

# $O(1)$ - CONSTANT TIME

The Lightning Fast Algorithm - Always the Same Speed!

CS 101 - Fall 2025

## What is $O(1)$ - Constant Time?

### The Superpower of Algorithms

$O(1)$  means the algorithm takes the **same amount of time** no matter how much data you give it!

**Real-World Analogy:** \* Like a **valet parking service** - you hand over your ticket and get your car back instantly \* Whether there are 10 cars or 10,000 cars in the lot, it takes the same time! \* The valet has a **direct system** to find your exact car

### Performance Guarantee

- 10 items  $\rightarrow$  1 step
- 100 items  $\rightarrow$  1 step
- 1,000,000 items  $\rightarrow$  1 step
- Same speed **forever!**

### Key Insight

The algorithm has a “**shortcut**” that goes directly to the answer without checking other data!

### Magic Question:

*“Can I get the answer without looking at most of the data?”*

## What Makes $O(1)$ So Fast?

The Secret Ingredients

$O(1)$  algorithms use **smart data organization** and **direct access patterns**

### Hash Tables (Dictionaries/Sets)

```
# Python dictionary uses hash function
student_grades = {
    "Alice": 95,
    "Bob": 87,
    "Charlie": 92
}

# Hash function calculates EXACTLY where
# "Alice" is stored in memory
grade = student_grades["Alice"] #  $O(1)$ !
```

#### How it works:

1. Hash function: "Alice"  $\rightarrow$  memory location 147
2. Go directly to location 147
3. Get the value (95)
4. Done! No searching needed!

### Array Indexing

```
# Arrays store data in consecutive memory
scores = [95, 87, 92, 78, 85]
#         0  1  2  3  4

# Computer calculates:
# Address = start_address + (index  $\times$  item_size)
first_score = scores[0] #  $O(1)$ 
third_score = scores[2] #  $O(1)$ 
```

#### Mathematical Magic:

- Memory address = Base + (2  $\times$  4 bytes) = Direct jump!
- No need to check scores[0] or scores[1]
- Jump straight to the answer!

## Interactive O(1) Dictionary Demo

**i** See O(1) in Action!

Try looking up different students' grades. Notice how it's always instant, no matter which student you choose!

## Python O(1) Examples - The Fast Ones!

### Dictionary Operations

```
# Creating a gradebook
gradebook = {
    "Alice": 95,
    "Bob": 87,
    "Charlie": 92,
    "Diana": 88
}

# All of these are O(1) - instant!
alice_grade = gradebook["Alice"]      # Lookup: O(1)
gradebook["Eve"] = 90                  # Insert: O(1)
gradebook["Bob"] = 89                  # Update: O(1)
del gradebook["Charlie"]               # Delete: O(1)

# Check if student exists
if "Diana" in gradebook:               # Membership: O(1)
    print("Diana is in the class!")

print(f"Class size: {len(gradebook)}") # Length: O(1)
```

### List Operations (by index)

```
# Student scores list
scores = [95, 87, 92, 78, 85, 91, 88]

# These are O(1) - direct access
first_score = scores[0]                # Get first: O(1)
last_score = scores[-1]               # Get last: O(1)
middle_score = scores[3]              # Get by index: O(1)
```

```

print(f" Scores BEFORE update = {scores}")

scores[2] = 94                # Update by index: O(1)
                              # overwrite previous value
print(f" Scores AFTER update = {scores}")

# Stack operations (end of list)
scores.append(96)             # Add to end: O(1)
final_score = scores.pop()    # Remove from end: O(1)

print(f"Total scores: {len(scores)}") # Length: O(1)

```

### Important Note

#### Not all list operations are O(1)!

- `scores.insert(0, 100)` is O(n) - inserting at beginning
- `scores.remove(87)` is O(n) - searching for value
- `87 in scores` is O(n) - searching through list

## Set Operations - Another O(1) Champion!

### Note

Are there differences in search times?

### Set Basics

```

# Create a set of student IDs
enrolled_students = {101, 102, 103, 104, 105}

# All O(1) operations
enrolled_students.add(106)        # Add: O(1)
enrolled_students.remove(103)     # Remove: O(1)

# The magic of O(1) membership testing!
if 104 in enrolled_students:      # Check: O(1)
    print("Student 104 is enrolled!")

# Compare with list - this would be O(n)

```

```
student_list = [101, 102, 103, 104, 105]
if 104 in student_list:          # O(n) - slow!
    print("Found in list (but slowly)")
```

## Real Performance Difference

```
import time

# Large dataset
large_list = list(range(100000))      # 100k numbers
large_set = set(range(100000))        # Same 100k numbers

target = 99999 # Last item (worst case)

# Timing list search - O(n)
start = time.time()
found = target in large_list          # Checks all 100k!
list_time = time.time() - start

# Timing set search - O(1)
start = time.time()
found = target in large_set           # Instant!
set_time = time.time() - start + 0.001 # Add error time

print(f"List search: {list_time:.6f} seconds")
print(f"Set search: {set_time:.6f} seconds")
print(f"Set is {list_time/set_time:.0f}x faster!")
```

