Discrete Structures: Programming Constructs CMPSC 102

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Week 3
Slides 02

Let's Discuss

Key Questions

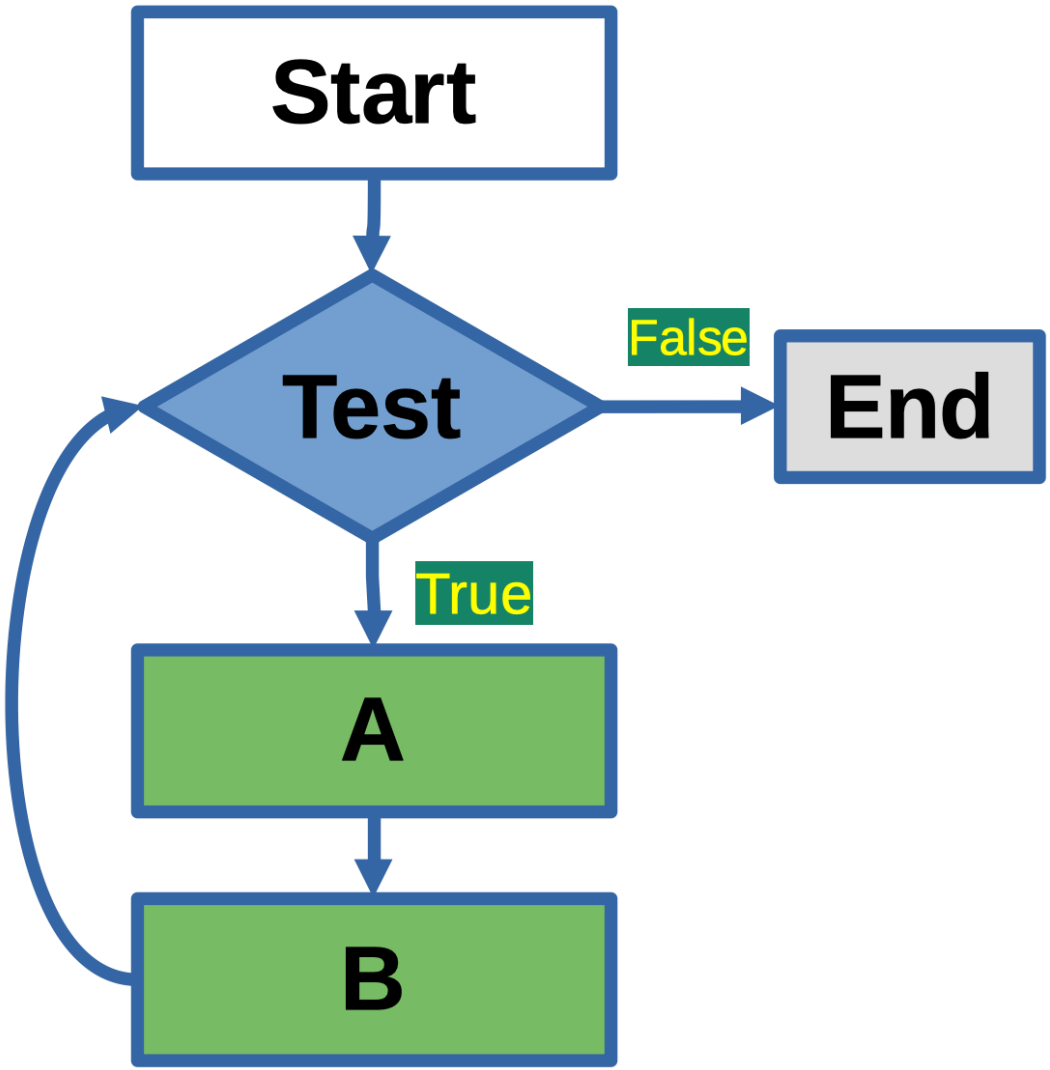
How do I use **iteration** and **conditional logic** in a Python program to perform computational tasks like processing a file's contents and mathematical tasks like using Newton's method to approximate the square root of a number?

Learning Objectives

To **remember** and **understand** some discrete mathematics and Python programming concepts, setting the stage for exploring of discrete structures.

