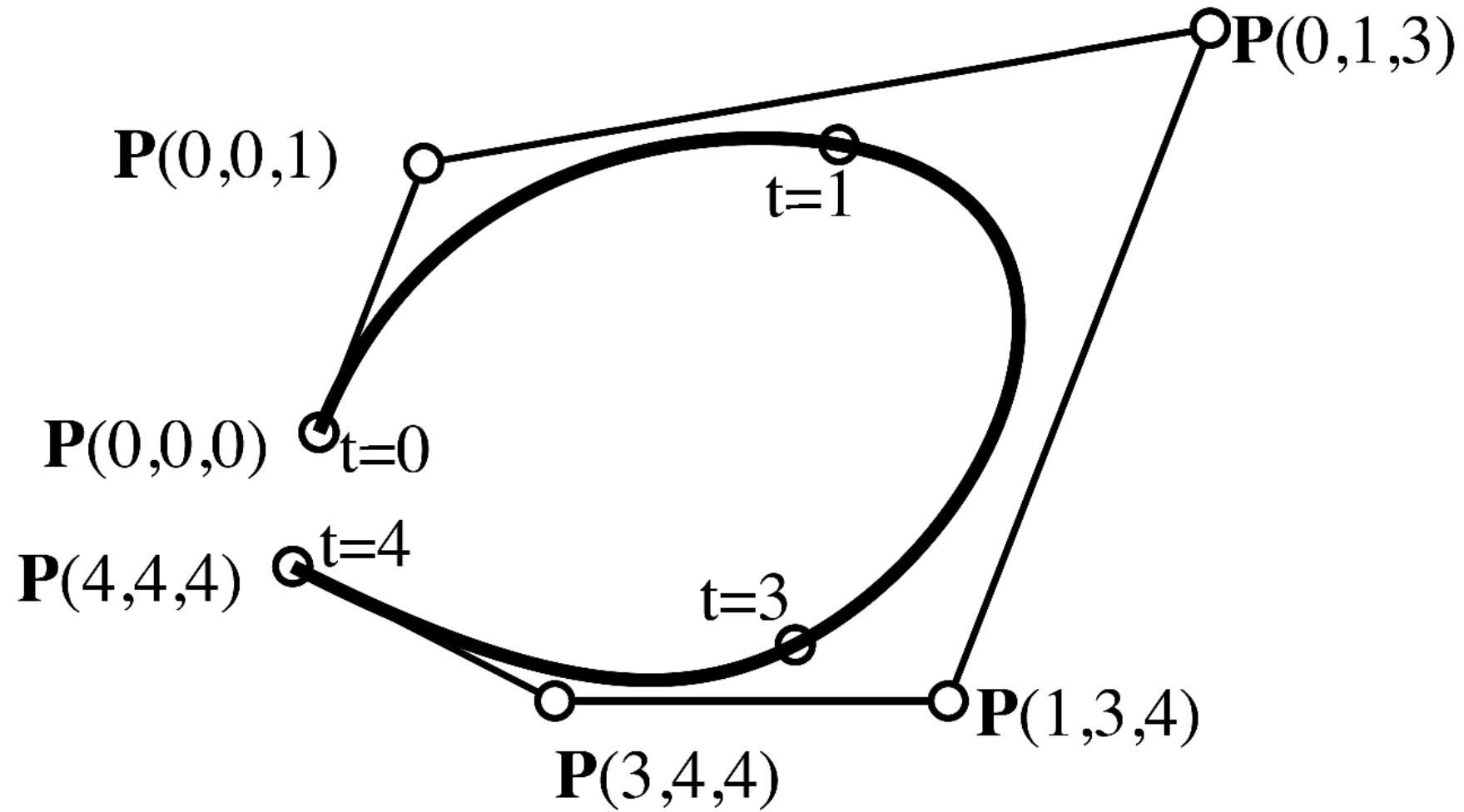
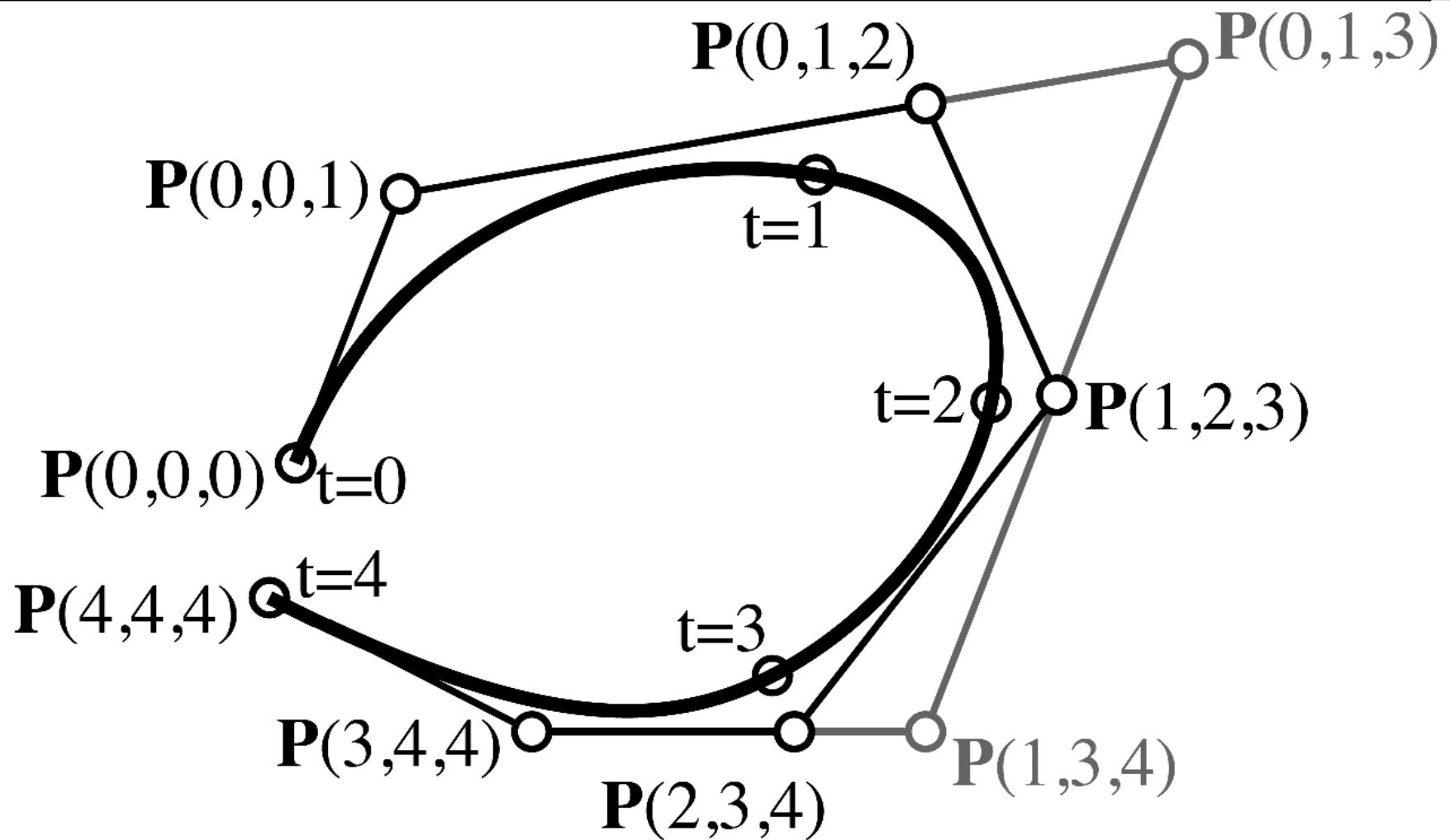


Refinement of B-splines and NURBS



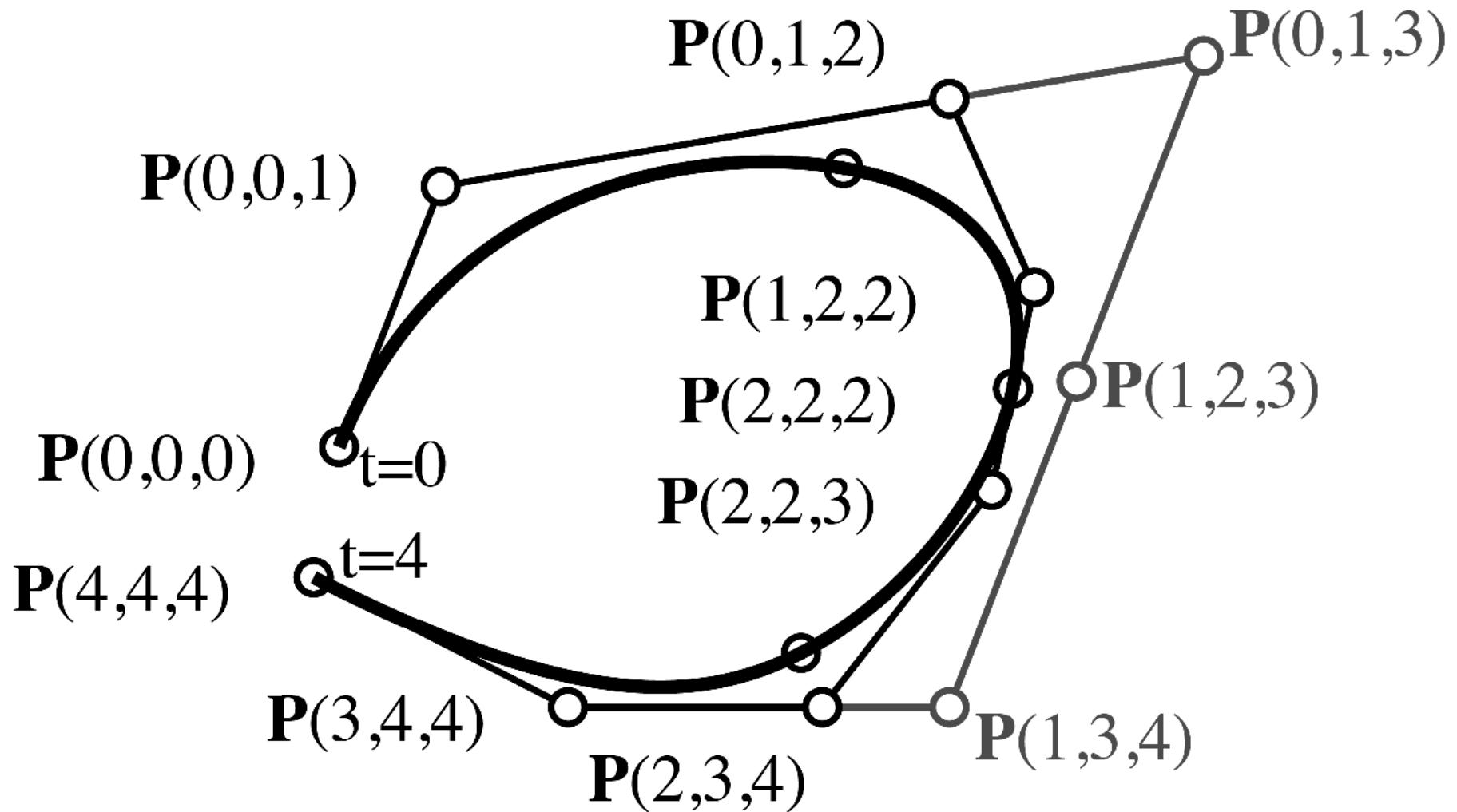
Knot Vector = [0,0,0,1,3,4,4,4]