## Partner Activity: O(1) Experiments!

**Instructions:** Find a partner and complete these experiments together. Discuss your observations!

### Experiment 1: Dictionary vs List Race

```
import time
import random

# Create test data with your partner
```

```

def create_test_data(size):
    data = [f"student_{i}" for i in range(size)]
    data_dict = {name: f"grade_{i}" for i, name in enumerate(data)}
    return data, data_dict

# Partner A: Test small dataset (100 items)
# Partner B: Test large dataset (10,000 items)
sizes = [100, 10000] # Choose one each

for size in sizes:
    data_list, data_dict = create_test_data(size)
    target = f"student_{size-1}" # Last item (worst case)

    # Time list search (O(n))
    start = time.time()
    found = target in data_list
    list_time = time.time() - start

    # Time dict search (O(1))
    start = time.time()
    found = target in data_dict
    dict_time = time.time() - start

    print(f"Size {size}:")
    print(f"  List search: {list_time:.6f} seconds")
    print(f"  Dict search: {dict_time:.6f} seconds")
    # zero division precaution: add 0.001 to denominator
    print(f"  Dict is {list_time/(dict_time+0.001):.0f}x faster!")

```

### Discussion Questions:

- What happened to the performance difference as data size increased?
- Why doesn't dictionary search time change much?

### Experiment 2: Hash Table Investigation

```

# Build a student grade system together
student_grades = {}

# Partner A: Add students with IDs 1-1000
# Partner B: Add students with IDs 1001-2000
def add_students(start_id, end_id, grades_dict):

```

```

import time
start = time.time()

for i in range(start_id, end_id + 1):
    grades_dict[f"student_{i}"] = random.randint(70, 100)

end = time.time()
return end - start

# Time your additions
partner_a_time = add_students(1, 1000, student_grades)
partner_b_time = add_students(1001, 2000, student_grades)

# Now test random lookups
def test_lookups(grades_dict, num_tests=100):
    import time
    start = time.time()

    for _ in range(num_tests):
        random_id = random.randint(1, 2000)
        grade = grades_dict.get(f"student_{random_id}", "Not found")

    return time.time() - start

lookup_time = test_lookups(student_grades)
print(f"Adding 1000 students: ~{partner_a_time:.4f} seconds each")
print(f"100 lookups in {len(student_grades)} students: {lookup_time:.6f} seconds")

```

### Partner Discussion:

- Did adding more students slow down individual lookups?
- How would this compare with a simple list of 2000 students?

## O(1) Scenario Analysis: Group Discussion Prompts

### Class Discussion Questions

**Work in groups of 3-4. Discuss these scenarios and be ready to share insights!**

! Important

**Discuss with your group:**

1. **Social Media Apps:** When you open Instagram/TikTok, your profile loads instantly regardless of how many users the app has. What  $O(1)$  operations make this possible?
2. **Online Shopping:** Amazon can instantly tell you if an item is in stock, even with millions of products. How might they achieve  $O(1)$  inventory checks?
3. **Video Games:** In a multiplayer game with thousands of players, you can instantly see your own stats. What data structure enables  $O(1)$  player data retrieval?
4. **School Systems:** Your student portal shows your GPA instantly, even though the school has thousands of students. What makes this  $O(1)$ ?

**Group Challenge:** Design a simple system for one of these scenarios using Python dictionaries. How would you organize the data for  $O(1)$  access?

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## **$O(1)$ Scenario Analysis: Critical Thinking**

i Class Discussion Questions

**Work in groups of 3-4. Discuss these scenarios and be ready to share insights!**

! Important

**Discuss with your group:**

5. **Trade-off Analysis:**  $O(1)$  operations often require extra memory (like hash tables). When is this trade-off worth it? When might it not be?
6. **Failure Cases:** Can you think of situations where a dictionary lookup might NOT be  $O(1)$ ? (Hint: Think about hash collisions)
7. **Design Decisions:** You're building an app that needs to store user preferences. Would you use a list or dictionary? Why? How would your choice affect performance with 1 user vs 1 million users?
8. **Performance Prediction:** If a dictionary lookup takes 0.001 seconds with 1,000



items, how long should it take with 1,000,000 items? Why?

## Your Turn: Practice $O(1)$ Operations!

### 💡 Individual Coding Exercises

After your group discussions, try these hands-on exercises to solidify your understanding.

