Find the sum of pentagonal numbers P_j and P_k such that both their sum and differenc is also pentagonal. Find the value for which D = | P_k - P_j| is minmised.

Problem 44 - Pentagonal Numbers

We are interested in the pentagonal numbers, P,

$$P_n = \frac{n}{2}(3n-1) = \frac{1}{2}(3n^2 - n)$$

we are interested in the case where both the sum and difference of two pentagonal numbers are also pentagonal.

Useful relations

$P_{n+1} = \frac{1}{2}(3n^2 + 5n + 2)$

Sums

We trivially get that the sum of two pentagonal numbers, P_n and P_m are given by

$$\frac{1}{2}(3m^2 - m + 3n^2 - n)$$

so for the case $m = n$

$$2P_n = 3n^2 - n$$

with sum of the next terms $m = n + 1$,

$$3n^2 + 2n + 1$$

or $m = n + 2$

$$3n^2 + 5n + 5$$

Differences

The differnce between two adjacent terms, $P_{n+1} - P_n$, is given by

$$P_{n+1} - P_n = 3n + 1$$

this means that

Solution Bounds