Discrete Structures: Programming Constructs CMPSC 102

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Week 4
Slides 02

Let's Discuss

Key Questions

How do I use **non-recursive** functions, **recursive** functions, and **lambda expressions** to perform mathematical operations such as computing the **absolute value** of a number and the **means** of a sequence of numbers?

Learning Objectives

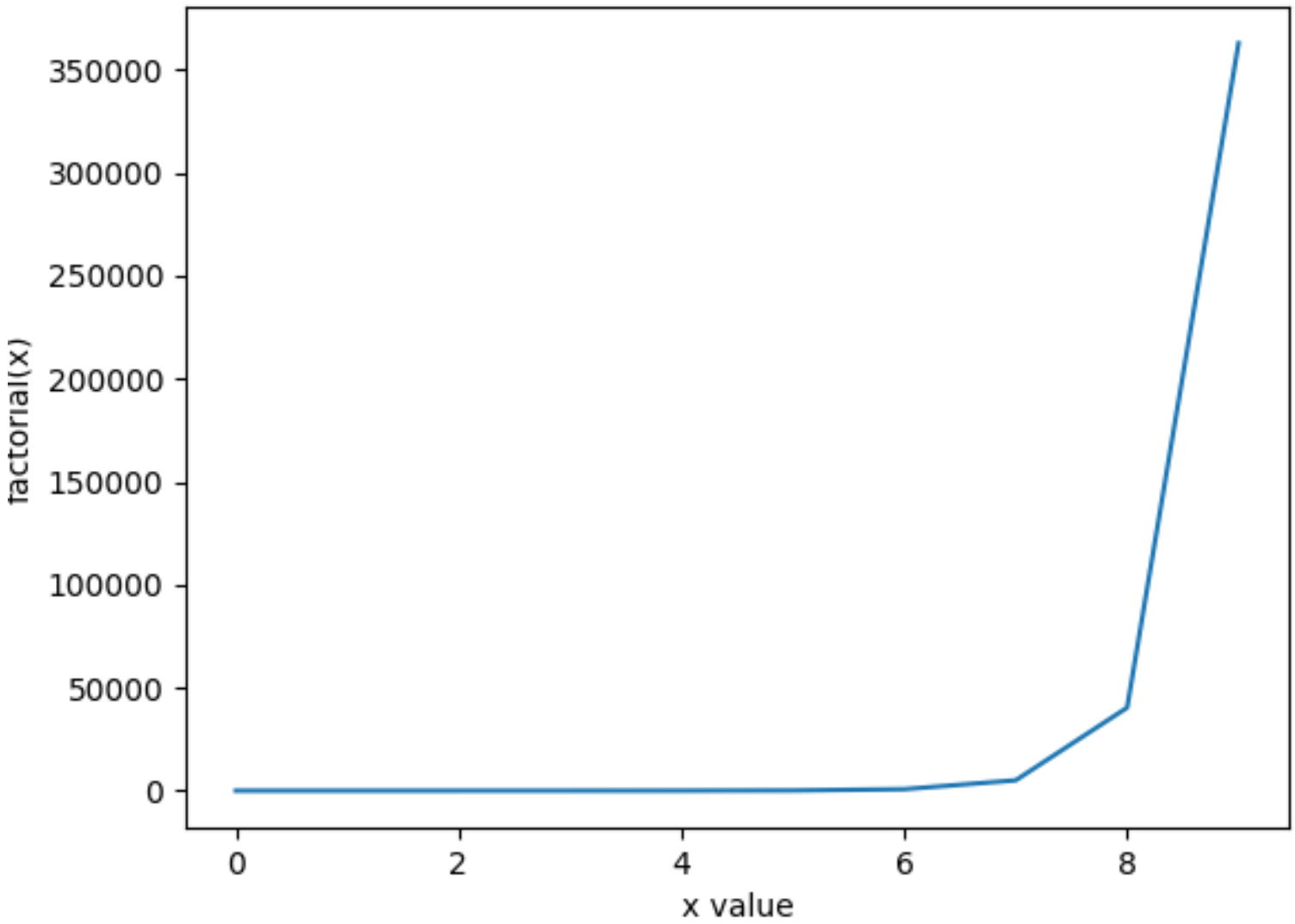
To **remember** and **understand** some discrete mathematics and Python programming concepts, setting the stage for exploring of discrete structures.

