

# Structural Causal Models (SCMs)

EC 421, Set 11

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Spring 2021

# Prologue

# Schedule

## Last time

Rubin-Neyman Causal Model

## Today

We're covering more complex causal relationships than we did last time - these are represented by what are known as Directed Acyclic Graphs (DAGs)

## Upcoming

Instrumental Variables

SCMs

# Causality is Complicated

Though we'd ideally be able to run **experiments** to identify causal effects, there are a number of ways experiments can fail. Just to name a few -

## Pre-administration

- Where can we 'intervene'?
- Do we have access to a truly representative sample?
- Is anyone willing to undertake the treatment/control?
- How close are we to a laboratory sample?

## Post-administration

- Placebo Effect
- Reverse causality
- Defiers and Always-Takers
- Mediators (ie. Is our experiment causing this or causing something else)?

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This means we have to think about

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Therefore it's useful to have a nice way of visualizing and understanding causal systems. Like **DAGs**.

- It's useful, however, to understand what a 'graph' actually is

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We can use the tools of formal graphs to organize our assumptions about the problem space into:

- Variables of interest
- Direct Causal Effects between them
- Potential exogeneity concerns

We can then find out...



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- whether or not the problem we are interested in is tractable
- If it is tractable, what variables do we need to measure and control for to estimate causal effects
- How existing experiments might get polluted by poor specification

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The graphs we are interested in for causal effects are

# Causality is Complicated

The graphs we are interested in for causal effects are

- directed
- acyclic

We restrict our problem space to these mostly because it reduces the complexity of the math.

So let's learn about graphs 📈

# Graphs

# Graphs

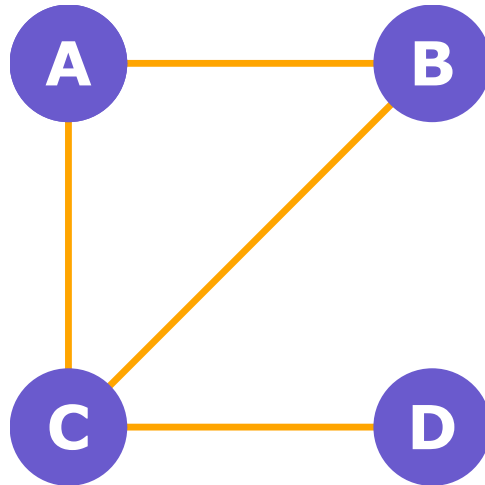
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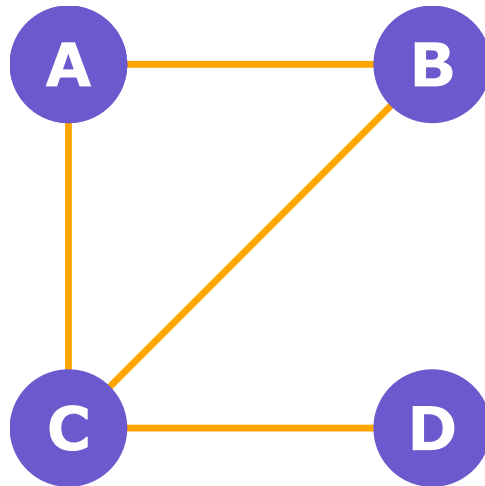




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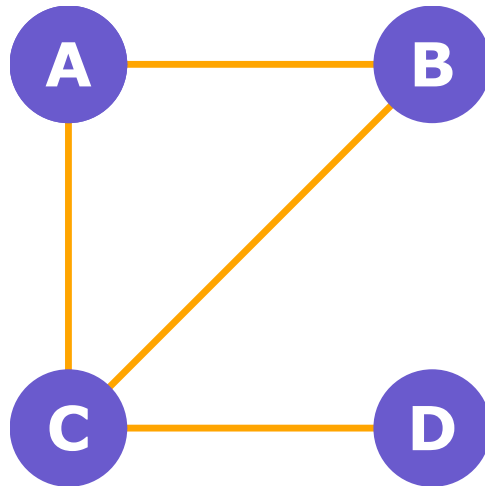


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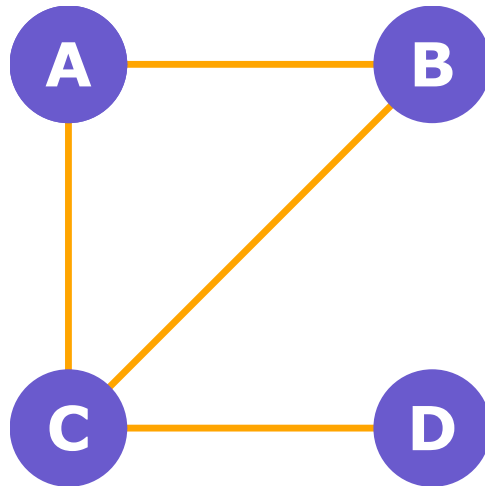


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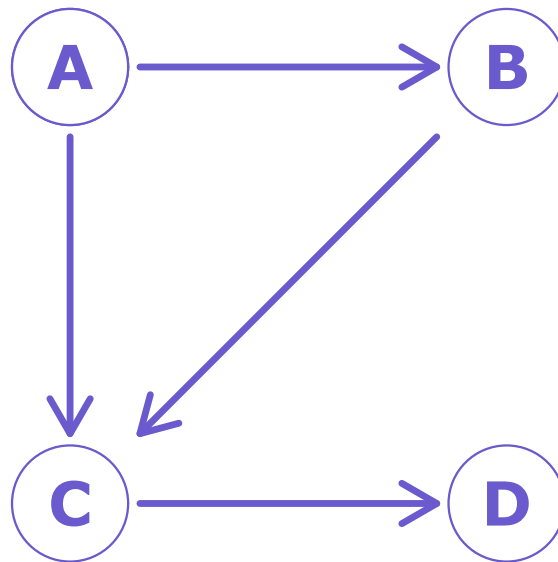


- Nodes connected by an edge are called adjacent.
- Paths run along adjacent nodes, e.g., **A – B – C**.
- The graph above is undirected, since the edges don't have direction. We can give our graphs more information by defining direction for the edges

# Graphs

## Directed

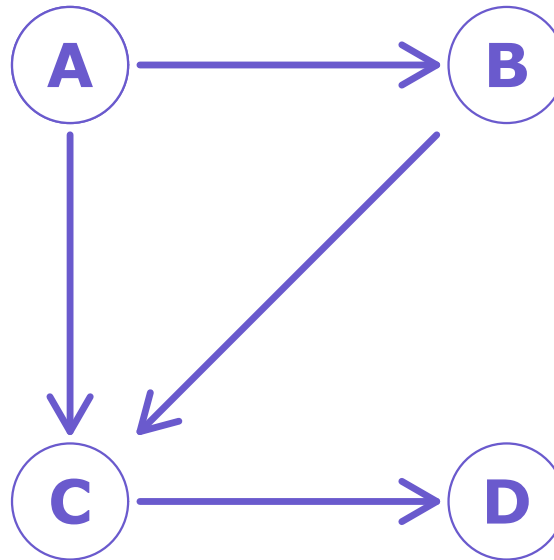
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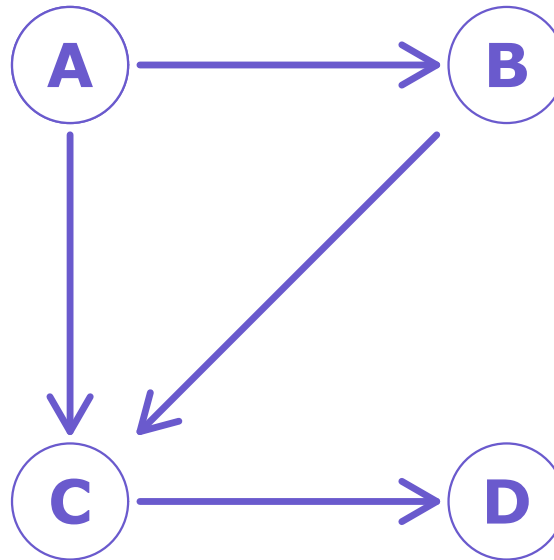


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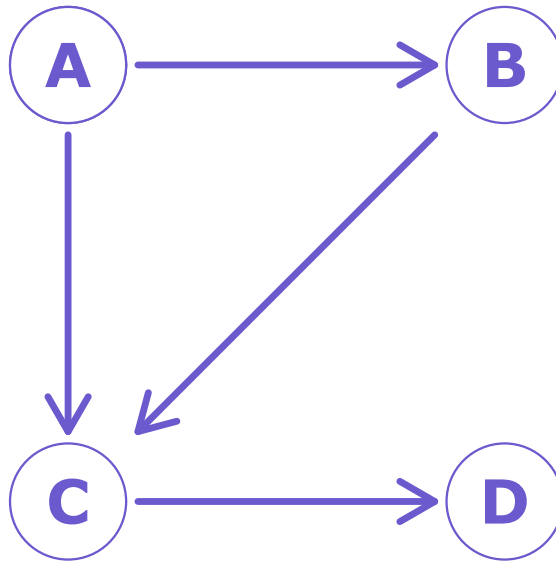


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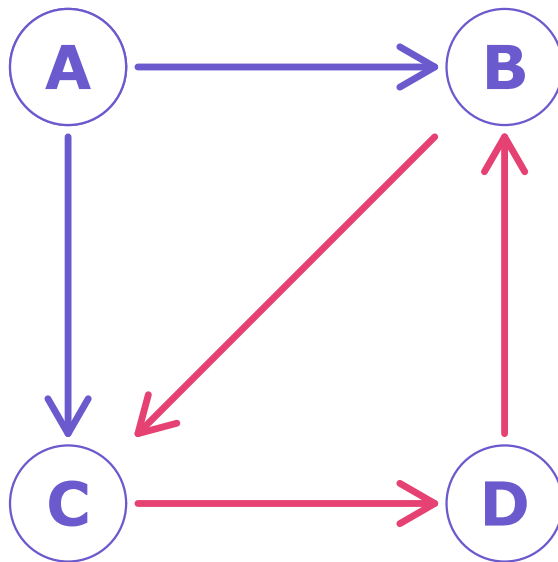


- Directed paths follow edges' directions, e.g.,  $A \rightarrow B \rightarrow C$ .
- Nodes that precede a given node in a directed path are its **ancestors**.
- The opposite: **descendants** come after the node, e.g.,  $D = \text{de}(C)$ .

# Graphs

## Cycles

If a node is its own descendant (e.g.,  $\text{de}(\mathbf{D}) = \mathbf{D}$ ), your graph has a **cycle**.

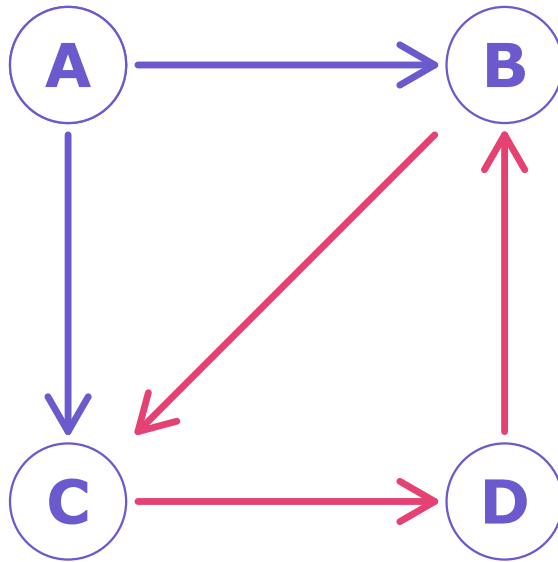




# Graphs

## Cycles

If a node is its own descendant (e.g.,  $\text{de}(\mathbf{D}) = \mathbf{D}$ ), your graph has a **cycle**.



If your directed graph does not have any cycles, then you have a **directed acyclic graph (DAG)**.

# DAGs

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DAGs are special graphs that help us understand causality in much the same way as microeconomic models help us understand markets

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A DAG is a graph that features a collection of **nodes** and **directed** edges

The graph illustrates and differentiates the causal associations and non-causal associations within a network of "random" variables.

# DAGs

## Why a DAG?

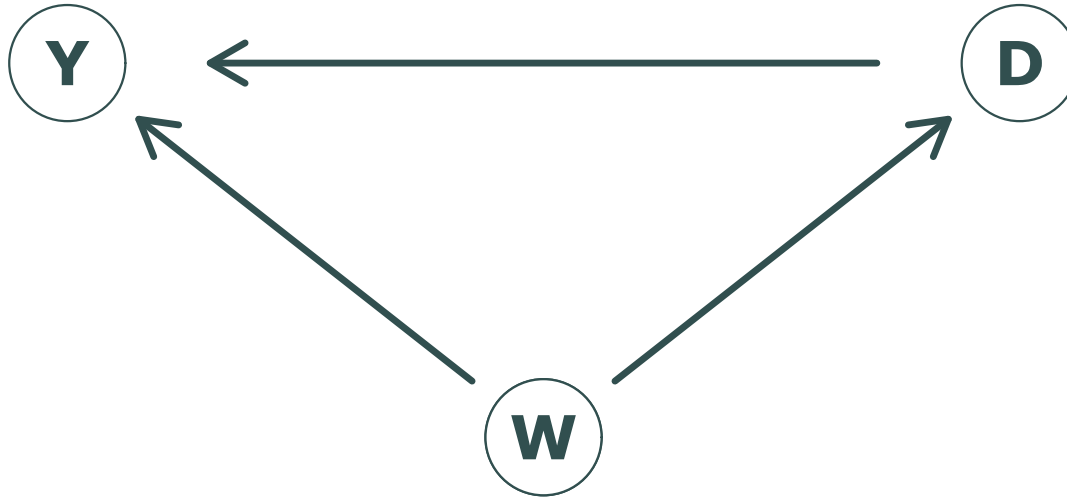
Following a fairly simple set of rules, DAGs allow researchers to visualize complex systems.

