

1 a. No, the runtime of  $2^n$  is definitely less than  $2^{n+1}$  at any points, that means  $2^{n+1} \notin O(2^n)$

1 b. No, the runtime of  $2^{2^n}$  is increasing more than  $2^n$  as  $n$  goes up. that means  $2^{2^n} \notin O(2^n)$   
( $2^n$  is growing faster than  $n$  as well)

1 c. No,  $n^{1.01}$  is linear increasing,  $\log^2 n$  is increasing in logarithmic, which is far slower than linear  
that means  $n^{1.01} \notin O(\log^2 n)$

1 d. Yes,  $\log^2 n$  is asymptotically dominated by  $n^{1.01}$  as just mentioned above, that means  $n^{1.01} \in \Omega(\log^2 n)$

1 e. No,  $\sqrt{n}$  is exponential function where  $\log n^3$  is logarithmic function, since exponential increase faster than logarithmic  
 $\sqrt{n}$  is asymptotically dominate  $\log n^3$ , so  
 $\sqrt{n} \notin O(\log n^3)$

1 f. Yes, as describe above,  $\sqrt{n} \in \Omega(\log n^3)$

1g. if  $\exists f(n) \in O(g(n))$ ,  $f(n) \in \omega(g(n))$

$$\Rightarrow f(n) \leq c \cdot g(n) \text{ and } f(n) \geq c \cdot g(n)$$

for every  $c > 0$ , that means  $c \cdot g(n) < f(n) < c \cdot g(n)$   
which is impossible

the set of it should be empty

2b. Find out the  $(x+1)^{\text{th}}$  number in the fibonacci sequence

$$\begin{aligned} 3b. \text{ work} &= O(n) + C \\ \text{span} &= O(n) + C \end{aligned}$$

$$\begin{aligned} 3d. \text{ work} &= O(2 \lg n) + C \\ \text{span} &= O(\lg n) + C \end{aligned}$$

$$\begin{aligned} 3f. \text{ work} &= O(2 \lg n) + C \\ \text{span} &= O(\lg n) + C \end{aligned}$$