Python Programming Retrospective

- Python code is to be **intuitive**
- Key components of Python programming include:
  - Function and their definitions
  - Input parameters for functions
  - The code block that completes the function's work
  - Return statements
  - Invocations of functions (*calls to functions*)
  - Collecting the returned values (function outpus).
- Investigate the ways to make the above commands possible with definitions and call using Python.

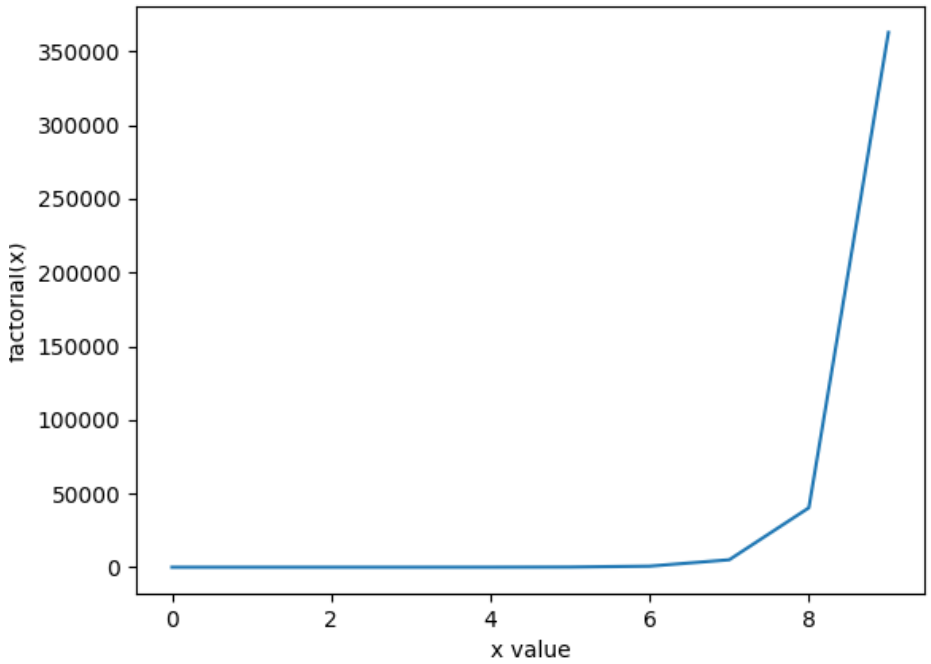
Factorials

Values get quickly get big



Factorials

Values get quickly get big



x	$fac(x)$
0	1
1	1
2	2
3	6
4	24
5	120
6	720
7	5040
8	40320
9	362880
10	3628800
11	39916800

Factorials

Factorials: one definition

$$N! = \prod_{i=1}^{N} i = 1 * 2 * .. * (N-1) * N$$

Factorials: another definition

$$N! = \frac{(N+1)!}{(N+1)} = \frac{(N+1)*N!}{(N+1)}$$

- Factorials are applied to integers

Factorials

Factorials

$N! = N * (N-1) * (N-2) * ... * (2) * (1)$

$5! = 5 * 4 * 3 * 2 * 1$

$4! = 4 * 3 * 2 * 1$

$3! = 3 * 2 * 1$

$2! = 2 * 1$

$1! = 1$

$0! = 1$ (Special case by convention)

