

# Capstone

Robot Kinematics and Dynamics

Prof. Jeff Ichnowski

Shahram Najam Syed

Yuemin Mao

# Contents

<b>1</b>	<b>Overview</b>	<b>3</b>
<b>2</b>	<b>Background</b>	<b>4</b>
2.1	Notation and Terminology . . . . .	4
2.2	Picking / Grasping . . . . .	4
2.3	Franka Kinematics . . . . .	4
2.4	Inverse Kinematics . . . . .	5
2.5	Trajectories . . . . .	6
2.5.1	Joint-Space Trajectories . . . . .	6
2.5.2	End-Effector Trajectories . . . . .	7
2.5.3	Trajectory Timing and Execution . . . . .	7
2.6	Placing . . . . .	8
2.7	Avoiding Collisions . . . . .	8
2.8	Avoiding Singularities . . . . .	9
<b>3</b>	<b>Instructions</b>	<b>10</b>
<b>4</b>	<b>Capstone</b>	<b>11</b>
4.1	Recovery policies and “Sensors” . . . . .	11
4.2	Home configuration . . . . .	11
4.3	Picking up a pen . . . . .	11
4.4	Moving the pen to the whiteboard . . . . .	12
4.5	Drawing a line . . . . .	12
4.6	Drawing an arbitrary curve . . . . .	12
4.7	Placing a pen in a drop location . . . . .	12
4.8	Human-in-the-loop . . . . .	13
<b>5</b>	<b>Checkpoint</b>	<b>14</b>
<b>6</b>	<b>Demo and Grading Rubric</b>	<b>15</b>
<b>7</b>	<b>Writeup</b>	<b>16</b>
<b>8</b>	<b>Submission Checklist</b>	<b>18</b>

# 1 Overview

In this capstone, you will form a team of 3 students to program a Franka Emika Panda robot to draw on a whiteboard.

## 2 Background

### 2.1 Notation and Terminology

Throughout this document we will use the following terms consistently:

**Configuration** (or config for short) specifies the settable degrees of freedom of a Robot. For Franka robots, this is a 7-element vector where the first coefficient is the angle of the first joint, etc.

**Cartesian** coordinates refers to the  $x, y, z$  position in workspace coordinates for translation, and a rotation relative to a fixed workspace identity rotation.

**Position** refers to the Cartesian translation of a an object.

**Orientation** refers to the rotation of an object.

**Pose** refers the the position and orientation of an object.

**Linear interpolation** given a start and end vector, this is an interpolation that starts at the start vector at  $t = 0$  and ends at the end vector at  $t = 1$ . Mathematically  $f(x_0, x_1, t) = x_0(1 - t) + x_1t$ , for  $x_0, x_1 \in \mathbb{R}^n$ .

### 2.2 Picking / Grasping

The picking/grasping task in the capstone is the same as in the hands-on section of Assignment 7: moving the robot to a config where the target object is at the center of the gripper and closing the gripper.