	Initial	After Knot Insertion
Knot Vector:	$[0,0,0,1,3,4,4,4]$	$[0,0,0,1,2,3,4,4,4]$
Control Points:	$\mathbf{P}(0,0,0)$	$\mathbf{P}(0,0,0)$
	$\mathbf{P}(0,0,1)$	$\mathbf{P}(0,0,1)$
		$\mathbf{P}(0,1,2)$
	$\mathbf{P}(0,1,3)$	$\mathbf{P}(1,2,3)$
	$\mathbf{P}(1,3,4)$	$\mathbf{P}(2,3,4)$
	$\mathbf{P}(3,4,4)$	$\mathbf{P}(3,4,4)$
	$\mathbf{P}(4,4,4)$	$\mathbf{P}(4,4,4)$

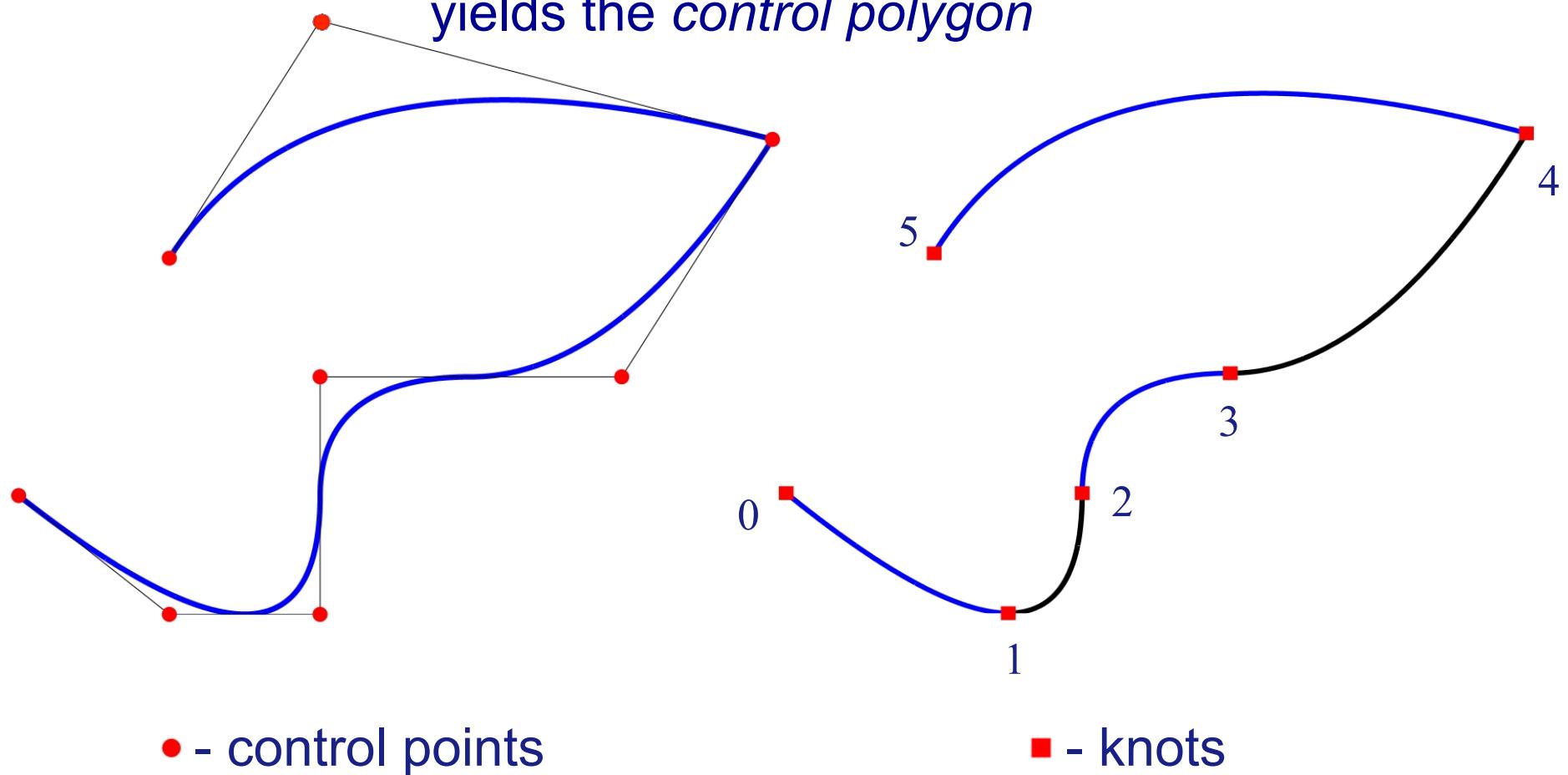


Old Knot Vector = [0,0,0,1,3,4,4,4]

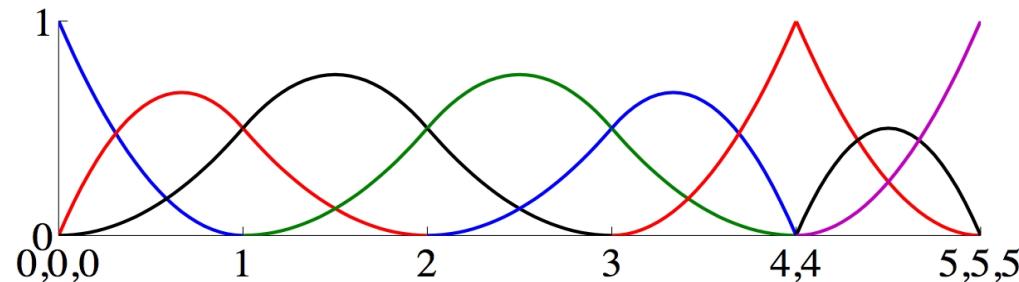
New Knot Vector = [0,0,0,1,2,3,4,4,4]



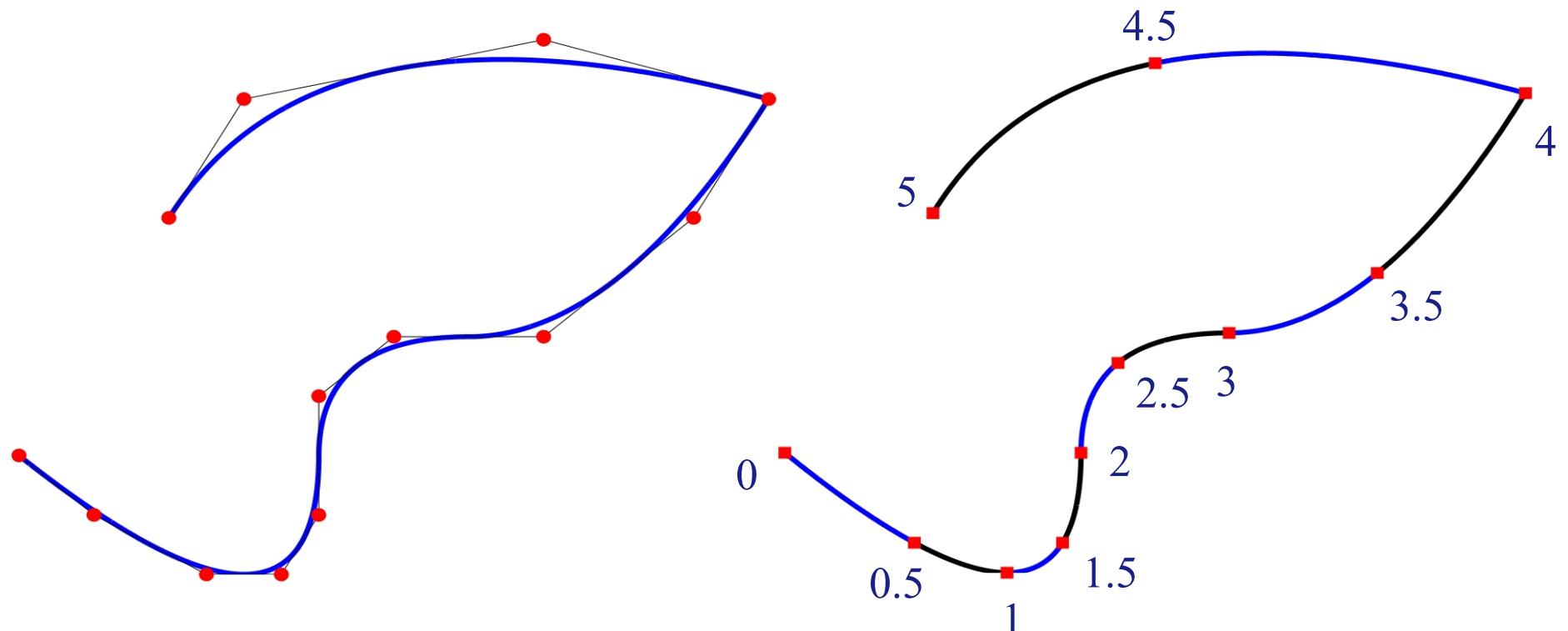
Linear interpolation of control points
yields the *control polygon*



