l
2^{-k}
k
$l >$
0
$_{l}^{d}$
$_{d}$
l
$(l\cdot)^d$
$$_{l}^{d} = \{[a_1, a_1+l]\times\cdots\times[a_d, a_d+l] : a_k \in l\cdot\}.$$
$E \subset^d$
$_{l}^{d}(E)$
$_{l}^{d}$
E
$$_{l}^{d}(E) = \{I \in_{l}^{d} : I \cap E = \emptyset\}.$$
$Z \subset^d$
$$(Z) = \liminf_{l\to 0} \frac{\log(\#_{l}^{d}(Z))}{\log(1/l)} and (Z) = \limsup_{l\to 0} \frac{\log(\#_{l}^{d}(Z))}{\log(1/l)}.$$
$0 \leq$
α
$\delta >$
0
$E \subset^d$
$$H_{\delta}^{\alpha}(E) = \inf\left\{\sum_{k=1}^{m} l_k^{\alpha} : E \subset \bigcup_{k=1}^{m} I_k and I_k \in_{l_k}^{d}, l_k \leq \delta for all\right.$$
$. The \alpha$
H^{α}
d
$H^{\alpha}(E) =$
$\lim_{\delta\to 0} H_{\delta}^{\alpha}(E)$
E
$(E) =$
$\inf\{\alpha \geq$
$0 :$
$H^{\alpha}(E) =$
$0\}$
$I \in_{l}^{dn}$
I
$I_1 \times$
$\cdots\times$
I_n
$I_1, \ldots, I_n \in_{l}^{d}$
I
$I_1, \ldots, I_n$
??
??
?
$\{U_k\}$
E
$E \subset$
$\limsup U_k$
E
U_k
??
α
μ
d
I
l
$\mu(I) l^{\alpha}$
μ
I
$$(E) = \sup\{\alpha : there is a Frostman measure of dimension dimension$$
α
E
$.(1)$
Z
??
X
Z
Z
X
??
$l \geq$
s
$l =$
l_n
$s =$
l_{n+1}
$E \subseteq$
$[0,1)^d$
d

(F)
$J_1 \times$
$\ldots \times$
$J_n \notin_s^{dn}$
(G)
Non-Concentration
$I' \in_r^d$
(E)
$J \in_s^d$
(F)
$J \subset$
I'
Large Size
$I \in_l^d$
(E)
$\#_s^d(F \cap$
$I) \geq$
$\#_r^d(I)/2 =$
$(l/r)^d/2$
Remark:
??
F
$_s^{dn}(G)$
??
??
$I \in_l^d$
(E)
$I' \in_r^d$
(I)
s
F
r

$$r \geq R := \left(2l^{-d} s^{dn} \#_s^{dn}(G)\right)^{\frac{1}{d(n-1)}}. \tag{4}$$

r
$r \in$
$[s,l]$
R
s
$r \geq$
s
R
l
r
$r \leq$
l
$I' \in_r^d$
(E)
$J_{I'}$
$_s^d(I)$
I'
$_r^d(E)$

$$U = \bigcup \left\{ J_{I'} : I' \in_r^d (E) \right\},$$

$$\mathcal{K}(U) = \{K \in_s^{dn} (G) : K \in U^n,$$

$stronglynon-$
$diagonal\}. Notethatthesetsandarerandomsets, inthesensethattheyaredependontherandomvariables\{_I\}$

$$F_U = U - \{\pi(K) : K \in \mathcal{K}(U)\}, \tag{5}$$

$\pi :^{dn} \to^d$
$(x_1, \cdots, x_n) \in$
$R^{dn} \mapsto$
$x_1 \in$
R^d
π
$K_1 \times$
$\ldots \times$
$K_n \in_s^{dn}$
$K_1 \in_s^d$
$J_1 \times$
$\cdots \times$
$J_n \in_s^{dn}$
(G)
$J_1 \times$
$\cdots \times$
$J_n \notin_s^{dn}$
(U^n)
$J_1 \times$
$\cdots \times$
$J_n \in_s^{dn}$