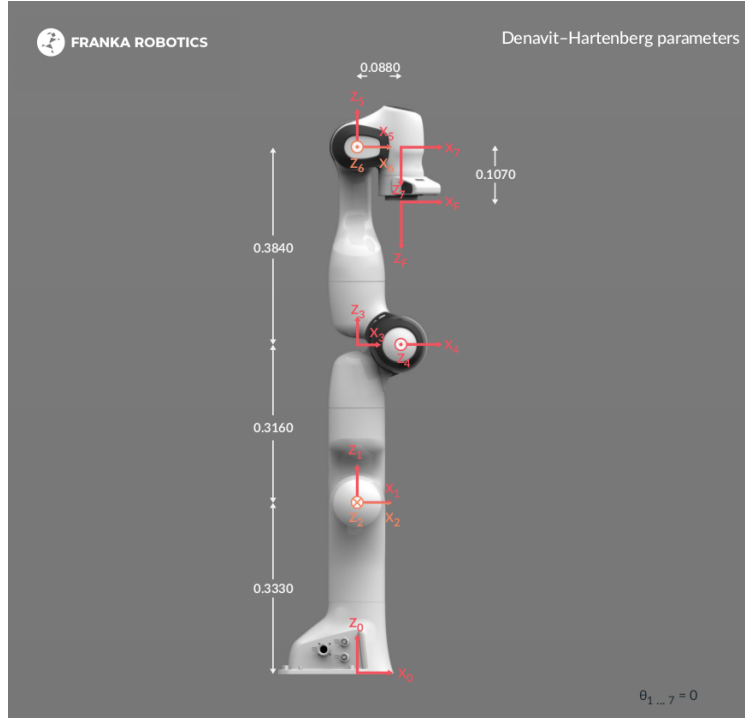
### 2.3 Franka Kinematics

Most robot manufacturers, including Franka Robotics, use DH parameters to represent the robot kinematics. DH parameters of the Franka Emika Panda robot see Fig. 1 and Table 1. (Ref: [Franka DH parameters](#), note that Panda and Research 3 have the same DH parameters.)

You will implement a forward kinematics function using the method introduced on Slide 16 of [DH parameter slides](#). While the given DH parameters allow you to compute the transformation from the robot's base frame to its flange frame, you can implement your forward kinematics function to operate to any arbitrary point by applying additional transformations. For instance, the distance between the center of grasp and the flange frame is 0.1034 m in the  $z$  direction, so you can apply an additional transformation using  $a = 0, d = 0.1034, \alpha = 0, \theta = 0$  to compute the transformation from the robot's base frame to its center of grasp. Similarly, you can also get the transformation from the robot's base frame to the tip of a grasped pen. Assuming you follow the slides, you will compute forward kinematic map in the form:

$$H_8^0 = H_1^0 H_2^1 \cdots H_8^7.$$

We will define  $\text{fk}(q)$  to be the forward kinematic function that, given a robot configuration  $q \in \mathbb{R}^7$ , outputs  $H_8^0$ .



**Figure 1:** Franka Emika Panda robot joint frames

Joint	$a_i$ (m)	$\alpha_i$ (rad)	$d_i$ (m)	$\theta_i$ (rad)
1	0	0	0.333	$\theta_1$
2	0	$-\pi/2$	0	$\theta_2$
3	0	$+\pi/2$	0.316	$\theta_3$
4	0.0825	$+\pi/2$	0	$\theta_4$
5	-0.0825	$-\pi/2$	0.384	$\theta_5$
6	0	$+\pi/2$	0	$\theta_6$
7	0.088	$+\pi/2$	0	$\theta_7$
Flange	0	0	0.107	0

**Table 1:** DH parameters of Franka Emika Panda robot

## 2.4 Inverse Kinematics

Inverse kinematics is the process of finding joint angles that position the robot's end effector at a specific point in Cartesian space. Gradient descent for IK starts from an initial guess of a configuration and iteratively follows the negative gradient to minimize the cost/error between the current end effector position and the target position. A good initial guess can significantly speed up convergence in gradient descent. You will need to define an inverse kinematic function that returns a configuration  $q$  based on this process as:

$$q = \text{ik}(x_{\text{target}}, q_{\text{init}}; \lambda),$$

where  $x_{\text{target}}$  is the target pose in  $SE(3)$ ,  $q_{\text{init}}$  is the initial guess, and  $\lambda \in \mathbb{R}^+$  is a gradient step size. We will often omit  $q_{\text{init}}$  and  $\lambda$  in most of the following text. You will have to tune  $\lambda$ , and have a strategy for selecting  $q_{\text{init}}$ .

**Table 2: Franka Panda Joint Limits**—Documented limits for the robot joints. Configuration limits have an upper and lower bound that cannot be exceeded due to hardware limits. Velocity, acceleration, and jerk limits list the upper bound in any direction—these may be enforced by software limits in the robot, and exceeding them may result in unexpected operation. Torque limits are limited by the capabilities of the motors.

