CMPS 2200 Assignment 1

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In this assignment, you will learn more about asymptotic notation, parallelism, functional languages, and algorithmic cost models. As in the recitation, some of your answer will go here and some will go in `main.py`. You are welcome to edit this `assignment-01.md` file directly, or print and fill in by hand. If you do the latter, please scan to a file `assignment-01.pdf` and push to your github repository.

1. **Asymptotic notation**

- 1a. Is $2^{n+1} \in O(2^n)$? Why or why not? .
  Yes, because $2^{n+1} = 2 \cdot 2^n = O(2^n)$
- 1b. Is $2^{2^n} \in O(2^n)$? Why or why not?
  No, because if we assume that $2^{2n} = O(2^n)$, then there is a constant c such that for n beyond some n_0, $2^{2n} <= c2^n$. If we have both sides divided by 2^n, we get $2^n < c$, which cannot be true in any case
- 1c. Is $n^{1.01} \in O(\log^2 n)$?
  No, if we use L'Hoptals rule, we can see that $\lim_{n\to\infty} \frac{n^{1.01}}{(\log n)^2} = \lim_{n\to\infty} \frac{n^{.01}}{2\log n} = \infty$. c cannot be $> \infty$ so it refutes the statement
- 1d. Is $n^{1.01} \in \Omega(\log^2 n)$?
  Yes, if we use L'Hopitals rule, we can see that $\lim_{n\to\infty} \frac{n^{1.01}}{(\log n)^2} = \lim_{n\to\infty} \frac{n^{.01}}{2\log n} = \infty$. This proves to be true as $g(n)$ asymptotically dominates $f(n)$
- 1e. Is $\sqrt{n} \in O((\log n)^3)$?
  Yes, If we use L'Hopitals rule, we can see that $\lim_{n\to\infty} \frac{\sqrt{n}}{(\log n)^3} = \frac{\frac{1}{2}n^{-1/2}}{3(\log n)^2} = \frac{1}{6\sqrt{n}(\log n)^2} = 0$. This clearly is zero and c can be ≥ 0 which means that $g(n)$ is dominated by $f(n)$.
- 1f. Is $\sqrt{n} \in \Omega((\log n)^3)$?
  No, If we use L'Hopitals rule, we can see that $\lim_{n\to\infty} \frac{\sqrt{n}}{(\log n)^3} = \frac{1/2\, n^{-1/2}}{3(\log n)^2} = \frac{1}{6\sqrt{n}(\log n)^2} = 0$. This refutes the statement because $c \geq 0$ and it means that $\sqrt{n}$ is dominated by $(\log n)^3$
- 1g. Consider the definition of "Little o" notation:

$g(n) \in o(f(n))$ means that for **every** positive constant c, there exists a constant n_0 such that $g(n) \leq c \cdot f(n)$ for all $n \geq n_0$. There is an analogous definition for "little omega" $\omega(f(n))$. The distinction between $o(f(n))$ and $O(f(n))$ is that the former requires the condition to be met for **every** c, not just for some c. For example, $10x \in o(x^2)$, but $10x^2 \notin o(x^2)$.

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Prove that $o(g(n)) \cap \omega(g(n))$ is the empty set.

If we assign o(g(n)) where f(n) ∈ o(g(n)) and assign ω(g(n)) where B(n) ∈ ω(g(n)), we can then use the definitions of f(n) < c · g(n) and B(n) > c · g(n).
We can rewrite the two statements together as
f(n) < c · g(n) < B(n). This shows that there are no shared values between f(n) and B(n)

2. **SPARC to Python**

Consider the following SPARC code:

```
foo x =
  if  x ≤ 1  then
    x
  else
    let  (ra, rb) = (foo (x − 1))  ,  (foo (x − 2))  in
      ra + rb
    end.
```

- 2a. Translate this to Python code – fill in the `def foo` method in `main.py`
- 2b. What does this function do, in your own words?

Fibonacci Sequence. The function recursively iterates through a Fibonacci sequence and it is a sequence that takes two previous values in a list starting at 1 and sums them.

3. **Parallelism and recursion**

Consider the following function:

```python
def longest_run(myarray, key)
    """
     Input:
       `myarray`: a list of ints
       `key`: an int
     Return:
       the longest continuous sequence of `key` in `myarray`
    """
```

E.g., `longest_run([2,12,12,8,12,12,12,0,12,1], 12) == 3`

- 3a. First, implement an iterative, sequential version of `longest_run` in `main.py`.
- 3b. What is the Work and Span of this implementation?

W(n) = O(n) → Linear

S(n) = O(n) → Linear

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- 3c. Next, implement a `longest_run_recursive`, a recursive, divide and conquer implementation. This is analogous to our implementation of `sum_list_recursive`. To do so, you will need to think about how to combine partial solutions from each recursive call. Make use of the provided class `Result`.
- 3d. What is the Work and Span of this sequential algorithm?

$W(\text{longest\_run\_recursive()}) = O(\log(n))$

$S(\text{longest\_run\_recursive()}) = O(\log(n))$

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- 3e. Assume that we parallelize in a similar way we did with `sum_list_recursive`. That is, each recursive call spawns a new thread. What is the Work and Span of this algorithm?

$W = O(\log(n))$

$S = 2w(n/2) + O(n)$

They are different now because we are parallelizing