Factorials defined

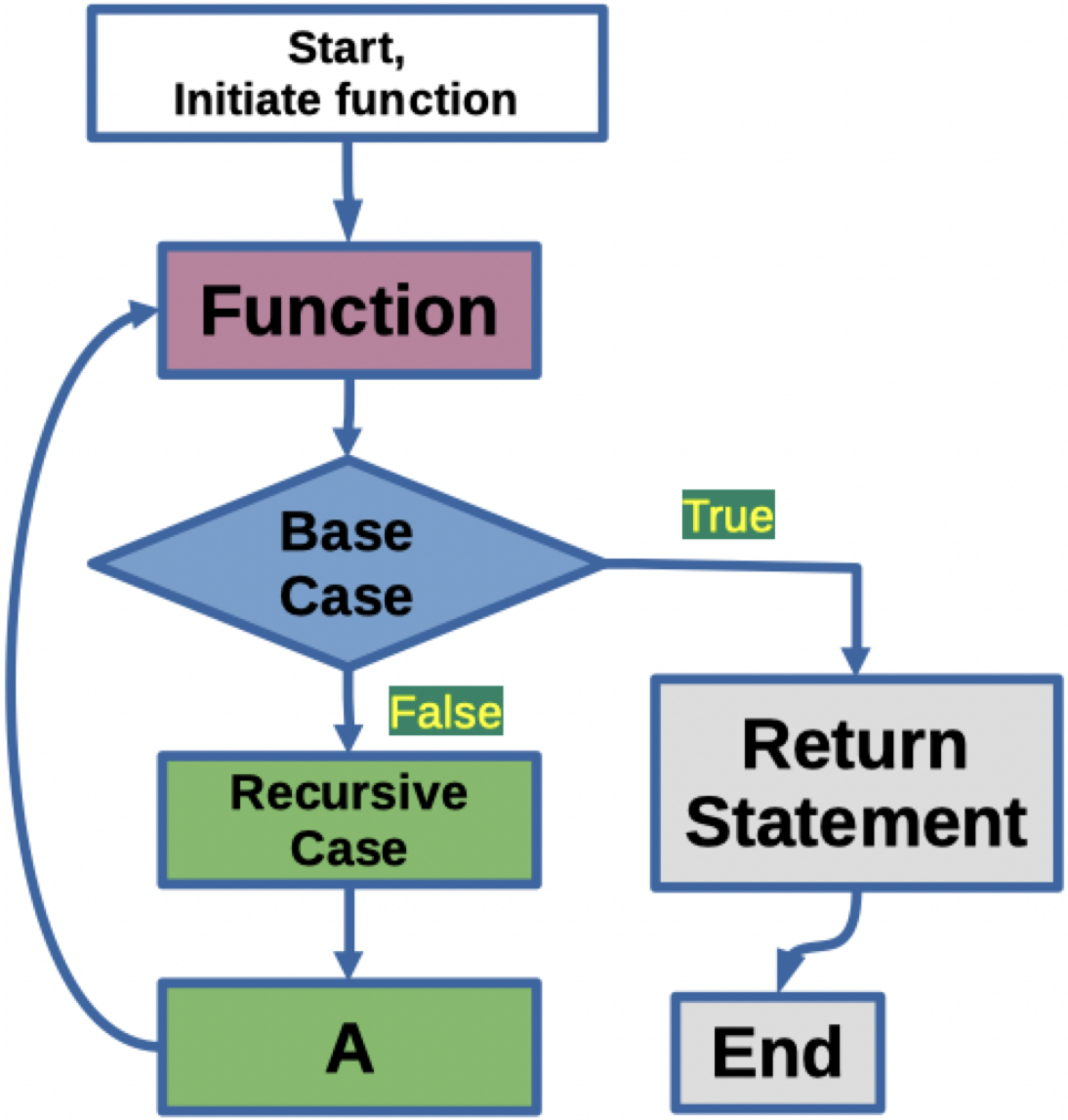
$N! = [\ (N-1)! + (N-2)!\ ] * (N-1)$

$7! = (6!\ +\ 5!\ )\ *\ 6$

$6! = (5! + 4!)\ *\ 5$

$5! = (4! + 3!)\ *\ 4$

Creating Solutions



Calculating Factorials by Recursion

```
def factorial(number: int):
    if number == 1:
        return 1
    return number * factorial(number - 1)

num = 5
print("The factorial of " + str(num)
+ " is " + str(factorial(num)))
```

- The recursive *factorial* function calls itself!
- How does this function ever stop executing?
- What are the benefits to using recursive functions?