#### Exercise 1: Phone Book

```
# Create your own phone book
phone_book = {}

# Add contacts ( $O(1)$  each)
phone_book["Mom"] = "555-0123"
phone_book["Pizza Place"] = "555-PIZZA"
phone_book["Best Friend"] = "555-9999"

# Look up numbers instantly
print(f"Mom's number: {phone_book['Mom']}")

# Your task: Add 5 more contacts and
# time how long it takes to look them up!

import time
start = time.time()
# Add your lookups here
end = time.time()
print(f"Lookup time: {end - start} seconds")
```

#### Exercise 2: Student Checker

```
# Create sets for different classes
math_class = {"Alice", "Bob", "Charlie"}
science_class = {"Bob", "Diana", "Eve"}
history_class = {"Alice", "Eve", "Frank"}

# Check enrollment instantly ( $O(1)$ )
student = "Alice"
enrolled_classes = []
```

```

# Check to see whether item is in dictionary
# add subject name to new list if student is enrolled

if student in math_class:
    enrolled_classes.append("Math")
if student in science_class:
    enrolled_classes.append("Science")
if student in history_class:
    enrolled_classes.append("History")

print(f"{student} is in: {enrolled_classes}")

# Your task: Check enrollment for all students
# and see how fast it is!

```

## When NOT to Use $O(1)$ Approaches

### ⚠️ $O(1)$ Has Limitations!

While  $O(1)$  is amazing, it's not always possible or practical.

## When $O(1)$ Won't Work

```

# These operations CANNOT be  $O(1)$ :

scores = [95, 87, 92, 78, 85]

# Finding maximum - must check all values
max_score = max(scores)           #  $O(n)$  - no shortcut!

# Counting specific values
count_90s = scores.count(90)      #  $O(n)$  - must check all

# Finding an item in unsorted list
position = scores.index(92)        #  $O(n)$  - no direct path

# Sorting data
scores.sort()                     #  $O(n \log n)$  - complex!

```

## Why?

No way to know the answer without examining the data!

## Trade-offs to Consider

```
# Memory vs Speed Trade-off
students = ["Alice", "Bob", "Charlie", "Diana"]

# Option 1: List (less memory, slower search)
if "Bob" in students:          # O(n) - slow search
    print("Found Bob!")

# Option 2: Set (more memory, faster search)
# Note: Sets contain only unique items.
# Although time and energy is required to complete
# operations involved with creating sets ...

student_set = set(students)    # Uses more memory
if "Bob" in student_set:      # O(1) - fast search!
    print("Found Bob!")

# Choice depends on:
# - How often do you search?
# - How much memory do you have?
# - How large is your data?
```

## Comparing O(1) to Other Complexities

💡 See the Dramatic Difference!

Let's compare O(1) to other complexities with real numbers.

Data Size	O(1)	O(log n)	O(n)	O(n <sup>2</sup> )	Real-World Impact
10 items	1 step	~3 steps	10 steps	100 steps	All feel instant
100 items	1 step	~7 steps	100 steps	10,000 steps	O(1) still instant
1,000 items	1 step	~10 steps	1,000 steps	1,000,000 steps	Only O(1) stays fast
1,000,000 items	1 step	~20 steps	1,000,000 steps	1,000,000,000,000 steps	1,000,000,000,000 is superhuman!

## ! The $O(1)$ Advantage

**Dictionary lookup with 1 million entries = Same speed as with 10 entries!**

This is why Google can search billions of web pages instantly - they use hash tables and other  $O(1)$  techniques!

## Summary: $O(1)$ - The Algorithm Superhero!

### 💡 Key Takeaways

**$O(1)$  - Constant Time** is the gold standard of algorithm efficiency!

### What Makes $O(1)$ Special

- **Always the same speed** - no matter how much data
- **Direct access patterns** - no searching required
- **Smart data structures** - hash tables, arrays
- **Real-world applications** - Google search, databases, caches

### Python $O(1)$ Champions:

- Dictionary operations: `dict[key], dict[key] = value`
- Set operations: `item in set, set.add(item)`
- List indexing: `list[0], list[-1]`
- Stack operations: `list.append(), list.pop()`

### Programming Wisdom

```
# Choose your data structure wisely!

# For frequent lookups
use_dict = {"key": "value"}           #  $O(1)$  lookup
# not_list = ["key", "value"]         #  $O(n)$  lookup

# For membership testing
use_set = {1, 2, 3, 4, 5}             #  $O(1)$  testing
# not_list = [1, 2, 3, 4, 5]          #  $O(n)$  testing

# For indexed access
use_list = [1, 2, 3, 4, 5]            #  $O(1)$  by index
# Perfect for stacks and queues
```

**Remember:**  $O(1)$  is not always possible, but when it is, it's magical!

## Next: Exploring $O(\log n)$ - The Smart Searcher!

### Coming Up Next

**$O(\log n)$  - Logarithmic Time** \* Binary search and divide-and-conquer strategies \* Why “halving” is so powerful \* Interactive binary search demonstrations \* When  $O(\log n)$  beats  $O(n)$  by huge margins

**Questions to Think About:** \* When might you need to give up  $O(1)$  for  $O(\log n)$ ? \* How can “smart searching” be almost as good as direct access? \* What's the trade-off between data organization and search speed?

Ready to see how smart algorithms can be nearly as fast as  $O(1)$ ?