CMPS 2200: Discussion 2

Thursday, Sep 3

We will work on the problem of *matrix multiplication* in this discussion. Matrix multiplication is an essential numerical operation that is ubiquitous. The simple problem of multiplying two matrices comes in thousands of problems in areas such as graphics, visualization, numerical analysis and many more. An n by n matrix A can be indexed just like an array: the element a_{ij} denotes the entry of A in the ith row and jth column. So,

$$A = \begin{pmatrix} a_{1,1} & a_{1,2} & \cdots & a_{1,n} \\ a_{2,1} & a_{2,2} & \cdots & a_{2,n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n,1} & a_{n,2} & \cdots & a_{n,n}. \end{pmatrix}$$

Consider two n by n matrices A and B with $A \cdot B = C$. The entry c_{ij} in C is defined as the dot product of row i of A and column j of B so that

$$c_{ij} = \sum_{k=1}^{n} a_{ik} \cdot b_{kj}.$$

We'll work on some algorithms to compute C.

1. One straightforward algorithm to compute C is to just compute each of the $O(n^2)$ elements and output them all. Write this algorithm in SPARC. As defined, what is the work and span of matrix multiplication? Is this approach parallelizable?

2. Let's devise a divide and conquer algorithm for matrix multiplication. First, view the matrices A and B as each consisting of 4 equally sized submatrices. So,

   $$A = \begin{pmatrix} A_{1,1} & A_{1,2} \\ A_{2,1} & A_{2,2} \end{pmatrix}$$

   and

   $$B = \begin{pmatrix} B_{1,1} & B_{1,2} \\ B_{2,1} & B_{2,2} \end{pmatrix}$$

   Can you define C in terms of these submatrices?

3. Now, come up with a divide and conquer algorithm suggested by this definition of C and provide the SPARC specification. What is the work and span of this algorithm?

4. How does the divide and conquer algorithm compare to our initial algorithm? Which seems easier to work with, to you?