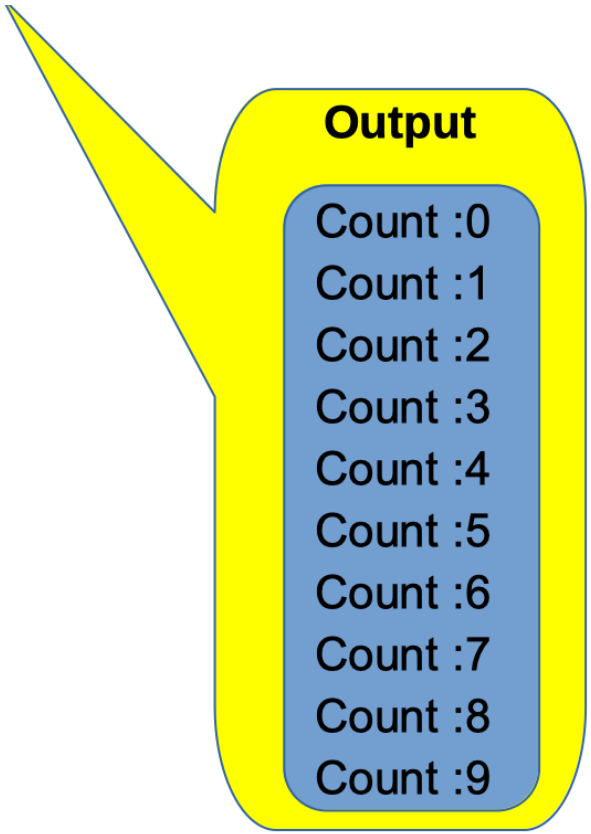
Loops for Iteration

A loop is a way to reuse code blocks



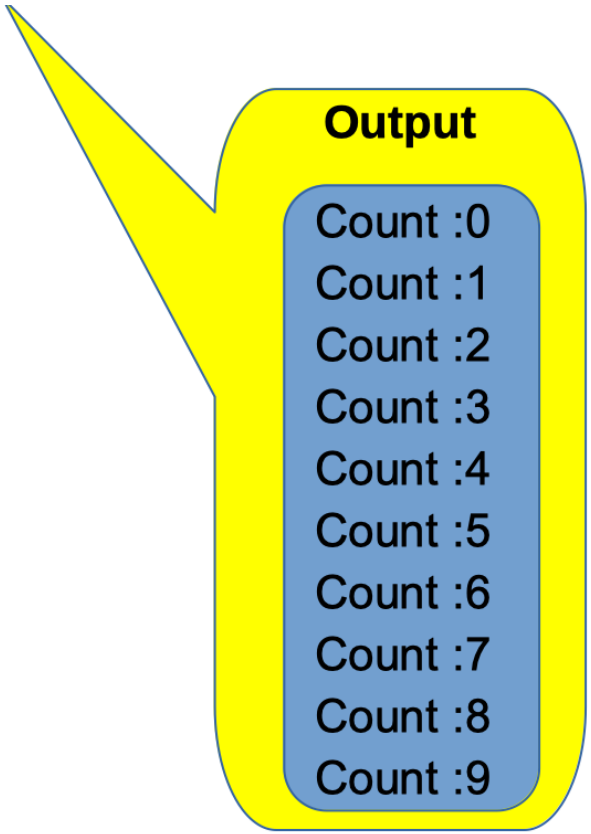
The While Loop

```
index = 0
while (index < 10): # condition
    print(f"Count :{index}")
    index += 1 # add one to index
```



The for ... in range() Loop

```
for index in range(10):
    print(f"Count :{index}")
```