It separates -

- causal 'associations'
- noncausal 'associations'

in any assumption space without cycles

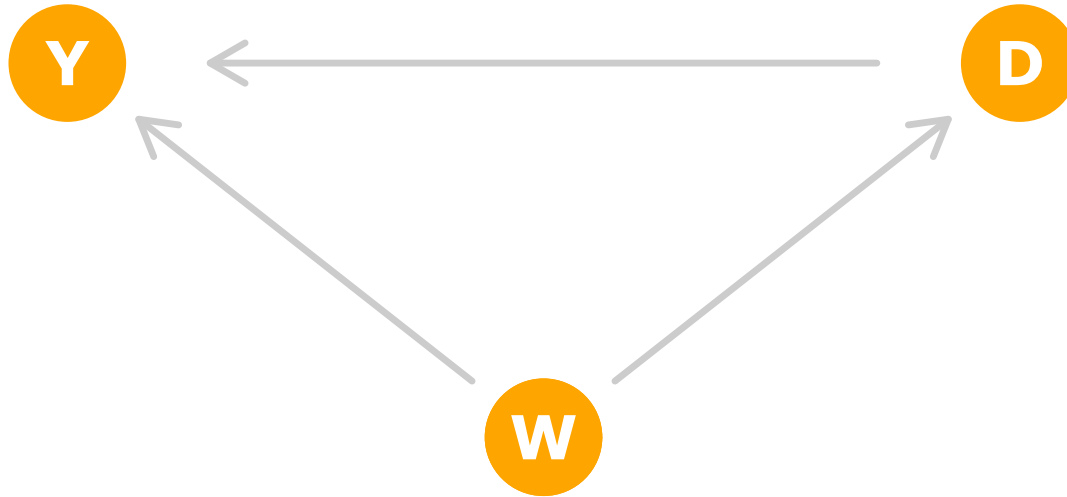
## Example Omitted-variable bias in a DAG



A pretty standard DAG.

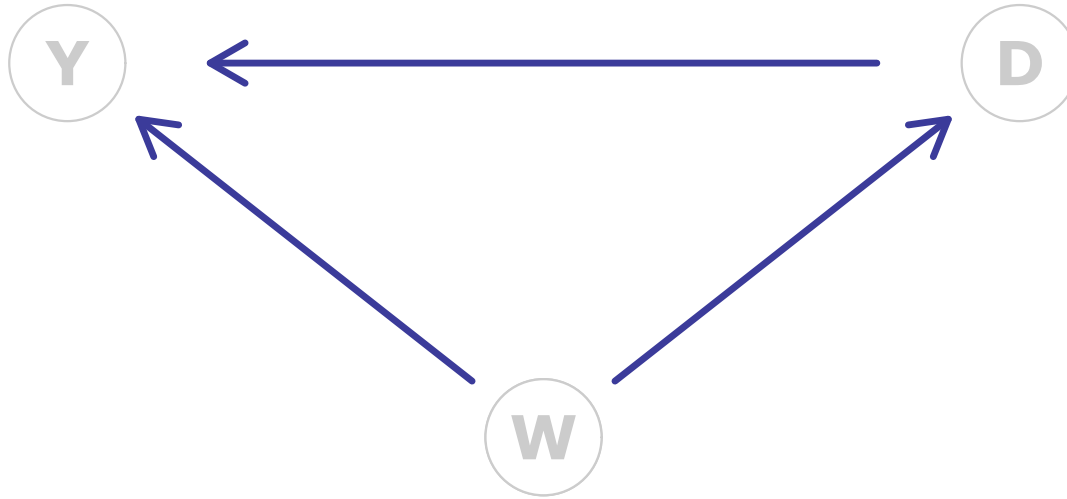


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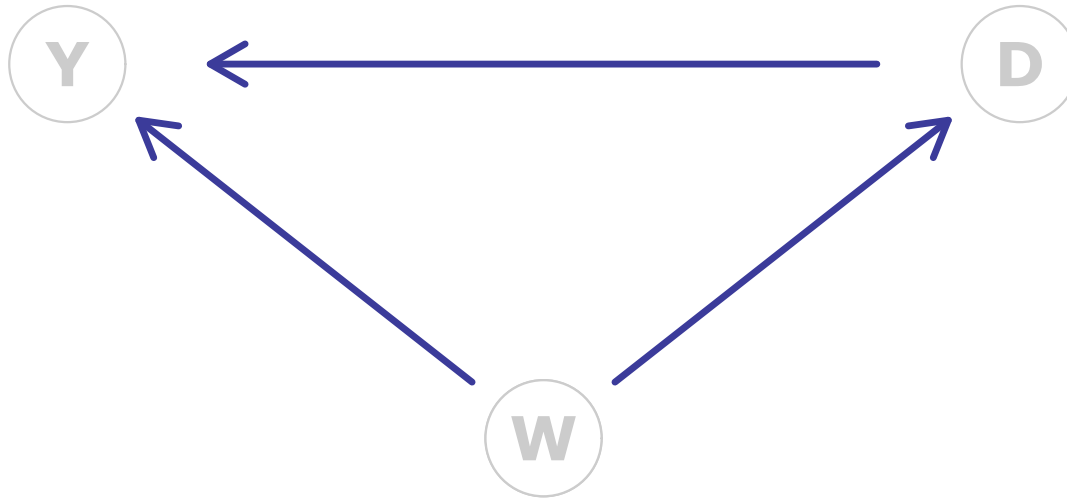
Nodes are random variables.

## Example Omitted-variable bias in a DAG



Edges depict causal links. Causality flows in the direction of the **arrows**.

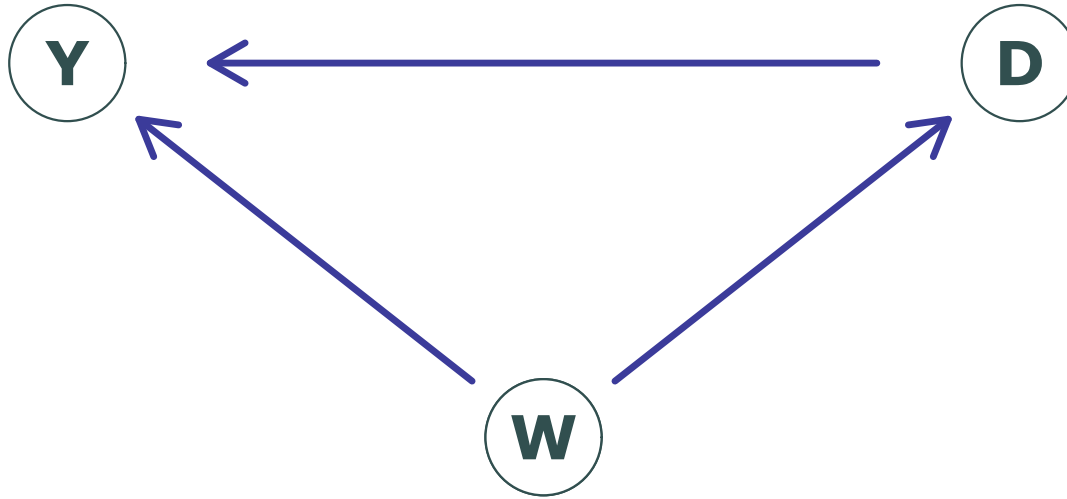
## Example Omitted-variable bias in a DAG



Edges depict causal links. Causality flows in the direction of the **arrows**.

- Connections are drawn between nodes that **directly** cause one another
- Direction matters (for causality).
- Non-connections also (sometimes) matter! (More on this topic soon.)

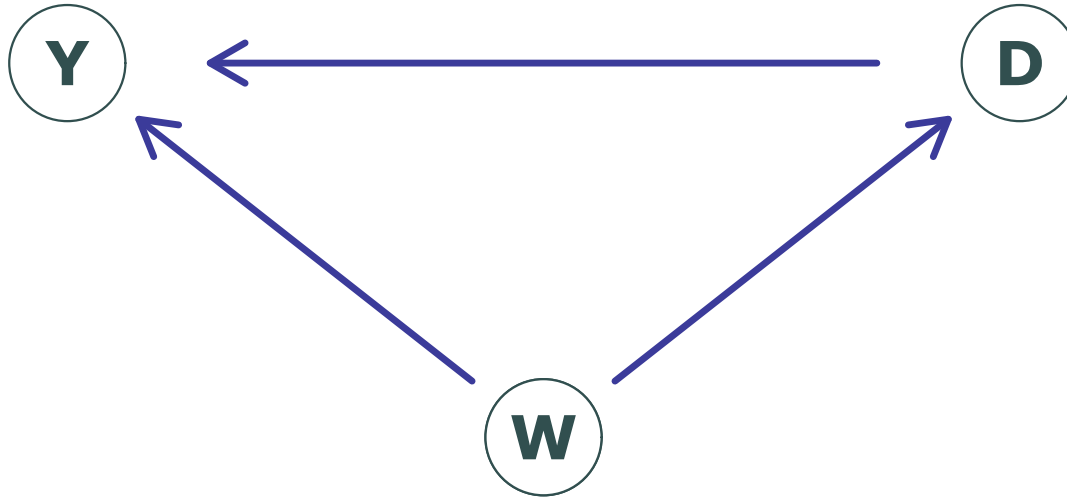
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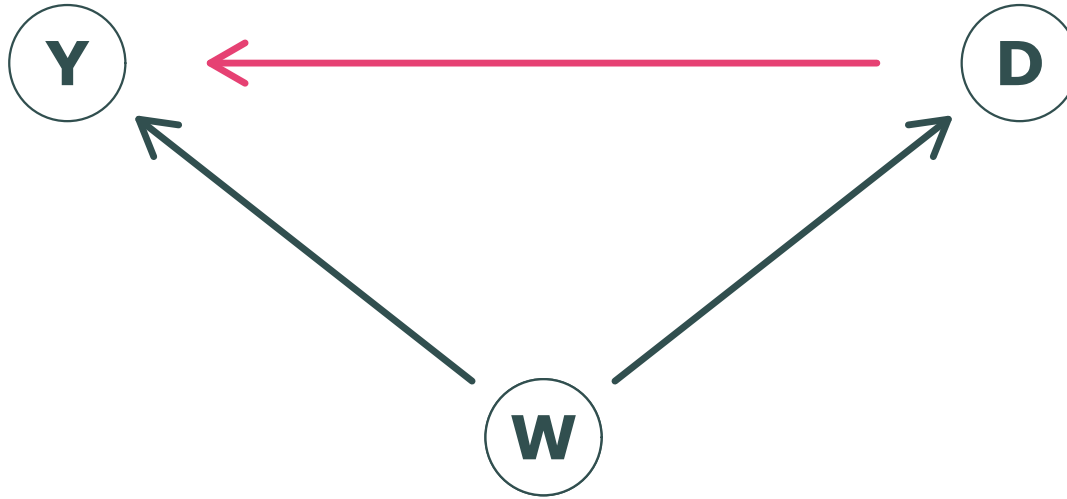


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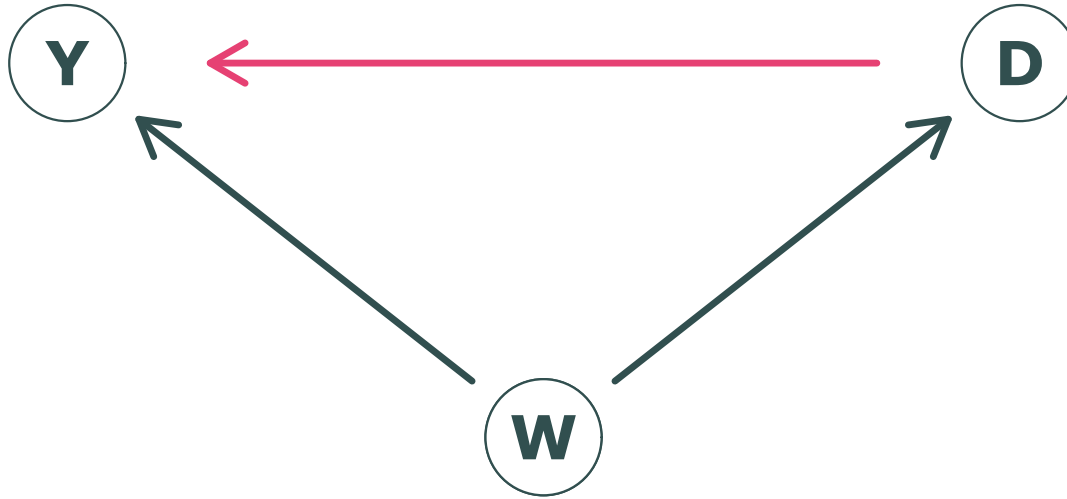
Q How does this graph exhibit OVB?