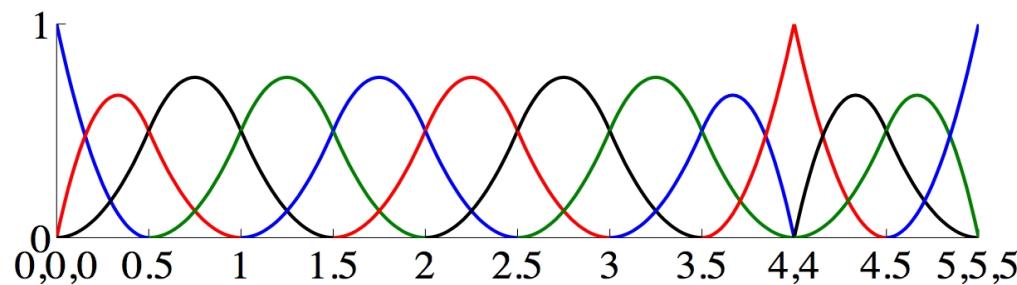
Quadratic basis



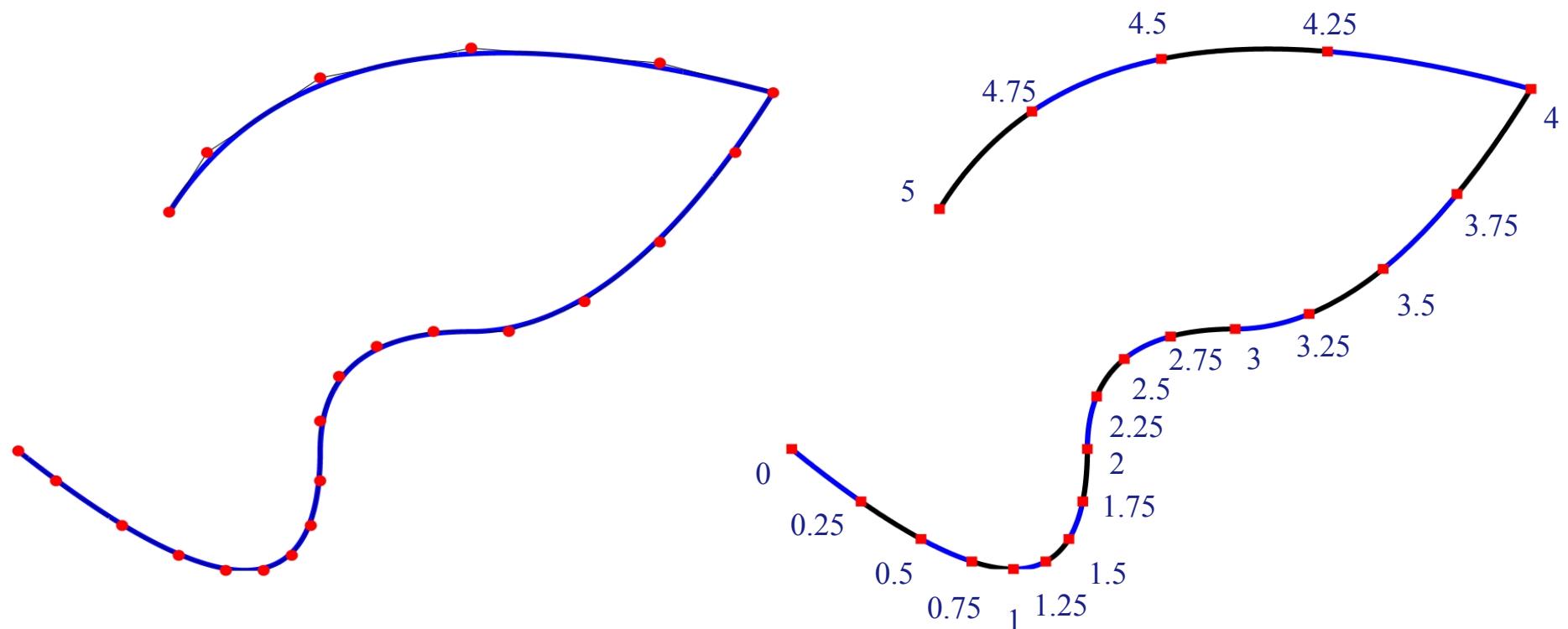
h -refined Curve



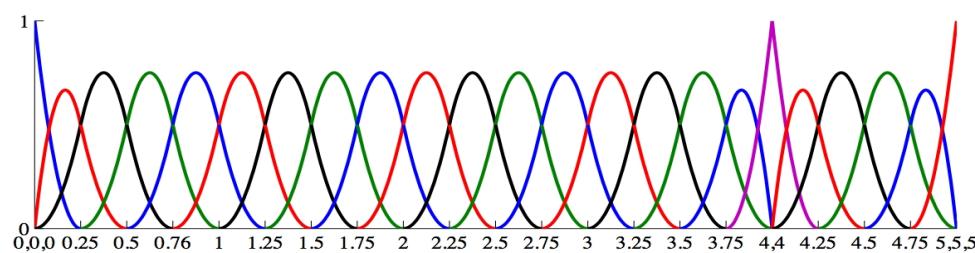
Quadratic basis



Further h -refined Curve

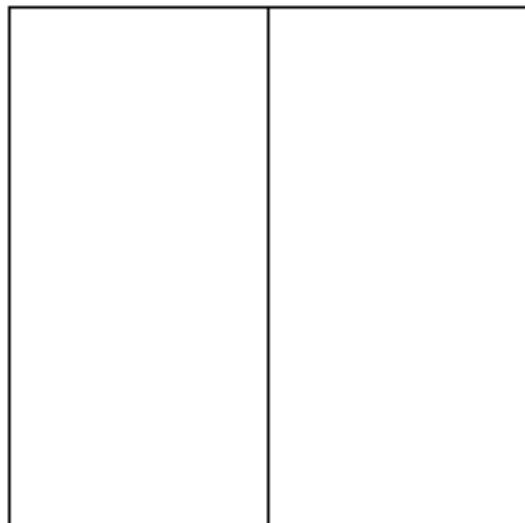


Quadratic basis



Coarsest Discretization

Patch $(0,1)^2$

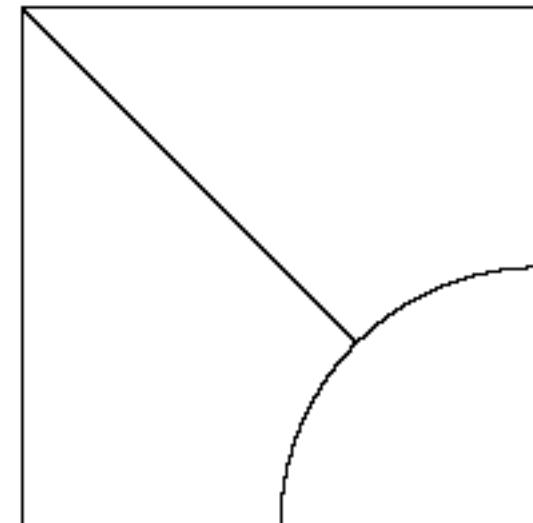


$$\left\{ \frac{w_i N_i}{w} \right\}_{i=1,\dots,n_0}$$

F

F, w – fixed

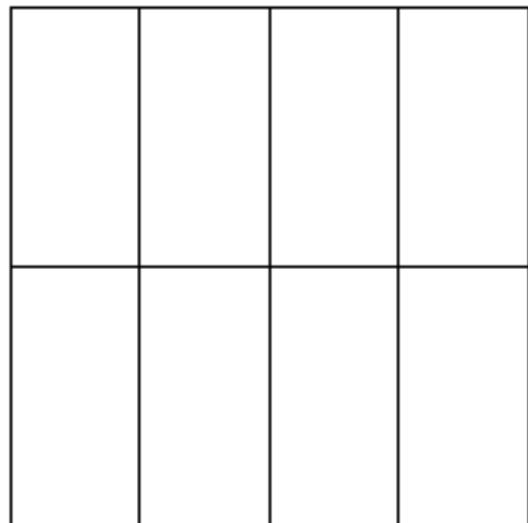
Physical domain



$$\left\{ \frac{w_i N_i}{w} \circ \mathbf{F}^{-1} \right\}_{i=1,\dots,n_0}$$

