Discrete Structures: Data Containers CMPSC 102

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Week 8
Slides 01

What to study

- Date: 25 Oct 2022, During Lab, open notes
- **Study: Slides, notes, with chapters to add detail to class material**
- Python basics and code
    - Determining output
    - Picking out bugs from code; fixing code
    - Study the code from the practicals and material covered in class to understand the how programs worked.
    - Lambda functions, lists, dictionaries, n-tuples
    - `for` and `while` loops
    - Iterations over sequences
    - Sequences, strings, sets
    - Conditional statements
    - And other concepts covered during class

Let's Discuss

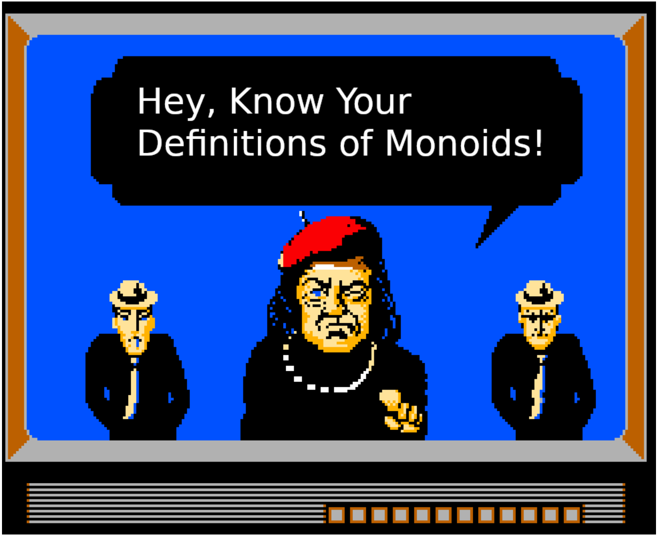
Key Questions

How do I employ the mathematical concepts of **sequences**, **monoids**, and **lists** to implement efficient Python programs that use functions with a **clearly specified behavior** to perform tasks like finding a name in a file or computing the arithmetic mean of data values?

Learning Objectives

To **remember** and **understand** some the concept of a **monoid**, seeing how it connects to **practical applications** with strings and sequences

And Now This TV



A Quick Definition

Monoid Definition

In Abstract Algebra, a **monoid** is a set equipped with an **associative** binary operation and an **identity** element. For example, the non-negative integers with addition form a monoid, the identity element being 0.

- A monoid is a combination of an object (a,b,c) and an operation (+) that meets the following conditions
  - the operation on two of the objects produces a new object of the same kind
    - int + int = int
  - associative operations
    - (a+b) + c = a + (b+c)
  - a null object e must exist, such that $e + a = a + e = a$
    - $n + 0 = n$

Ref: `https://en.wikipedia.org/wiki/Monoid#Definition`

Examples of Sequences in Python

- Sequences are commonly found in Python programs!
- Examples of the **sequence discrete structure** in Python:
  - A string is a sequence of individual characters
  - The `range(20)` function returns a sequence of numbers
  - Files are sequences of lines containing content
  - Each line in a file is a sequence of individual characters
  - Each individual character is a sequence of numbers
  - Each individual number is a sequence of binary digits
- Do these sequences all have properties in common?
- Can we **generalize**?

Licensed to Sequence

A demonstration

Make a Sequence

```
first = "James"
last = "Bond"
print(f"The name is, {last}, {first}-{last}")
```

What is an n-Tuple

Make an n-tuple

```
myStuff = ()
type(myStuff) # is tuple
item_1 = "Omega Watch"
item_2 = "Aston Martin"
item_3 = "Spy Manual"
myStuff = list(myStuff) # conv to list
type(myStuff) # is list
myStuff.append(item_1)
myStuff.append(item_2)
myStuff.append(item_3)
myStuff = tuple(myStuff)
type(myStuff) # is tuple
print(myStuff)
```

What the difference?

What is the difference between **sequences** and **tuples**?

Comparing Sequences and n-Tuples

- Are sequences and n-tuples the same? Are they different?
- Understanding the properties of n-tuples and sequences:
  - Both n-tuples and sequences are **ordered collections**
  - Sequences are normally composed of the same type of data
  - n-tuples are not required to contain the same type of data
  - Sequences are not "theoretically bounded" in their size
  - n-tuples are "theoretically bounded" in their size
  - Both sequences and n-tuples are **practically bounded** in size
- Do different types of sequences have common properties and behavior?
- Can we more generally understand these discrete structures?
- Generalization aids in understanding different discrete structures!

String Concatenations in Python

```
hello = "hello"
world = "world"
space = " "
message = hello + space + world
print(f"The message is: {message}")
```

- You can concatenate or "glue together" strings
- What would happen if you picked a different order?

- `hello + space + world`
- `space + hello + world`
- `world + space + hello`

Reversed String Concatenation


```
hello = "hello"
world = "world"
space = " "
message = world + space + hello
print(f"The message is: {message}")
```

- What is the **output** of this program segment?
- How does Python **represent** a string in memory?
- What are the different **types** of strings?
- What is an **empty string** in Python?
- How is an empty string different from " "?

