Discrete Structures: CMPSC 102

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Week 07

Let's Discuss

Key Questions

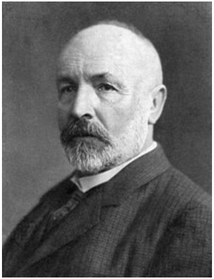
How do I use the mathematical concepts of **sets** and **Boolean logic** to design Python programs that are easier to implement and understand?

Learning Objectives

To **remember** and **understand** some concepts about the **set**,exploring how its use can simplify the implementation of programs.

Georg Ferdinand Ludwig Philipp Cantor

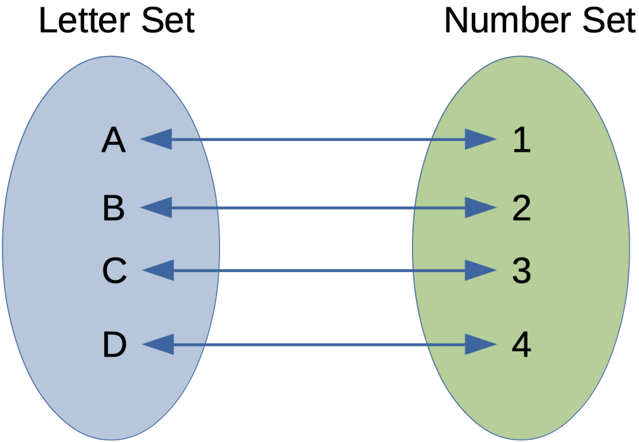
Creator of Set theory



- German mathematician: 19 February 1845 - 6 January 1918
- Function definition: established the importance of one-to-one correspondence between the members of two sets (more on that in a moment!)
- Defined infinite and well-ordered sets
- Proved that the real numbers (*rational* and *irrational*) are more numerous than the natural numbers (*counting* numbers)

Functions as Sets

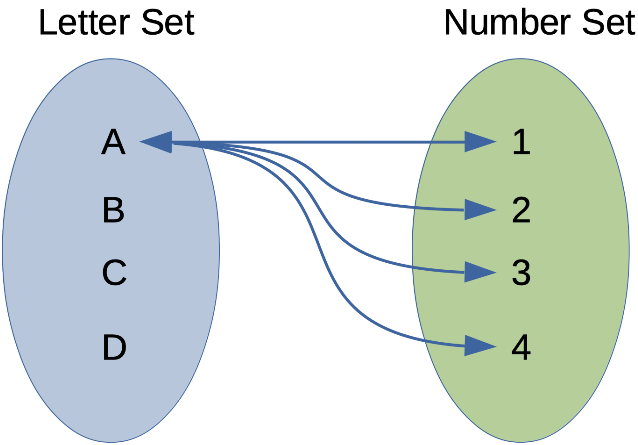
Regular Set: one-to-one relationship maintained



- The Letter set maps to the Number set.
- $LetterSet(x) \rightarrow NumberSet(y)$

Functions Sets

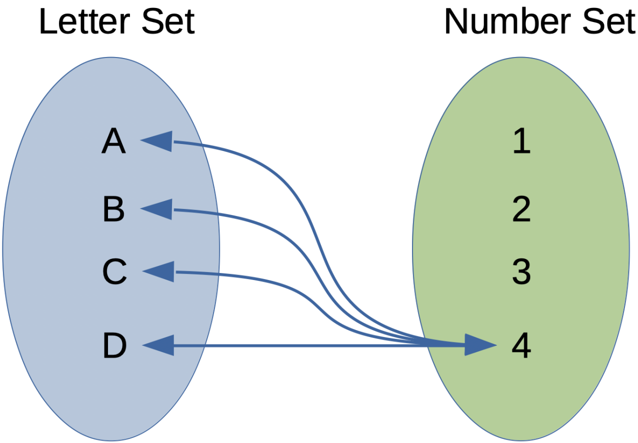
Regular Set: one-to-one-*ism* is maintained



- The Letter set maps to the Number set.
- $LetterSet(x) \rightarrow NumberSet$

Functions as Sets

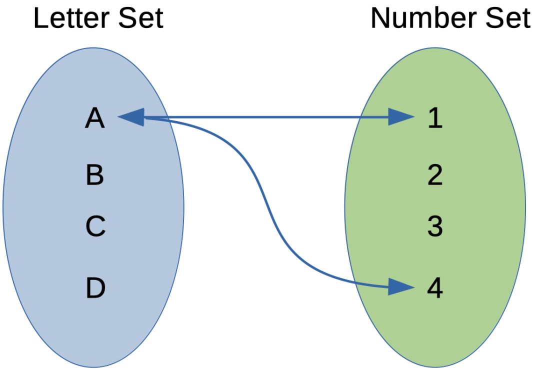
One-to-one-*ism* is NOT maintained!



