Assignment 0

16-384: Robot Kinematics and Dynamics

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Contents

1 Overview

Welcome to Robot Kinematics and Dynamics! This initial assignment aims to familiarize you with the tools we will be using this semester and provide a brief review of the prerequisite knowledge we expect you to have. The topics and processes in this assignment are essential and will be frequently utilized throughout the course. If you experience any challenges with any part of this assignment, please reach out to the course staff (Shahram Najam Syed or Yuemin Mao) so we can ensure you are well-prepared for the upcoming assignments.

2 Background

This section provides a brief overview of the key concepts discussed in lectures as they relate to the labs and the assignments.

2.1 Matrices

Throughout this course, we will extensively use matrices and matrix multiplication. While a deep understanding of linear algebra is not required, you should be comfortable with the following basics:

- Matrix multiplication
- Transposition
- Inversion
- Rank
- Vector norms
- Determinants
- Cross products
- Dot products

2.2 Calculus

You are expected to be familiar with the following calculus concepts:

- Derivatives (including trigonometric functions)
- Partial derivatives

2.3 Course Logistics

In this class you would be using the following websites regularly:

- Canvas: The main course web page. This links to everything relevant to the class (including the next two links).
- Piazza: This is the best way to ask course staff questions, and see course announcements.
- Gradescope: This is where you will be turning in your assignment. You should have already been added to the course. If you haven't, please contact the course staff via email or Piazza post.

3 Instructions

- The deadline for this assignment is 29th August, 2024 09:00 P.M.
- The LaTEX folder and the relevant code can be downloaded from the following GitHub repository.
- The LaTEX format has been designed and tested on Overleaf, and **we strongly recommend using the Overleaf** to avoid any formatting errors as you fill out your answers.
- Zip your code into a single file named `<AndrewId>.zip`. See the complete submission checklist at the end, to ensure you have everything. Submit your PDF file to Gradescope.
- Each question (for points) is marked with a **points** heading.
- **Start early!** This homework may take a long time to complete.
- **During submission indicate the answer/page correspondence carefully when submitting on Gradescope.** If you skip a written question, just submit a blank page for it. This makes our work much easier to grade.
- If you have any questions or need clarifications, please post in Piazza or visit the TAs during the office hours.
- Unless otherwise specified, **all units are in radians, meters, and seconds**, where appropriate.

4 Theory Questions

4.1 Matrix and Vector Operations

Question a (10 points) Matrix Validity and Operations: Determine whether the following matrix expressions are *valid* or *invalid*. If valid, provide the *dimensions* of the output matrix. If invalid, explain why the operation, relating to the properties of matrix dimensions, invertibility, and multiplication rules.

(i) $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \cdot \left(\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \right)^T$

(ii) $\left(\left(\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} \right)^T \cdot \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} \right)^{-1}$

(iii) $\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$

(iv) $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \cdot \left(\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \right)^T$

(v) $\left(\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} \right)^T \cdot \left(\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} \right)^{-1}$

(vi) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 5 \\ -5 \end{bmatrix}$

(vii) $\left(\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 1 \end{bmatrix} \right)^T$

(viii) $\left(\begin{bmatrix} 2 & 3 \\ 5 & 7 \end{bmatrix} \right)^{-1} \cdot \begin{bmatrix} 2 & 3 \\ 5 & 7 \end{bmatrix}$

(ix) $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \cdot \left(\begin{bmatrix} 1 & 1 \\ 1 & 1 \\ 1 & 1 \end{bmatrix} \right)^{-1}$

(x) $\begin{bmatrix} 1 & 2 & 3 \end{bmatrix} \cdot \left(\left(\begin{bmatrix} 1 & 2 & 3 \end{bmatrix} \right)^T \cdot \begin{bmatrix} 1 & 2 & 3 \end{bmatrix} \right)^{-1}$

Question b (10 points) Matrix Multiplications:

(i) Multiply a 3×2 matrix with a 3×1 vector: $\begin{bmatrix} a & b \\ c & d \\ e & f \end{bmatrix} \cdot \begin{bmatrix} 4 \\ 2 \\ 9 \end{bmatrix}$

(ii) Multiply a 2×2 matrix with a 2×2 matrix: $\begin{bmatrix} 10 & 3 \\ 0 & 6 \end{bmatrix} \cdot \begin{bmatrix} 2 & 1 \\ 5 & 3 \end{bmatrix}$

Question c (15 points) Rotation Matrix and Its Properties: Given the rotation matrix

$$R = \begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix},$$

(i) Determine R^{-1}.

(ii) Determine R^T.

(iii) Explain the relationship between R^{-1} and R^T.

Question d (20 points) Determinants and Ranks: Given $A = \begin{bmatrix} 1 & 5 \\ 6 & 18 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 4 \\ 6 & 12 \end{bmatrix}$,

(i) Determine $\det(A)$.