## Example Omitted-variable bias in a DAG



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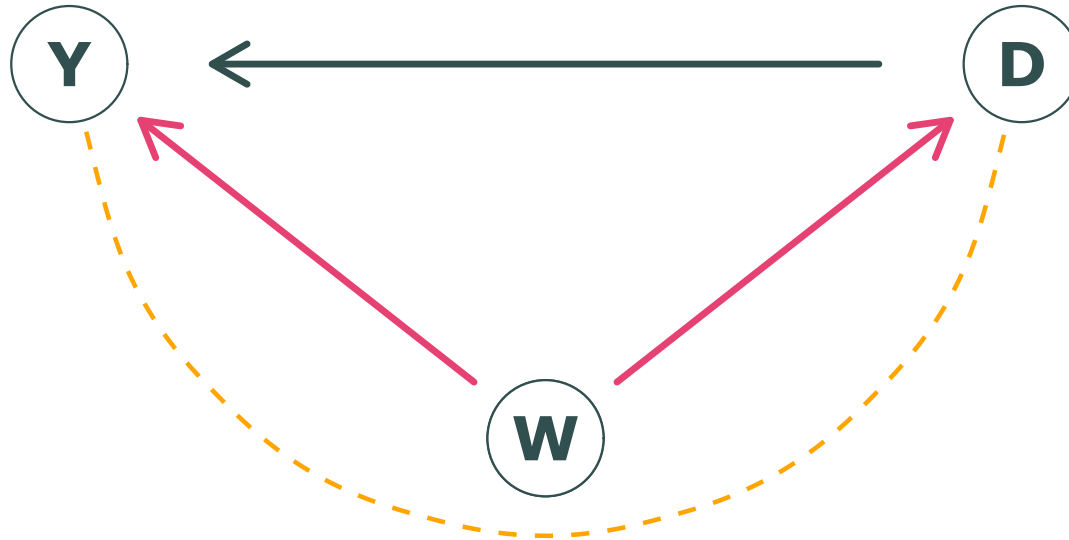
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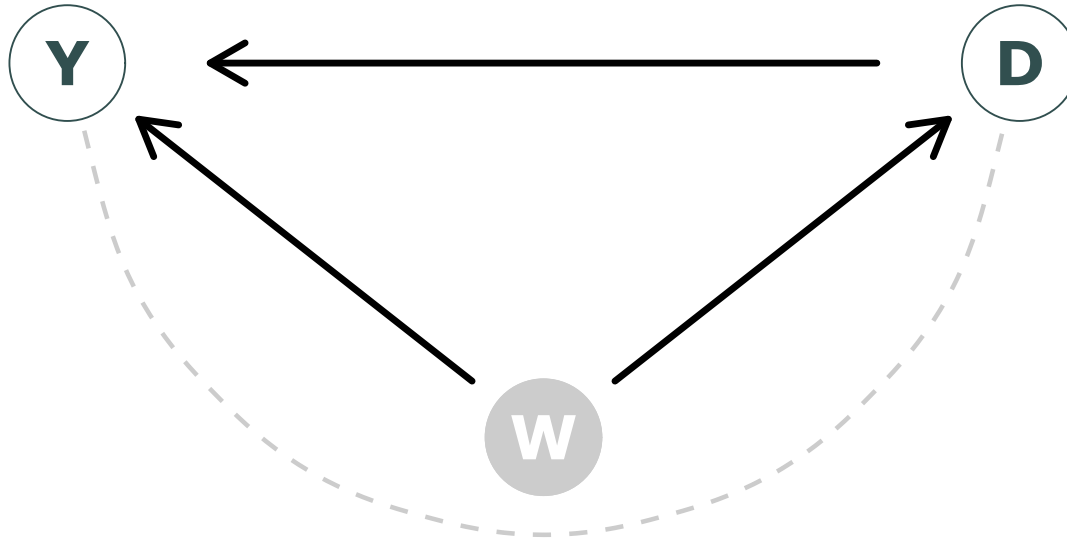


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2. The path ( $Y \leftarrow W \rightarrow D$ ) creates a **non-causal association** btn D and Y.

To shut down this pathway creating a non-causal association, we must condition on W. Sound familiar?

# DAGs

## The origin story

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I won't require you guys to learn the formal math behind Dags, though it is totally doable.

If you go to the end of the lecture, there will be an extensive section covering the theory/proofs.

# DAGs

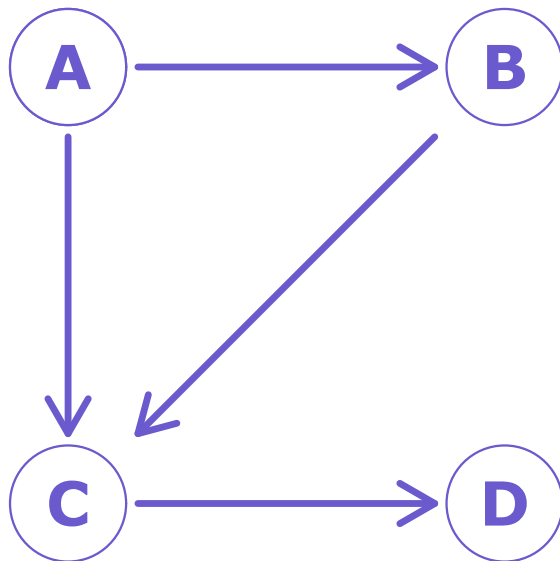
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DAGs help us think through how we can break down complex causal systems of many variables - simplifying  $P(x_k | x_{k-1}, x_{k-2}, \dots, x_1)$ .

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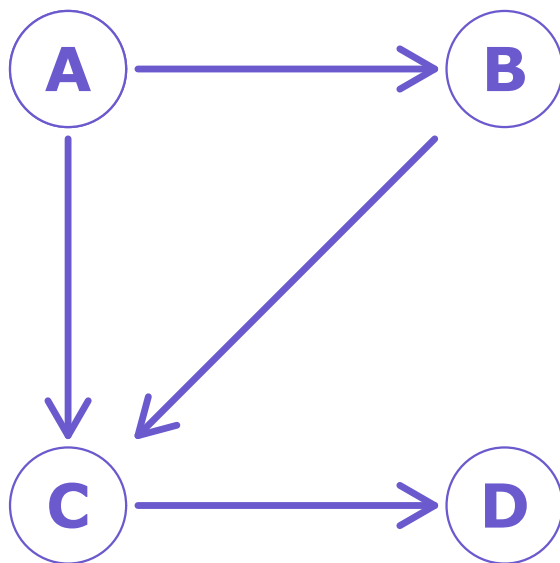


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Given a prob. dist. and a DAG, can we assume some independencies?

Given **C**, is it reasonable to assume **D** is independent of **A** and **B**?



## Local Markov

This intuitive approach is the [Local Markov Assumption](#)

Given its parents in the DAG, a node  $X$  is independent of all of its non-descendants.

# DAGs

## Local Markov

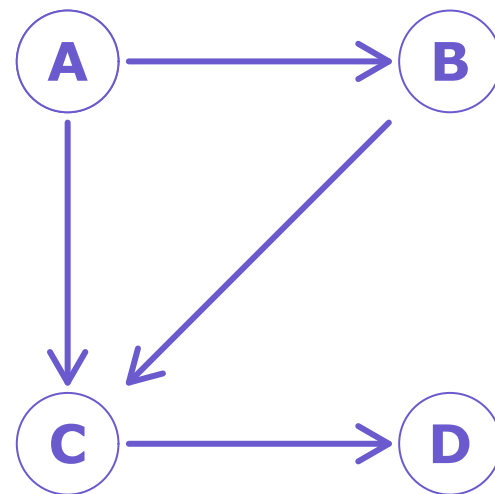
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Ex. Consider the DAG to the right:

With the Local Markov Assumption,  
 $P(D|A, B, C)$  simplifies to  $P(D|C)$ .

Conditional on its parent (C),  
D is independent of A and B.



## Independence

What have we learned so far? (Why should you care about this stuff?)

Local Markov tells us about independencies within a probability distribution implied by the given DAG.

You're now able to say something about which variables are **independent**.

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You're now able to say something about which variables are **independent**.

There's more: Great start, but there's more to life than independence. We also want to say something about **dependence**.

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The **Minimality Assumption**<sup>†</sup>

1. **Local Markov** Given its parents in the DAG, a node  $X$  is independent of all of its non-descendants.
2. (NEW) Adjacent nodes in the DAG are dependent.

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With the minimality assumption, we can learn both **dependence** and **independence** from connections (or non-connections) in a DAG.

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### Strict Causal Edges Assumption

Every parent is a direct cause of each of its children. For  $Y$ , the set of direct causes is the set of variables to which  $Y$  responds.

This assumption actually strengthens the second part of Minimality:

2. Adjacent nodes in the DAG are dependent.

## Assumptions

Thus, we only need two assumptions to turn DAGs into causal models:

1. **Local Markov** Given its parents in the DAG, a node  $X$  is independent of all of its non-descendants.
2. **Strict Causal Edges** Every parent is a direct cause of each of its children.

## Assumptions

Thus, we only need two assumptions to turn DAGs into causal models:

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2. **Strict Causal Edges** Every parent is a direct cause of each of its children.

Not bad, right?

# DAGs

## Flows

Brady Neal emphasizes the flow(s) of association and causation in DAGs, and I find it to be a super helpful way to think about these models.

Flow of association refers to whether two nodes are associated (statistically dependent) or not (statistically independent).

We will be interested in unconditional and conditional associations.

# DAGs

## Building blocks

We will run through a few simple building blocks (DAGs) that make up more complex DAGs.

For each simple DAG, we want to ask a few questions:

1. Which nodes are unconditionally or conditionally **independent**?<sup>†</sup>
2. Which nodes are **dependent**?
3. What is the **intuition**?

<sup>†</sup> To prove **A** and **B** are conditionally independent, we can show  $P(\mathbf{A}, \mathbf{B} | \mathbf{C})$  factorizes as  $P(\mathbf{A} | \mathbf{C})P(\mathbf{B} | \mathbf{C})$ .

## Building block 1: Two unconnected nodes





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Intuition:

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**Proof:**

## Building block 1: Two unconnected nodes



**Intuition:** A and B appear independent—no link between the nodes.

**Proof:** By Bayesian network factorization,

$$P(A, B) = P(A)P(B)$$

(since neither node has parents). ✓

## Building block 2: Two connected nodes



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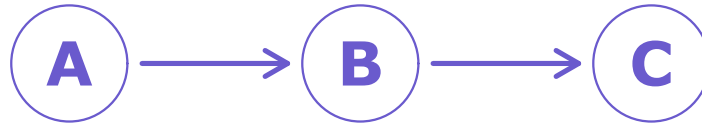


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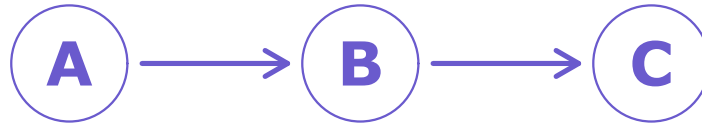


**Intuition:** A "is a cause of" B: there is clear (causal) dependence. **Proof:** By the **Strict Causal Edges Assumption**, every parent (here, A) is a direct cause of each of its children (B). ✓

## Building block 3: Chains



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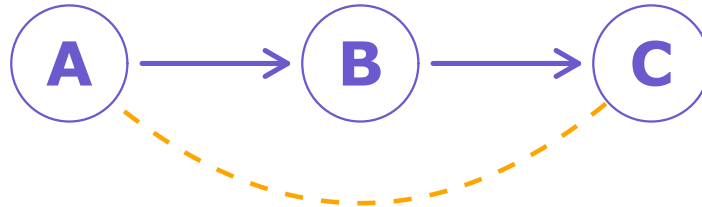
**Intuition:** We already showed two connected nodes are dependent:

- **A** and **B** are dependent.
- **B** and **C** are dependent.

The question is whether **A** and **C** are dependent:

Does association flow from **A** to **C** through **B**?

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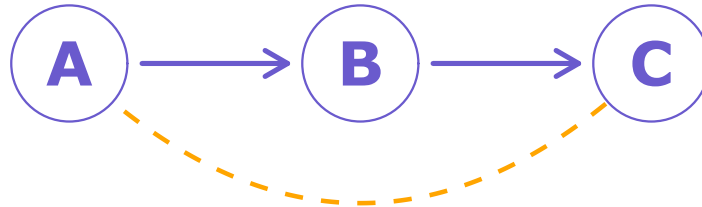
The question is whether **A** and **C** are dependent:

Does association flow from **A** to **C** through **B**?

The answer generally<sup>†</sup> is "yes": changes in **A** typically cause changes in **C**.

<sup>†</sup> Section 2.2 of [Pearl, Glymour, and Jewell](#) provides a "pathological" example of "intransitive dependence". It's basically when **A** induces variation in **B** that is not relevant to **C**' outcome.

## Building block 3: Chains

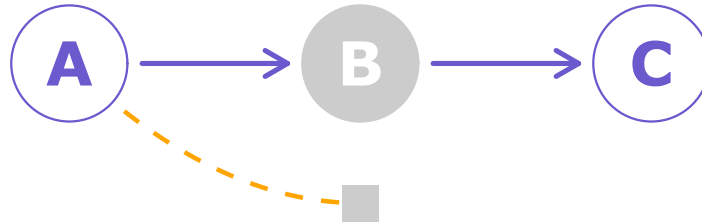


**Proof:** Here's the unsatisfying part.

Without more assumptions, we can't *prove* this association of **A** and **C**.

We'll think of this as a potential (even likely) association.

## Building block 3: Chains with conditions



Q How does conditioning on **B** affect the association between **A** and **C**?

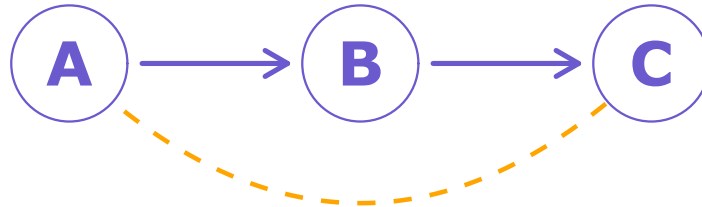
Intuition:

1. **A** affects **C** by changing **B**.
2. When we hold **B** constant, **A** cannot "reach" **C**.

We've **blocked** the path of association between **A** and **C**.

Conditioning blocks the flow of association in chains. ("Good" control!)

## Building block 3: Chains

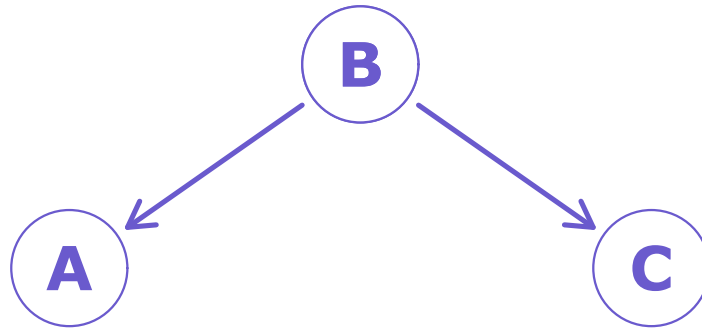


Note This **association of A and C** is not directional. (It is symmetric.)

On the other hand, causation is directional (and asymmetric).

As you've been warned for years: Associations are not necessarily causal.