Square Roots

Mathematical loops to find quadruple roots

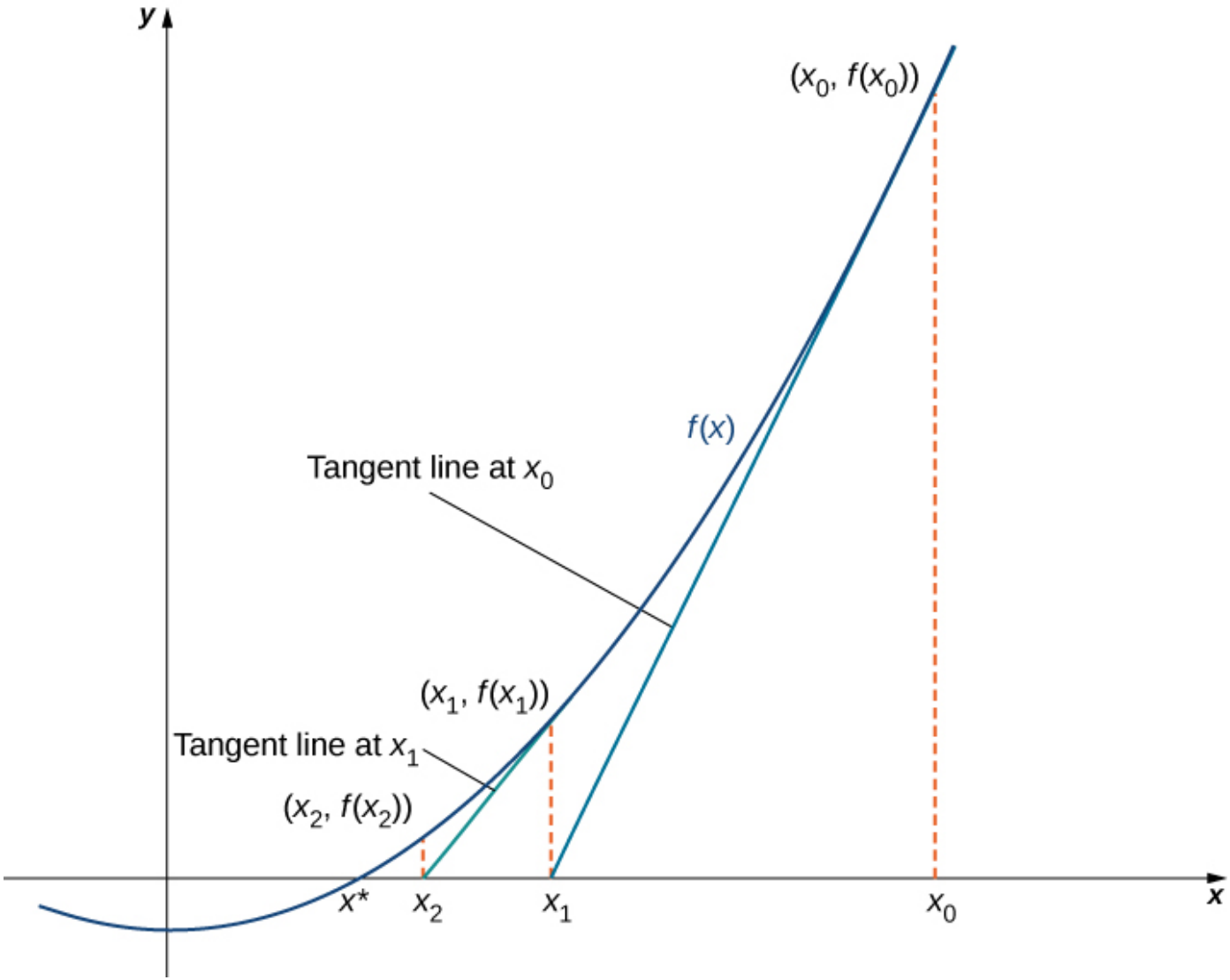
How to compute : $\sqrt{x}$?

Method

The function initializes the guess for the square root and iteratively refines it using an approximation formula until the approximation is within the specified tolerance.

Newton's Method

Mathematical loops to find square roots



The `While` Loop Application

Finding a Square Root

```
n = 4
guess = 1.0
while abs(n - guess*guess) > 0.0001:
    guess = guess - (guess*guess - n)/(2*guess)
    print(f"n = {n} : guess = {guess}")
```

- Iteratively **guesses** the square root until **within tolerance**
- The `while` loop uses 'abs' for computing an absolute value
- This loop computes the root as 2.000000929222947
- The `math.sqrt(n)` function confirms this approximation!
- Any questions about this way to approximate a square root?

The While Loop Application

Finding a Square Root

```
n = 4
guess = 1.0
while abs(n - guess*guess) > 0.0001:
    guess = guess - (guess*guess - n)/(2*guess)
    print(f"n = {n} : guess = {guess}")
```

Output

```
n = 4 : guess = 2.5
n = 4 : guess = 2.05
n = 4 : guess = 2.000609756097561
n = 4 : guess = 2.000000929222947
```

The For Loop Application

Finding a Square Root


```
n = 4
guess = 1.0
for i in range(5):
    abs(n - guess*guess) > 0.0001
    guess = guess - (guess*guess - n)/(2*guess)
    print(f"n = {n} : guess = {guess}")
```