- Multiple elements of Number set map to Letter set.

Functions as Sets

One-to-one-*ism* is NOT maintained!



- Multiple elements of Number set map to Letter set.

General Sets

What is a set?

- For example, the numbers 1, 2, and 3 are distinct objects when considered separately, but when they are considered **collectively** they form a single set of size three, written $\{1,2,3\}$.
- Set theory is now a ubiquitous part of mathematics,
- May be used as a foundation from which nearly all of mathematics can be derived (From 19^{th} century mathematical thinking!)

Types of Sets

Intensional and Extensional

- **Question**: What kind of set do we have?
- **Answer**: We can provide two main definitions of sets.

Intentional definition of sets: *I intend this set to be ...*

- Defines a set by specifying the necessary and sufficient conditions for when the set should be used.

Extensional definition of sets: *Logically this set is ...*

- Defines a set by some definition of a concept or a term.

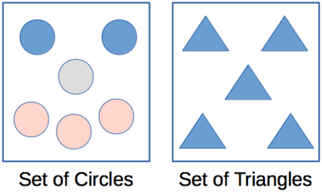
Intensional Sets

A list of characters in Sherlock Holmes

- {Sherlock Holmes, Dr. John Watson, D.I. Greg Lestrade, Mrs. Hudson, Mycroft Holmes, Irene Adler, Mary (Morstan) Watson}

Types of Sets

Intentional: One decides which elements make up a set



Intentional definition of sets: *I intend that these set be ...*

- The set of blue, grey and pink circles
- The set of blue triangles
- The set of colors of the Union Jack (i.e., the British flag)

Types of Sets

Extensional: Sets of members in curly brackets

Extensional definition of sets

- $A_2 = \{4, 2, 1, 3\}$
  - The first four positive numbers
- $B_2 = \{\text{Blue, Red and White}\}$
  - The set of colors of the Union Jack (the British flag)
- $F = \{n^2 - 4 : n \text{ is an integer; and } 0 \leq n \leq 19\}$
  - The set of all values gained from plugging in n between 0 and 19 into the equation $n^2 - 4$

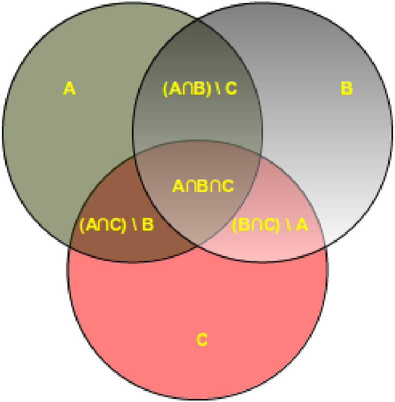
Types of Sets

Extensional definition of sets: a list of its members in curly brackets

- **Intentional Definition:**
  - A_1 is the set are the first four positive integers.
  - B_1 is the set of colors of the Union Jack
- **Extensional Definition:**
  - $A_2 = \{4, 2, 1, 3\}$
  - $B_2 = \{\text{Blue, Red and White}\}$

Specify a set *intensionally* or *extensionally*

In the examples above, for instance, $A_1 = A_2$ and $B_1 = B_2$



- $\cup$, Union: $A \cup B$ of a collection of sets A and B is the set of all elements in the collection
- $\cap$, Intersection $A \cap B$ of two sets A and B is the set that contains all elements of A that also belong to B

Create your own Venn diagram of TWO sets!!

```
#!/usr/bin/env python3
# -*- coding: utf-8 -*-
# ------------------------------------------------------------------
# setup a python virtual environment
# python3 -m venv myVenv
# source myVenv/bin/activate
# pip install matplotlib_venn


import matplotlib.pyplot as plt
from matplotlib_venn import venn2

# Define the two sets
set1 = set([1, 2, 3, 4, 5])
set2 = set([3, 4, 5, 6, 7])

# Create a Venn diagram
venn2([set1, set2],('Group1', 'Group2'))

# Add a title
plt.title('Venn Diagram of Two Sets')

# Show the plot
plt.show()
```

Note: you may need to run this code in a virtual envirnment with `numpy` and `matplotlib` installed!

