Contributions and Statements

Part I - Contributions to Research and Development

a. Articles Published or Accepted in Peer-Reviewed Journals

[1] Denson, J., and Pramanik, M., and Zahl, J. (2021) Large Sets Avoiding Rough Patterns. Springer Optimization and Its Applications. 168: 59-75 (MSc Work).

b. Articles Submitted to Peer-Reviewed Journals

[2] Denson, J. (Submitted October, 2021) Large Salem Sets Avoiding Nonlinear Configurations. Submitted to Analysis & PDE: 38 Pages. (PhD Work)

c. Other Peer-Reviewed Contributions

[3] Denson, J. (2019). Cartesian Products Avoiding Patterns. University of British Columbia. (MSc Work)

d. Non Peer-Reviewed Contributions

[4] Denson, J. (2021). Capacity of Rank Decreasing Operators. Brascamp Lieb Inequalities Summer School, Kopp: 80-83. (PhD Work)

[5] Denson, J*. (2020-2021). Salem Sets Avoiding Patterns. (Oral Presentation given Institutionally at the UW-Madison Analysis Student Seminar in 2020, and Nationally at the Ottawa Math Conference in 2020, and at the Ohio River Analysis Meeting in 2021).. (PhD Work)

[6] Denson, J*. (2018-2019). Fractals Avoiding Fractal Sets. (Oral Presentation given Nationally at the 2018 CMS Winter Meeting, and Internationally at the 2018 Mid-Atlantic Analysis Seminar, and GAHA 2019). (Poster Presented Nationally at the 2018 CMS Winter Meeting, and Internationally at the 2019 February Fourier Talks and 2019 Madison Lectures in Fourier Analysis). (Msc Work)

[7] Denson, J. (2018). Proofs in Three Bits or Less. CMS Notes From the Margin. 13: 1-3. (MSc Work)

Part II - Most Significant Contributions to Research

Large Sets Avoiding Rough Patterns [1]

This paper forms a major part of my MSc Thesis research project, and thus a summary of the results of this paper can be found in the Thesis section of the report (this paper proves Theorem 27 of that thesis). The results of this paper evolved over a year and a half of weekly conversations with my two advisors, Malabika Pramanik and Joshua Zahl. But I was mainly responsible for writing up the paper describing our work, meeting with my advisors weekly to review the current draft and obtain a style we were all happy with. This process forced me to tighten up my mathematical writing style. Before we started writing the paper, we had agree to publish as a chapter in the Springer Volume "Harmonic Analysis and Applications". This was because the volume was being organized by a reputable publisher with a peer review process, we knew the deadline for submission to the book, which gave us a precise schedule to write up the paper, and the deadline implied that if accepted, our article would be published without delay.

Large Salem Sets Avoiding Nonlinear Configurations [2]

After graduating from UBC, I tried to improve the Fourier dimension constructions in my thesis [3], since these constructions had smaller dimension than the Hausdorff dimension construction given the same assumptions. Most results about patterns and Fourier dimension in the literature concern patterns formed by linear equations, so improving Fourier dimension constructions for nonlinear patterns is quite groundbreaking. Using more sophisticated probabilistic concentration bounds than standard results used in the construction of sets avoiding linear patterns, I was able to reduce the study of the pattern avoidance problem to the study of a family of oscillatory integrals with 'nonsmooth' amplitude functions. These oscillatory integrals are natural, but don't seem to have been considered in harmonic analysis or probability, and it does not seem we have the tools to estimate them completely. Thus the techniques of this paper are primed to produce further interesting results in the field. We were able to obtain a nontrivial, but imperfect analysis of these oscillatory integrals in various cases, and we can avoid them entirely for certain patterns exhibiting a very weak translation-invariance property, thus completely removing the barrier between Fourier and Hausdorff dimensions in this case.

This project is solely my own. I started developing the ideas behind the project after I finished my Master's degree, and my current PhD advisor does not study pattern avoidance problems. I have submitted the paper for publication in the journal "Analysis & PDE", since many members of the editorial board are interested in geometric measure theory and harmonic analysis, including Izabella Łaba, who has even published multiple papers on the existence and avoidance of patterns in sets. Moreover, various analogues of the result of [2] for Hausdorff dimension have been published in this journal (e.g. Pramanik and Fraser, 2018), so the journal seems like a natural fit.

Part III - Applicant's Statement

Research Experience

Being able to translate questions between discrete and continuous settings has proved very useful in my work on pattern avoidance. But it is also incredibly useful to study radial multipliers, where one converts the problem into the study of L^p bounds on a sum, which is highly related to the incidence geometry of spheres. During my undergraduate degree in theoretical computing science, and my master's thesis, I have obtained a wide background not only in harmonic analysis, but also in various aspects of discrete mathematics useful to harmonic analysis, such as incidence geometry and the polynomial method. I have taken courses on the polynomial method while at UBC which involved reading new results in the setting, and I organized a reading group in the polynomial method at UW-Madison. As described in my outline, current research indicates these techniques might prove very useful into the study of radial multipliers.

Relevant Activities

During my Master's thesis, I attended a conference where a colleague invited me to attend the February Fourier Talks, a talk attended mainly by applied mathematicians. Presenting a poster thus seemed quite intimidating as a pure mathematicians. I thus prepared by reviewing aspects of geometric measure theory related to the more applied theory. One mathematician commented that "fractals been a dead field for 40 years", so I talked to him to the study of Kakeya sets, their relation to pattern-avoidance problems, and how they arise in the study of the instability of natural cutoff filters that occur in signals processing. I surpassed my expectations on presenting my work to a different audience, since I was awarded the 2nd place prize for a poster presentation at the conference. I also learned the probabilistic techniques I used

to develop the results in [2] by taking a class geared to study problems in data science. Being able to communicate between pure and applied variants of the same subject thus seems like a very profitable