Output

```
n = 4 : guess = 2.5
n = 4 : guess = 2.05
n = 4 : guess = 2.000609756097561
n = 4 : guess = 2.000000929222947
```

Cube Roots

Mathematical loops to find cube roots

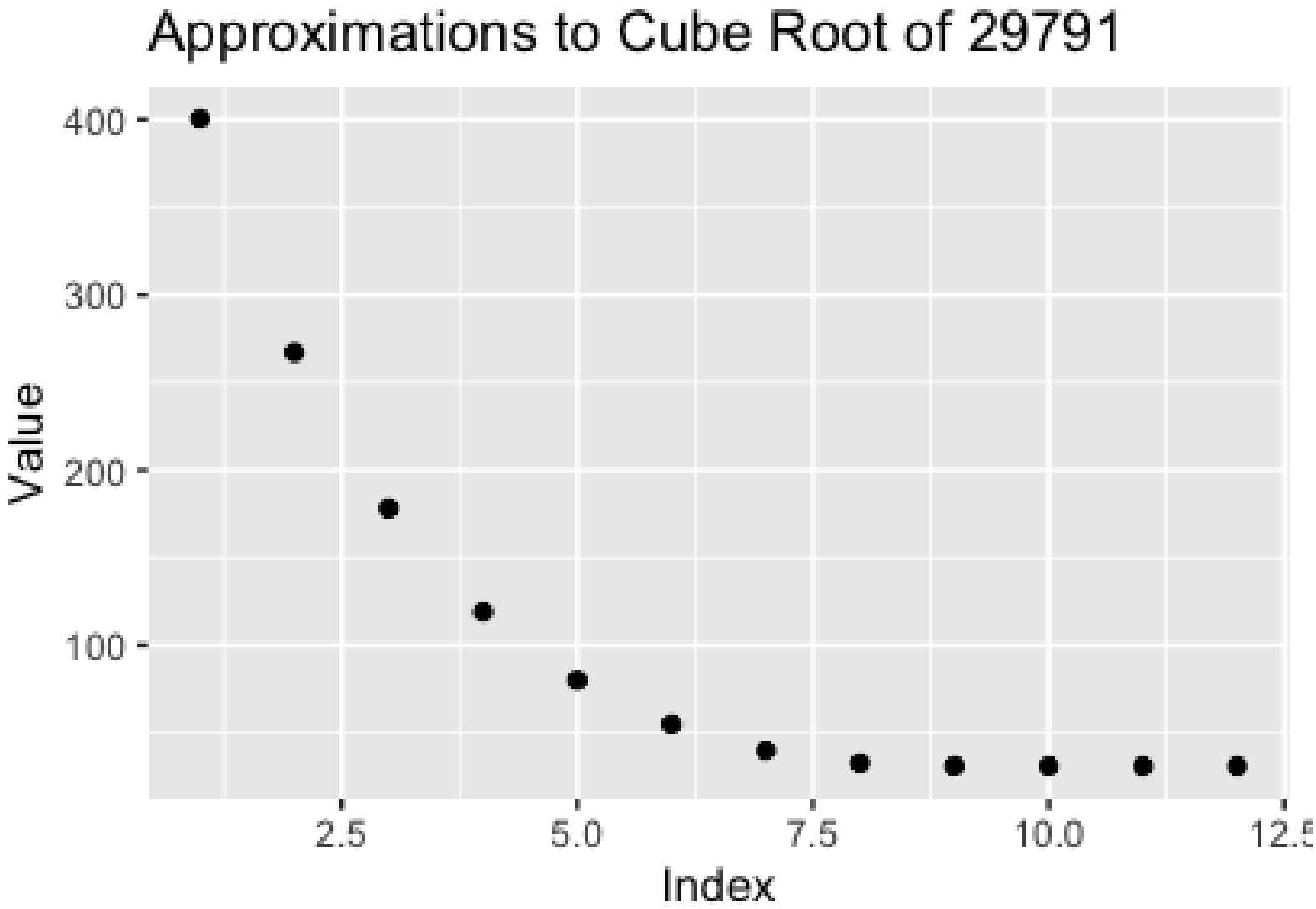
How to compute : $\sqrt[3]{x}$?

Method

The function initializes the guess for the cube root and iteratively refines it using an approximation formula until the approximation is within the specified tolerance.