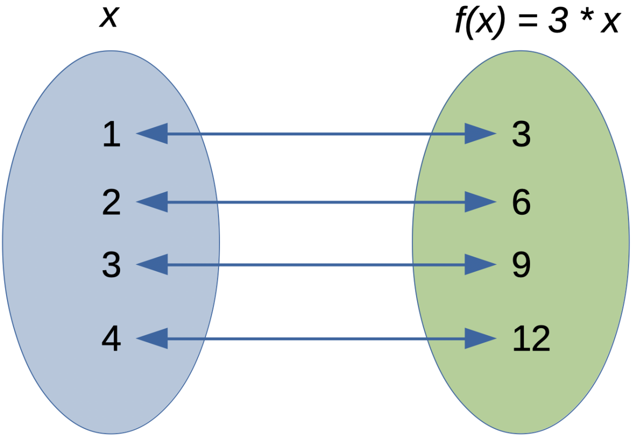
Create your own Venn diagram of THREE sets!!

```
#!/usr/bin/env python3
# -*- coding: utf-8 -*-
# -----------------------------------------------------------------
# setup a python virtual environment
# python3 -m venv myVenv
# source myVenv/bin/activate
# pip install matplotlib_venn

import matplotlib.pyplot as plt
from matplotlib_venn import venn3

set1 = set(['A', 'B', 'C'])
set2 = set(['A', 'B', 'D'])
set3 = set(['A', 'E', 'F'])

venn3([set1, set2, set3], ('Group1', 'Group2', 'Group3'))

plt.show()
```

Note: you may need to run this code in a virtual envirnment with `numpy` and `matplotlib` installed!

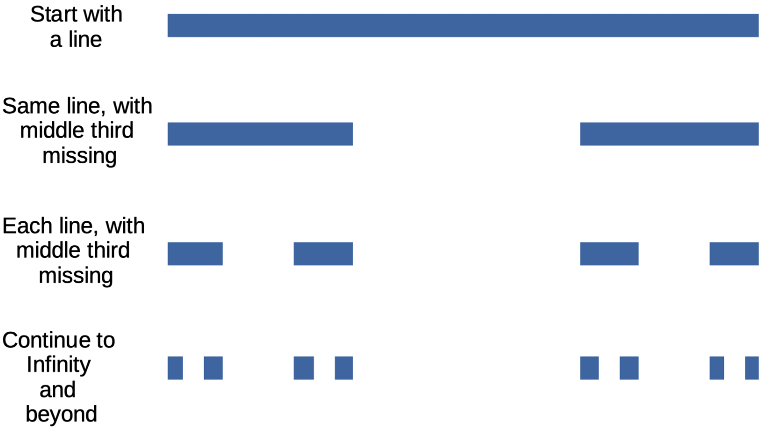
Infinite Sets: An Extensional Set Example

Sets that go on forever



Infinite Sets: An Extensional Set Example

See File `sandbox/cantorSet.py`



Create your own Cantor set!!

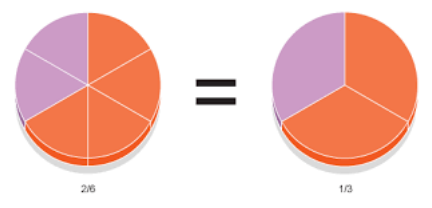
```
import numpy as np
import matplotlib.pyplot as plt
import matplotlib.colors as mcolors
import random
# -----------------------------------------------------------------
myColors = mcolors.TABLEAU_COLORS
line = [0,1]
depth = 6

def divide(line, level=0):
    """ partition the lines to form the sets. """
#     thisColour = "k" # black
    thisColour = random.choice(list(myColors.values()))
    plt.plot(line,[level,level], color=thisColour, lw=5, solid_capstyle="butt")
    if level < depth:
        s = np.linspace(line[0],line[1],4)
        divide(s[:2], level+1)
        divide(s[2:], level+1)

divide(line)
plt.gca().invert_yaxis()
plt.show()
```

Note: you may need to run this code in a virtual envirnment with `numpy` and `matplotlib` installed!

