

Graph Tidbits

- $G = (V, E)$
 - V = vertices
 - E = edges
 - Can be undirected or directed
 - Range of $|E|$?
- Why graphs?
- Representations:
 - Adjacency matrix
 - Adjacency list

- How do you explore a labyrinth?

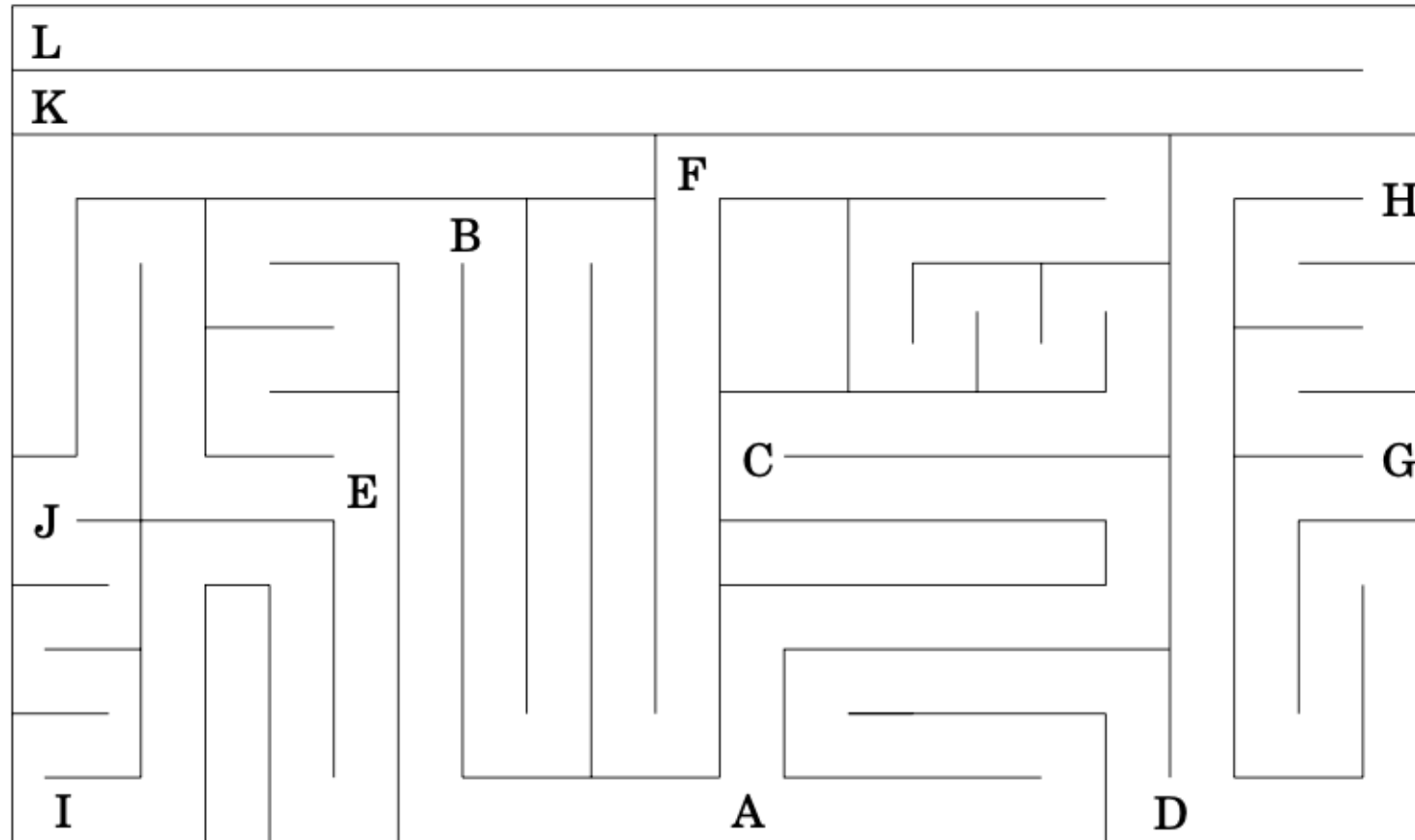


Figure 3.3 Finding all nodes reachable from a particular node.

procedure `explore`(G, v)

Input: $G = (V, E)$ is a graph; $v \in V$

Output: `visited`(u) is set to true for all nodes u reachable from v

`visited`(v) = true

`previsit`(v)

for each edge $(v, u) \in E$:

 if not `visited`(u): `explore`(u)

