# Supplementary Material - The Code Lists of Two Case Studies

**Wangchuan Feng, Guanglu Zhang, Jonathan Cagan**

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**Supplementary Material A - Code List of the Bird's Function Case Study**

The bird's function [1] is employed to construct the constraint in the case study in Section 5.1. The continuous constraint satisfaction problem is defined as:

Find all

$$\boldsymbol{X} = [x_1 \quad x_2], \tag{A1}$$

within

$$-2\pi \leq x_1 \leq 2\pi, \tag{A2}$$

$$-2\pi \leq x_2 \leq 2\pi, \tag{A3}$$

subject to

$$g_1(\boldsymbol{X}) = (x_1 - x_2)^2 + \sin(x_1) \cdot e^{[1-\cos(x_2)]^2} + \cos(x_2) \cdot e^{[1-\sin(x_1)]^2} \leq 0, \tag{A4}$$

where $x_1$ and $x_2$ are two variables, and $g_1(x)$ is the only inequality constraint function.

The code list used to compute the bounds for the value of the bird's function (i.e., the constraint function $g_1(x)$) based on interval arithmetic is provided as follows:

$$G_{1-1} = x_1 - x_2, \tag{A5}$$

$$G_{1-2} = G_{1-1}^2, \tag{A6}$$

$$G_{1-3} = \cos x_2, \tag{A7}$$

$$G_{1-4} = 1 - G_{1-3}, \tag{A8}$$

$$G_{1-5} = G_{1-4}^2, \tag{A9}$$

$$G_{1-6} = e^{G_{1-5}}, \tag{A10}$$

$$G_{1-7} = \sin x_1, \tag{A11}$$

$$G_{1-8} = G_{1-7} \times G_{1-6}, \tag{A12}$$

$$G_{1-9} = 1 - G_{1-7}, \tag{A13}$$

$$G_{1-10} = G_{1-9}^2, \tag{A14}$$

$$G_{1-11} = e^{G_{1-10}}, \tag{A15}$$

$$G_{1-12} = G_{1-3} \times G_{1-11}, \tag{A16}$$

$$G_{1-13} = G_{1-2} + G_{1-8}, \tag{A17}$$

$$g_1 = G_{1-12} + G_{1-13}. \tag{A18}$$

The code list defined by Eqs. (A5) - (A18) is used in the PITSA software. Notably, the code list of the bird's function is not unique, and a different code list could be generated if necessary. Since subscripts and hyphens cannot be included in the symbolic names of variables in Python, $G_{1\text{-}j}$ in Eqs. (A5) - (A18) is defined as G1_*j* in the PITSA software, where $j \in \{1, 2, \ldots, 13\}$. For example, $G_{1\text{-}1}$ is defined as G1_1 in the PITSA software.

**Supplementary Material B - Code Lists of Welded Beam Design Case Study**

The welded beam design problem [2-4] is employed as the case study in Section 5.2. As illustrated in Figure B1, a rectangular beam is designed to be welded onto a primary structure. The design variables are defined as:

$$\boldsymbol{X} = [x_1 \quad x_2 \quad x_3 \quad x_4], \tag{B1}$$

where $x_1$ is the weld height, $x_2$ is the length of the beam that is welded onto the primary structure, $x_3$ is the height of the beam, and $x_4$ is the thickness of the beam. The allowable ranges of these design variables are defined as follows:

$$0.001\ in \leq x_1 \leq 1\ in, \tag{B2}$$

$$0.001\ in \leq x_2 \leq 8\ in, \tag{B3}$$

$$5\ in \leq x_3 \leq 30\ in, \tag{B4}$$

$$0.001\ in \leq x_4 \leq 1\ in. \tag{B5}$$

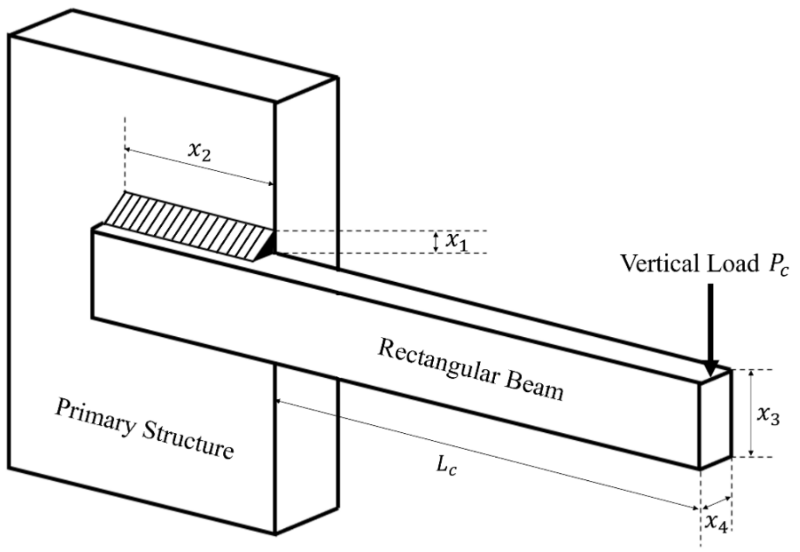


*Fig. B1 Welded beam design problem*

The design problem includes eight inequality constraints defined by Eqs. (6) - (23). Six constants involved in the welded beam design problem are defined in Table B1.

