

# Discrete Structures: Data Containers CMPSC 102

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Fall 2022  
Week 8  
Slides 02

# What to study

- **Date:** 26 Oct 2022, After Gator Day
- During class, open notes
- **Study: Slides, notes, with chapters to add detail to class material**
- Python basics and code
  - Determining output
  - Picking out bugs from code; fixing code
  - Study the code from the practicals and material covered in class to understand the how programs worked.
  - Lambda functions, lists, dictionaries,  $n$ -tuples
  - for and while loops
  - Iterations over sequences
  - Sequences, strings, sets
  - Conditional statements
  - And other concepts covered during class

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## Key Questions

How do I employ the mathematical concepts of **sequences**, **monoids**, and **lists** to implement efficient Python programs that use functions with a **clearly specified behavior** to perform tasks like finding a name in a file or computing the arithmetic mean of data values?

## Learning Objectives

To **remember** and **understand** some the concept of a **monoid**, seeing how it connects to **practical applications** with strings and sequences

# A Quick Definition

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## Monoid Definition

In Abstract Algebra, a **monoid** is a set equipped with an **associative** binary operation and an **identity** element. For example, the non-negative integers with addition form a monoid, the identity element being 0.

- A monoid is a combination of an object  $(a,b,c)$  and an operation  $(+)$  that meets the following conditions
  - the operation on two of the objects produces a new object of the same kind
    - $\text{int} + \text{int} = \text{int}$
  - associative operations
    - $(a+b) + c = a + (b+c)$
  - a null object  $e$  must exist, such that  $e + a = a + e = a$ 
    - $n + 0 = n$

# What are the benefits of the monoid concept?

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Ask yourself

- Generalizes the behavior of structures
- Offers an archetype for understanding
- Logical foundation for approach to code
- **And provides a better and more logical flow to your code for others to follow?!**

# Summations

## Adding

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Ask yourself

```
standard_list = [1, 2, 3, 4, 5]
```

```
reversed_list = [5, 4, 3, 2, 1]
```

```
sum_list = sum(standard_list)
```

```
sum_reversed_list = sum(reversed_list)
```

- Summation(i.e, **adding**): Remember that the order does not matter for positive values being added
- `sum` is a built-in function provided by Python and is used for lists
- What is the output of this program segment?

# Products

## Multiplying

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Ask yourself

```
import math
standard_list = [1, 2, 3, 4, 5]
reversed_list = [5, 4, 3, 2, 1]
product_list = math.prod(standard_list)
product_reversed_list = math.prod(reversed_list)
```

- Products (i.e, **multiplying**): Remember that the order does not matter for positive values being added
- `math.prod` is a built-in function provided by Python's math library and is used for lists
- What is the output of this program segment?

## CSV File Containing Population Data

```
1972-01-01,84.700
1973-01-01,85.500
1974-01-01,86.100
1975-01-01,87.000
1976-01-01,87.600
1977-01-01,87.600
1978-01-01,88.000
```

- CSV file stores ordered pairs of **dates** and **population counts**
- Both lists and tuples are examples of **sequences**
- A tuple is an **immutable** data container
- A list is a **mutable** data container
- What are the **trade-offs** when using these containers?



# Using Mutable Lists in Python

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Ask yourself

```
data_number_list = []  
    for line in data_text.splitlines():  
        ordered_pair = line.split(",")  
        data_number_list.append(float(  
            ordered_pair[1]))  
    return data_number_list
```

- This source code parses the CSV file and extracts content
- What is the purpose of `ordered_pair[1]` ?
- Does this source code use a tuple or a list?
- What are the differences between lists and tuples?

# Playing With Code

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Ask yourself

## Data from the "file"

```
data_text = ""1972-01-01,84.700
1973-01-01,85.500
1974-01-01,86.100
1975-01-01,87.000
1976-01-01,87.600
1977-01-01,87.600
1978-01-01,88.000
""
print(data_text)
data_text
```

- What does this code do?

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Ask yourself

```
for line in data_text.splitlines():  
    print(f"line:: {type(line)}") #str
```

- What does this code do?

## Separate the string

```
data_number_list = []  
for line in data_text.splitlines():  
    ordered_pair = line.split(",")  
    print(f"ordered_pair = {ordered_pair}")
```

- What does this code do?

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Ask yourself

## Make a list of data

```
data_number_list = []
for line in data_text.splitlines():
    ordered_pair = line.split(",")
    data_number_list.append(float(ordered_pair[1]))
print(f"data_number_list == {data_number_list}")
```

- What does this code do?