`postvisit`(v)

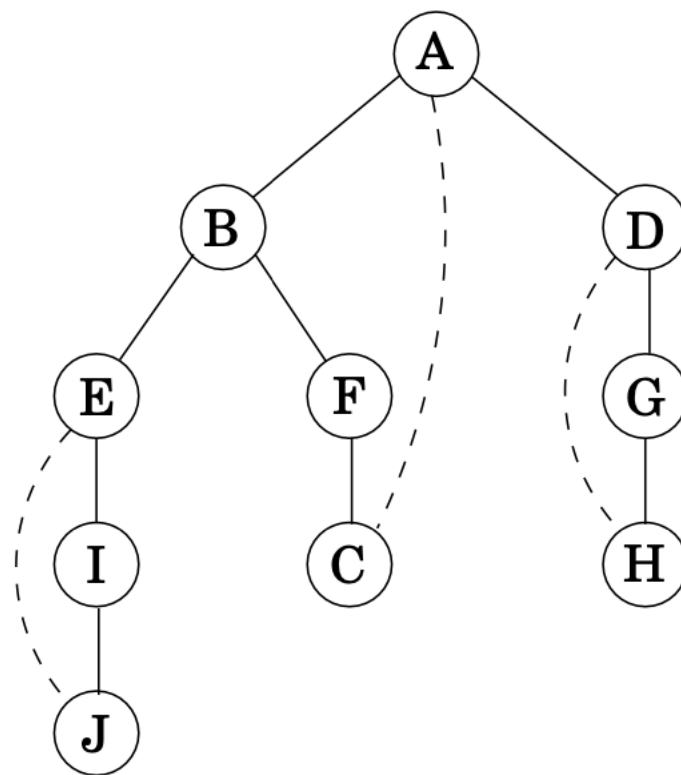
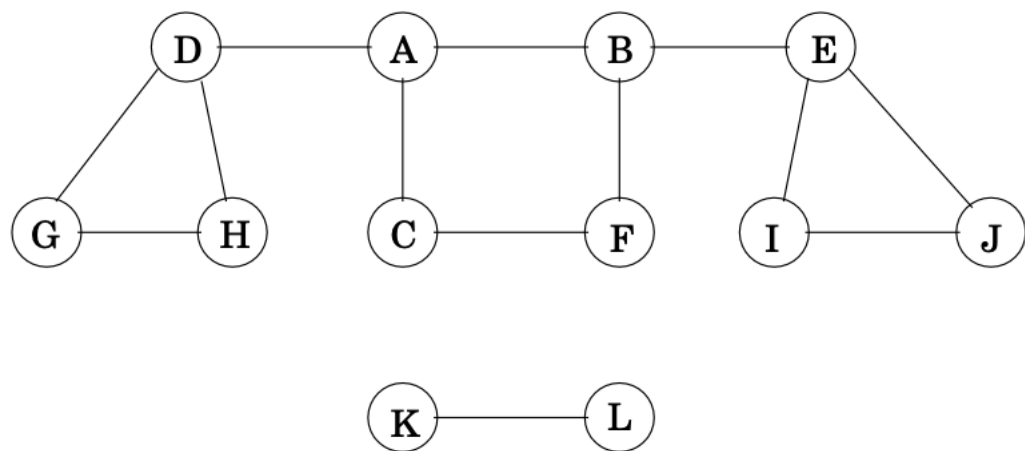
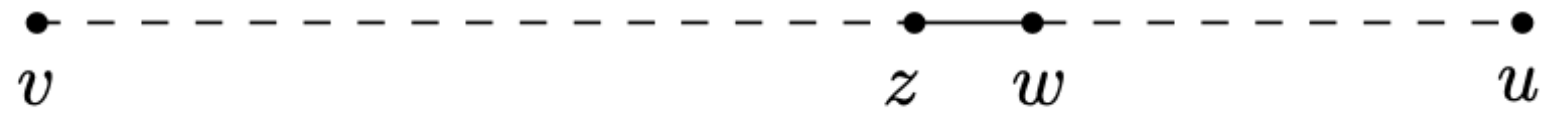


Figure 3.5 Depth-first search.

procedure `dfs`(G)

for all $v \in V$:
 `visited`(v) = `false`

for all $v \in V$:
 if not `visited`(v): `explore`(v)



- Connectivity

Figure 3.6 (a) A 12-node graph. (b) DFS search forest.