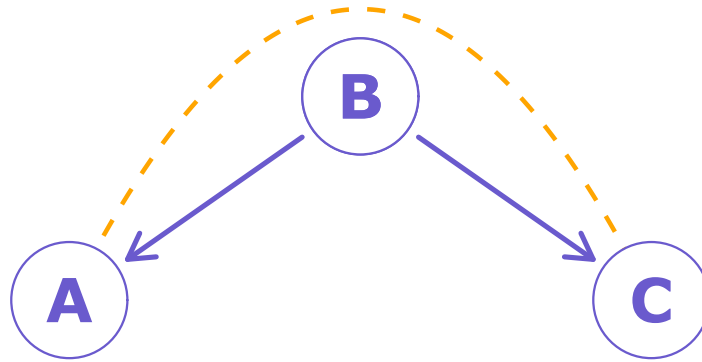
## Building block 4: Forks



**Forks** are another very common structure in DAGs:  $A \leftarrow B \rightarrow C$ .



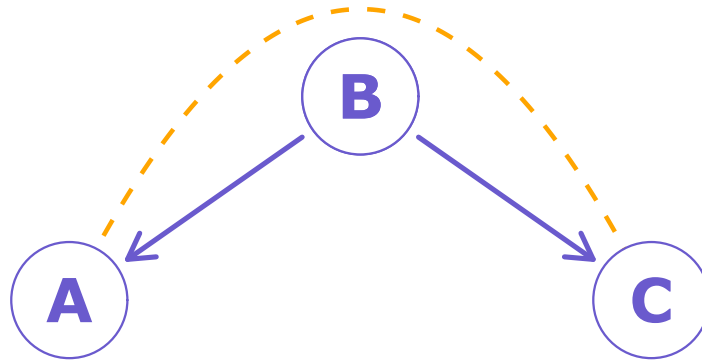
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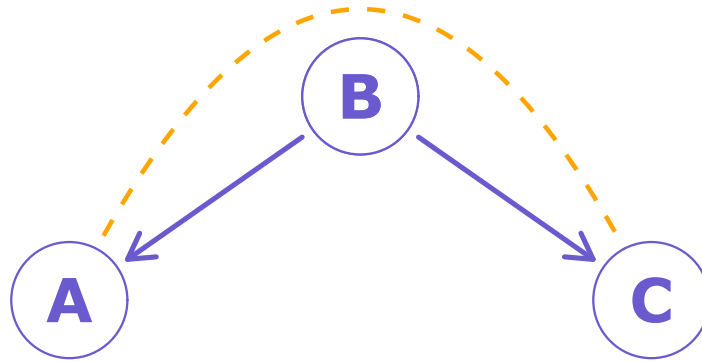


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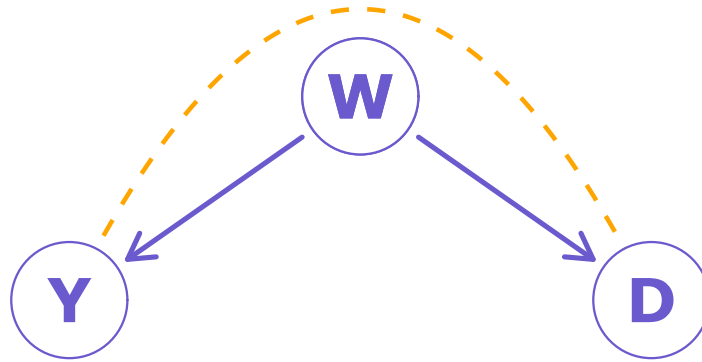


**A** and **C** are *usually* associated in forks. (As with chains.)

This chain of association follows the path  $\mathbf{A} \leftarrow \mathbf{B} \rightarrow \mathbf{C}$ .

**Intuition:** **B** induces changes in **A** and **C**. An observer will see **A** change when **C** also changes—they are associated due to their common cause.

## Building block 4: Forks

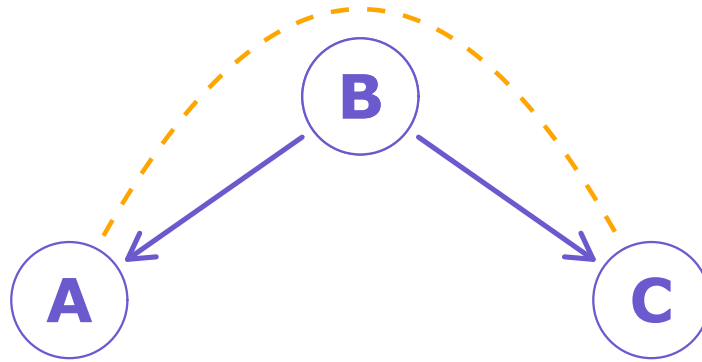


Another way to think about forks:

OVB when a treatment **D** does not affect the outcome **Y**.

Without controlling for **W**, **Y** and **D** are (usually) non-causally associated.

## Building block 4: Forks

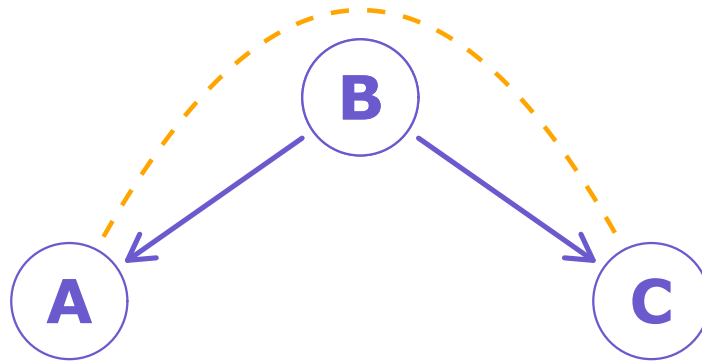


$A$  and  $C$  are *usually* associated in forks. (As with chains.)

This chain of association follows the path  $A \leftarrow B \rightarrow C$ .

Proof:

## Building block 4: Forks

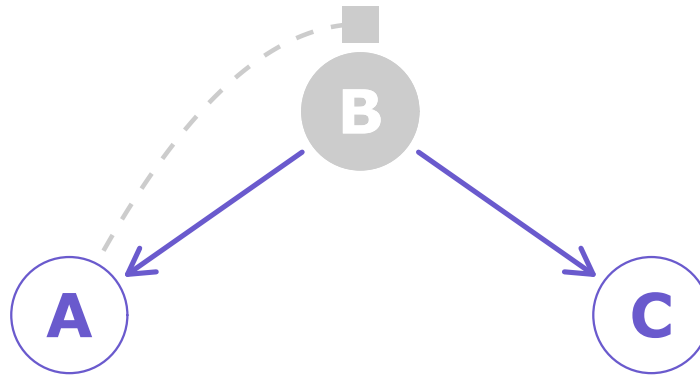


A and C are *usually* associated in forks. (As with chains.)

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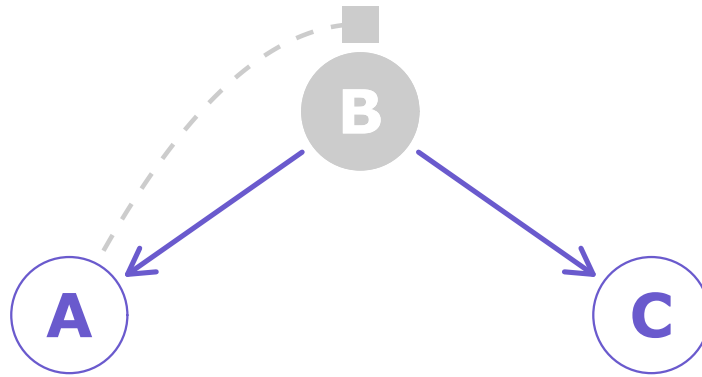
**Proof:** Same problem as chains: We can't show A and C are independent, so we assume they're likely (potentially?) dependent.

## Building block 4: Blocked forks



Conditioning on **B** makes **A** and **C**

## Building block 4: Blocked forks

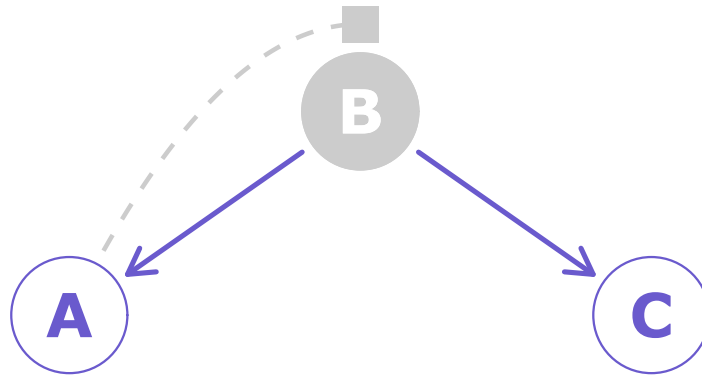


Conditioning on **B** makes **A** and **C** independent. (As with chains.)

Intuition:



## Building block 4: Blocked forks

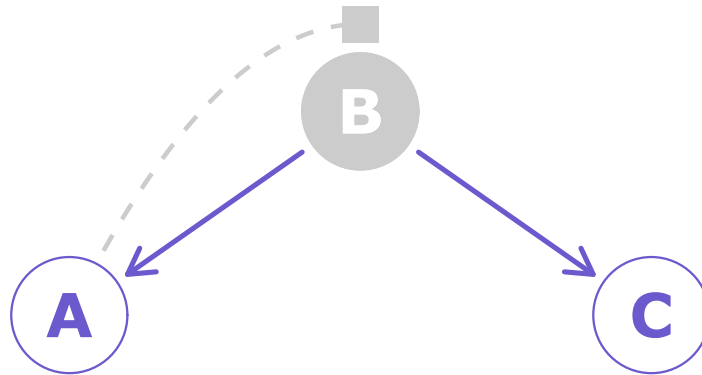


Conditioning on **B** makes **A** and **C** independent. (As with chains.)

**Intuition:** **A** and **C** are only associated due to their common cause **B**.

When we shutdown (hold constant) this common cause (**B**),  
there is way for **A** and **C** to associate.

## Building block 4: Blocked forks



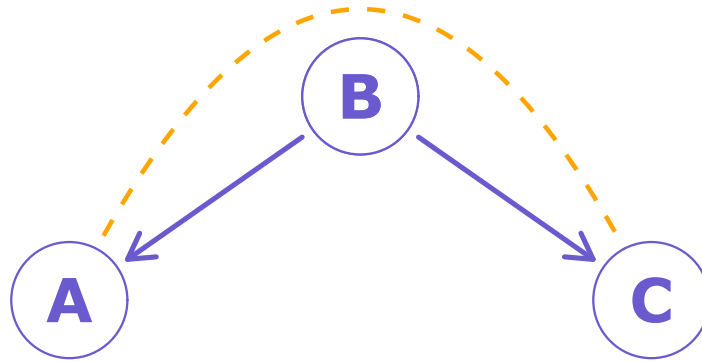
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Also: Think about Local Markov. Or think about OVB.

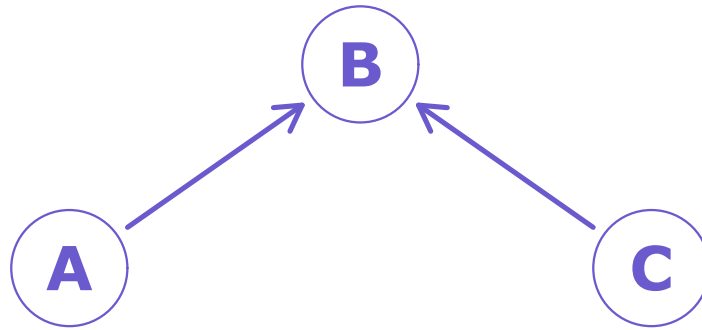
## Building block 4: Forks



Two more items to emphasize:

1. **Association** need not follow paths' directions, *e.g.*,  $A \leftarrow B \rightarrow C$ .
2. **Causation** follows directed paths.

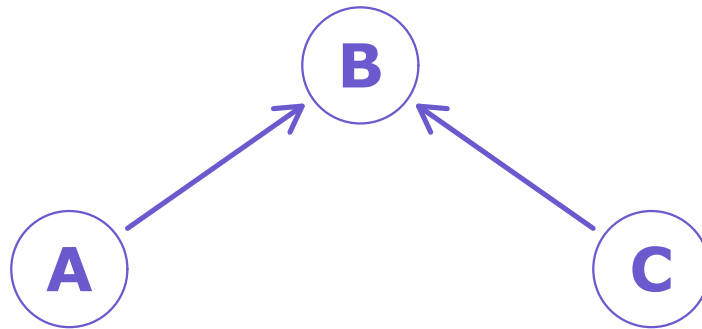
## Building block 5: Immoralities



An **immorality** occurs when two nodes share a child without being otherwise connected.<sup>†</sup>  $A \rightarrow B \leftarrow C$

<sup>†</sup> Yes, this is a stupid term, and I hate it. I way prefer just using 'collider'

## Building block 5: Immoralities

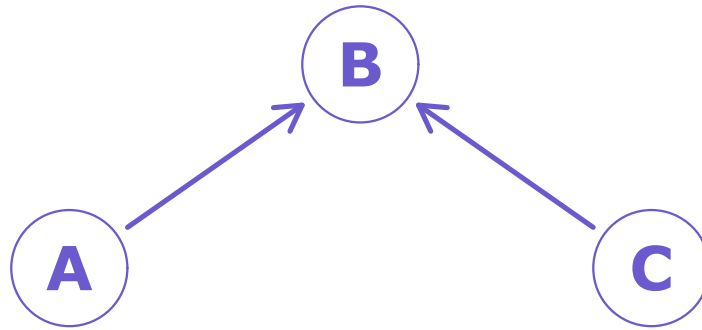


An **immorality** occurs when two nodes share a child without being otherwise connected.<sup>†</sup>  $A \rightarrow B \leftarrow C$

The child (here: **B**) at the center of this immorality is called a **collider**.