Joint	$q_{\min}$ [rad]	$q_{\max}$ [rad]	$\dot{q}_{\max}$ [ $\frac{\text{rad}}{\text{s}}$ ]	$\ddot{q}_{\max}$ [ $\frac{\text{rad}}{\text{s}^2}$ ]	$\dddot{q}_{\max}$ [ $\frac{\text{rad}}{\text{s}^3}$ ]	$\tau_{j_{\max}}$ [N m]	$\dot{\tau}_{j_{\max}}$ [ $\frac{\text{N m}}{\text{s}}$ ]
1	-2.8973	2.8973	2.1750	15	7500	87	1000
2	-1.7628	1.7628	2.1750	7.5	3750	87	1000
3	-2.8973	2.8973	2.1750	10	5000	87	1000
4	-3.0718	-0.0698	2.1750	12.5	6250	87	1000
5	-2.8973	2.8973	2.6100	15	7500	12	1000
6	-0.0175	3.7525	2.6100	20	10 000	12	1000
7	-2.8973	2.8973	2.6100	20	10 000	12	1000

Source: [https://frankaemika.github.io/docs/control\\_parameters.html#limits-for-panda](https://frankaemika.github.io/docs/control_parameters.html#limits-for-panda)

**Table 3: Franka Panda Cartesian Limits**—Documented limits of the robot’s motion. Trajectories that exceed these limits may not run properly, so try to keep the trajectories within these limits.

	Translation	Rotation	Elbow
$\dot{p}_{\max}$	1.7 $\frac{\text{m}}{\text{s}}$	2.5 $\frac{\text{rad}}{\text{s}}$	2.1750 $\frac{\text{rad}}{\text{s}}$
$\ddot{p}_{\max}$	13.0 $\frac{\text{m}}{\text{s}^2}$	25.0 $\frac{\text{rad}}{\text{s}^2}$	10.0 $\frac{\text{rad}}{\text{s}^2}$
$\dddot{p}_{\max}$	6500.0 $\frac{\text{m}}{\text{s}^3}$	12 500.0 $\frac{\text{rad}}{\text{s}^3}$	5000.0 $\frac{\text{rad}}{\text{s}^3}$

Source: [https://frankaemika.github.io/docs/control\\_parameters.html#limits-for-panda](https://frankaemika.github.io/docs/control_parameters.html#limits-for-panda)

When computing the gradient descent, ensure that the joint configurations stay within the limits of the Franka Emika Panda robot. See the  $q_{\min}$  and  $q_{\max}$  columns of Table. 2. The easiest way to do this is to `clamp/clip` the configuration in every iteration of the gradient descent.

## 2.5 Trajectories

For the capstone, we recommend switching, as appropriate, between running joint-space or Cartesian-space interpolations. In almost all cases, the start configuration ( $q_0$ ) or pose ( $x_0 = \text{fk}(q_0)$ ) will be given from the robot’s current configuration or forward kinematics. The end configuration or pose will be task-based. You can also compute a sequence of interpolations by chaining the end-configuration of one interpolation into the start of the next.

### 2.5.1 Joint-Space Trajectories

Joint-space **linear interpolation** (LERP) is straight-forward. Compute

$$\text{lerp}(q_0, q_1, t) = q_0(1 - t) + q_1t,$$

where  $q_0 \in \mathbb{R}^7$  is the start configuration,  $q_1 \in \mathbb{R}^7$  is the end configuration, and  $t \in [0, 1]$ . This hopefully seems simple, as it is just standard vector times scalar and a vector addition.

