

CMPS 2200 Assignment 1

Name: _____

In this assignment, you will learn more about asymptotic notation, parallelism, functional languages, and algorithmic cost models. As in the recitation, some of your answer will go here and some will go in `main.py`. You are welcome to edit this `assignment-01.md` file directly, or print and fill in by hand. If you do the latter, please scan to a file `assignment-01.pdf` and push to your github repository.

1. Asymptotic notation

- 1a. Is $2^{n+1} \in O(2^n)$? Why or why not?

$$\text{if } 2^{n+1} \in O(2^n), \text{ then } 2^{n+1} \leq C \cdot (2^n).$$

$$2^n \cdot 2 \leq C \cdot 2^n \text{ for some value of } C$$

$$C = 3, 2^n \cdot 2 \leq 2^n \cdot 3 \checkmark \text{ therefore } 2^{n+1} \in O(2^n)$$

- 1b. Is $2^{2^n} \in O(2^n)$? Why or why not?

$$2^{2^n} \in O(2^n) = 2^{2^n} \in O(2^n) \cdot C, \therefore 2^{2^n} \leq 2^n \cdot C$$

$$\text{for } n=1 \text{ and } C=3$$

$$2 \leq 2 \cdot 3 \checkmark, \therefore 2^{2^n} \in O(2^n)$$

- 1c. Is $n^{1.01} \in O(\log^2 n)$?

$$\text{if } n^{1.01} \in O(\log n)^2, \text{ then for } f(n) \leq C \cdot g(n) \text{ where } f(n) = n^{1.01}, g(n) = \log n^2$$

$$\text{if } f(n) \leq C \cdot g(n), \lim_{n \rightarrow \infty} f(n) = \infty \text{ using L'Hopital's rule, } \lim_{n \rightarrow \infty} f'(n) = 1.01 n^{0.01}$$

$$\text{then } \lim_{n \rightarrow \infty} \frac{f(n)}{g(n)} \leq C \lim_{n \rightarrow \infty} g(n) = \infty$$

$$\text{if } n^{1.01} \in \Omega(\log^2 n), \lim_{n \rightarrow \infty} \frac{f(n)}{g(n)} = \infty$$

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- 1d. Is $n^{1.01} \in \Omega(\log^2 n)$?

$$\text{From above, we know:}$$

$$f'(n) = 1.01 n^{0.01}$$

$$\text{if } n^{1.01} \in \Omega(\log^2 n), \text{ then } \lim_{n \rightarrow \infty} \frac{f(n)}{g(n)} \geq C \text{ and } \infty \geq C, n^{1.01} \in \Omega(\log^2 n)$$

- 1e. Is $\sqrt{n} \in O((\log n)^3)$?

$$f(n) \leq C \cdot g(n), \text{ then } \lim_{n \rightarrow \infty} \frac{f(n)}{g(n)} \leq C \text{ using L'Hopital's rule}$$

$$\lim_{n \rightarrow \infty} f(n) = \infty, \lim_{n \rightarrow \infty} g(n) = \infty$$

$$\lim_{n \rightarrow \infty} f'(n) = \frac{1}{2\sqrt{n}}, \lim_{n \rightarrow \infty} g'(n) = 3 \ln n \cdot \frac{1}{x}$$

$$\lim_{n \rightarrow \infty} \frac{\frac{1}{2\sqrt{n}}}{3 \ln n \cdot \frac{1}{x}} = \frac{\infty}{\infty} = \infty \geq C$$

$$\sqrt{n} \text{ is not in } O(\log n)^3$$

- 1f. Is $\sqrt{n} \in \Omega((\log n)^3)$?

$$\text{if } \lim_{n \rightarrow \infty} \frac{f(n)}{g(n)} = \infty \text{ then } \infty \geq C \text{ therefore}$$

$$\sqrt{n} \in \Omega((\log n)^3)$$

- 1g. Consider the definition of "Little o" notation:

$g(n) \in o(f(n))$ means that for every positive constant c , there exists a constant n_0 such that $g(n) \leq c \cdot f(n)$ for all $n \geq n_0$. There is an analogous definition for "little omega" $\omega(f(n))$. The distinction between $o(f(n))$ and $O(f(n))$ is that the former requires the condition to be met for every c , not just for some c . For example, $10x \in o(x^2)$, but $10x^2 \notin o(x^2)$.

Prove that $o(g(n)) \cap \omega(g(n))$ is the empty set.

$o(g(n)) = O(g(n)) \setminus \Omega(g(n))$ which is all numeric functions less than $g(n)$.

$\omega(g(n)) = \Omega(g(n)) \setminus O(g(n))$ which is all numeric functions greater than $g(n)$.

If $o(g(n))$ is all functions less than $g(n)$ and $\omega(g(n))$ is all functions greater than $g(n)$, then there are no values possible to be in $o(g(n)) \cap \omega(g(n))$.

2. SPARC to Python

Consider the following SPARC code:

```
foo x =
    if x ≤ 1 then
        x
    else
        let (ra,rb) = (foo (x - 1)), (foo (x - 2)) in
            ra + rb
    end.
```

- 2a. Translate this to Python code – fill in the `def foo` method in `main.py`
- 2b. What does this function do, in your own words?

3. Parallelism and recursion

Consider the following function:

```
def longest_run(myarray, key)
"""
Input:
    `myarray`: a list of ints
    `key`: an int
Return:
    the longest continuous sequence of `key` in `myarray`
"""
```

E.g., `longest_run([2,12,12,8,12,12,12,0,12,1], 12) == 3`

- 3a. First, implement an iterative, sequential version of `longest_run` in `main.py`.
- 3b. What is the Work and Span of this implementation?

$$W(n) \in O(n^2)$$

$$S(n) \in O(n^2)$$

- 3c. Next, implement a `longest_run_recursive`, a recursive, divide and conquer implementation. This is analogous to our implementation of `sum_list_recursive`. To do so, you will need to think about how to combine partial solutions from each recursive call. Make use of the provided class `Result`.
- 3d. What is the Work and Span of this sequential algorithm?

$$W(n) = \Theta(n \log n) + cn$$

$$S(n) = \Theta(n)$$

$$W(n) = 4W(n/2) + cn$$

$$S(n) = S(n/2) + cn$$

$$W(n) = \Theta(n \log n)$$

$$S(n) = \Theta(n \log n)$$

- 3e. Assume that we parallelize in a similar way we did with `sum_list_recursive`. That is, each recursive call spawns a new thread. What is the Work and Span of this algorithm?

$$W(n) = 4W(n/2) + cn$$

$$S(n) = S(n/2) + cn$$