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## Building block 5: Immoralities



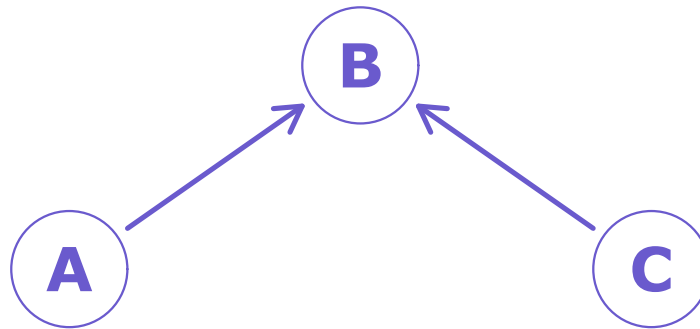
An **immorality** occurs when two nodes share a child without being otherwise connected.<sup>†</sup>  $A \rightarrow B \leftarrow C$

The child (here: **B**) at the center of this immorality is called a **collider**.

Notice: An immorality is a fork with reversed directions of the edges.

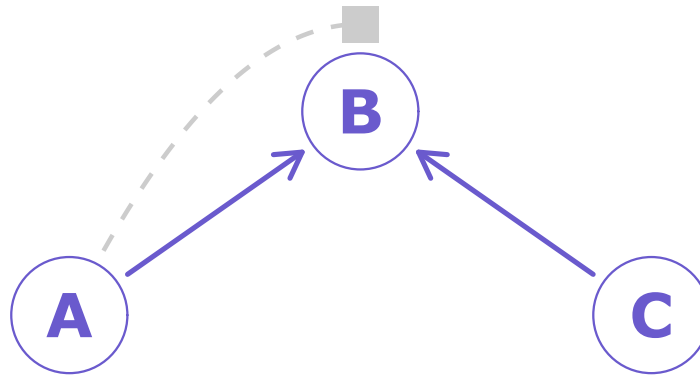
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## Building block 5: Immoralities



Q Are **A** and **C** independent?

## Building block 5: Immoralities

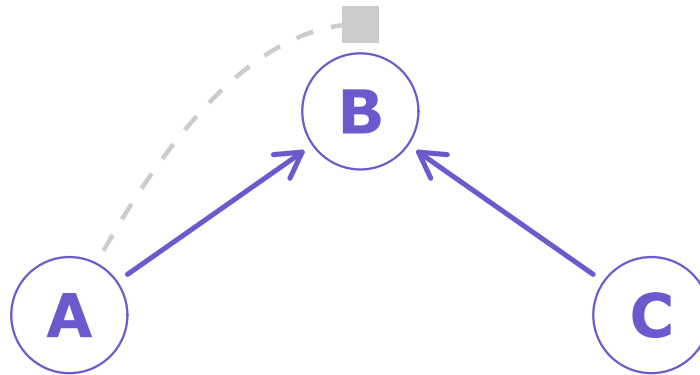


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A Yes.  $A \perp\!\!\!\perp C$ .



## Building block 5: Immoralities



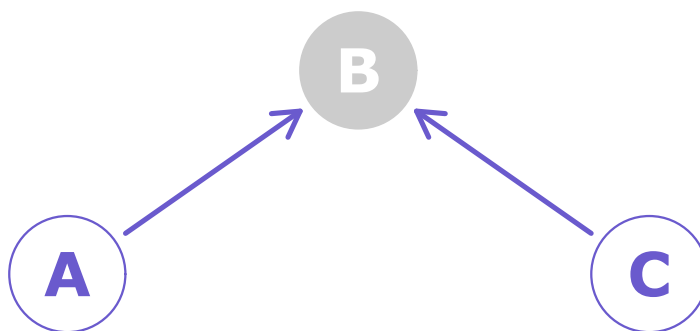
Q Are **A** and **C** independent?

A Yes.  $A \perp\!\!\!\perp C$ .

**Intuition:** Causal effects flow from **A** and **C** and stop there.

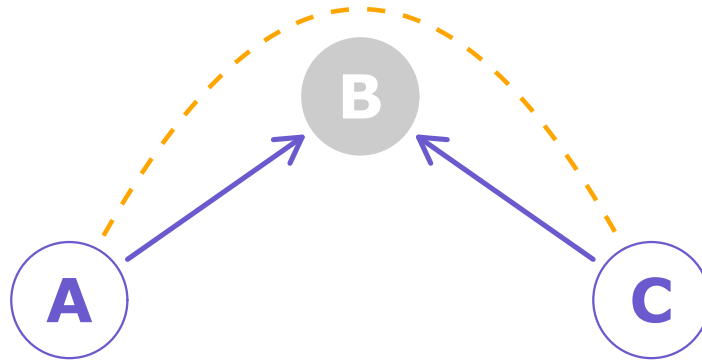
- Neither **A** nor **C** is a descendant of the other.
- **A** and **C** do not share any common causes.

## Building block 5: Immoralities with conditions



Q What happens when we condition on **B**?

## Building block 5: Immoralities with conditions

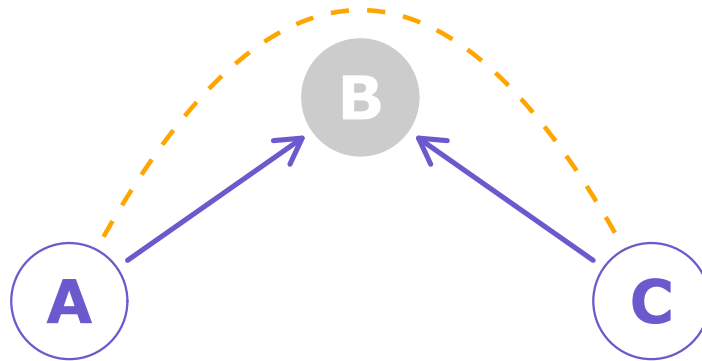


Q What happens when we condition on **B**?

A We **unlock** (or **open**) the previously blocked (closed) path.

While **A** and **C** are independent, they are **conditionally dependent**.

## Building block 5: Immoralities with conditions



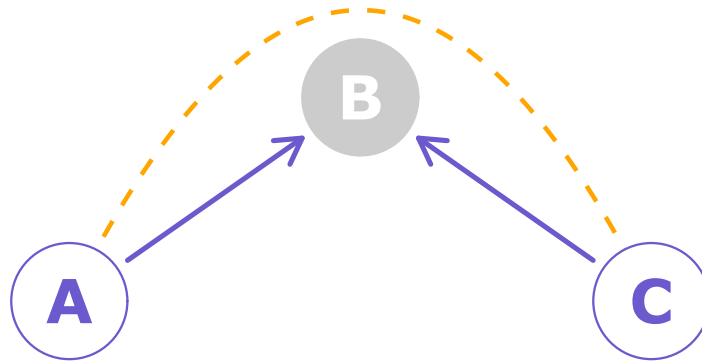
Q What happens when we condition on **B**?

A We **unblock** (or **open**) the previously blocked (closed) path.

While **A** and **C** are independent, they are **conditionally dependent**.

Important: When you condition on a collider, you open up the path.

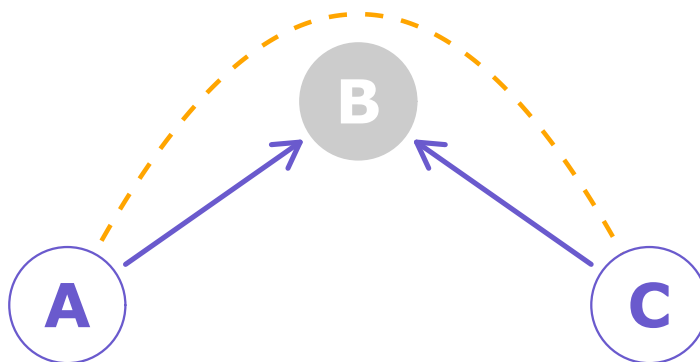
## Building block 5: Immoralities with conditions



**Intuition:** **B** is a combination of **A** and **C**.

Conditioning on a value of **B** jointly constrains **A** and **C**—they can no longer move independently.

## Building block 5: Immoralities with conditions



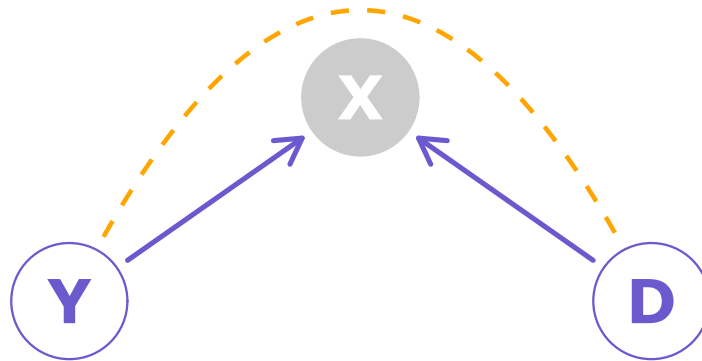
**Intuition:** **B** is a combination of **A** and **C**.

Conditioning on a value of **B** jointly constrains **A** and **C**—they can no longer move independently.

Example: Let **A** take on  $\{0, 1\}$  and **C** take on  $\{0, 1\}$  (independently).

Conditional on **B** = 1, **A** and **C** are perfectly negatively correlated.

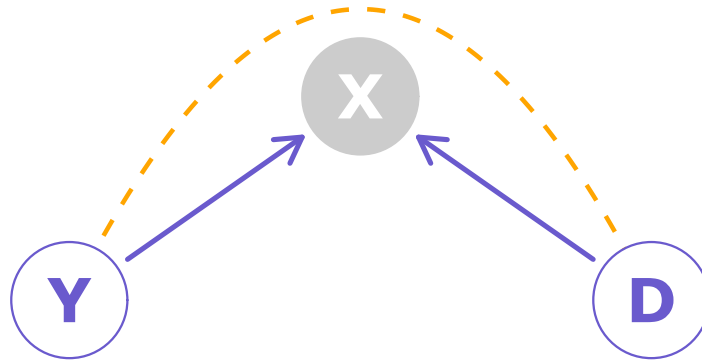
## Building block 5: Immoralities with conditions



In *MHE* vocabulary: The collider  $X$  is a *bad control*.

$X$  is affected by both your treatment  $D$  and outcome  $Y$ .

## Building block 5: Immoralities with conditions



In *MHE* vocabulary: The collider **X** is a *bad control*.

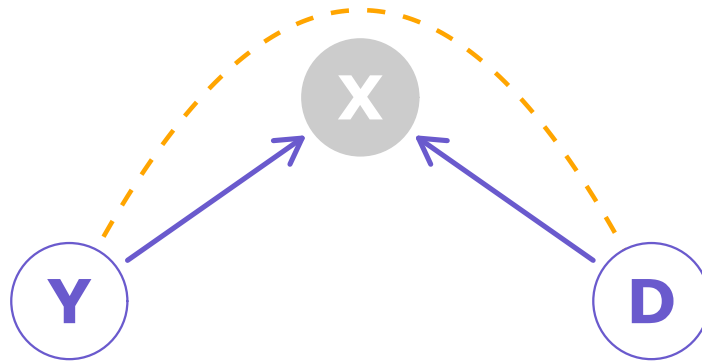
**X** is affected by both your treatment **D** and outcome **Y**.

The result: A spurious relationship between **Y** and **D**

Remember: they're actually (unconditionally) independent.



## Building block 5: Immoralities with conditions



In *MHE* vocabulary: The collider **X** is a *bad control*.

**X** is affected by both your treatment **D** and outcome **Y**.

The result: A spurious relationship between **Y** and **D**

Remember: they're actually (unconditionally) independent.

This spurious relationship is often called **collider bias**.

Example Obesity, Mortality Factors and Cardiovascular Disease.

Define **O** as obesity,

Example Obesity, Mortality Factors and Cardiovascular Disease.

Define **O** as obesity, **A** as age of patient,

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Suppose for the moment obesity and age

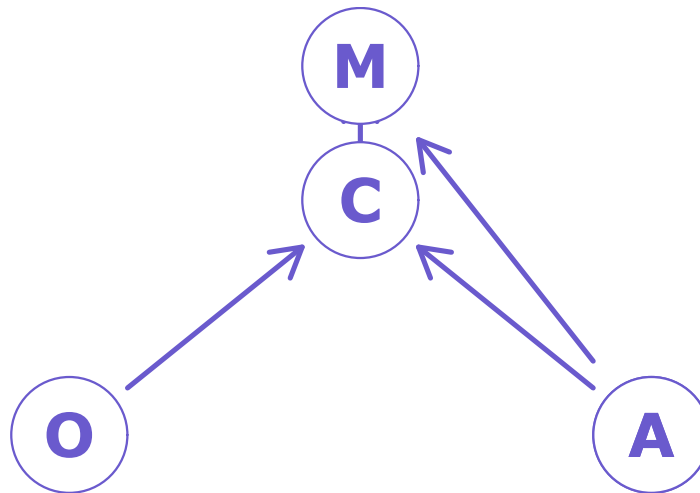
1. are independent of each other
2. each cause cardiovascular disease and  $\text{cvd} \uparrow \mathbf{M}$

Example Obesity, Mortality Factors and Cardiovascular Disease.

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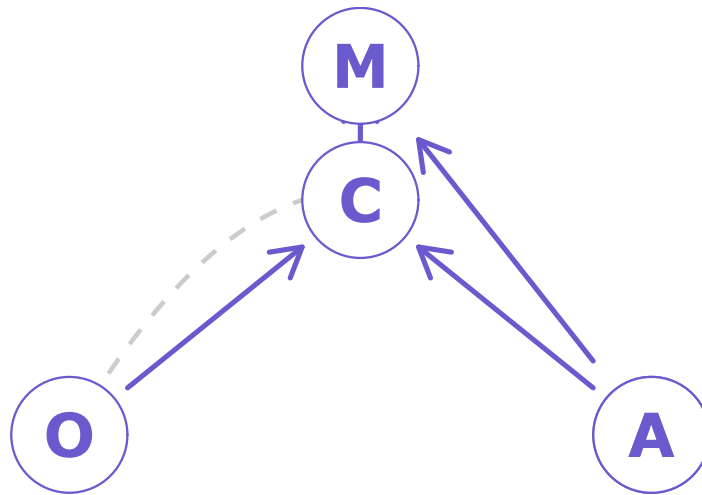
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The implied DAG.

Example Obesity, Cardiovascular Disease and Mortality.

