Assignment 8: 3D Jacobians and Manipulability

Robot Kinematics and Dynamics

Prof. Jeff Ichnowski
Shahram Najam Syed
Yuemin Mao

Contents

1 Overview

This assignment reinforces the following topics:

- Angular and Linear Velocity Jacobians
- Manipulability

2 Background

2.1 Angular and Linear Velocity Jacobians

As a reminder, each column of the Jacobian is influenced by each joint. The Jacobian for each joint varies based on the type of the joint as shown in the table below.

*i*th joint	Revolute	Prismatic
Linear	$z_{i-1}^0 \times (o_n - o_{i-1})$,	z_{i-1}^0
Angular	z_{i-1}^0	0

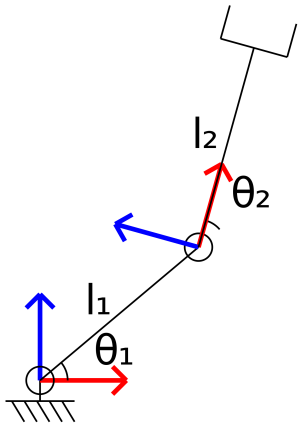
2.2 Manipulability

Recall that manipulability is a measure of the Jacobian and represents how far away we are from a singularity. We will be using the Yoshikawa Manipulability measure which is defined as:

$$\mu = \sqrt{det(JJ^T)}$$

3 In Class Question

The following question will be done in class, as a part of a group. Your group's answer will still need to be turned in with the rest of your assignment, however unlike the rest of the work this is allowed to be done in groups.



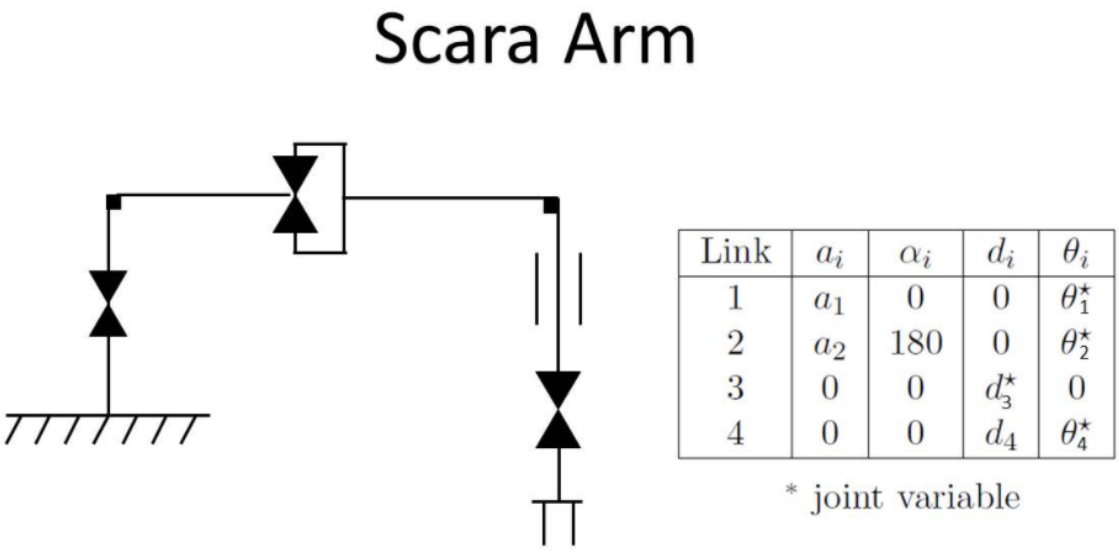
Using a RR robot shown above, find a joint configuration with maximal manipulability.

4 Written Questions

For all of the following questions, please fully compute all answers and show your work for full credit, unless otherwise specified. Submit your answers in a PDF entitled `writeup.pdf` in your handin directory. All answers must be typed, but diagrams may be hand-drawn and scanned in. However, they must be tidy and fully legible! Consider drawing them in a black or blue pen. All units are in radians and meters, where appropriate.

1) Intermediate Transformations

Using the DH parameters bellow for the SCARA arm find the bellow homogeneous transformation matrices.



Link	a_i	α_i	d_i	θ_i
1	a_1	0	0	$\theta_1^\star$
2	a_2	180	0	$\theta_2^\star$
3	0	0	$d_3^\star$	0
4	0	0	d_4	$\theta_4^\star$

* joint variable

(1) [2 points] H_1^0

(2) [2 points] H_2^1

(3) [2 points] H_3^2

(4) [2 points] H_4^3

2) Origin frames and Z axes

Calculate the Origin frames and Z axes at each joint.

(1) [2 points] Joint 1

(2) [2 points] Joint 2

(3) [2 points] Joint 3

(4) [2 points] Joint 4

3) 3D Jacobians

1 Calculate the Jacobian to each joint. Using these values: $a_1 = 5, a_2 = 10, d_4 = 2$
All thetas are 0 and d_3 is 2.

(1) [2 points] Joint 1

(2) [2 points] Joint 2

(3) [2 points] Joint 3

(4) [2 points] Joint 4

4) Gravity Compensation

(1) [5 points] Using your Jacobians from the previous question and given the following joint masses and assuming the links are magically massless, determine the torques that each joint needs to output in order to resist the force of gravity where g = 9.8 m/s^2.
joint masses = [0.5, 0.5, 1, 0.5]

5) Manipulability

(1) [10 points] Determine the Yoshikawa Manipulability of end-effector of the capstone arm at the following 2 joint configurations:

- $\theta_1 = \frac{\pi}{2}, \theta_2 = \frac{\pi}{2}, d_3 = 4, \theta_4 = 0$

- $\theta_1 = 0, \theta_2 = 0, d_3 = 4, \theta_4 = 0$

(2) [4 points] Which of the following above joint configurations is better and why?

5 Submission Checklist

- ☐ Upload `writeup.pdf` to Gradescope.