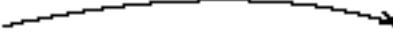
First Knot Insertion

Patch $(0,1)^2$

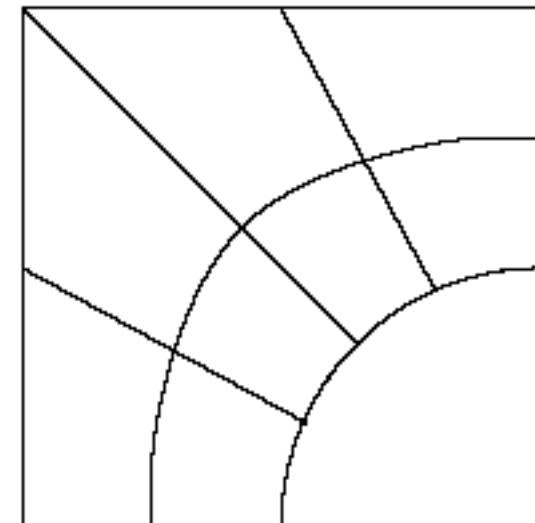


$$\left\{ \frac{w_i N_i}{w} \right\}_{i=1,\dots,n_1}$$

F



Physical domain

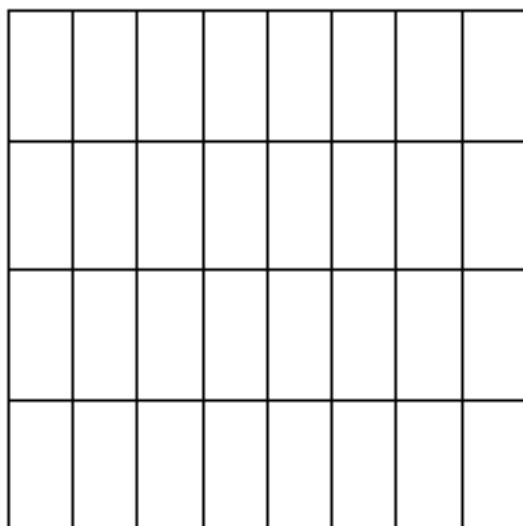


F, w – fixed

$$\left\{ \frac{w_i N_i}{w} \circ \mathbf{F}^{-1} \right\}_{i=1,\dots,n_1}$$

Second Knot Insertion

Patch $(0,1)^2$

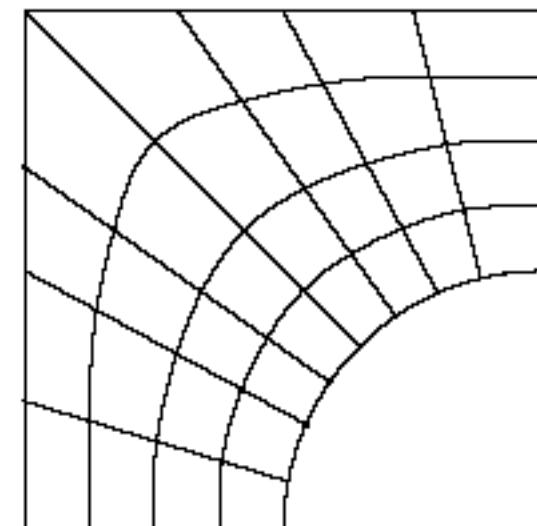


$$\left\{ \frac{w_i N_i}{w} \right\}_{i=1,\dots,n_2}$$

F

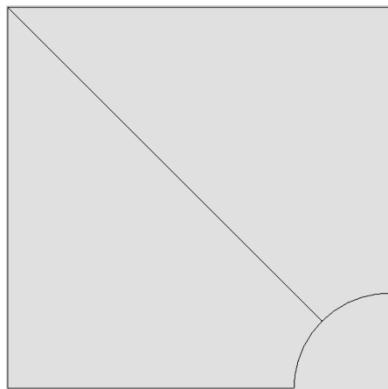
F, w – fixed