Cube Roots

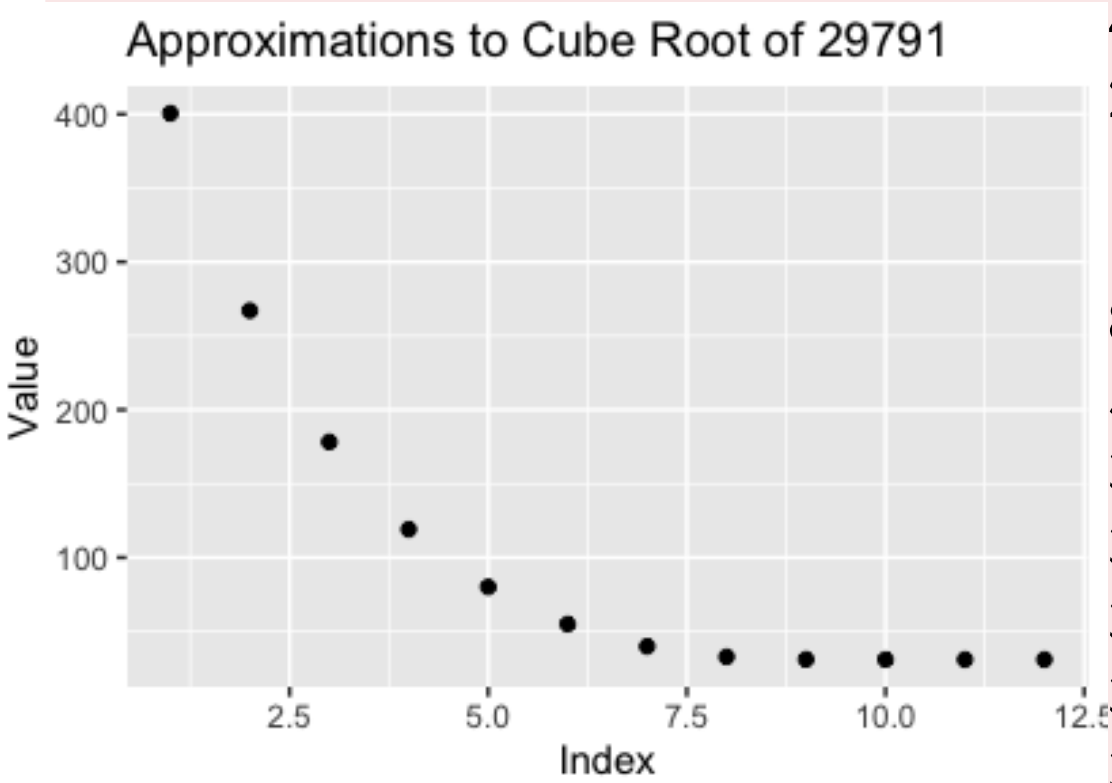
Approximations to find cube roots; result = 31



Cube Roots

Approximations to find cube roots; result = 31

Approximations



400.5466666666667,
267.0930063983859,
178.2012041240575,
119.11351331724008,
80.10891816753315,
54.95334366560159,
39.92389386164385 5,
32.84607262847925 6,
31.10181238464575 4,
31.00033292113973,
31.00000000357532,
31.0

Cube Roots

Code

```
def cube_root_approximation(number, tolerance=1e-6):
    # Initial guess for the cube root
    # guess = number / 2.0 # one way to start
    guess = 5
    # Iterate until the approximation
    # is within the specified tolerance
    while abs(guess**3 - number) > tolerance:
        # Update the guess using the approximation formula
        guess = (2 * guess + number / (guess**2)) / 3.0
        print(f" guess = {guess}")
    return guess

# Example: Calculate the cube root of 29791
input_number = 29791
result = cube_root_approximation(input_number)

# Display the result
print(f"The cube root of {input_number}")
print(f" is approximately: {result}")
```