### 2.5.2 End-Effector Trajectories

Cartesian-space interpolation is a bit more complicated. If considering just the translation components, you can hold the rotation fixed and linearly interpolate from start to end translation, then use inverse kinematics to get the joint angles. If you want to include rotation too, you will need to interpolate both translation and rotation. Fortunately, there is a really good way to interpolate rotations: **spherical linear interpolation** or SLERP. SLERP requires converting the rotations to quaternions, interpolating a quaternion, then converting the quaternion back.

$$\text{slerp}(p_0, p_1, t) = \frac{\sin((1-t)\phi)}{\sin\phi} p_0 + \frac{\sin(t\phi)}{\sin\phi} p_1,$$

where  $p_0, p_1 \in \mathbb{R}^4$  are the coefficients of unit quaternions stacked into vectors, and  $\cos\phi = p_0 \cdot p_1$  (dot product). SLERP can trace the short way or the long way. To ensure taking the shortest path, test if  $\cos\phi < 0$ , and if so, negate one of the quaternions (remember  $p_0$  and  $-p_0$  are the same rotation), and recompute  $\cos\phi$ . You can take a shortcut to compute  $\sin\phi$  using the identity  $\sin^2\phi = 1 - \cos^2\phi$ . Note also that  $\phi$  is the angular distance between rotations—you can use this to decide how many steps to take along the interpolation.

Putting together, interpolating in Cartesian space, given  $x_0 = [d_0, p_0]$  and  $x_1 = [d_1, p_1]$  are the translations (displacements)  $(d_0, d_1)$  and rotations  $(p_0, p_1)$ . Note that  $x_0, x_1 \in SE(3)$  and can be expressed as homogeneous matrices and converted to and from a [translation, quaternion] tuple (we leave these conversions as implicit here). Compute interpolation of a configuration  $q_t$  at time  $t \in [0, 1]$  as:

$$q_t = \text{ik}(x_t) = \text{ik}([d_t, p_t]) = \text{ik}([\text{lerp}(d_0, d_1, t), \text{slerp}(p_0, p_1, t)]).$$

When computing an interpolation, the most sensible setting for  $q_{\text{init}}$  in the inverse kinematics computation is the previous configuration computed. Thus, switching the subscript  $i$  to reflect the  $i^{\text{th}}$  waypoint along the interpolation:

$$q_{i+1} = \text{ik}(x_{i+1}, q_i),$$

where  $q_0$  is the start of the interpolation.

### 2.5.3 Trajectory Timing and Execution

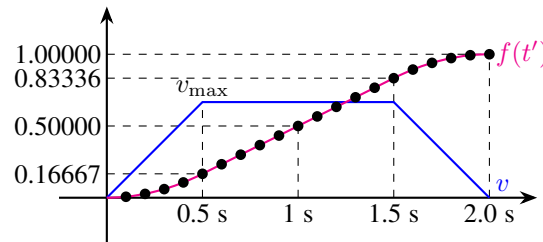
When computing an interpolation-based trajectory, the next thing to consider is velocity and acceleration, thus timing. Robots cannot instantaneously accelerate, and the Franka robot has velocity and acceleration limits, both in joint space (Table. 2) and end-effector/Cartesian space (Table. 3). You should thus smoothly vary the time that you pass to the interpolation function to smoothly accelerate and avoid instantaneous changes. You can usually stay within the documented limits in the tables by trial-and-error or rough estimations.

We recommend starting with a trapezoidal profile, but you are free to try other profiles. Example: suppose you want to interpolate a line in joint space from  $q_0$  to  $q_1$ , where  $q_0, q_1 \in \mathbb{R}^7$ . You want to interpolate over 2 seconds, accelerating for 0.5 seconds and decelerating for 0.5 seconds. First, define the number of waypoints you want to compute. We'll say 20 waypoints, thus 5 for accelerating, then 10 at constant velocity, and 5 for decelerating.

Our goal is to modify our interpolation from:

$$\text{lerp}(q_0, q_1, t) \quad \text{to} \quad \text{lerp}(q_0, q_1, f(t')),$$

where  $f(t')$  maps from  $t' \in [0, 2]$  to  $f(t') \in [0, 1]$ . The interpolation will look something like the following:



With  $f(\cdot)$  defined, you can then interpolate  $t'$  from 0 to 2 seconds and send the waypoints to the robot to run. You can apply the same method to interpolate in Cartesian space.