Physical domain

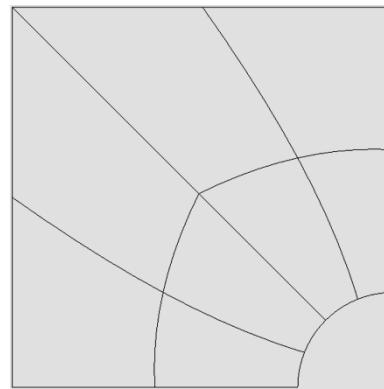


$$\left\{ \frac{w_i N_i}{w} \circ \mathbf{F}^{-1} \right\}_{i=1,\dots,n_2}$$

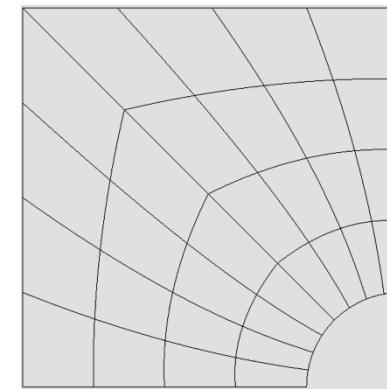
Uniformly Refined NURBS Meshes



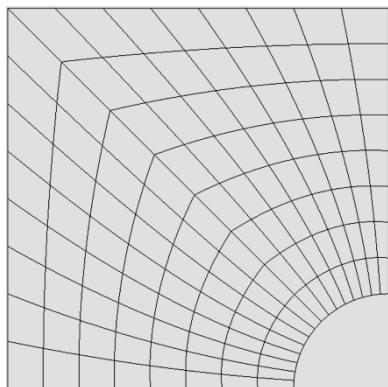
Mesh 1



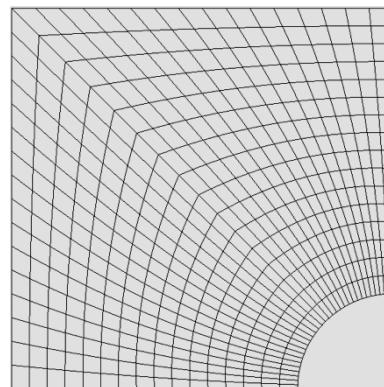
Mesh 2



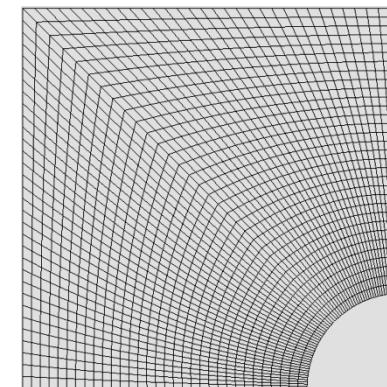
Mesh 3



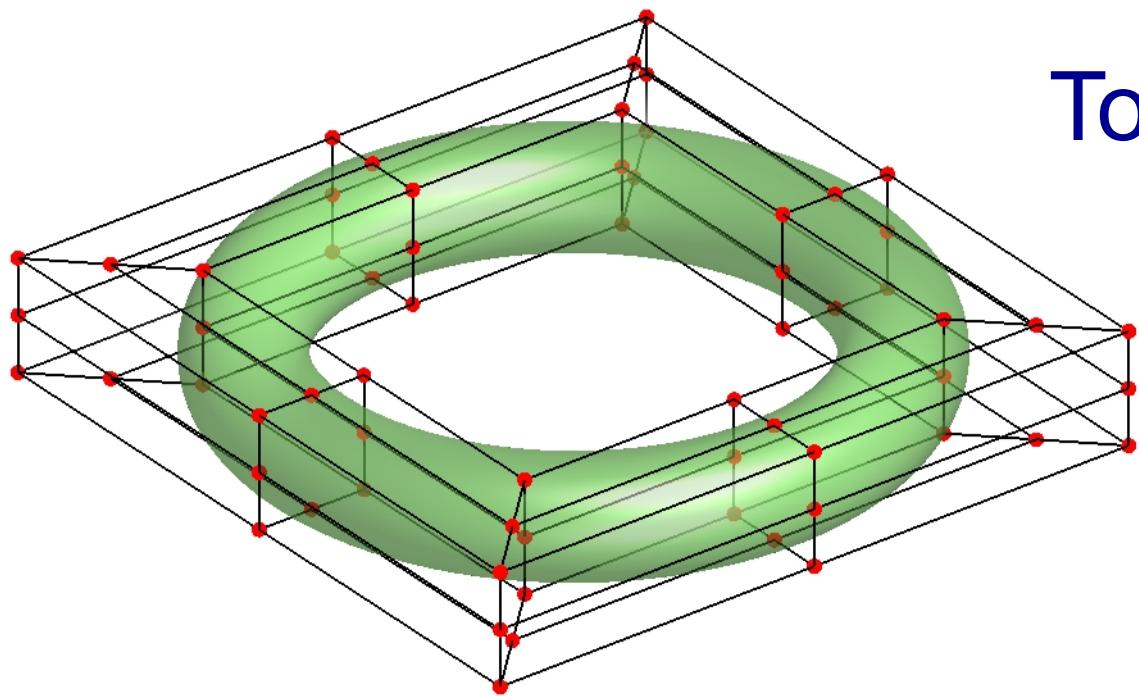
Mesh 4



Mesh 5

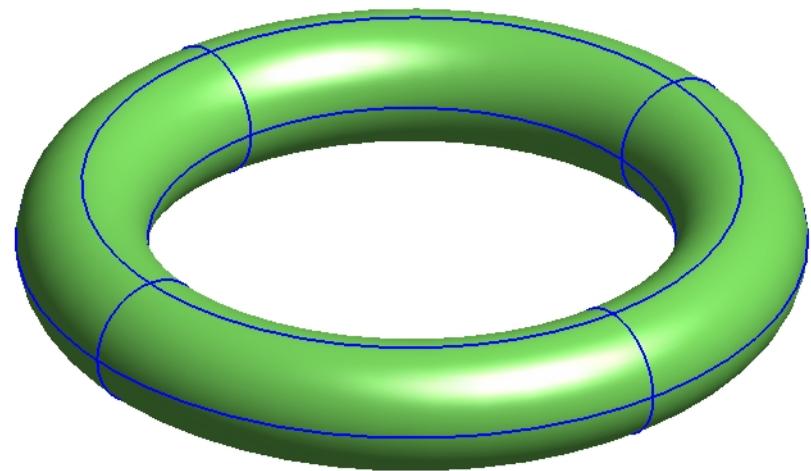


Mesh 6



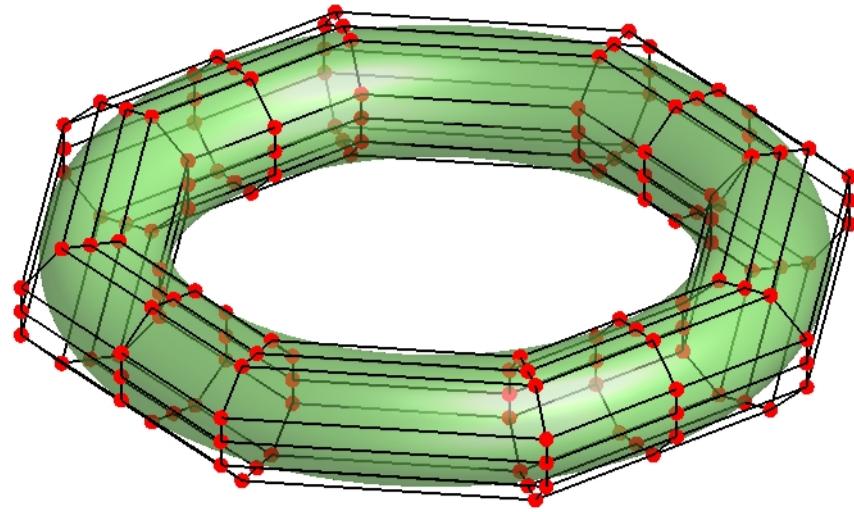
Toroidal Surface