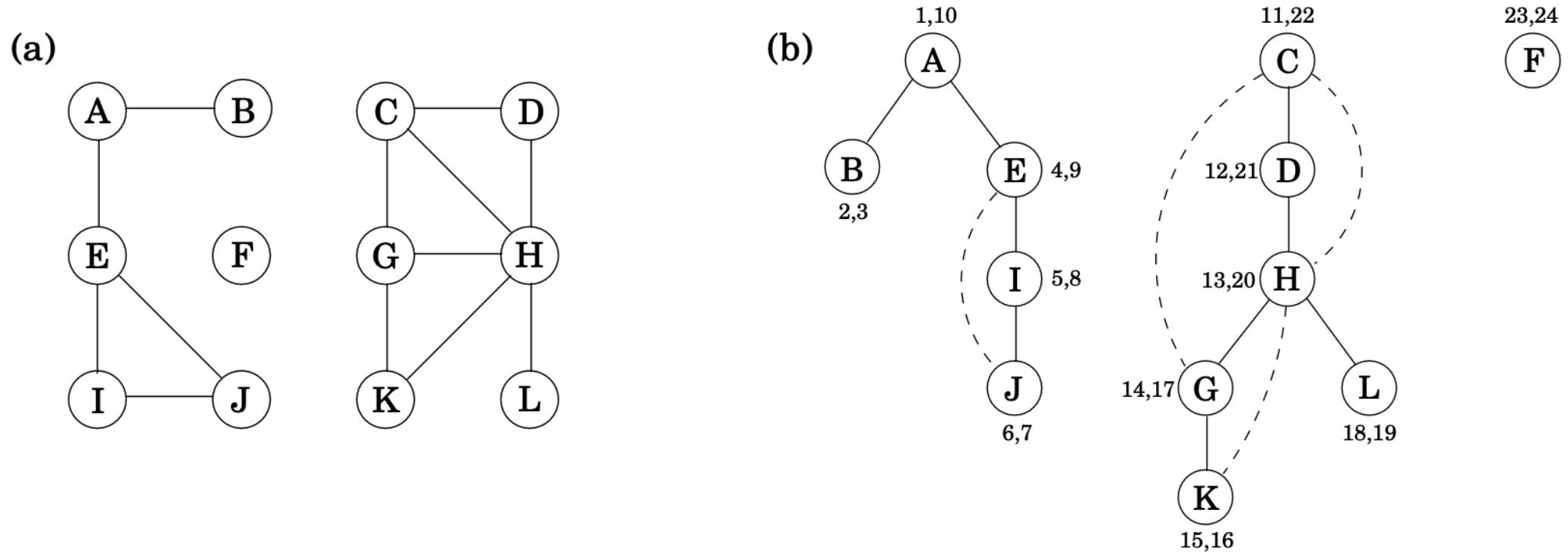
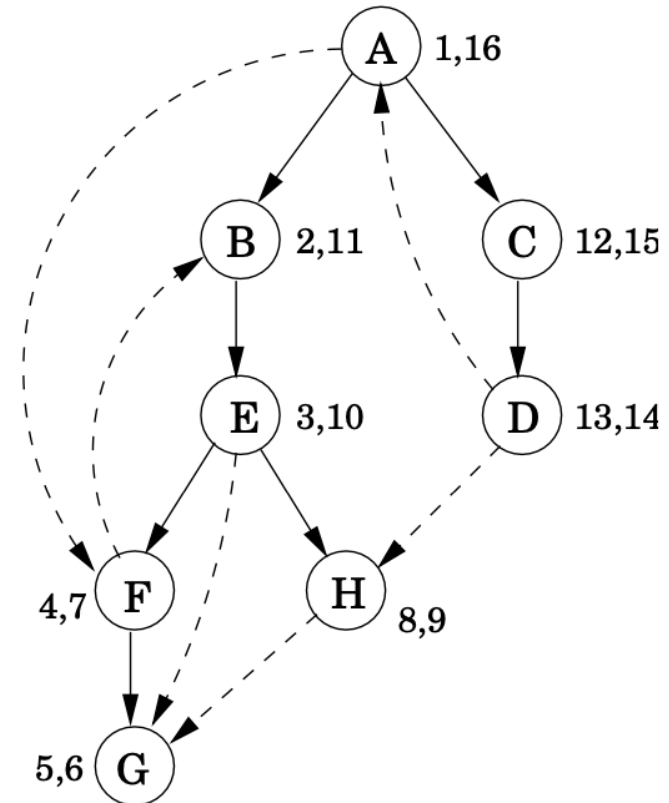
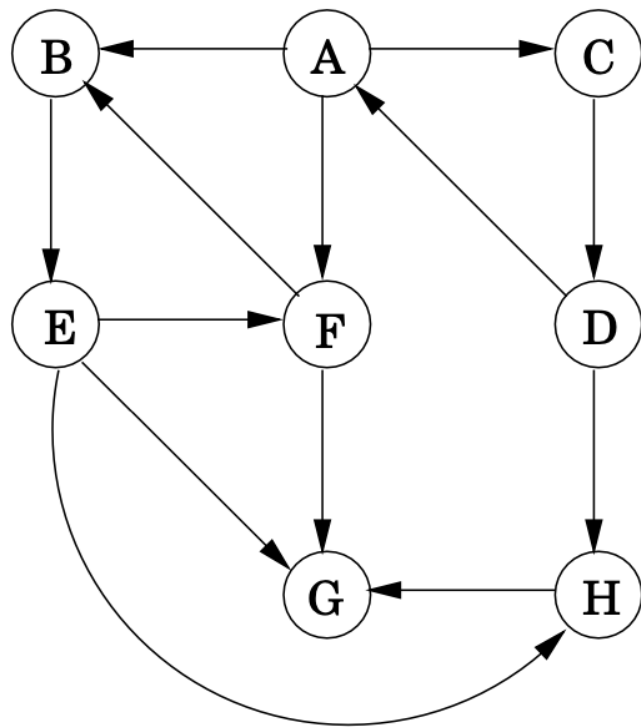