Listing Elements in Sets

- In extensionally defined sets, members in braces can be listed two or more times,
  - For example, $\{11, 6, 6\}$ is identical to the set $\{11, 6\}$
- Order of members is not important
  - For example, $\{6, 11\} = \{11, 6\} = \{11, 6, 6, 11\}$

Similar to the equivalence of these pie charts: the content is the same in both cases



Sets in Python

An array of non-redundant elements

Creating a set of chars

```
x_st = set("This is a set")
x_st   # or print(x_st)
   # the unordered chars are the elements
   # {'s', 'T', ' ', 'e', 't', 'h', 'i', 'a'}
print(type(x_st))
   # <class 'set'>
```

Creating a set of string(s)

```
x_st = set(["This is a set"])
x_st   # or print(x_st)
   # only one element in set; the string itself
   #{'This is a set'}
x_st = set(["This", "is", "a", "set"])
   # each word is an element
   #{'This', 'is', 'set', 'a'}
```

Sets in Python

```
# next line on one line
cities_st = set(("Paris", "Lyon",
     "London","Berlin","Birmingham", "Paris"))
print(cities_st)
   # {'Berlin', 'Paris', 'Birmingham', 'London', 'Lyon'}
```

Adding new elements

```
cities_st = set(["Frankfurt", "Basel", "Freiburg"])
cities_st.add("Meadville")
cities_st # or print(cities_st)
   # {'Freiburg', 'Meadville', 'Basel', 'Frankfurt'}
```

Sets in Python

Removing elements

```
cities_st = set(["Frankfurt", "Basel", "Meadville"])
cities_st.remove("Meadville")   # Meadville is a key
cities_st   # or print(cities_st)
   # {'Basel', 'Frankfurt'}
```

Frozensets cannot be changed

```
cities_st = frozenset(["Frankfurt", "Basel", "Freiburg"])
cities_st.add("Meadville")
   # AttributeError:
   # 'frozenset' object has no attribute 'add'
cities_st # or print(cities_st)
   # frozenset({'Freiburg', 'Basel', 'Frankfurt'})
type(cities_st)
   # <class 'frozenset'>
```

Sets in Python

Removing all elements of set

```
cities_st = {"Stuttgart", "Konstanz", "Freiburg"}
cities_st
   # {'Freiburg', 'Konstanz', 'Stuttgart'}
cities_st.clear()
cities_st
   # set()
```

Determining difference between sets

```
x = {"a","b","c","d","e"}
y = {"b","c"}
z = {"c","d"}
x.difference(y)  # {'a', 'e', 'd'}
x.difference(y).difference(z)  # {'a', 'e'}
```

- Returns the characters which are never repeated across {x, y, y}

Sets in Python

Difference and subtraction

```
x = {'c', 'a', 'd', 'b', 'e'}
y = {'c', 'b'}
x.difference_update(y)
print(x) # {'a', 'd', 'e'}
print(y) # {'c', 'b'}

print(x)  # {'a', 'e', 'd'}
x = {"a","b","c","d","e"}
y = {"b","c"}
x = x - y
print(x)   # {'e', 'd', 'a'}
```

- Top: Returns an updated set of x of the characters which are never repeated across {x, y, y}

Sets in Python

Cloning and removing from original

```
x = {'e', 'd', 'a'}
v = x
print(x)   # {'a', 'e', 'd'}
print(v)   # {'a', 'e', 'd'}

x.remove('a')
x   #  {'e', 'd'}
v   #  {'e', 'd'}

v.remove('d')
x   # {'e'}
v   # {'e'}
```

- $x = v$ does not make a copy of x. Instead this is a reference from one object to another.

Checking for Particular Elements

Is an element in a List?

```
x = {"a","b","c","d","e"}
"e" in x   # True
"e" and "a" in x  # True
"e" and "i" in x  # False
```

Iterating Through Elements in Sets

Iteration

```
abc_set = {"a","b","c","d","e"}
for i in abc_set:
  print(i)
```

Note

- Since there is no order control in the set, you cannot know which element will be printed first (from above).

Creating Solutions

Go check out the fun code about sets in the `sandbox/`!