Berkeley Preliminary Exam

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Chapter 1

2007 Preliminary Exam

Exercise 1.1. *Let f and g be entire functions for which $f' = g$, $g' = -f$, and $f(2z) = 2f(z)g(z)$ for all $z \in \mathbf{C}$. Find all possiblities for f.*

Proof. Consider the differential equation $f'' = -f$. If $f = e^{Kiz}$. Then $f'' = -K^2 e^{Kiz}$, so $K = \pm 1$, and the general solution is

$$f(z) = Ae^{iz} + Be^{-iz}$$

$$f(2z) = Ae^{2iz} + Be^{-2iz} = 2(Ae^{iz} + Be^{-iz})(Aie^{iz} - Bie^{-iz}) = 2(A^2 ie^{2iz} - B^2 ie^{-2iz})$$

This means that $A = 2A^2 i$, $B = -2B^2 i$, so either $A = 0, -i/2$, $B = 0, i/2$. Thus either

$$f(z) = 0 \quad f(z) = i/2e^{-iz} \quad f(z) = -i/2e^{iz} \quad f(z) = i/2(e^{-iz} - e^{iz}) = \sin(z)$$

$\square$