Control net

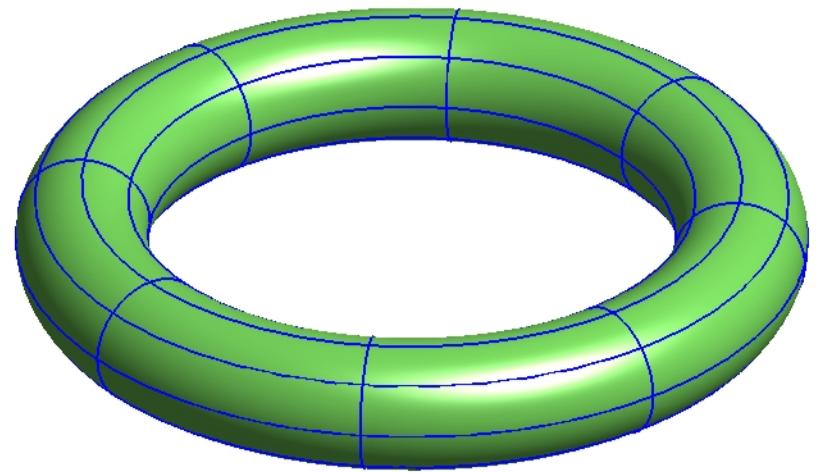


Mesh

h-refined Surface

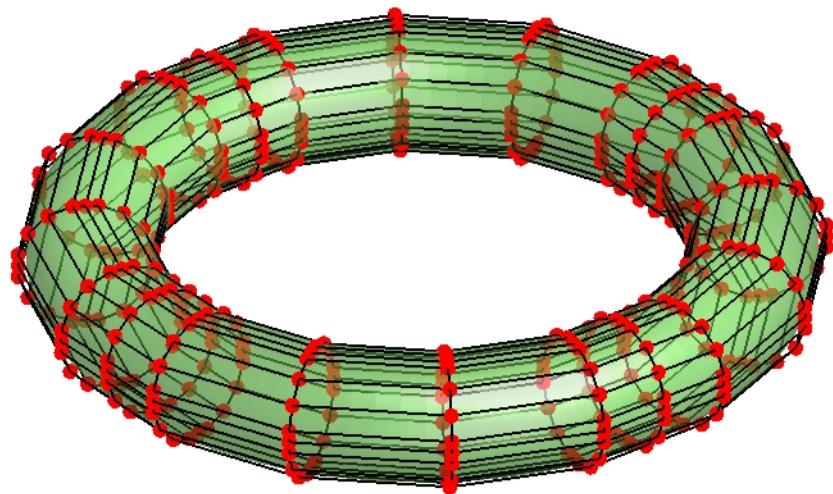


Control net

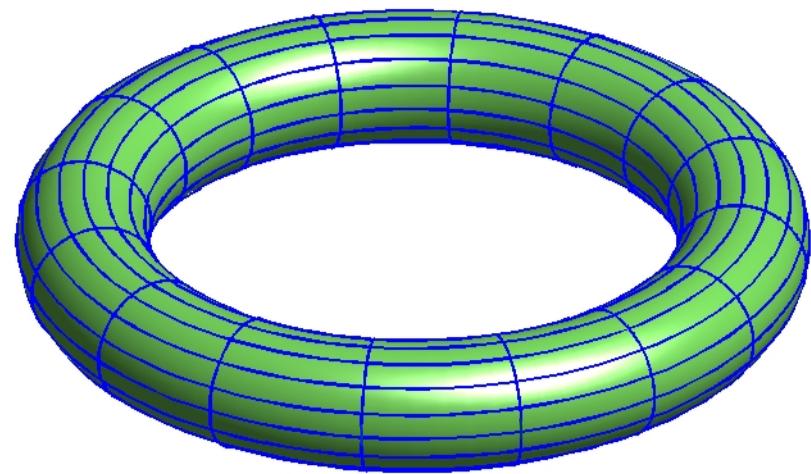


Mesh

Further h -refined Surface

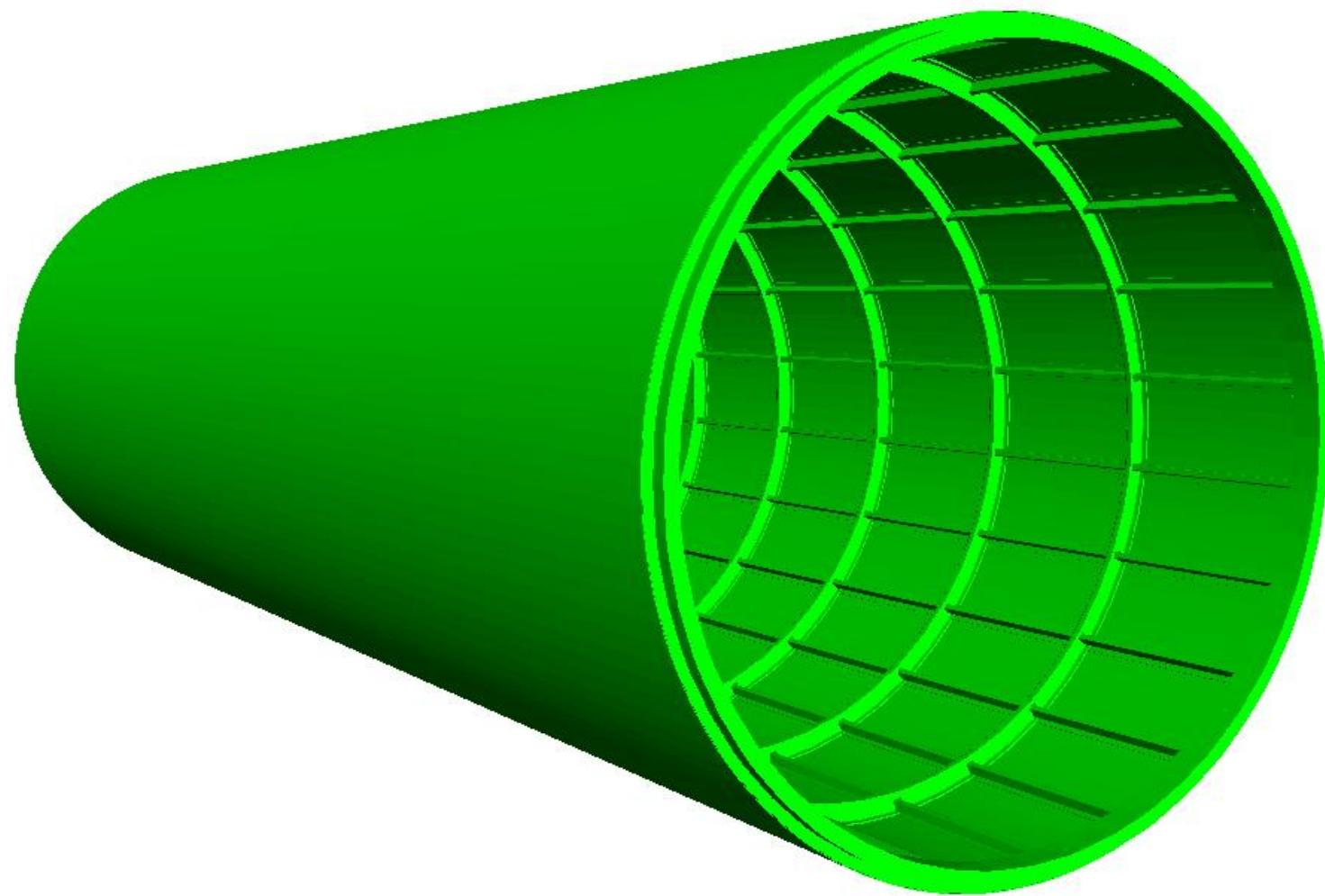


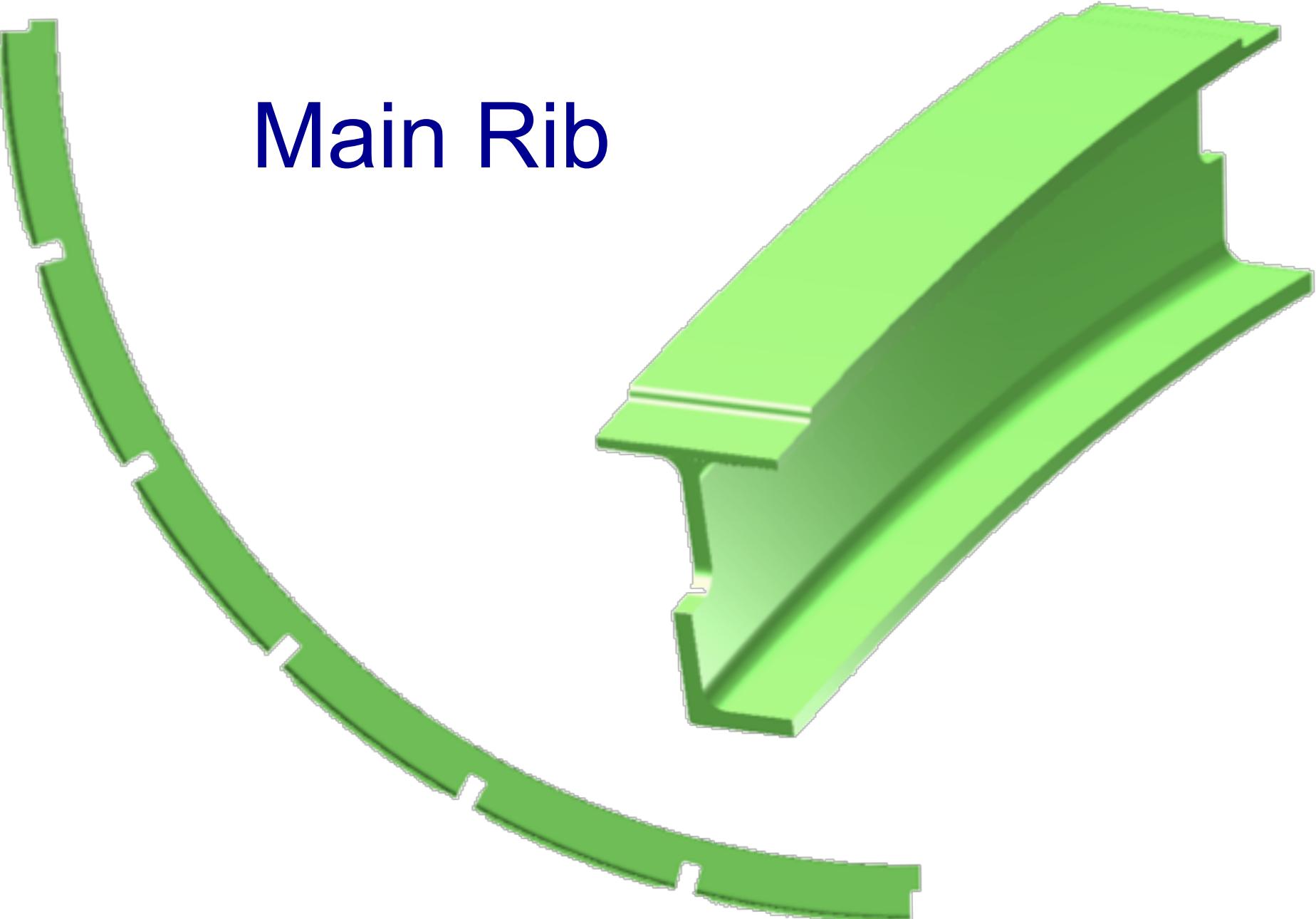
Control net



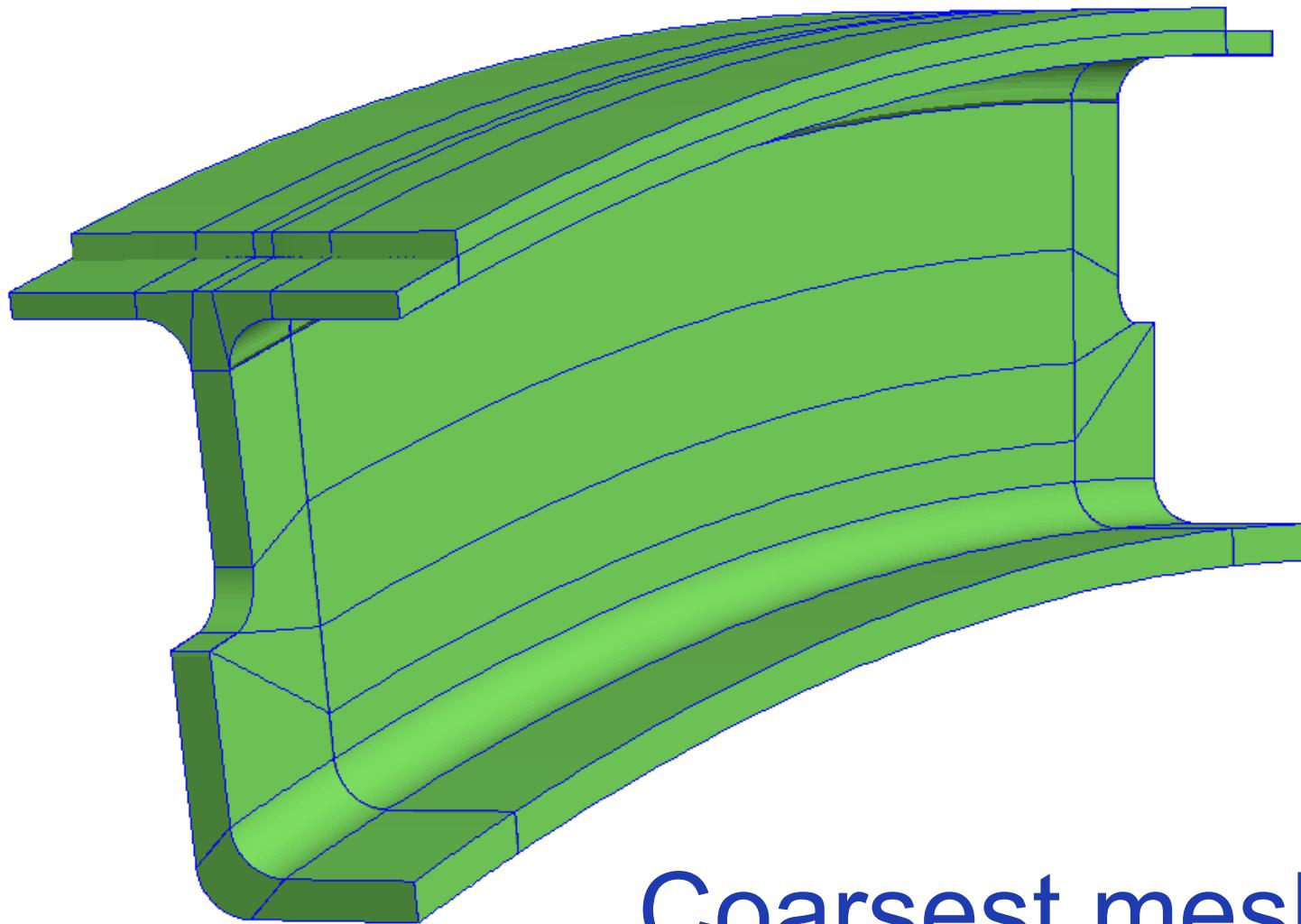
Mesh

NASA ATC Frame and Skin

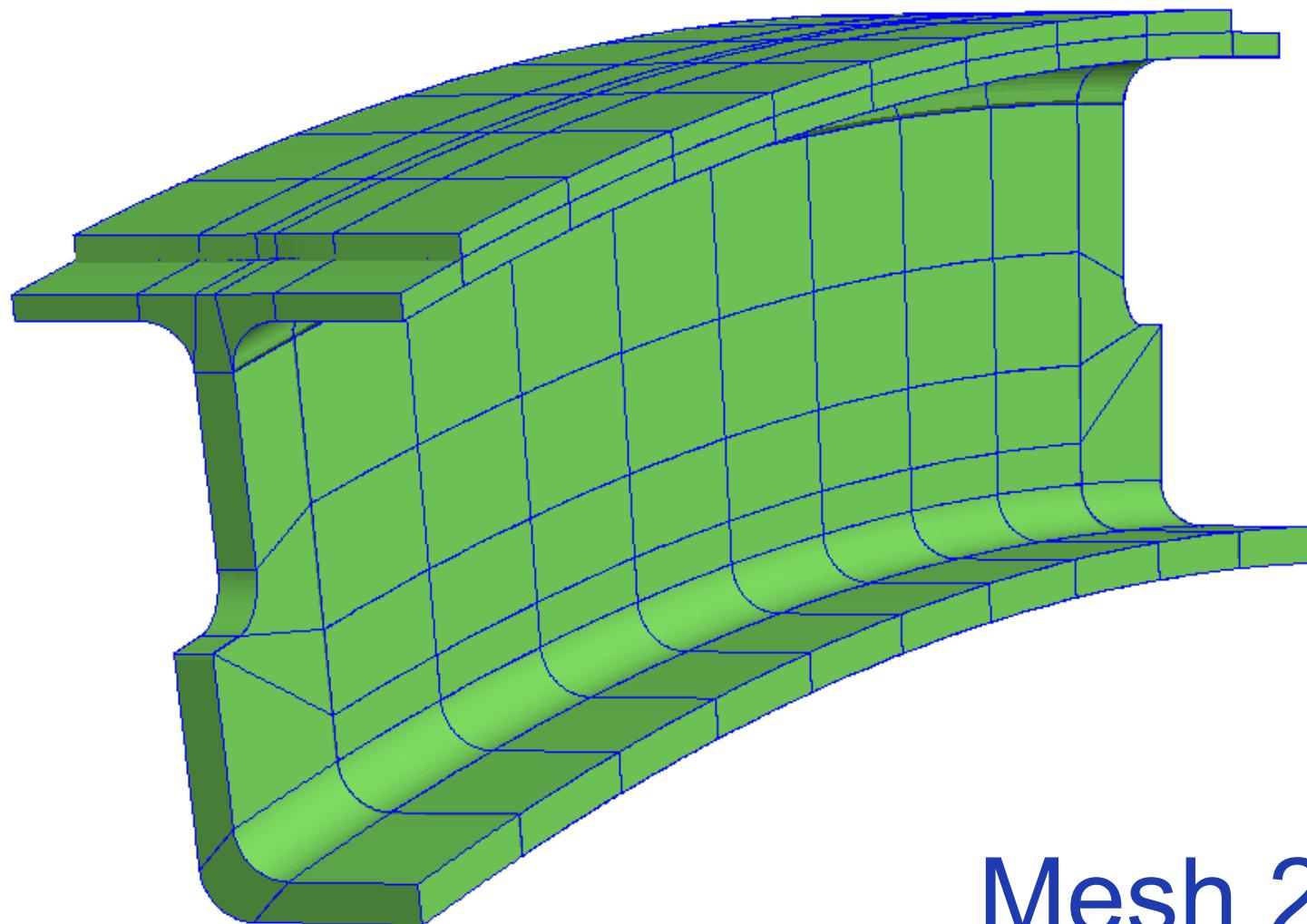




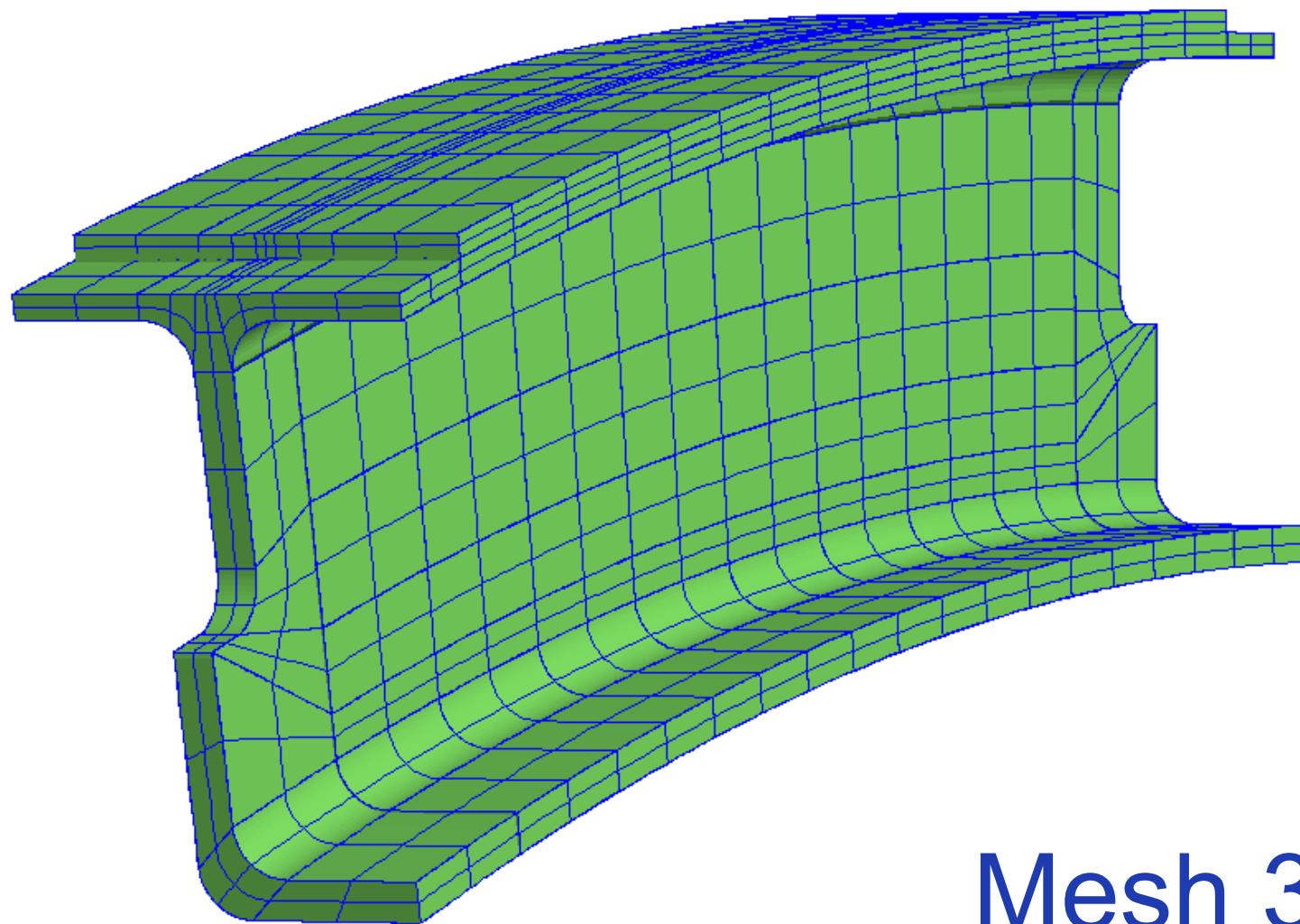
Main Rib



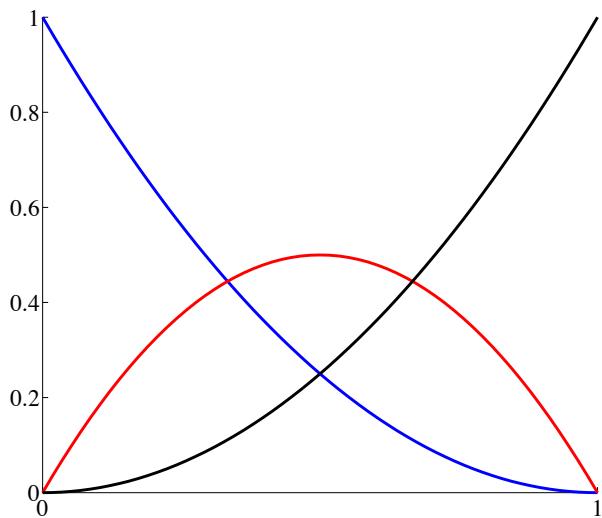
Coarsest mesh
