CMPS 2200 Assignment 1

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In this assignment, you will learn more about asymptotic notation, parallelism, functional languages, and algorithmic cost models. As in the recitation, some of your answer will go here and some will go in `main.py`. You are welcome to edit this `assignment-01.md` file directly, or print and fill in by hand. If you do the latter, please scan to a file `assignment-01.pdf` and push to your github repository.

1. **Asymptotic notation**

- 1a. Is $2^{n+1} \in O(2^n)$? Why or why not? .

Yes, because $2^{n+1} = 2 \cdot 2^n$, and since 2^n asymptotically dominates 2, we can say $2^{n+1} \in O(2^n)$

- 1b. Is $2^{2^n} \in O(2^n)$? Why or why not?

No, because there's no constant term that can be "pulled out" to get a scalar (like in 1a) or a lower-order term. That is to say, there exists no point $n = n_0$ where $2^{2^n} \leq c \cdot 2^n$ for all $n > n_0$.

- 1c. Is $n^{1.01} \in O(\log^2 n)$?

$\lim_{n\to\infty} \left(\frac{n^{1.01}}{\log^2 n}\right) = \lim_{n\to\infty} \left(\frac{1.01\, n^{-0.01}}{2/\log n}\right) = \lim_{n\to\infty} \left(\frac{1.01/n^{0.01}}{2/\log n}\right) = \lim_{n\to\infty} \left(\frac{1.01 \log n}{2 n^{0.01}}\right) = \frac{1.01}{2} \lim_{n\to\infty} \left(\frac{1/n}{1/100n^{99/100}}\right)$

Since both functions asymptotically dominate one another, $n^{1.01} \in \Theta(\log^2 n) \in O(\log^2 n)$. So, yes.

- 1d. Is $n^{1.01} \in \Omega(\log^2 n)$?

Yes, since $n^{1.01} \in \Theta(\log^2 n) \in \Omega(\log^2 n)$.

- 1e. Is $\sqrt{n} \in O((\log n)^3)$?

$\lim_{n\to\infty} \left(\frac{\sqrt{n}}{\log^3 n}\right) = \lim_{n\to\infty} \left(\frac{1/2\sqrt{x}}{3/\log n}\right) = \ldots$, we get the same pattern, so yes, $\sqrt{n} \in O((\log n)^3)$.

- 1f. Is $\sqrt{n} \in \Omega((\log n)^3)$?

Yes, since like before $\sqrt{n} \in \Theta((\log n)^3) \in \Omega((\log n)^3)$

- 1g. Consider the definition of "Little o" notation:

$g(n) \in o(f(n))$ means that for **every** positive constant c, there exists a constant n_0 such that $g(n) \leq c \cdot f(n)$ for all $n \geq n_0$. There is an analogous definition for "little omega" $\omega(f(n))$. The distinction between $o(f(n))$ and $O(f(n))$ is that the former requires the condition to be met for **every** c, not just for some c. For example, $10x \in o(x^2)$, but $10x^2 \notin o(x^2)$.

Prove that $o(g(n)) \cap \omega(g(n))$ is the empty set.

Let $o(g(n)) \cap \omega(g(n)) \neq \emptyset$. Then: $\forall_{c \in \mathbb{R}^+} \exists_{n_0} \left(\forall_{n \geq n_0} \left(c \cdot g(n) \leq g(n) \leq c \cdot g(n) \right) \right)$.
Thus $\overset{\text{for } n \geq n_0}{\vee}$ $g(n) = c \cdot g(n)$, which is only true when $c = 1$ and not true for all $c \in \mathbb{R}^+$.
Therefore it is not the case that $\forall_{c \in \mathbb{R}^+} \exists_{n_0} (\forall_{n \geq n_0} (c \cdot g(n) \leq g(n) \leq c \cdot g(n)))$, and thus $o(g(n)) \cap \omega(g(n)) \neq \emptyset$ is false, meaning $o(g(n)) \cap \omega(g(n)) = \emptyset$.

2. **SPARC to Python**

Consider the following SPARC code:

```
foo x =
  if x ≤ 1 then
    x
  else
    let (ra, rb) = (foo (x − 1)) , (foo (x − 2)) in
      ra + rb
    end.
```

- 2a. Translate this to Python code – fill in the `def foo` method in `main.py`
- 2b. What does this function do, in your own words?

Given an integer, it outputs the next number in the Fibonacci sequence. It's a recursive definition of the Fibonacci sequence.

3. **Parallelism and recursion**

Consider the following function:

```
def longest_run(myarray, key)
  """
   Input:
     `myarray`: a list of ints
     `key`: an int
   Return:
     the longest continuous sequence of `key` in `myarray`
  """
```

E.g., `longest_run([2,12,12,8,12,12,12,0,12,1], 12) == 3`

- 3a. First, implement an iterative, sequential version of `longest_run` in `main.py`.
- 3b. What is the Work and Span of this implementation?

$W_{\text{longest\_run\_seq}}(n) \in O(n)$
$S_{\text{longest\_run\_seq}}(n) \in O(n)$

- 3c. Next, implement a `longest_run_recursive`, a recursive, divide and conquer implementation. This is analogous to our implementation of `sum_list_recursive`. To do so, you will need to think about how to combine partial solutions from each recursive call. Make use of the provided class `Result`.
- 3d. What is the Work and Span of this sequential algorithm?

$W_{longest\_run\_recursive\_seq}(n) \in O(\log n)$

$S_{longest\_run\_recursive\_seq}(n) \in O(\log n)$

- 3e. Assume that we parallelize in a similar way we did with `sum_list_recursive`. That is, each recursive call spawns a new thread. What is the Work and Span of this algorithm?

$W_{longest\_run\_recursive\_par}(n) \in O(\log n)$

$S_{longest\_run\_recursive\_par}(n) \in O(\log n)$