1a. $f(n) \to g(n)$ $\lim_{n\to\infty} \frac{2^{n+1}}{2^n} <= C$ $\frac{2 \cdot 2^n}{2^n} <= C$

yes, it is $2 <= C$ ✓

1b. $\lim_{n\to\infty} \frac{2^{2n}}{2^n} <= C$ $\log 2^{2n} - \log 2^n <= \log c$

$\lim_{n\to\infty} 2n - n <= \log c$ $\lim_{n\to\infty} n = \log c$

FALSE

1c. $\lim_{n\to\infty} \frac{n^{1.01}}{\log^2 n} <= C$ $\cancel{n^{1.01} <= C \log(\log(n))}$ $n^{1.01} = C\log(\log(n))$

$n^{1.01} = \log(\log(n))^C$

$\cancel{n^{1.01} = \log(\log(n))^C}$ $\frac{1.01 n^{.01}}{2\log n}$ $2^{n^{1.01}}$

$\cancel{2^{n^{1.01}} = (\log(n))^C}$

$\frac{n^{1.01}}{C} = \log(\log(n))$ $2^{\frac{n^{1.01}}{C}} = \log(n)$

$n^{1.01} = C \cdot \log(\log(n))$

$\frac{2^{\frac{n^{1.01}}{C}}}{\log(n)} = 1$ L'H $\frac{C \cdot 1.01 \cdot n^{.01} - n^{1.01} \cdot 0}{C^2}$

1c. $\lim_{n\to\infty} \frac{n^{1.01}}{\log^2 n} \overset{L'H}{\to} \frac{1.01 \cdot n^{.01}}{2\log n \cdot \frac{1}{n}} \to \frac{1.01 \cdot n^{1.01}}{2\log n}$

$\frac{1.01^2 \cdot n^{.01}}{2 \cdot \frac{1}{n}} = \frac{1.01^2}{2} \cdot n^{1.01} = \infty$

FALSE

1d. This is true b/c the limit is infinite.

1e. $\lim \frac{\sqrt{n}}{(\log n)^3} \to \frac{\frac{1}{2} n^{-\frac{1}{2}}}{3(\log n)^2 \cdot \frac{1}{n}} \to \frac{\sqrt{n}}{6(\log n)^2}$

$\to \frac{n}{24 \log n \cdot \sqrt{n}} \Rightarrow \frac{\frac{1}{2} n^{-\frac{1}{2}}}{24 \cdot \frac{1}{n}} = 48 \cdot \sqrt{n} \Rightarrow \infty$

False, limit is ∞

1f. This is true b/c the limit ~~the~~ is infinite.

1g. $O(g(n)) = \{ f(n) \mid f(n) \le c \cdot g(n), \forall n \ge n_0 \}$

$\omega(g(n)) = \{ g(n) \mid g(n) \ge c \cdot g(n), \forall n \ge n_0 \}$

$f(n) < c \cdot g(n) < g(n)$, since $O(g(n)) <$ $\omega(g(n))$, the set is empty

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2b. The recursive function adds the returned value of the previous two functions. For instance, b/c foo(2) = 1 and foo(3) = 2, foo(4) = 1 + 2 = 3, foo(5) = 3 + 2 = 5, and so on.

3b. Work $\to O(n)$

Span $\to O(n)$

3d. Work $\to O(n)$

Span $\to O(n)$

3e. Work $\to O(n)$

Span $\to O(\log n)$