**Note:** The robot controller receives new waypoints every 0.02 seconds, so your interpolation function should ensure that all waypoints are spaced 0.02 seconds apart. Each successive waypoint must be reachable within 0.02 seconds from the previous one—the robot should not exceed its joint limits during the 0.02-second motion to avoid triggering errors. The final output of your interpolation function must be an  $n \times 7$  array, where each row represents a waypoint defined by the 7 joint angles.

## 2.6 Placing

Placing means positioning an object with the robot, which can have different levels of precision. Dropping an object roughly within an area, as in the hands-on section of Assignment 7, is a simple placing task with low precision requirements. In contrast, inserting a peg into a hole is difficult due to the noise in actuation and sensing, thus requiring a feedback loop to deal with imprecision. In the capstone, you will complete placing tasks with multiple levels of precision without a feedback loop.

## 2.7 Avoiding Collisions

When planning robot motions, it's essential to consider the risk of collisions, especially during joint-space interpolation. Joint-space interpolation causes the robot to sweep arcs through space as the joints rotate, which may inadvertently sweep through an obstacle, such as the table, the whiteboard, the bin, or the robot itself.

To minimize collision risk:

- **Use smaller increments** This allows for more control when the waypoints are close to obstacles.
- **Use tested interpolations** You can find a safe trajectory first and optimize it by applying minor changes to its waypoints and speed.



## 2.8 Avoiding Singularities

Singularities can result in erratic or unpredictable motions, which occur more frequently with Cartesian interpolation. When the end effector moves along waypoints defined in Cartesian space, the robot may pass through its singular configurations as the inverse kinematics solver only guarantees each individual configuration is within joint position limits. If you are interpolating in Cartesian space, singularities will show up as the inverse kinematics solver not finding a solution (within the specified tolerance).

To avoid singularities:

- **Avoid high-risk end-effector positions** Avoid moving the end effector to positions where the arm has to fully extend or fold.
- **Use tested interpolations** Similar to collision avoidance, using tested trajectories with small variations can ensure smoother transitions and avoid singularities.

### 3 Instructions

- The deadline for this project is in-class on Dec. 5, the last day off class. There is no way to extend this.
- There is a checkpoint on Nov. 19. Aim for much earlier than this! If you are just scraping together the checkpoint on the due date, you should consider your team behind schedule.
- See the submission section for details on checkpoint and final submission.
- Pay attention to the grading rubric.
- **Start early!** The capstone has many parts, and may take a long time to complete.
- If you have any questions or need clarifications, please post in Piazza or visit the TAs during the office hours.
- Unless otherwise specified, **all units are in radians, meters, and seconds where appropriate.**

## 4 Capstone

The robot must pick up an uncapped pen from a holder at a known location, move it to place the tip on a whiteboard, draw a shape on the board, then place the pen in a discard location. It must do this sequence at least 3 times, choosing a different pen each time.

Some important points:

- The robot must complete several tasks that involve picking, placing, forward kinematics and inverse kinematics.
- Everyone on the team must participate in the capstone. It is up to the team to determine how to distribute the effort, but **all team members must know all parts of the code**.
- During the capstone presentation, **one team member, selected at random** by the instructor staff, will run the demo—thus every team member must be prepared to demo from **their own account**.
- Be careful about collisions! Before running any program, have your hand hovering over the e-stop button. Stop the robot as soon as you think something might go wrong—rerunning the program takes a minute, fixing a broken robot takes months.

### 4.1 Recovery policies and “Sensors”

A.k.a. At first you don’t succeed, try, try again. Throughout this explanation, there will be times where something fails. The approach we will take is to allow the robot to try again. Since sensing is not part of this class, the “sensor” input will be someone running the robot code. On a failure, the robot operator can run tell the robot to try again. It is up to you to determine when failures are likely to happen and what recovery steps to take.