Figure 3.7 DFS on a directed graph.



Tree edges are actually part of the DFS forest.

Forward edges lead from a node to a *nonchild* descendant in the DFS tree.

Back edges lead to an ancestor in the DFS tree.

Cross edges lead to neither descendant nor ancestor; they therefore lead to a node that has already been completely explored (that is, already postvisited).

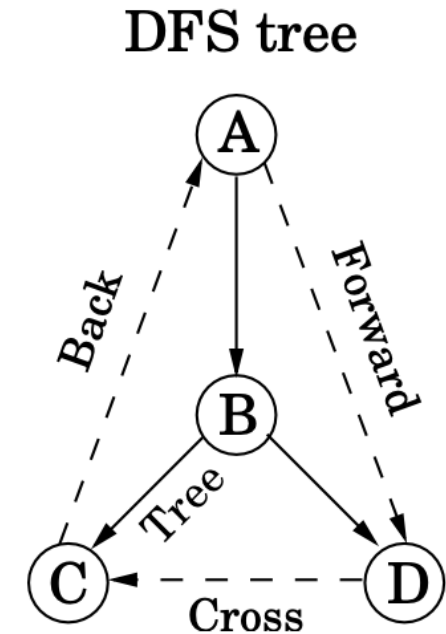
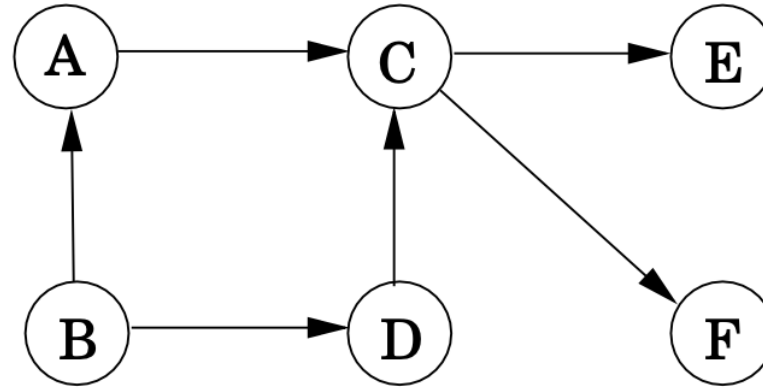


Figure 3.8 A directed acyclic graph with one source, two sinks, and four possible linearizations.



Property *A directed graph has a cycle if and only if its depth-first search reveals a back edge.*

Prove: If a directed graph G is such that every node has an incoming edge, then G must contain a cycle.

