PB&J: Peanut Butter and Joints for Damped Articulation - Parameter Estimation Protocol

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The parameters of the joint damping model (μ_k, μ_d, and b) were fit to experimental angular position data from pendulum drop tests using a bootstrapping method. These pendulum drop tests are described in our main manuscript. μ_k and μ_d were simultaneously fit to data from experiments without the damper, and then, given those parameters, b was fit to data from experiments with the damper.

To fit the frictional parameters: For a single iteration, the following algorithm was used:

1. Sample (with replacement) 5 dataset from the 15 no-damper experiments.
2. Sample an initial condition for μ_k and μ_d from a uniform distribution over the range $[0, 1e-2]$ and $[0, 1e-5]$ respectively.
3. Minimize the objective function ϕ_1 for both parameters using *fminsearch*.

The objective function ϕ was the total squared error across all five experiments, modified by penalty terms that penalize negative parameters:

$$\phi_1(\boldsymbol{\mu}) = \sum_{i=1}^{5} \left(\sum_j ((\theta_j^{\text{data}} - \theta_j^{\text{model}}(\boldsymbol{\mu}))^2) \right) + (10^9) \Big[(\boldsymbol{\mu} < 0) \Big] \tag{1}$$

where $\boldsymbol{\mu} = [\mu_k, \mu_d]$. $\theta_j^{\text{model}}(\boldsymbol{\mu})$ was found by integrating the equation of motion from Appendix B of our manuscript. The initial model state was determined from the corresponding dataset. Because the model had a higher temporal resolution than the data, the model output was re-interpolated at the dataset time points. This procedure was repeated for $N = 300$ iterations to obtain distributions on μ_k and μ_d.

To fit the damping parameter: At each iteration:

1. Sample (with replacement) 5 dataset from the 15 no-damper experiments.
2. Sample a value of $\boldsymbol{\mu}$ from the obtained parameter distribution.

3. Sample an initial condition for b from a uniform distribution over the range $[0, 1e5]$.

4. Minimize the objective function ϕ_2 for b using *fminsearch*.

The objective function ϕ_2 was of the same form as ϕ_1 except that it was parameterized by b. To account for the variability introduced by randomly sampling the frictional coefficients, this procedure was repeated for $N = 3000$ iterations. The resampled μ_k and μ_d were saved for each value of b to obtain the full joint distribution of parameters.