Quadruple Roots

Mathematical loops to find quadruple roots

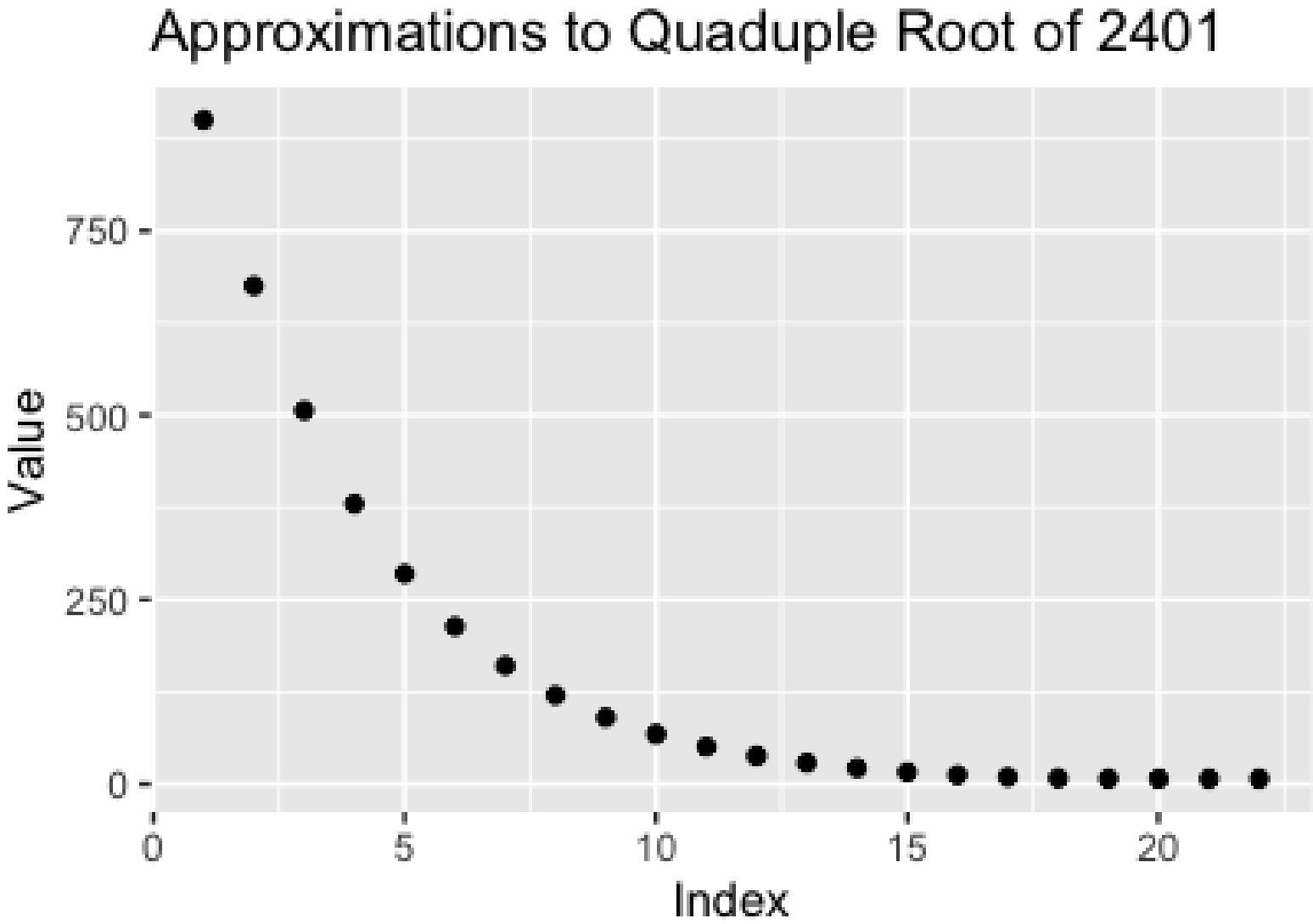
How to compute : $\sqrt[4]{x}$?

Method

The function initializes the guess for the quadruple root and iteratively refines it using an approximation formula until the approximation is within the specified tolerance.