15° segment of main rib



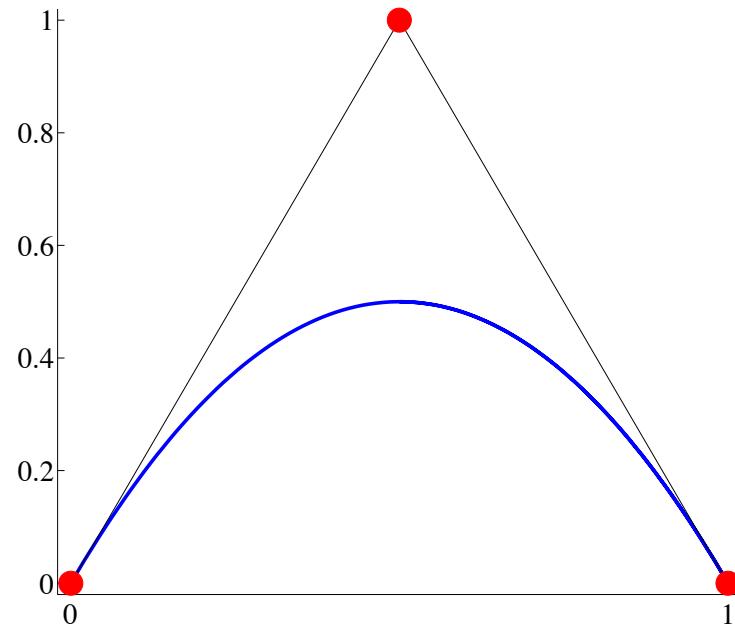
Mesh 2



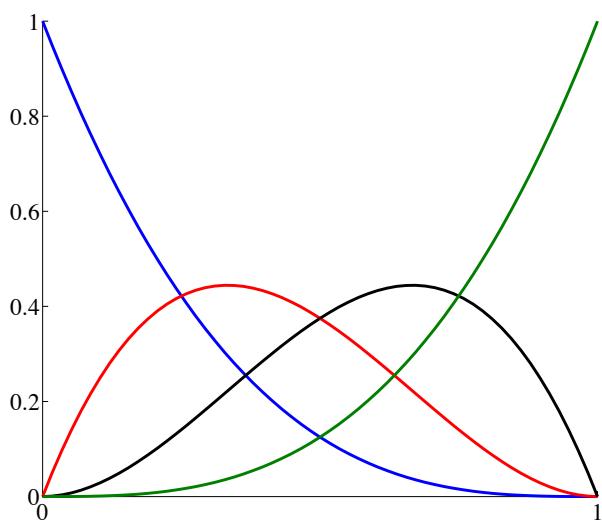
Mesh 3



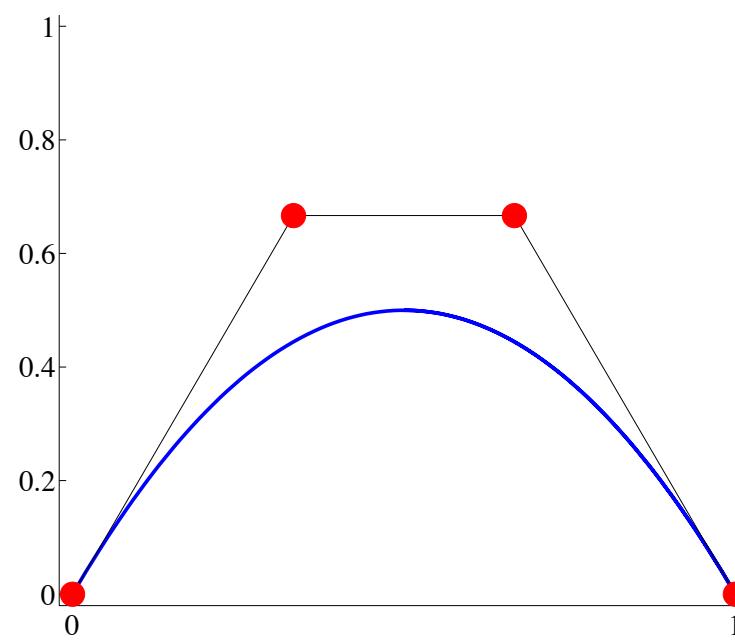
Quadratic basis



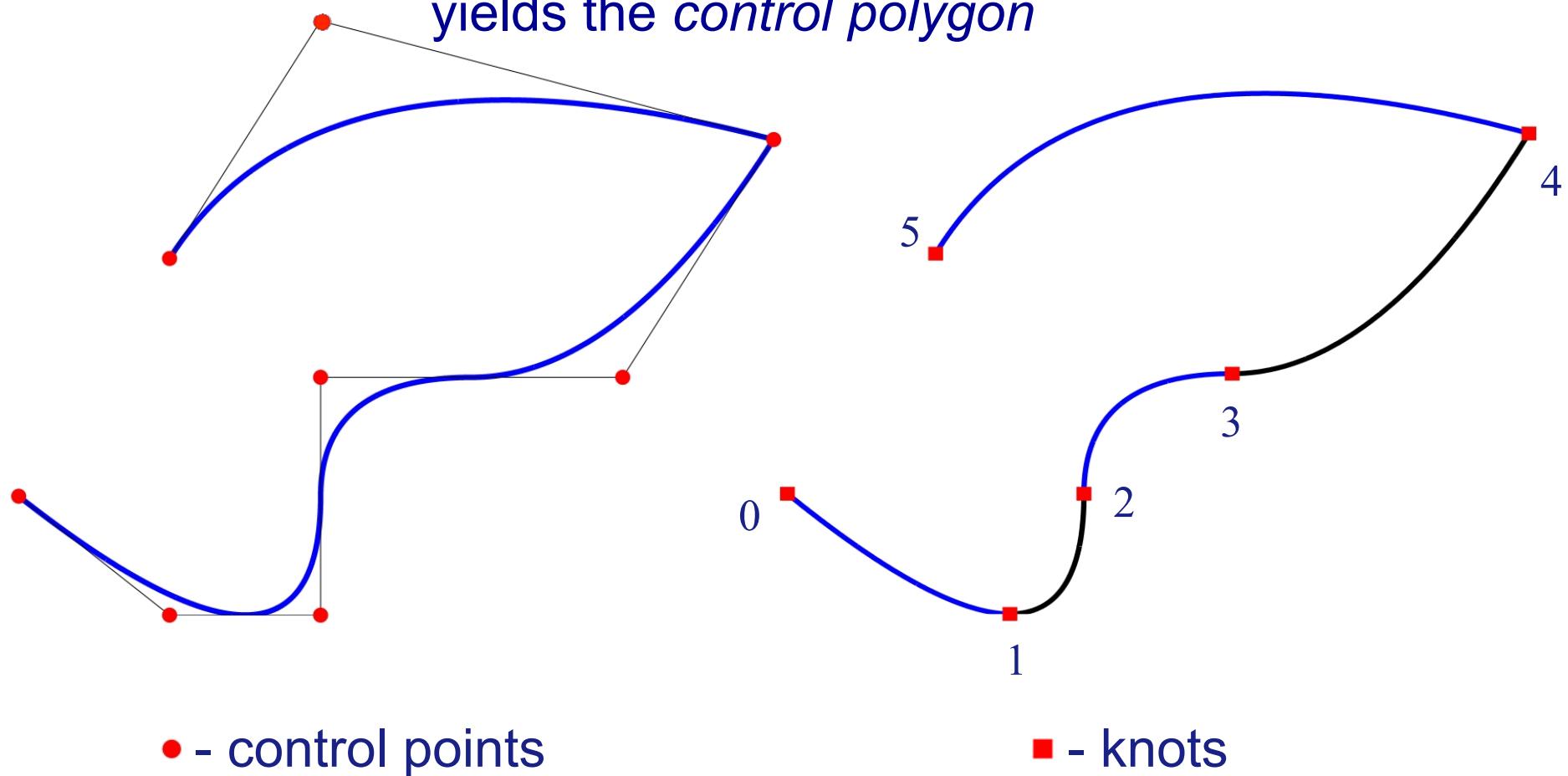
● - control points



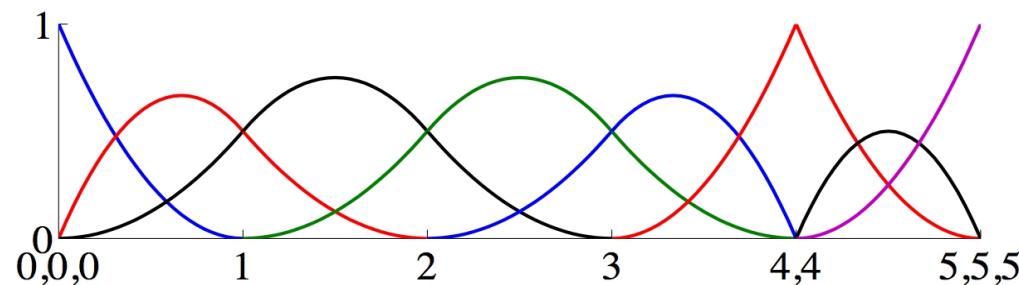
Cubic basis



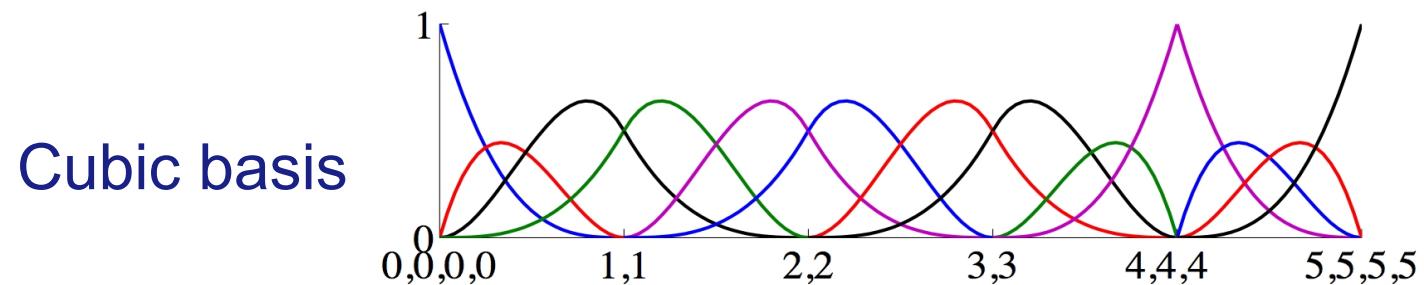
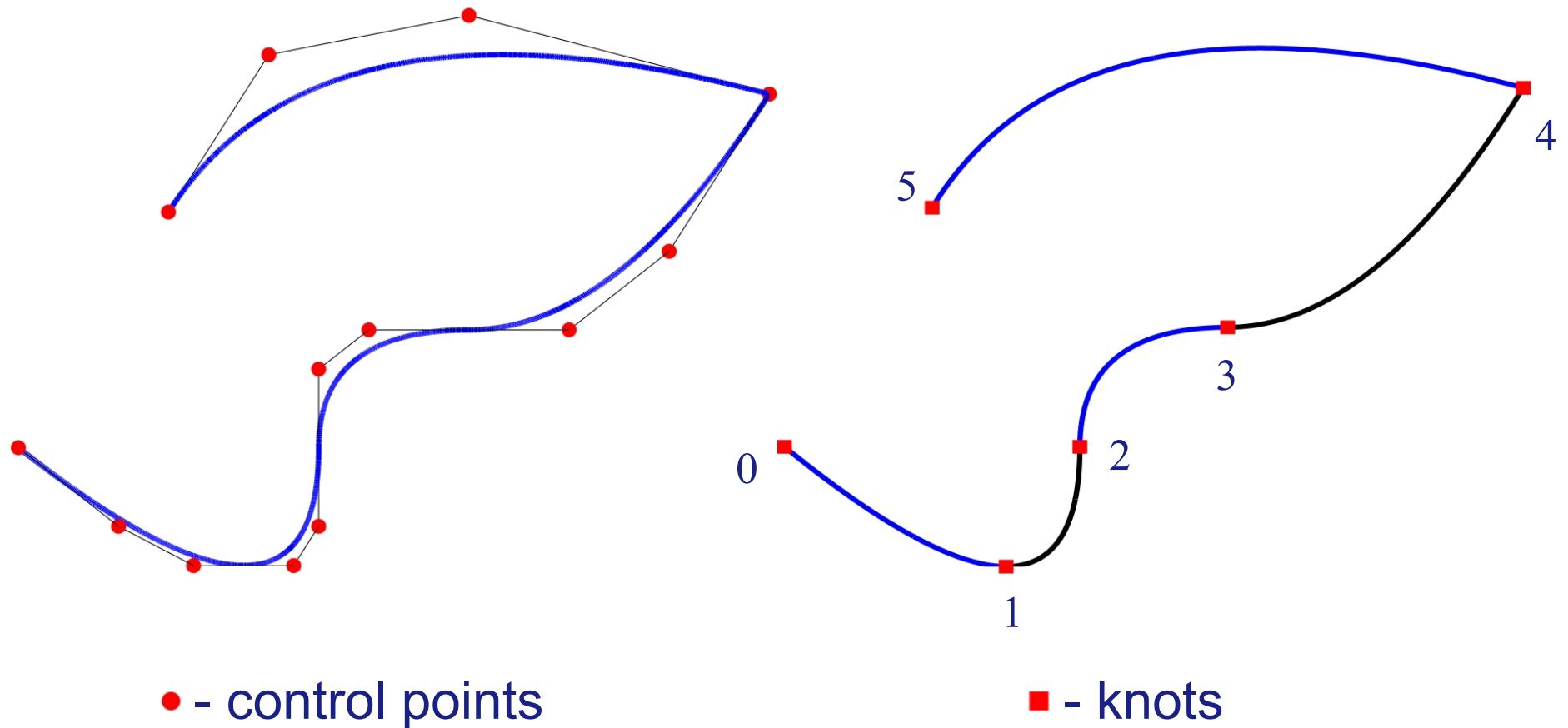
Linear interpolation of control points
yields the *control polygon*



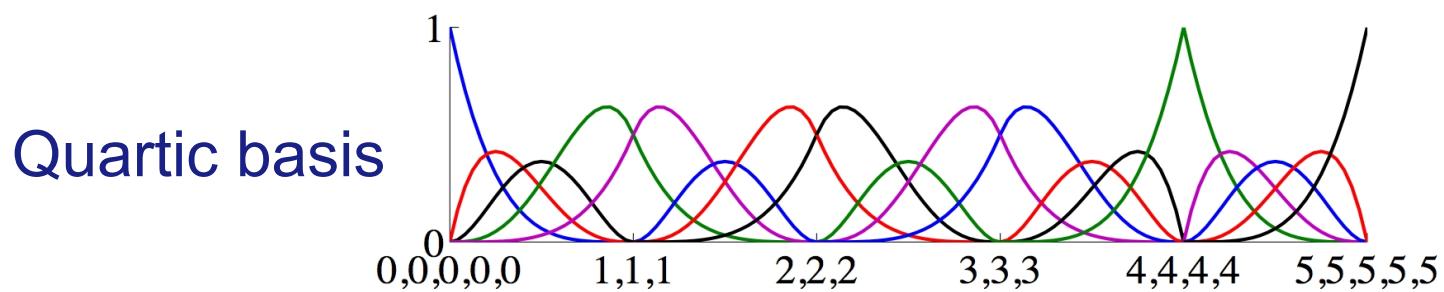
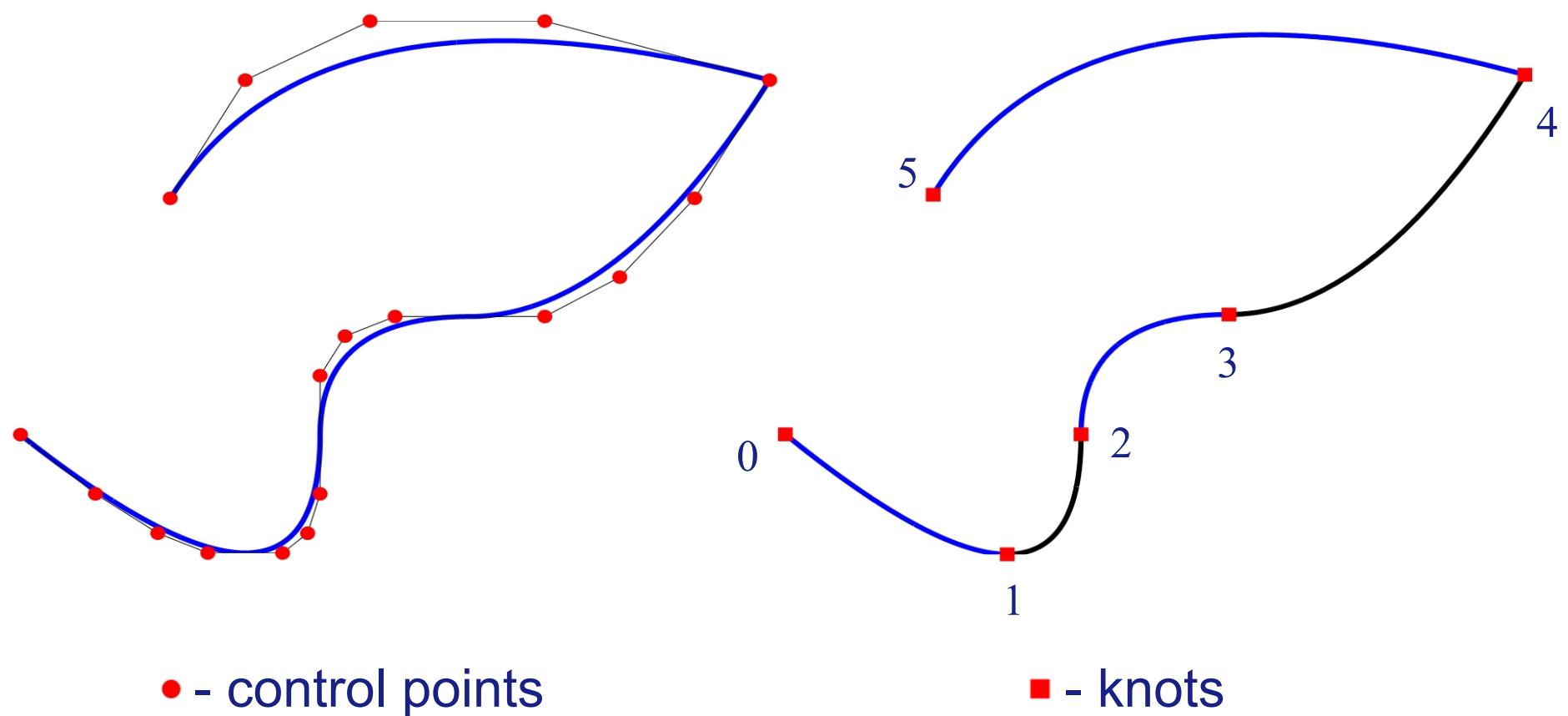
Quadratic basis