You may also provide “sensor” input to tell the robot what to draw (e.g., if you want the robot to play an interactive drawing game). Just make sure you hit all parts of the rubric.

The “sensor” input must be either an observation or a simple command. For example, you can tell the robot that “the pen did not drop” to have it try to drop the pen again. You can also tell the robot to “pick up pen 1” if it failed to pick up the planned pen and you want to have it try another. You can also tell the robot to “draw an X at 0,0.”

If in doubt, ask the TAs on Piazza.

### 4.2 Home configuration

The robot should first be placed in a home configuration (important, this is not a home pose, it should be specified in joint angles!). This configuration will allow for predictable/repeatable motions from home to the first pick. We suggest using a linear interpolation in configuration space from the home to a pre-pick configuration.

### 4.3 Picking up a pen

When setting up the environment, we will place a pen holder at a known location. Note: the TAs will provide a range of locations where the pen holder will be during the final demo. The Teams may decide which pen color to place in each pen holder.

To pick up a pen, first ensure that the gripper is open. Then move the gripper to place it vertically over the pen so that when the gripper closes, the pen will be between the fingers. **Note: when the robot approaches the pen, it can collide with things!** Plan a motion that will not.

After securing the pen in the gripper, the robot should lift the pen vertically out of the pen holder. This will require a linear interpolation in Cartesian space using inverse kinematics to resolve the robot configurations along the interpolation.

**Note: The pen holder's orientation is constant but its position varies, which will be provided during the final demo. You must make sure your code can adapt to different penholder positions.**

## 4.4 Moving the pen to the whiteboard

Once the pen is above the pen holder, determine the coordinate of the point on the board on where to place the pen tip. You can try to do a joint-space interpolation, however, be careful, this can fail with a collision. You may instead want to explore strategies that include: (a) moving the robot to a known safe (e.g., “home 2”) position, or (b) moving to a pre-draw position, or (c) some combination or other idea.

**Note: The whiteboard's position and orientation vary, which will be provided during the final demo. You must make sure your code can adapt to different whiteboard positions and orientations.**

## 4.5 Drawing a line

Drawing a line requires tracing a linear interpolation in world space and converting to joint configurations along the way using inverse kinematics. The new twist is that you have to trace the inverse kinematics for the tip of the pen, instead of the end-effector. You'll want to interpolate the motion at a smooth pen drawing rate.

Between drawing lines, you'll need to lift the pen to move it to the next place. You can use the same interpolation, just offset from the board (and you can move faster).

## 4.6 Drawing an arbitrary curve

Once you've got the hang of drawing a line, the next thing to try is to trace a curve, such as a circle, an arc, a polynomial, etc. It's much like tracing a line, but the biggest difference is timing the path. Keep the drawing motion as even as possible. Don't go too fast or too slow. Note: the white board pens have a chisel tip—you can use it to draw wide and narrow lines for optional extra points.

## 4.7 Placing a pen in a drop location

After you're done drawing with a single pen, the next thing you'll need to do is put down the pen. Unfortunately, placing the pen back in the holder is likely not to work—go ahead and try if you're interested! So, instead, you should place the pen in a drop bin. Since dropping the pen on the tip repeatedly will wear down the pen, you need to place the pen on its side (so it is parallel to the table surface).

**Note:** The drop bin's orientation is constant but its position varies, which will be provided during the final demo. You must make sure your code can adapt to different drop bin positions.

*Bonus Point:* if you can successfully place the pen back in the holder, that will count as a successful drop and earn bonus points, however you must demonstrate dropping in the drop bin at least once.

## 4.8 Human-in-the-loop

Humans must not touch or interact with the robot or the environment with the following exceptions:

- Human input on command line for a recovery policy or “sensor” input/command is okay.
- A human can pick up a dropped pen and replace it in the holder or drop bin.