Licensed to Sequence

Does this have the same sense?!

Make a Sequence

```
first = "James"
last = "Bond"
print(f"The name is, {first}, {last}-{first}")
```

- Are concatentated sequences still monoids?

Empty String Concatenation in Python


```
world = "world"
empty = ""
message = empty + world
print(f"The message is: {message}")
```

- The ‘empty‘ variable is an identity string
- What is the output of this program segment?
- What if we switched the order of the concatenation?
- How is the ‘empty‘ variable different from “” ”‘ ?
- What is an ”identity content” for other data types and operators?

Reversed Empty String Concatenation

```
world = "world"
empty = ""
message = world + empty
print(f"The message is: {message}")
```

- What is the output of this program segment?
- Why does the order of operations not matter in this case?
- Can we generalize these observations about strings?
- Can we define a general discrete structure with predictable properties?
- If you get confused, revisit what you know about working with `str`'s in Python!

Characterizing String Concatenations

- Define S to be the set of all possible strings
- What properties of S are always true?
  - For $s_1, s_2 \in S$ and the concatenation operator "$+$", $s_1 + s_2 \in S$
  - For $s_1, s_2, s_3 \in S$, "$+$" is associative: $(s_1 + s_2) + s_3 = s_1 + (s_2 + s_3)$
  - For $s_1, s_2, \in S$, "$+$" is not commutative: $(s_1 + s_2) \neq s_2 + s_1$
  - For $s_1, s_2, \in S$, if $s_1 = s_2$ or either $s_1 = \epsilon$, then "$+$" is commutative
- These properties of strings help us to **generalize** and **understand** their behavior! Let's this concept explore further!
- The **monoid** discrete structure generalizes data that "behaves like strings"

Properties (of real numbers)

Said in a different way from previous slide

Property	Addition	Multiplication
Commutative	$a + b = b + a$	$a \cdot b = b \cdot a$
Associative	$a + (b + c) = (a + b) + c$	$a \cdot (b \cdot c) = (a \cdot b) \cdot c$
Distributive	$a \cdot (b + c) = a \cdot b + a \cdot c$	$a \cdot (b + c) = a \cdot b + a \cdot c$
Identity	$a + 0 = a$	$a \cdot 1 = a$
Inverse	$a + (-a) = 0$	$a \cdot \frac{1}{a} = 1$

- Remember that strings do not behave like numbers when using these properties.

Properties of Strings and Integers

String

- Concatenation through the use of the + operator
- Identity: exists in the "" string
  - "this" + "" = "this"
  - `len("this" +"")`
- Concatenation is associative but **is not** commutative

Integers

- Two integers separated by an + operator creates another integer.
- Addition of integers is the associative property.
- Identity: exists as a 0
  - $n + 0 = n$
- Concatenation is associative and commutative

Monoid Classes :: `__init__`

File: `sandbox/base_permutations.py`


```
#!/usr/bin/env python3
# -*- coding: utf-8 -*-

class Monoid:
    def __init__(self, null, typeify, operator):
        # __init__ allows class variables to be definded
        # when the class is initiated
        self.null = null
        self.typeify = typeify
        self.operator = operator
```

- Sets up the class in terms of object's variables

Monoid Classes :: `__call__`

File: `sandbox/base_permutations.py`


```
def __call__(self, *args):
    # __call__ method enables classes for which
    # the instances behave like functions and
    # can be called as such
    result = self.null
    for arg in args:
        arg = self.typeify(arg)
        result = self.operator(result, arg)
    return result
```

- Sets up ability for the class to be *called* as a function to simplify programming

Main Function :: cartesian_prod()

File: sandbox/base_permutations.py


```
def cartesian_prod(a_list,b_list):
    print(f"my a_list and my b_list : {a_list} && {b_list}")
    # input()
    c = []
    for a in a_list:
        for b in b_list:
            c.append(a+b)
    return c
```

- Function to Calculate Cartesian product

Command

File: sandbox/base_permutations.py


```
cartesian_product_monoid  =
  Monoid([''],
  lambda x: x,
  cartesian_prod)
# define class
```

- Command to initiate class and pass in list variables for permutation calculation

Command

File: `sandbox/base_permutations.py`

```
base_list = ['A','C','G','T']

print("Length 2 cartesian products")
permutations_list = cartesian_product_monoid(base_list, base_list)
print(f"\t [+] Length 2 Permutations_list = {permutations_list}")
print(f"\t [+] Number of permutations : {len(permutations_list)}")
```

- Prepare the list of characters
- Call `cartesian_product_monoid()`, assign all results to `permutations_list for length 2 products`

Command

File: `sandbox/base_permutations.py`

```
print("Length 4 cartesian products")
permutations_list = cartesian_product_monoid(base_list, base_list, base_list, base_list)
print(f"\t [+] Length 4 Permutations_list = {permutations_list}")
print(f"\t [+] Number of permutations : {len(permutations_list)}")
```

- Prepare the list of characters
- Call `cartesian_product_monoid()`, assign all results to `permutations_list for length 4 products`