*Table B1 Six constants involved in the welded beam design problem*

| | Definition | Value |
|---|---|---|
| $P_c$ | The vertical load applied on the beam | 6,000 pounds |
| $L_c$ | Distance from load to the point of support | 14 inches |
| $E_c$ | Young's modulus of the beam material | $30\times10^6$ psi |
| $G_c$ | Shear modulus of the beam material | $12\times10^6$ psi |
| $S_1$ | Weld material cost per unit volume | 0.10471 dollar/in$^3$ |
| $S_2$ | Beam material cost per unit volume | 0.04811 dollar/in$^3$ |

1. The weld shear stress is less than or equal to 13,600 psi.

$$g_1(\boldsymbol{X}) = \tau(\boldsymbol{X}) - 13600 \leq 0, \tag{B6}$$

$$\tau(\boldsymbol{X}) = \sqrt{(\tau')^2 + 2\tau'\tau"\frac{x_2}{2R} + (\tau")^2}, \tag{B7}$$

where $\tau'$ is the primary stress acting over the weld throat area defined by Eq. (B8), and $\tau"$ is the secondary torsional stress defined by Eqs. (B9) - (B12).

$$\tau' = \frac{P_c}{\sqrt{2}x_1x_2}, \tag{B8}$$

$$\tau" = \frac{MR}{J}, \tag{B9}$$

$$M = P_c\left(L_c + \frac{x_2}{2}\right), \tag{B10}$$

$$R = \sqrt{\frac{x_2^2}{4} + \left(\frac{x_1 + x_3}{2}\right)^2}, \tag{B11}$$

$$J = 2\sqrt{2}x_1x_2\left[\frac{x_2^2}{12} + \left(\frac{x_1 + x_3}{2}\right)^2\right]. \tag{B12}$$

2. The normal stress in the beam is less than or equal to 30,000 psi.

$$g_2(\boldsymbol{X}) = \sigma(\boldsymbol{X}) - 30000 \leq 0, \tag{B13}$$

$$\sigma(\boldsymbol{X}) = \frac{6P_cL_c}{x_4x_3^2}. \tag{B14}$$

3. The weld height cannot exceed beam thickness.

$$g_3(\boldsymbol{X}) = x_1 - x_4 \leq 0. \tag{B15}$$

4. The overall material cost should not exceed \$5.

$$g_4(\boldsymbol{X}) = S_1 x_1^2 x_2 + S_2 x_3 x_4 (L_c + x_2) - 5 \leq 0. \tag{B16}$$

5. The weld height should be larger than or equal to 0.125 inches.

$$g_5(\boldsymbol{X}) = 0.125 - x_1 \leq 0. \tag{B17}$$

6. The maximum allowable beam-end deflection is 0.25 inches.

$$g_6(\boldsymbol{X}) = \delta(\boldsymbol{X}) - 0.25 \ \leq 0, \tag{B18}$$

$$\delta(\boldsymbol{X}) = \frac{4 P_c L_c^3}{E_c x_3^3 x_4}. \tag{B19}$$

7. The 6,000 lb vertical load does not cause the beam to buckle.

$$g_7(\boldsymbol{X}) = 6000 - B(\boldsymbol{X}) \leq 0, \tag{B20}$$

where the beam buckling load, $B(\boldsymbol{X})$, is derived by

$$B(\boldsymbol{X}) = \frac{4.013\sqrt{E_c G_c \frac{x_3^2 x_4^6}{36}}}{L_c^2}\left(1 - \frac{x_3}{2L_c}\sqrt{\frac{E_c}{4G_c}}\right). \tag{B21}$$

8. The overall fabrication cost cannot exceed \$3.5.

$$g_8(\boldsymbol{X}) = F(\boldsymbol{X}) - 3.5 \leq 0, \tag{B22}$$

$$F(\boldsymbol{X}) = (1 + S_1) x_1^2 x_2 + S_2 x_3 x_4 (L_c + x_2). \tag{B23}$$

The code list used to compute the bounds for the value of each constraint function based on interval arithmetic is provided as follows.