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Ask yourself

```
import
```

```
import math  
print(f"data_number_list == {data_number_list}")
```

```
Sum
```

```
sum(data_number_list)
```

```
Product
```

```
math.prod(data_number_list)
```

- What does this code do?

# Challenges When Using CSF Files?

What could possibly go wrong?!

## Data from the "file"

```
data_text = ""1972-01-01,84.700
1973-01-01,85.500
1974-01-01|86.100
1975-01-01;87.000
1976-01-01,
87.600
87.600;1977-01-01
1978-01-01,88.000
""
print(data_text)
data_text
```

- Handling missing values or values with delimiters
- Parsing files with corrupted data values
- Difficult to efficiently parse large CSV files

# Higher-Order Sequence Functions

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Ask yourself

- Functions that *work* for **any sequence**?
- These **Higher Order** functions should work for lists, ordered pairs, tuples:
  - **map**: Apply a function to every element of a sequence
  - **filter**: Apply a boolean function to every element of a sequence, returning only those matching the filter's rules
  - **reduce**: Apply a function that acts like a binary operator to a sequence of values, combining them to a single value
- These three operators give a **vocabulary** for implementing complex, yet easy-to-ready programs in a functional programming style
- These functions are **higher-order** because they accept function as input

# Map Function with a Literal Tuple

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Ask yourself

```
def square(value):  
    return value * value  
  
def map(callFunction, sequence):  
    result = ( )  
    for element in sequence:  
        result += ( callFunction(element), )  
    return result  
  
squared = map(square, (2, 3, 5, 7, 11))  
print(squared)
```

- What does this code do?



# Map Function with a Range Sequence

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Ask yourself

```
def square(value):  
    return value * value  
  
def map(callFunction, sequence):  
    result = ( )  
    for element in sequence:  
        result += ( callFunction(element), )  
    return result  
  
squared_range = map(square, range(10))  
print(squared_range)
```

- What does this code do?

# Filtering Even Numbers from a Tuple

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Ask yourself

```
def is_even(value):  
    if value % 2 == 0:  
        return True  
    return False
```

```
filtered_even = filter(is_even,  
                        (2, 3, 4, 5, 7, 11))  
print(list(filtered_even))
```

- What does this code do?

# Filtering Odd Numbers from a Tuple

One way to do it ...

```
def is_even(value):  
    if value % 2 != 0:  
        return True  
    return False
```

```
filtered_even = filter(is_even,  
                        (2, 3, 4, 5, 7, 11))  
print(list(filtered_even))
```

- What does this code do?
- How to modify this code to find **another** way?

# Summations By Using Reduce

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Ask yourself

```
def plus(number_one, number_two):  
    return number_one + number_two
```

```
def reduce(callFunction, sequence, initial):  
    result = initial  
    for value in sequence:  
        result = callFunction(result, value)  
    return result
```

```
numbers = [1, 2, 3, 4, 5]  
added_numbers = reduce(plus, numbers, 0)  
print(f"Added numbers: {added_numbers}")
```

- What does this code do?

# Monoids and Map-Filter-Reduce

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Ask yourself

- **Higher-order sequence functions** are **independent** and free of *side effects* and thus can be **parallelized**
- Since a **monoid** has the associativity property, can use **map**, **filter**, and **reduce** operators in **parallel** and then combine the solution, often achieving a **speedup**. This makes the program more efficient!

# Monoids and Map-Filter-Reduce

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Ask yourself

- These three operators give a **vocabulary** for implementing complex, yet easy-to-read, programs in a **functional** programming style
- Map-Filter-Reduce enables **parallel** computation, which is important given the **diminishing** returns associated with sequential computation
- If you can prove that a structure and operation is a **monoid** then you can use **map**, **reduce**, and **filter** to **parallelize** its computations

# Monoids and Map-Filter-Reduce

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Ask yourself

- Monoids are frequently used in Python programs
- Python programs can use higher-order sequence functions
- Using **monoids** and **higher-order** sequence functions:
  - ① What is the difference between a list and a tuple?
  - ② How does a monoid generalize strings and integers?
  - ③ How do higher-order sequence functions use monoids?
  - ④ How can map-filter-reduce support parallel programming?
  - ⑤ What type of speedup will a parallel program achieve?
- What are the ways in which the mathematical concept of a monoid connects to a wide variety of **practical applications** in the area of **parallel computing**?
- How does the concept of a **monoid** create an **archetype** in our minds?