A Closer Look at the Code

```
def factorial(number: int):
    if number == 1:
        return 1
    return number * factorial(number - 1)

num = 5
print("The factorial of " + str(num) +
        " is " + str(factorial(num)))
```

- Where is the base case?
- Where is the recursive case?
- How could this code work without these two functions?

Recursive Factorial Function

To consider

- As an equation: $n! = n \times (n-1) \times (n-2) \times \ldots \times 1$
  - What are the **parts** of a recursive function in Python?
  - Defined by **cases** using conditional logic (*a case to go, and one to force a stop*)
  - A mathematical function defined to **call itself**
  - A recursive call that makes progress to a **base case**
  - A **base case** that **stops** the **recursive function calls**
  - Repeatedly perform an operation through (*self*) function calls
  - What would happen if you input a **negative number**?
- How could you write this function with **iteration**?

A Solution Using Basic Conditions

No numbers less than zero

```
def factorial(number: int):
    if number == 1:
        return 1
    if number < 0: #Catch negative numbers
        print("cannot compute")
    if number > 1:
        return number * factorial(number - 1)

num = -5
print("The factorial of " + str(num)
+ " is " + str(factorial(num)))
```

A Solution Using `While`

No numbers less than zero

```
def factorial(number: int):
    while number > 0:
        if number == 1:
            return 1
        if number > 1:
            return number * factorial(number - 1)
    print("cannot compute")
```

Higher-Order Functions

```
def square(number: int):
    print(f"Called square({number})")
    print(f"  returning {number*number}")
    return number * number
```

```
def call_twice(f, number: int):
    print(f"Calling twice {f} with number {number}")
    return f(f(number))
```

- Higher-order functions can accept and call functions as their input!
- What are the benefits of this function type?

What Can YOU Do With Higher-Order Functions

- You can pass a **function** as an **argument** to a **function**!

Higher-Order Functions

```
def square(number: int):
    print(f"Called square({number})")
    print(f"  returning {number*number}")
    return number * number
```

- The behavior of **higher-order** functions in Python:
- `square()` is a function computes `number*number` and returns value.

Higher-Order Functions

```
def call_twice(f, number: int):
    print(f"Calling twice {f} with number {number}")
    return f(f(number))
```

- `call_twice()` is a function that calls a function `f` twice
- First, `call_twice()` calls `f` with `number`
- Then, `call_twice()` calls `f` with `f(number)`
- Finally, `call_twice()` returns result off`( f(number) )`
- Can you predict the output of the`call_twice()` function?
- How would you test the `call_twice()` function? Can you express it differently?

Why Do We Care About Higher-Order Functions!?

- Supports general-purpose function creation
- Allows executable functions as function input
- Supports both code reuse and modularity

Lambda Expressions

```
def call_twice(f, number: int):
    print(f"Calling twice {f} with number {number}")
    return f(f(number))
```

```
square = lambda x: x*x
number = 5
result = call_twice(square, number)
print("Calling square lambda twice " +
      "with "  +  str(number) +
      " is " + str(result))
```

- Functions are values in the Python programming language
- square is an expression that has a function as its value

Lambda Expressions

```
def call_twice(f, number: int):
    print(f"Calling twice {f} with number {number}")
    return f(f(number))
```

```
square = lambda x: x*x
number = 5
result = call_twice(square, number)
print("Calling square lambda twice " +
      "with "  +  str(number) +
      " is " + str(result))
```

```
Calling twice <function <lambda> at 0x37500c8> with number 5
Calling square lambda twice with 5 is 625
```

- Lambda functions are known as **anonymous functions** and add simplicity in programming
- Useful for small function input to other functions

Consider this...

ToDo

- Try completing the following `Lambda` function to compute factorials

General form

```
factorial = lambda x: 1 if x == 0 else x * fact(?-?)
```