Property *In a dag, every edge leads to a vertex with a lower post number.*

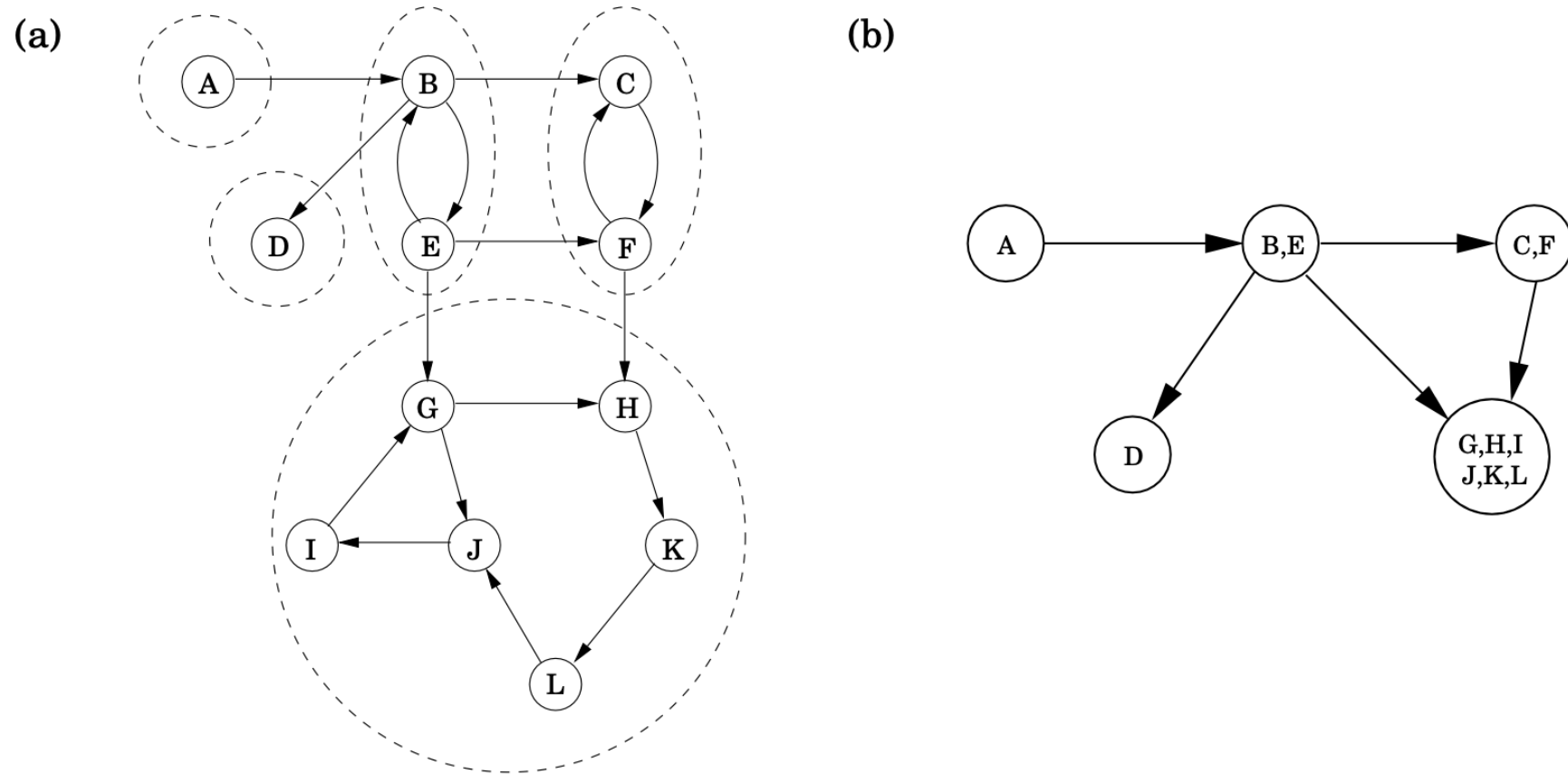
Property *Every dag has at least one source and at least one sink.*

The guaranteed existence of a source suggests an alternative approach to linearization:

Find a source, output it, and delete it from the graph.

Repeat until the graph is empty.

Figure 3.9 (a) A directed graph and its strongly connected components. (b) The meta-graph.



Property 1 *If the explore subroutine is started at node u , then it will terminate precisely when all nodes reachable from u have been visited.*

Property 2 *The node that receives the highest post number in a depth-first search must lie in a source strongly connected component.*

Property *In a dag, every edge leads to a vertex with a lower post number.*