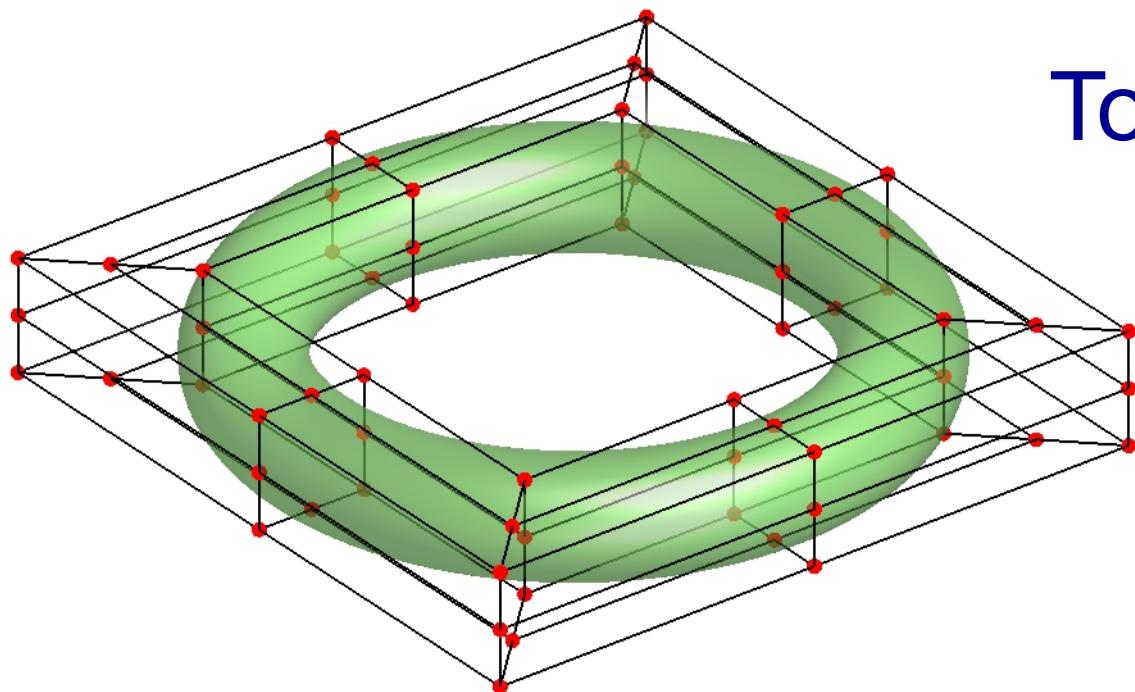


Cubic p -refined Curve



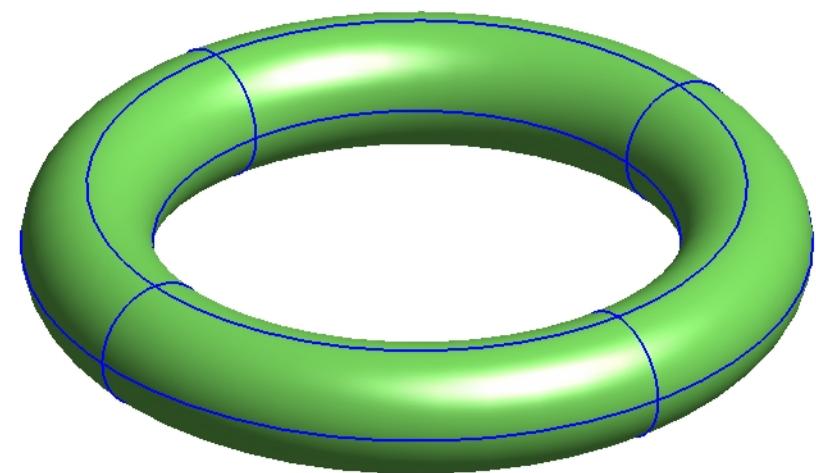
Quartic p -refined Curve





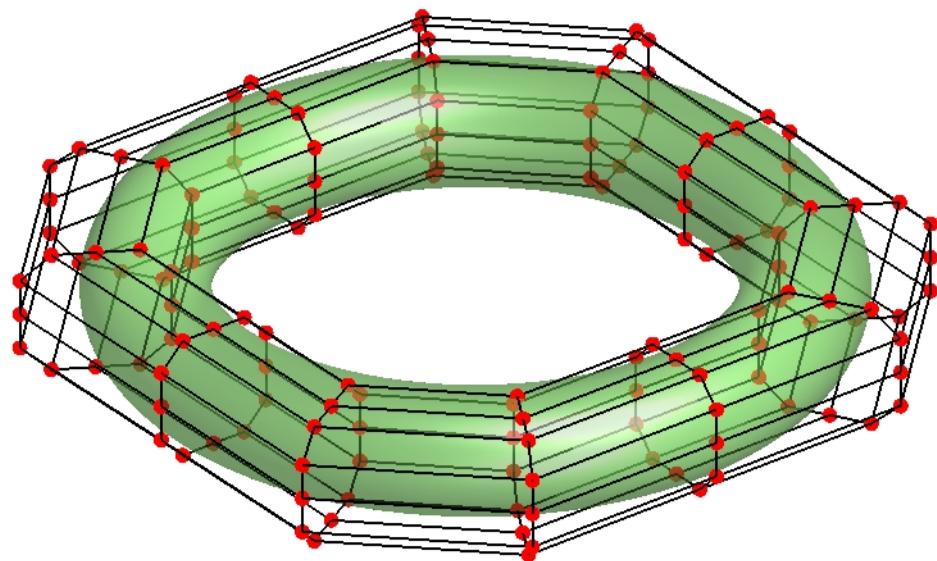
Toroidal Surface

Control net

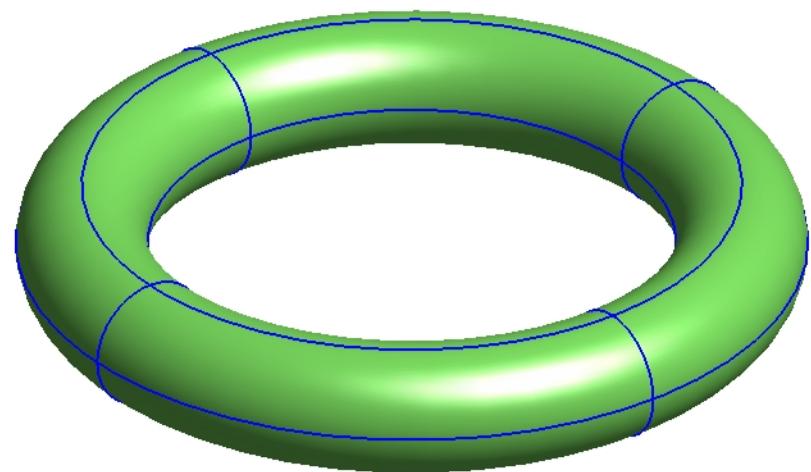


Mesh

Cubic p -refined Surface

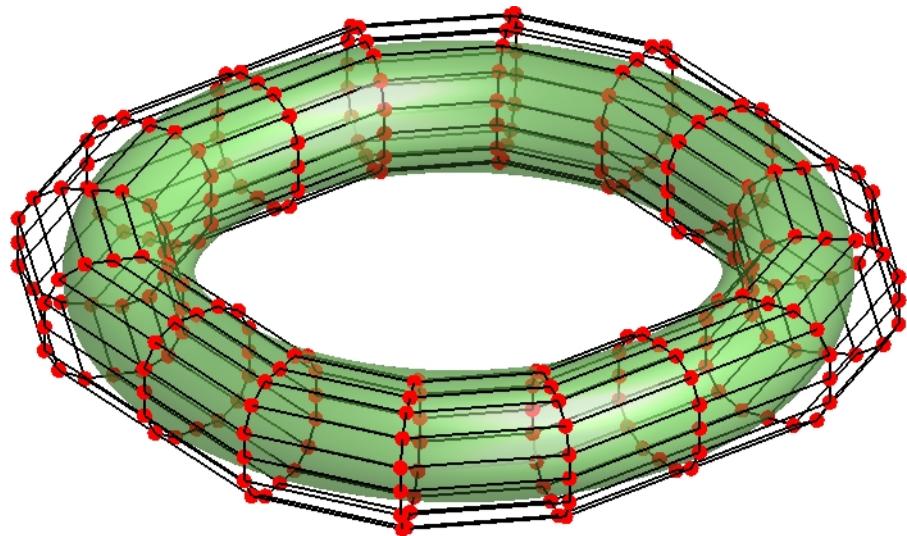


Control net

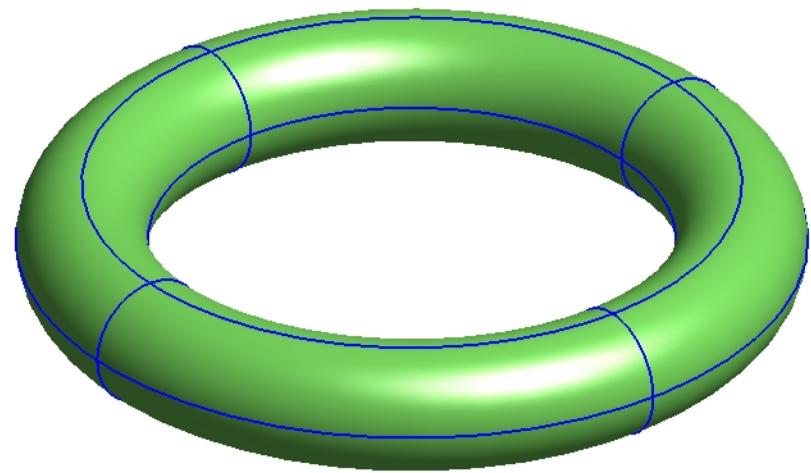


Mesh

Quartic p -refined Surface



Control net



Mesh