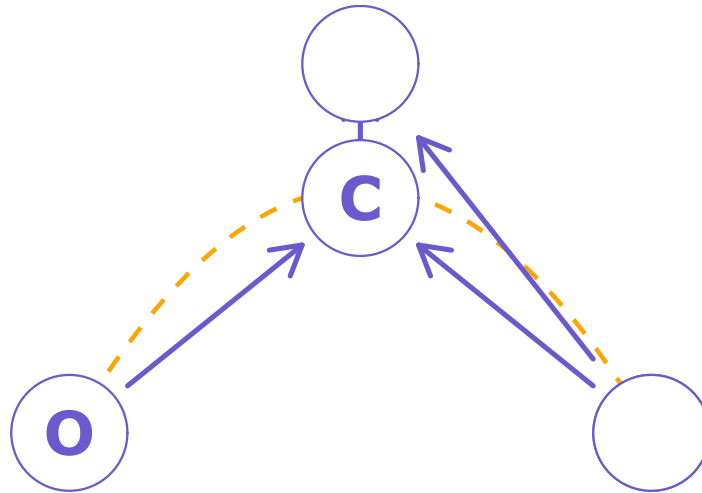
Define **O** as obesity, **A** as Age, **C** as Cardiovascular Disease, and **M** as Mortality.



If we **do not condition on Cardiovascular Disease**,  $S \rightarrow H \leftarrow A$  is **blocked**,  
but  $O \rightarrow C \rightarrow M$  is not

Example Obesity, Cardiovascular Disease and Mortality.

Define **O** as obesity, **A** as Age, **C** as Cardiovascular Disease, and **M** as Mortality.



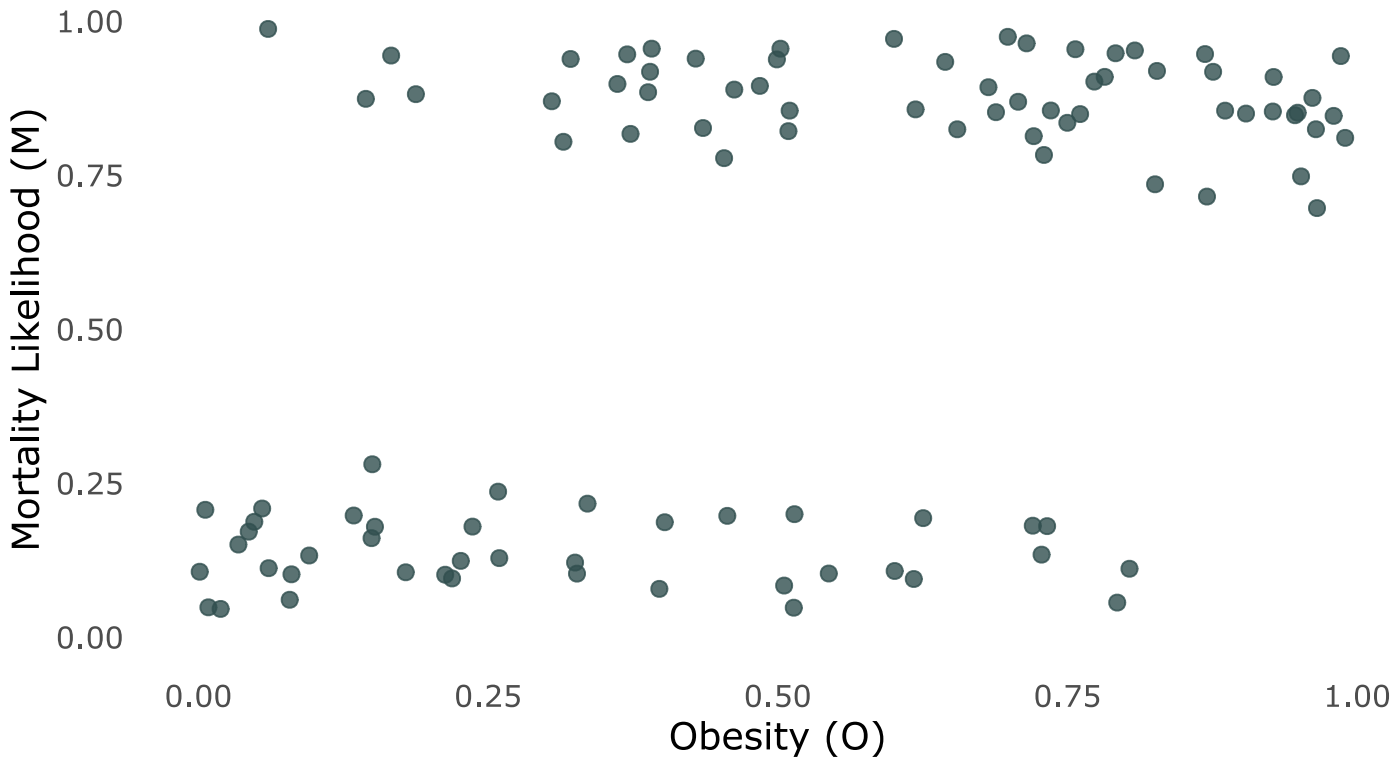
Our data conditions on cardiovascular disease, which opens  $O \rightarrow C \leftarrow A$ .



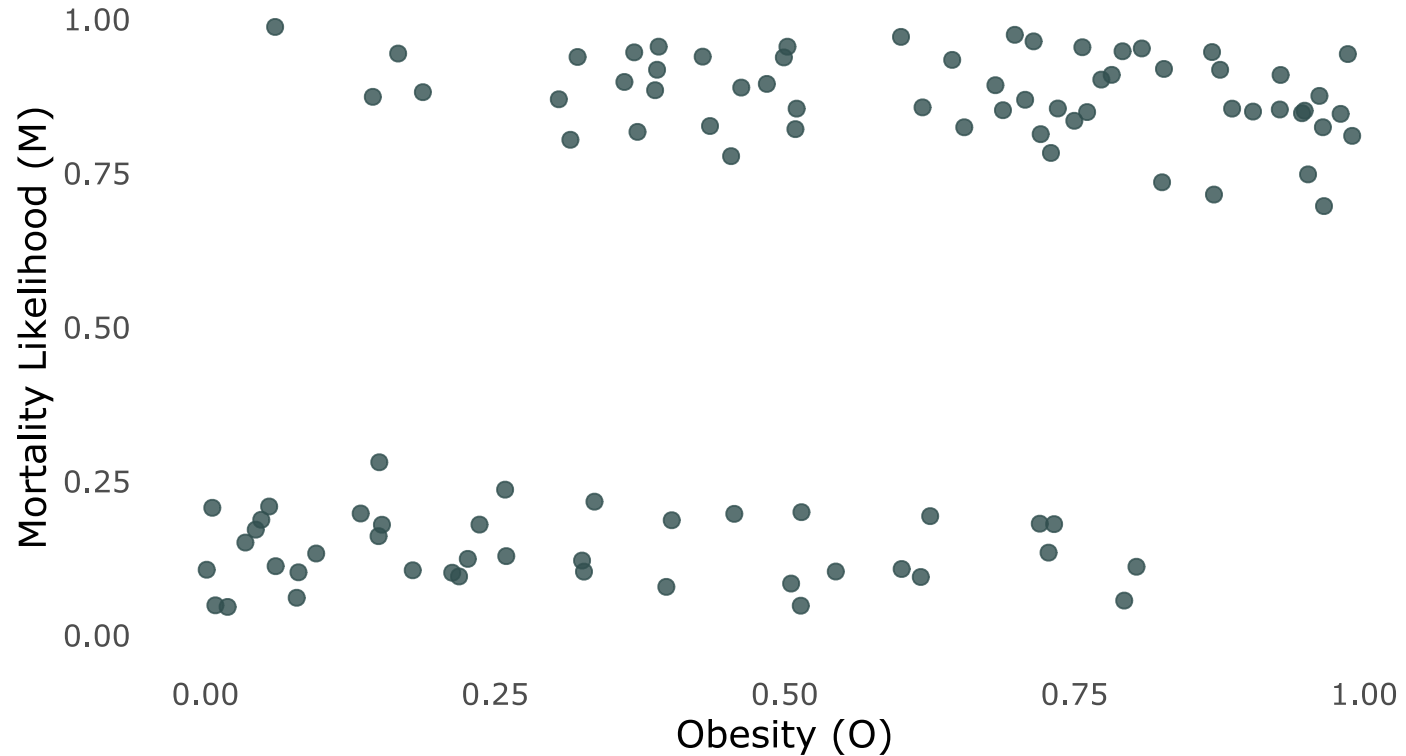
You can also see this example graphically...

Example Obesity, Cardiovascular Disease and Mortality.

Example Obesity, Cardiovascular Disease and Mortality.

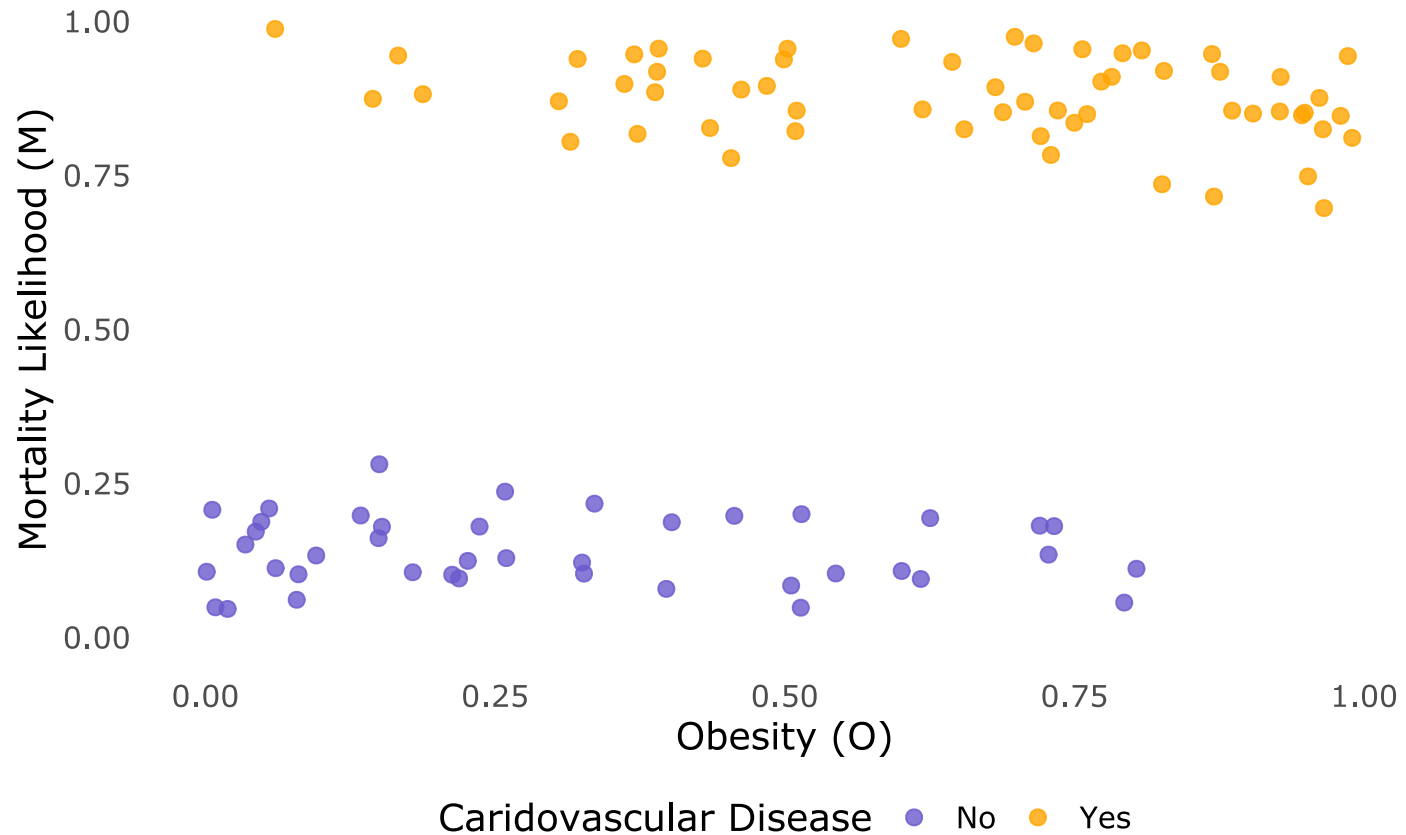


## Example Obesity, Cardiovascular Disease and Mortality.



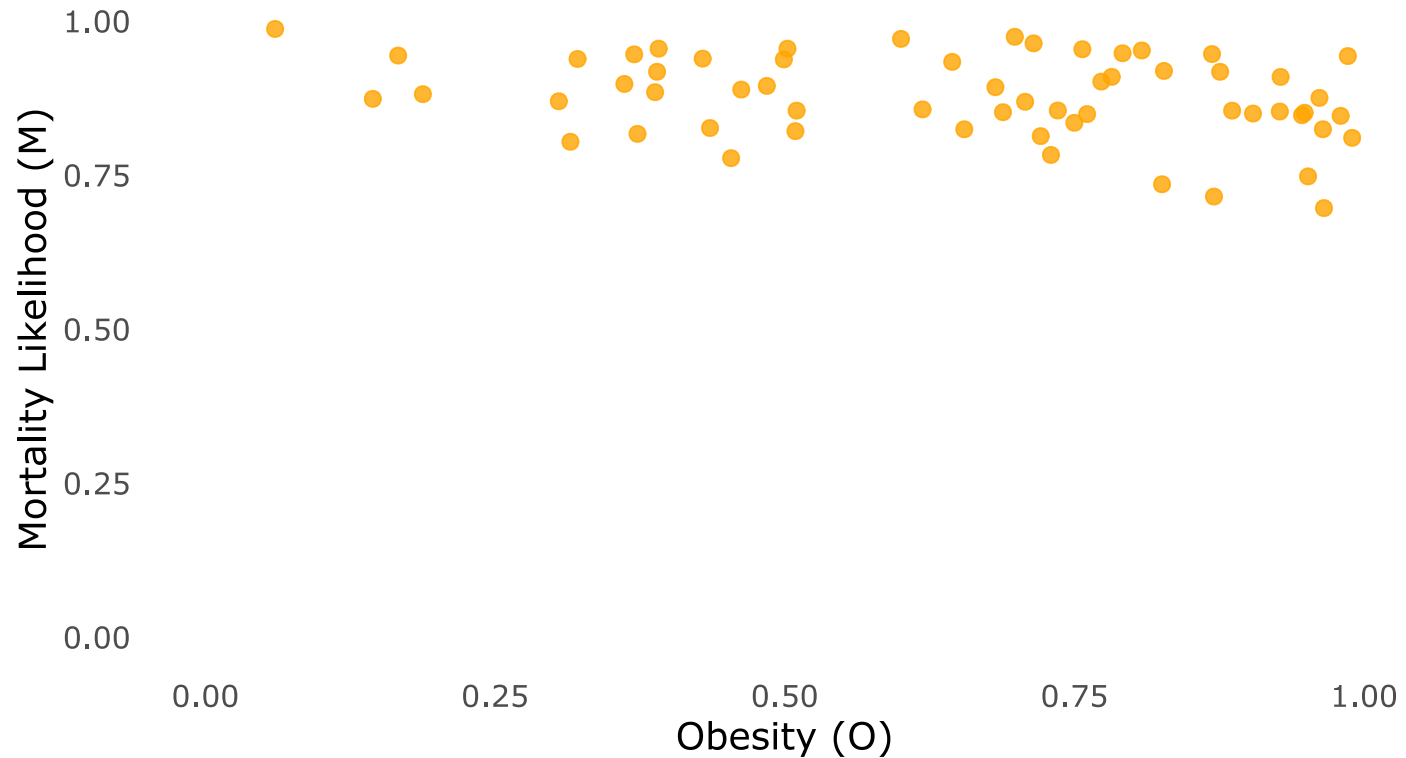
Without conditioning: Positive relationship between obesity and mortality.