(ii) Determine $\operatorname{rank}(A)$. Is A full rank?

(iii) Determine $\det(B)$.

(iv) Determine $\operatorname{rank}(B)$. Is B full rank?

(v) Explain the relationship between the rank and the determinant of these matrices.

Question e (10 points) **Vector Operations:** Given the vectors $\mathbf{x} = \begin{bmatrix} 2 \\ 4 \\ 6 \end{bmatrix}$ and $\mathbf{y} = \begin{bmatrix} 3 \\ 6 \\ 9 \end{bmatrix}$

(i) Determine $\mathbf{x} \cdot \mathbf{y}$ (dot product).

(ii) Determine $\mathbf{x} \times \mathbf{y}$ (cross product).

(iii) Determine $\|\mathbf{x}\|_2$ and $\|\mathbf{y}\|_2$ (L2 norm).

(iv) Prove whether x and y are linearly independent.

Question f (10 points) Matrix Inversion and Systems of Equations:

(i) Invert the matrix $\begin{bmatrix} 3 & 1 \\ 1.5 & 1 \end{bmatrix}$.

(ii) Solve the linear system $\begin{bmatrix} 3 & 4 \\ 6 & 8 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 12 \\ 24 \end{bmatrix}$.

4.2 Calculus

Question a (5 points) Advanced Derivative Computation: Find the derivative with respect to x of the function $f(x) = x^2 \cos(x^3) e^{x^2}$. **Note:** Provide a detailed solution including any relevant differentiation rules used. This task will require you to apply the product rule, chain rule, and possibly the exponential rule effectively. Please detail each step of your calculation process to demonstrate your understanding of these rules in the context of derivative computation.

Question b (10 points) Partial Derivatives: Given the function

$$f(x, y, z) = z \sin(x^2 y) + y^3 e^{xz},$$

(i) Find the partial derivative with respect to x

(ii) Find the partial derivative with respect to y

(iii) Find the partial derivative with respect to z

Note: Carefully perform and document each step in finding the partial derivatives. Explain the implications of these derivatives in terms of changes in f when varying one variable while holding the others constant. This will test your ability to understand and apply the chain rule in a multivariable context.

Question c (10 points) Gradient Analysis: Consider the function $g(x, y, z) = x^2 e^y + z \ln(y)$:

i. Find the gradient vector ∇g.
ii. Determine the critical points of g and classify them using the second derivative test.

Notes: Explain each part of your answer, detailing how the gradient relates to the function's optimization. Classify the critical points and discuss whether they represent local minima, maxima, or saddle points.

5 Code Questions

This section aims to reinforce the theoretical foundations through practical coding exercises in Python, focusing on linear algebra and calculus operations that are pivotal in robotics kinematics and dynamics. Ensure Python and the NumPy library are correctly installed for handling matrix operations, and consider using SymPy for calculus operations.

For Python installation, visit:

https://www.python.org/downloads/

For NumPy and SymPy, install via pip:

```
pip install numpy sympy
```

After setting up, download the necessary files from the course website and navigate to the Code Handout folder which contains initial scripts.

1) Exercise 01 *40 points*

Implement matrix operations using NumPy to verify your understanding:

- Define two matrices `A` (2×2) and `B` (2×3) and compute their dot product where applicable.
- Compute the transpose, determinant, and inverse of `A` (ensure `A` is invertible).
- Calculate the rank and vector norm of `B`.
- Multiply `A` by a vector and compute the resulting vector's norm.

Attach screenshot of your result in the write up submission.

2) Exercise 02 *40 points*

Create a script using SymPy to work with calculus concepts:

- Define a symbolic function $f(x, y) = x^2 \sin(y) + y^3 \cos(x)$ and compute its partial derivatives with respect to x and y.
- Evaluate the derivatives at specific points (e.g., $(x, y) = (\pi, \pi/2)$).
- Compute the directional derivative of f along the vector $[1, 1]$ at the point $(1, 2)$.

3) Exercise 03 *20 points*

Simulate basic vector operations important in robotics:

- Define vectors `u` and `v` in 3D space.
- Compute and interpret the dot product and cross product of `u` and `v`.
- Analyze the geometric relationship between `u` and `v` based on the dot and cross product results (angle between vectors, perpendicularity).

4) Submission

To submit, run `create_submission.py`. This script checks that your files run without error and performs basic output consistency checks. Note, this script does not grade your submission but ensures it is prepared correctly. The script will generate a file called `handin.zip`. Upload this to Canvas to complete your submission.

6 Submission Checklist

- ☐ Create a PDF of your answers to the written questions using the template provided. Enter your responses where prompted.
- ☐ Run `create_submission.py` in the python terminal.
- ☐ Upload `<andrew_id>_hw0.zip` to Gradescope.
- ☐ Upload `<andrew_id>_hw0.pdf` to Gradescope.
- ☐ Coding questions are autograded, with no limit on number of submissions within the mandated deadline.