Quadruple Roots

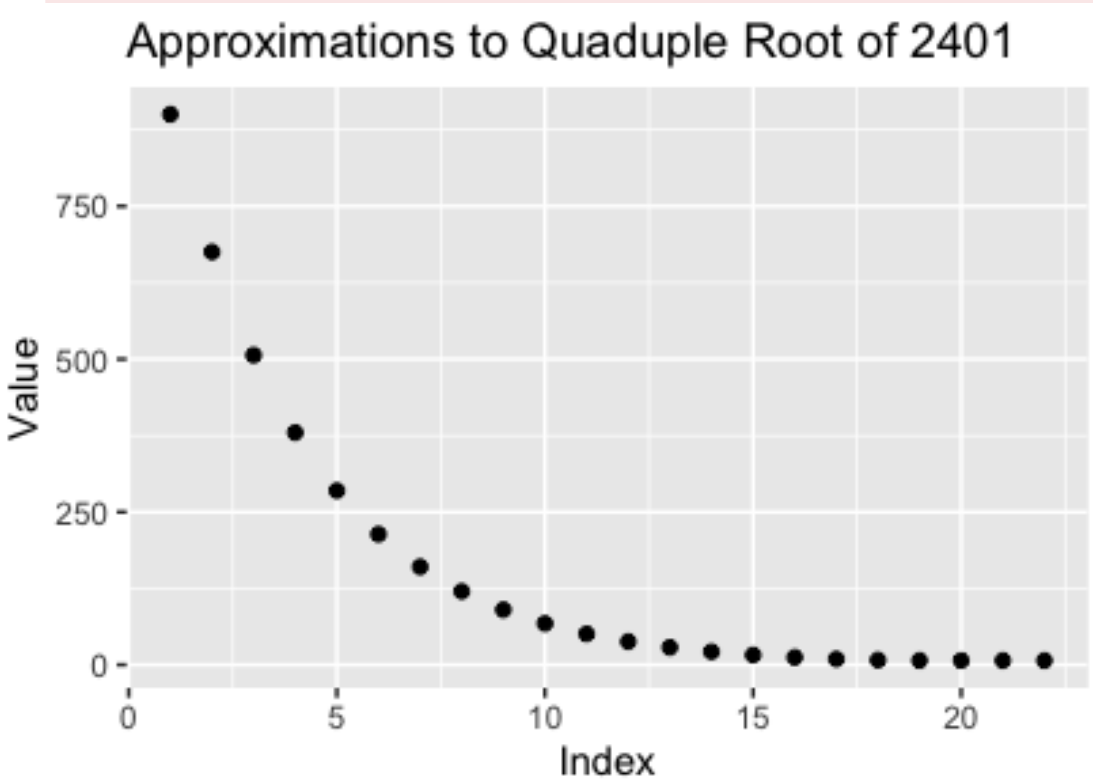
Approximations to find quadruple roots, Result = 7



Quadruple Root Approximations

Approximations to find quadruple roots, Result = 7

Approximations



900.375000346933,
675.2812510825596,
506.46094026121705,
379.8457098164694,
284.8842933147822,
213.66324594739956,
160.24749599845413,
120.18576786629791,
90.13967165836334,
67.60557331037666,
50.70612258852655,
38.03419610727984,
28.5365566959587,
21.428247703754217,
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12.242116142093641,
9.508748977118197,
7.82973307939813,
7.122821698405513,
7.00314043464742,
7.000002111777238,
7.000000000000956

Quadruple Roots
Code


```
def quadruple_root_approximation(number, tolerance=1e-6):
    # Initial guess for the fourth root
    guess = number / 2.0
    # Iterate until the approximation
    # is within the specified tolerance
    while abs(guess**4 - number) > tolerance:
        # Update the guess using the approximation formula
        guess = (3 * guess + number / (guess**3)) / 4.0
        print(f" guess = {guess}")
    return guess

# Example: Calculate the fourth root of 2401
input_number = 2401
result = quadruple_root_approximation(input_number)

# Display the result
print(f"The fourth root of {input_number}")
print(f" is approximately: {result}")
```

Quadratic Roots
The Problem Defined

To solve

$$x^2 + 3x - 4 = 0 \quad (1)$$

Want to have roots

$$x_1 =? \text{ and } x_2 =?$$

Quadratic Root Calculation

Quadratic Equation

$$ax^2 + bx + c = 0 \tag{2}$$

Quadratic Formula

$$x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \tag{3}$$

Special Note

Note the $x_{1,2}$ to imply that there are **two** solutions (i.e., x_1 and x_2) to find for a second degree equation as observed from the x^2.

Programmed Solution

Function `calc_quad_eqn_roots()`

```
def calc_quad_eqn_roots(
a: float, b: float, c: float) -> float:
    """Calculate roots of quadratic equation."""
    D = (b * b - 4 * a * c) ** 0.5
    x_one = (-b + D) / (2 * a)
    x_two = (-b - D) / (2 * a)
    return x_one, x_two

print(f"{calc_quad_eqn_roots(1,2,1)}")
```

- Three floating-point inputs: a, b, and c
- Two floating-point outputs: x_{one} and x_{two}
- How does it calculate the roots of a quadratic equation?

- **Wait, how do we test functions to ensure correctness??**

Creating Solutions