## Example Obesity, Cardiovascular Disease and Mortality.



Recall: Our sample excludes individuals without cv-disease.

## Example Obesity, Cardiovascular Disease and Mortality.



Conditioning on **C**: Obesity now only increases mortality through Age and there are disproportionately large numbers of young obese patients with cvd.

Across the general population...

Across the general population...

```
lm(data = cb_dt, M ~ O) %>% tidy() %>% filter(term == 'O')
```

<b>term</b>	<b>estimate</b>	<b>std.error</b>	<b>statistic</b>	<b>p.value</b>
O	0.64	0.106	6.04	2.74e-08



Across the general population...

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term	estimate	std.error	statistic	p.value
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Only on patients with CVD...

```
lm(data = cb_dt %>% filter(C == 1), M ~ O) %>% tidy() %>% filter(term == 'O')
```

term	estimate	std.error	statistic	p.value
O	-0.0792	0.0339	-2.34	0.0229

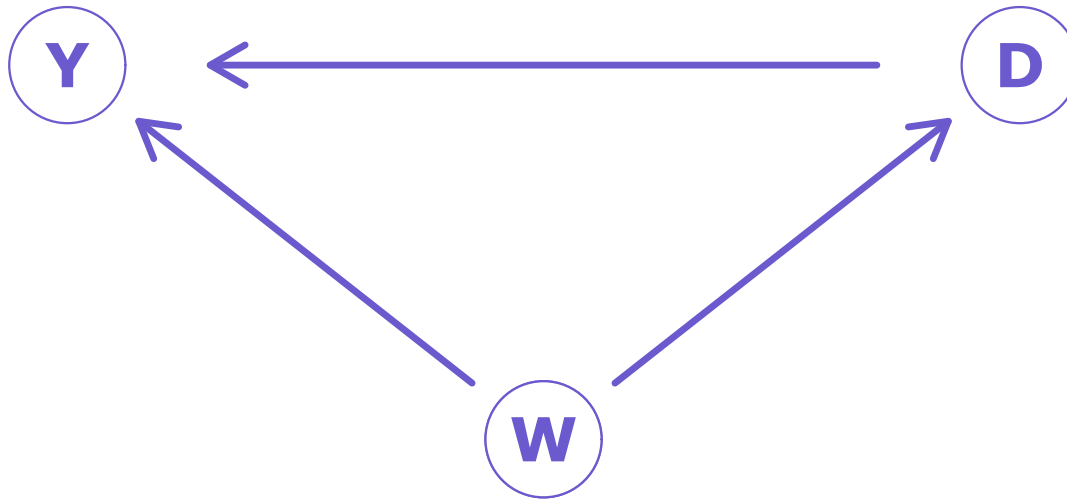
I like this example because it reminds us that conditioning occurs both explicitly (e.g., "controlling for") and implicitly (e.g., sample inclusion/exclusion).

This example of collider bias in hospitalization data comes from V. Viallon & M. Dufournet's 2016 paper [Can collider bias fully explain the obesity paradox?](#).

More generally: You'll hear this called selection bias or Berkson's paradox.

# Examples

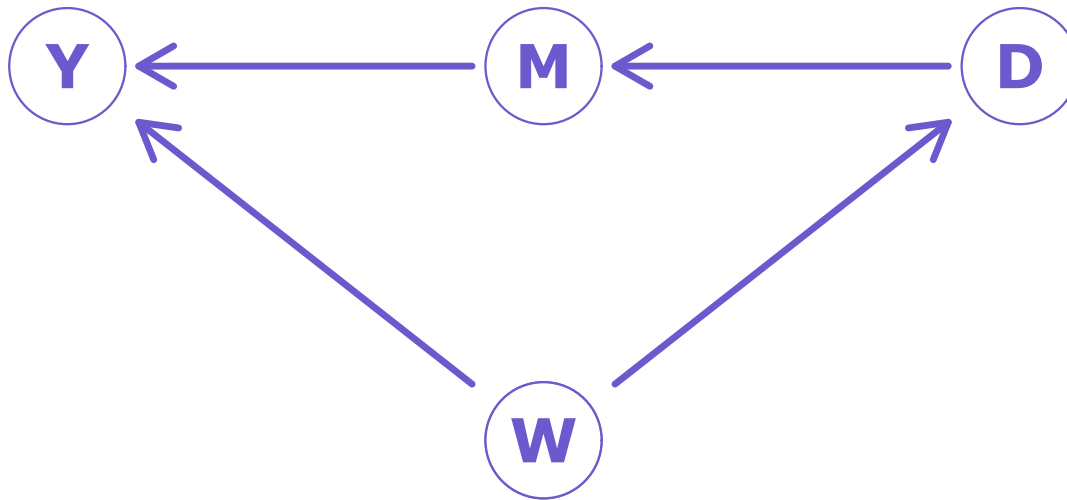
## Example 1: OVB



Q OVB using DAG fundamentals: When can we isolate causal effects?

## Example 2: Mediation

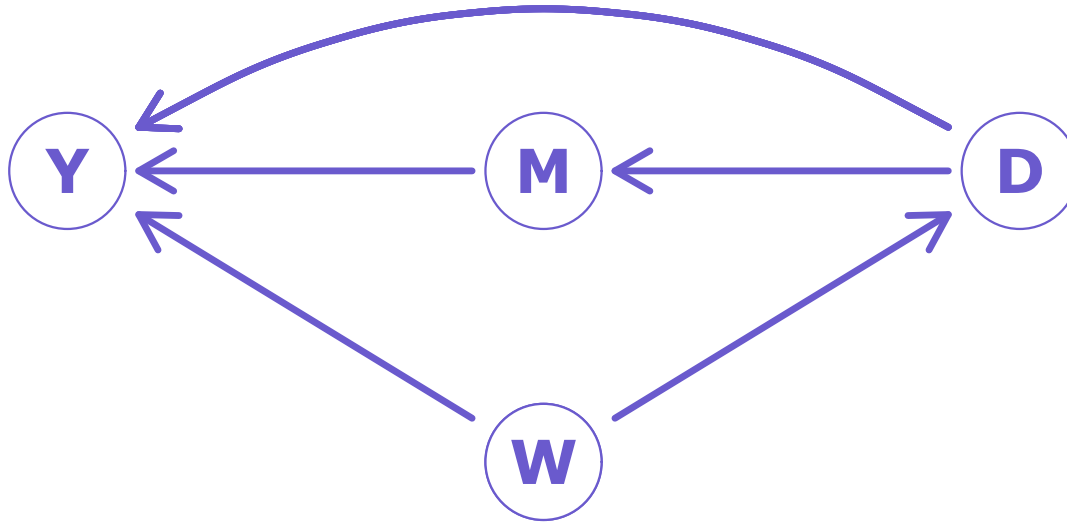
Here **M** is a **mediator**: it **mediates** the effect of **D** on **Y**.



Q<sub>1</sub> What do we need to condition on to get the effect of **D** on **Y**?

Q<sub>2</sub> What happens if we condition on **W** and **M**?

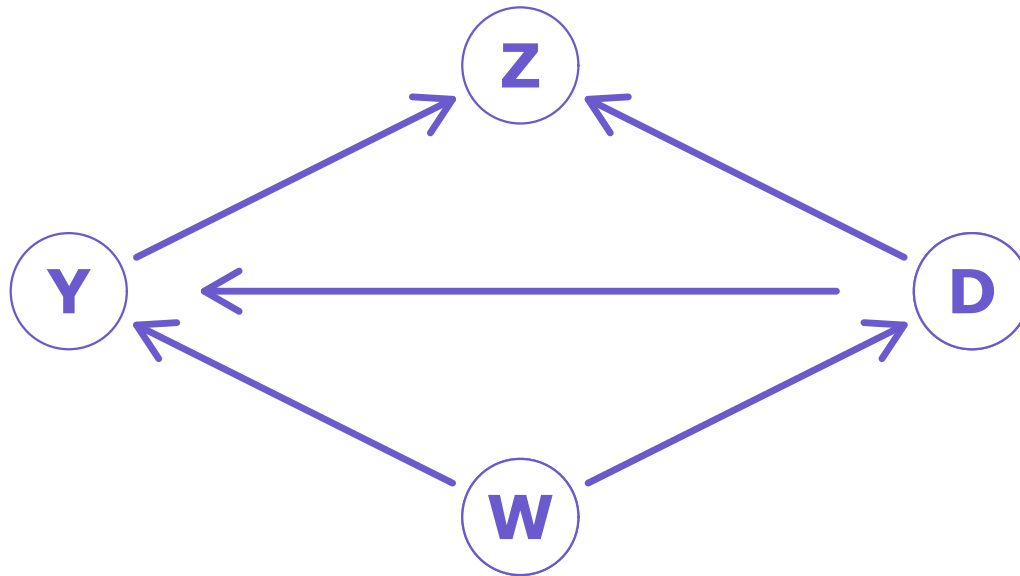
### Example 3: Partial mediation



Q<sub>1</sub> What do we need to condition on to get the effect of **D** on **Y**?

Q<sub>2</sub> What happens if we condition on **W** and **M**?

#### Example 4: Non-mediator descendants

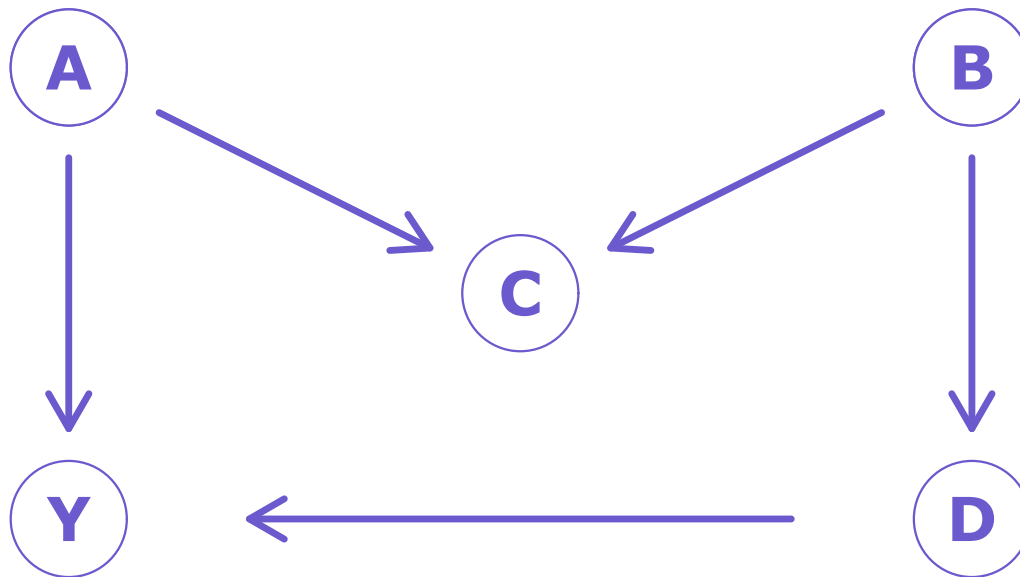


Q<sub>1</sub> What do we need to condition on to get the effect of **D** on **Y**?

Q<sub>2</sub> What happens if we condition on **W** and/or **Z**?

## Example 5 M-Bias

Notice that **C** here is not a result of treatment (could be "pre-treatment").



Q<sub>1</sub> What do we need to condition on to get the effect of **D** on **Y**?

Q<sub>2</sub> What happens if we condition on **W** and/or **Z**?



One more note:

DAGs are often drawn without "noise variables" (disturbances).

But they still exist—they're just "outside of the model."

# Experiments in SCM

# Experiments

So - how do we think about a randomized experiment with a SCM?

- Recall, an experiment induces random noise in a variable that is unrelated to other causal factors

We can think about an experiment **deleting** the edges out of the experiment. Let's return to our fertilizer example.