## 5 Checkpoint

On Thur, Nov. 19 at 9:00 PM EST, we will have a checkpoint to verify that you're on the right track. For this checkpoint, you must have the robot go to the home position, pick up a pen and remove it from the holder (rubric items 1, 2, 3, and 4). For the checkpoint, we do not expect inverse kinematics, thus you may use any method to remove the pen, even ones that displace the pen holder in the process.

To turn in the checkpoint, submit a zip file named `<team-name>-checkpoint.zip` with code, and a movie `<team-name>-checkpoint.mp4` (or `<team-name>-checkpoint.MOV` or equivalent) of the motion.

## 6 Demo and Grading Rubric

We will run the capstone projects in a lecture. Each team will have 10 minutes to run their demo. As mentioned above, one team member will be chosen at random to be the robot operator. The operator will have to log in and run all the code from their account. Other team members can give advice and root on the team. The main task and points are listed in bold below. The subtasks are not bold and sum to the main tasks. This rubric is evaluated 3 times, once for each cycle with a different pen. Note that you must draw at least 3 lines and 3 curves, but it does not matter with which pen (but each pen must draw at least one line or curve). So you can draw 10 lines with pen 1, 1 curve with pen 2, and 1 curve with pen 3, and still satisfy the rubric. The sum of lengths of all lines must be 30 cm or more. The sum of lengths of all curves must be 30 cm or more.

The pen holder and the drop bin will be placed at arbitrary locations on the top board of the robot workcell. The whiteboard will be placed at an arbitrary location and orientation in the robot's work space. We will provide the necessary positions and orientations during the final demo—you must make sure your code can adapt to small variations from what you tested your code on.

Task	Points
<b>1. Go to home</b>	<b>5</b>
<b>2. Go to pre-pick location</b>	<b>5</b>
Avoid collisions	2
Place gripper in correct location	3
<b>3. Close gripper and grasp pen</b>	<b>5</b>
<b>4. Lift pen vertically from holder</b>	<b>10</b>
Pen holder does not move	3
Pen tip clears holder	5
<b>5. Move pen to whiteboard</b>	<b>10</b>
Avoids collisions	2
Tip in contact with whiteboard	8
<b>6. Draw line</b>	<b>10</b>
Pen tip remains in contact with whiteboard	5
Pen tip motion is straight	5
<b>7. Lift and move pen to next drawing</b>	<b>10</b>
No extraneous contact at lift point	3
Avoid collisions / contact	3
Place pen tip at start of next drawing	4
<b>8. Draw curve</b>	<b>10</b>
Pen tip remains in contact with whiteboard	5
Pen motion is smooth (no instantaneous accelerations/jerks)	5
<b>9. Move pen to drop location</b>	<b>10</b>
Avoid collisions	2
Over target location	3
At desired orientation	5
<b>10. Open gripper to drop pen</b>	<b>5</b>
<b>Subtotal (passes × points)</b>	<b>3 × 80</b>
<b>Bonus point opportunities</b>	<b>10</b>
Use a pen's chisel tip to draw a wide and narrow lines	3
Use four colors in your drawing	3
Place a pen in the holder after drawing (must complete 9 at least once)	4
<b>Total (possible/subtotal)</b>	<b>250/240</b>

## 7 Writeup

In the final submission, each team member must submit an individual 1-page writeup of their (1) contributions to the project, and (2) insights they gained during the capstone. Example insights:

- What did you learn about timing trajectories?
- How accurate/repeatable were particular parts of the process?
- What special trick did you do to avoid collisions or singularities?
- Was there something special you needed to do to tune inverse kinematics?



## Submission

1. Upload your video to Youtube and attach the link in your writeup.
2. Include the script you write in your code submission.

## 8 Submission Checklist

- ☐ Upload your writeup <andrew\_id>.pdf to Gradescope.
- ☐ Upload the code in a zipped folder name <team-name>.zip to Gradescope.