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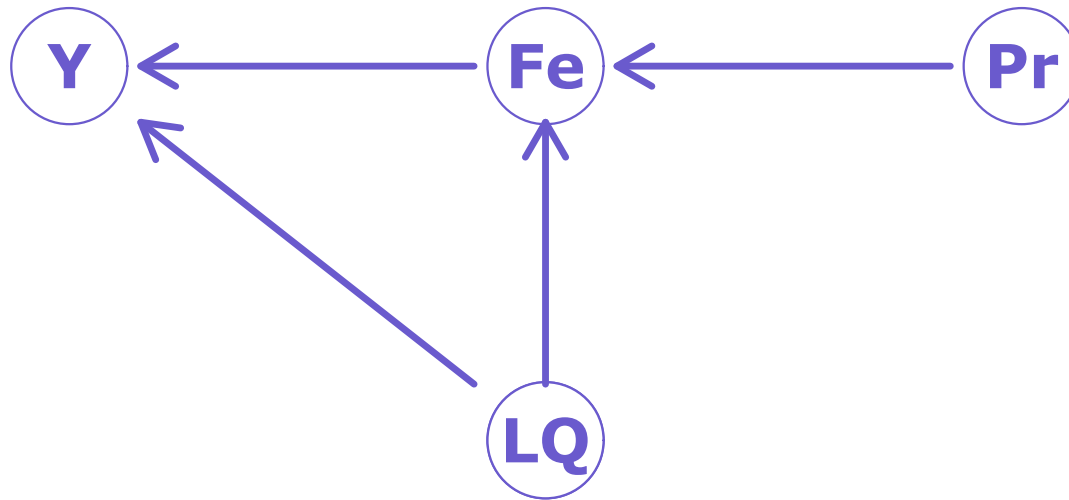
We can think about an experiment **deleting** the edges out of the experiment. Let's return to our fertilizer example.

If we induce random noise in fertilizer - we know that no effect that **causes** fertilizer placement is related to the random placement

But if our experiment works, then things **caused** by fertilizer should still be impacted

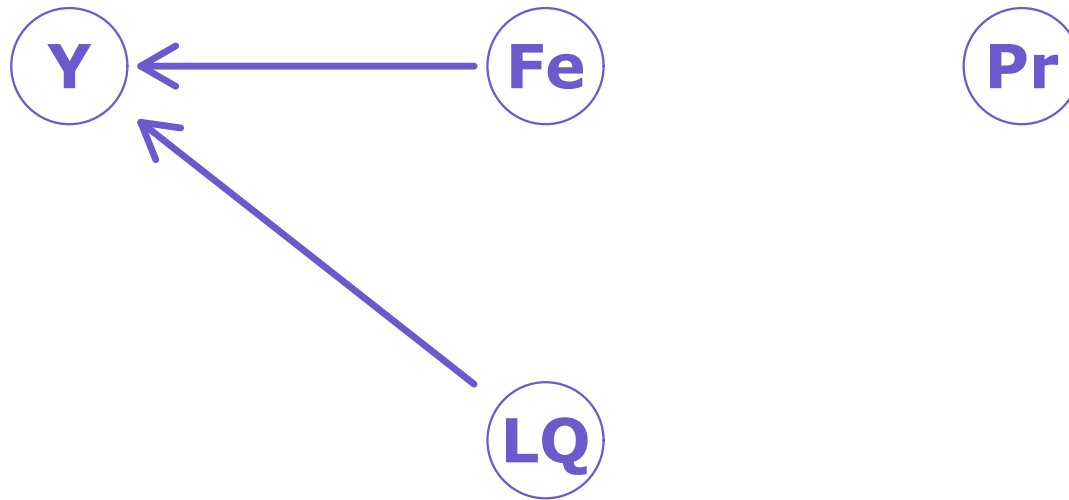
# Experiments

Fertilizer on Yield: pre-experiment



# Experiments

Fertilizer on Yield: post-experiment



# DAGs

## Limitations

So what can't DAGs do (well)?



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Philosophy  $\rightarrow$  DAGs/Epidemiology  $\leftarrow$  Economics

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Philosophy  $\rightarrow$  DAGs/Epidemiology  $\leftarrow$  Economics

- Functional Form: All of the existing tools are functionally indifferent - but functional forms are useful (as we've seen.)

# Sources

## Thanks

These notes rely heavily upon Brady Neal's *Introduction to Causal Inference*.

I also borrow from Scott Cunningham's *Causal Inference: The Mixtape*.

I found the Sackett (1978) example on the "Catalog of Bias" website.

# Table of contents

## Admin

- Today and upcoming

## Other

- Sources

## DAGs

- What's a DAG?
- Example
- Graphs
  - Definition/undirected
  - Directed
  - Cycles

## DAGs continued

# Math and Probability

- Let's quickly review some probability concepts (we'll need them for this section)



# Theory

## Brief Probability Review

We can decompose any joint probability into a conditional probability and its product

a.)  $P(A \cap B) = P(A|B) P(B)$

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$$\text{b.) } P(A|B) = \frac{P(A \cap B)}{P(B)}$$

We can substitute a into b and get **bayes rule** - ditching the intersection notation

$$P(B|A) = \frac{P(A|B) P(B)}{P(A)}$$

# Theory

## Product Rule

The important piece of this however is the **product rule**.

By using our definitions from before, we can decompose larger and larger joint probability distributions, like so -

$$2 \quad P(x_1, x_2) = P(x_1)P(x_2|x_1)$$

$$3 \quad P(x_1, x_2, x_3) = P(x_1)P(x_2, x_3|x_1) = P(x_1)P(x_2|x_1)P(x_3|x_2, x_1)$$

$$\vdots$$

$$n \quad P(x_1, x_2, \dots, x_n) = P(x_1) \prod_{i=2}^n P(x_i|x_{i-1}, \dots, x_1)$$

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$$\vdots$$

$$n \quad P(x_1, x_2, \dots, x_n) = P(x_1) \prod_{i=2}^n P(x_i|x_{i-1}, \dots, x_1)$$

This final product can include *a lot* of terms.

E.g., even when  $x_i$  are binary,  $P(x_4|x_3, x_2, x_1)$  requires  $2^3 = 8$  parameters.

# Theory

## Local Markov

This intuitive approach is the [Local Markov Assumption](#)

Given its parents in the DAG, a node  $X$  is independent of all of its non-descendants.

# Theory

## Local Markov

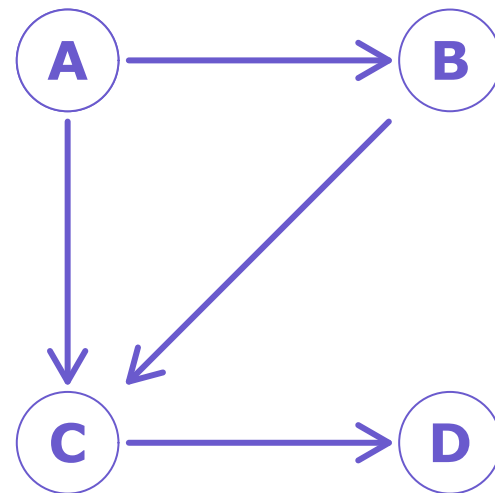
This intuitive approach is the [Local Markov Assumption](#)

Given its parents in the DAG, a node  $X$  is independent of all of its non-descendants.

Ex. Consider the DAG to the right:

With the Local Markov Assumption,  
 $P(D|A, B, C)$  simplifies to  $P(D|C)$ .

Conditional on its parent (C),  
D is independent of A and B.



# Local Markov and factorization

The Local Markov Assumption is equiv. to [Bayesian Network Factorization](#)

For prob. dist.  $P$  and DAG  $G$ ,  $P$  factorizes according to  $G$  if

$$P(x_1, \dots, x_n) = \prod_i P(x_i | \text{pa}_i)$$

where  $\text{pa}_i$  refers to  $x_i$ 's parents in  $G$ . Bayesian network factorization is also called *the chain rule for Bayesian networks* and *Markov compatibility*.

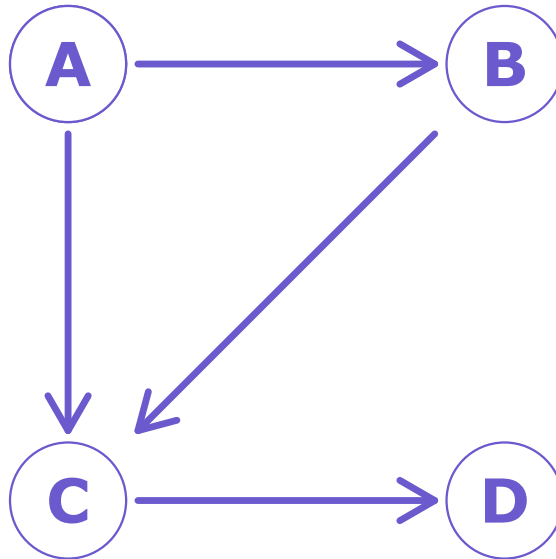


# Theory

## Factorize!

You can now (more easily) factorize a causal system!

- Let's think about this graph...



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- Let's think about this graph...

Factorization via B.N. chain rule

$$\begin{aligned} P(A, B, C, D) \\ &= \prod_i P(x_i | \text{pa}_i) \\ &= P(A)P(B|A)P(C|A, B)P(D|C) \end{aligned}$$

# Theory

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Association flows along unblocked paths.

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## d-separation and d-connected(-ness)

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**Connection:** If there is at least **one path** between  $\mathbf{X}$  and  $\mathbf{Y}$  that is **unblocked**, then  $\mathbf{X}$  and  $\mathbf{Y}$  are **d-connected**.



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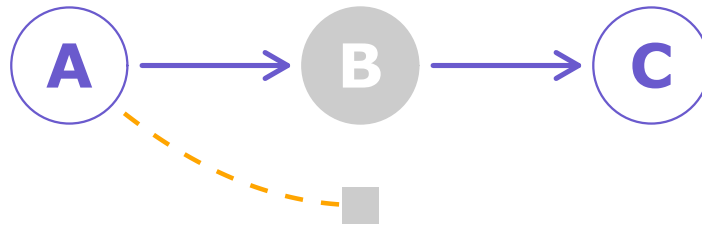
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# Theory

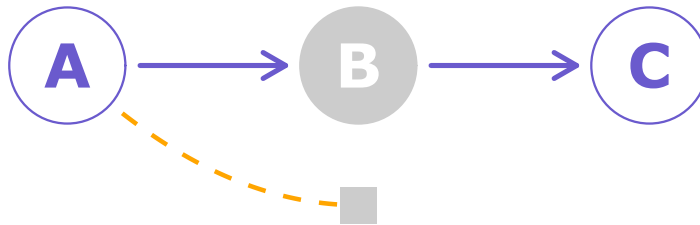
Building block 3: Chains with conditions



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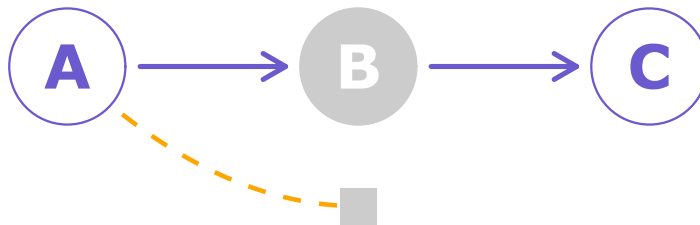


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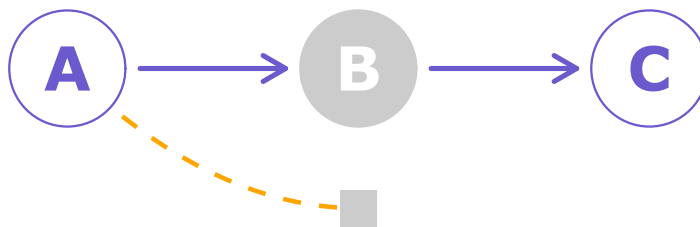


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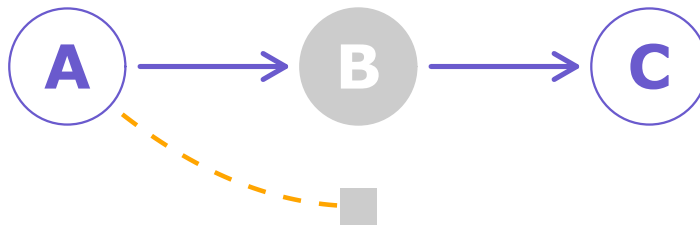
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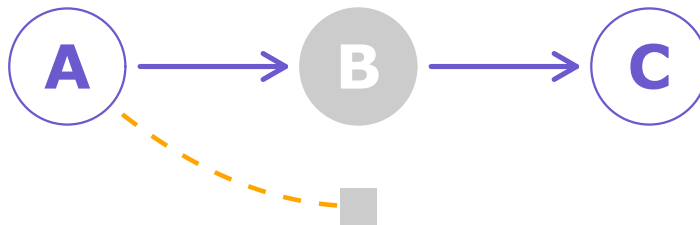
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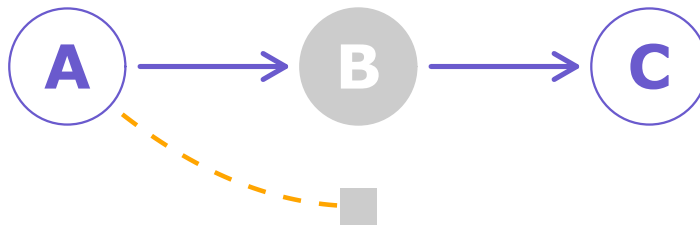
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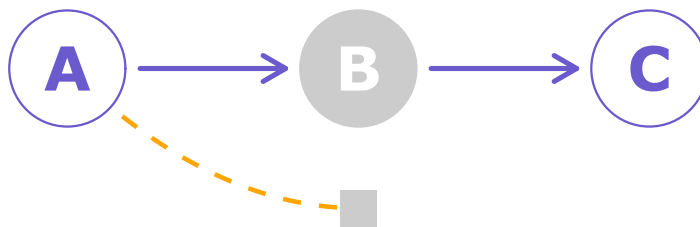
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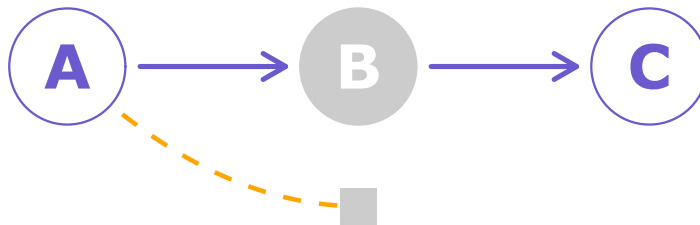
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