Constraint function $g_1$:

$$G_{1-1} = x_1 x_2, \tag{B24}$$

$$t_1 = \frac{P_c}{\sqrt{2} G_{1-1}}, \tag{B25}$$

$$G_{1-2} = 0.5 x_2, \tag{B26}$$

$$G_{1-3} = G_{1-2} + L_c, \tag{B27}$$

$$M = G_{1-3}P_c, \tag{B28}$$

$$G_{1-4} = x_2^2, \tag{B29}$$

$$G_{1-5} = 0.25G_{1-4}, \tag{B30}$$

$$G_{1-6} = x_1 + x_3, \tag{B31}$$

$$G_{1-7} = G_{1-6}^2, \tag{B32}$$

$$G_{1-8} = 0.25G_{1-7}, \tag{B33}$$

$$G_{1-9} = G_{1-5} + G_{1-8}, \tag{B34}$$

$$R = \sqrt{G_{1-9}}, \tag{B35}$$

$$G_{1-10} = x_2^2, \tag{B36}$$

$$G_{1-11} = \frac{1}{12}G_{1-10}, \tag{B37}$$

$$G_{1-12} = x_1x_2, \tag{B38}$$

$$G_{1-13} = 2\sqrt{2}G_{1-12}, \tag{B39}$$

$$G_{1-14} = G_{1-11} + G_{1-8}, \tag{B40}$$

$$J = G_{1-14}G_{1-13}, \tag{B41}$$

$$G_{1-15} = M \cdot R, \tag{B42}$$

$$t_2 = \frac{G_{1-15}}{J}, \tag{B43}$$

$$G_{1-16} = t_1^2, \tag{B44}$$

$$G_{1-17} = t_2^2, \tag{B45}$$

$$G_{1-18} = G_{1-16} + G_{1-17}, \tag{B46}$$

$$G_{1-19} = t_1t_2, \tag{B47}$$

$$G_{1-20} = \frac{x_2}{R}, \tag{B48}$$

$$G_{1-21} = G_{1-19}G_{1-20}, \tag{B49}$$

$$G_{1-22} = G_{1-18} + G_{1-21}, \tag{B50}$$

$$t = \sqrt{G_{1-22}}, \tag{B51}$$

$$g_1 = t - 13600. \tag{B52}$$

Constraint function $g_2$:

$$G_{2-1} = x_3^2, \tag{B53}$$

$$G_{2-2} = x_4 G_{2-1}, \tag{B54}$$

$$G_{2-3} = \frac{(6P_c L_c)}{G_{2-2}}, \tag{B55}$$

$$g_2 = G_{2-3} - 30000. \tag{B56}$$

Constraint function $g_3$:

$$g_3 = x_1 - x_4. \tag{B57}$$

Constraint function $g_4$:

$$G_{4-1} = x_1^2, \tag{B58}$$

$$G_{4-2} = G_{4-1} x_2, \tag{B59}$$

$$G_{4-3} = S_1 G_{4-2}, \tag{B60}$$

$$G_{4-4} = x_3 x_4, \tag{B61}$$

$$G_{4-5} = S_2 G_{4-4}, \tag{B62}$$

$$G_{4-6} = L_c + x_2, \tag{B63}$$

$$G_{4-7} = G_{4-5} G_{4-6}, \tag{B64}$$

$$G_{4-8} = G_{4-3} + G_{4-7}, \tag{B65}$$

$$g_4 = G_{4-8} - 5. \tag{B66}$$

Constraint function $g_5$:

$$g_5 = 0.125 - x_1. \tag{B67}$$

Constraint function $g_6$:

$$G_{6-1} = x_3^3, \tag{B68}$$

$$G_{6-2} = G_{6-1}x_4, \tag{B69}$$

$$G_{6-3} = \frac{1}{G_{6-2}}, \tag{B70}$$

$$G_{6-4} = \left(\frac{4P_c L_c^3}{E_c}\right)G_{6-3}, \tag{B71}$$

$$g_6 = G_{6-4} - 0.25. \tag{B72}$$

Constraint function $g_7$:

$$G_{7-1} = x_3^2, \tag{B73}$$

$$G_{7-2} = x_4^6, \tag{B74}$$

$$G_{7-3} = G_{7-1}G_{7-2}, \tag{B75}$$

$$G_{7-4} = \frac{G_{7-3}}{36}, \tag{B76}$$

$$G_{7-5} = \sqrt{G_{7-4}}, \tag{B77}$$

$$G_{7-6} = \left(\frac{4.013\sqrt{E_c G_c}}{L_c^2}\right)G_{7-5}, \tag{B78}$$

$$G_{7-7} = \left(\frac{\sqrt{\frac{E_c}{4G_c}}}{2L_c}\right)x_3, \tag{B79}$$

$$G_{7-8} = 1 - G_{7-7}, \tag{B80}$$

$$G_{7-9} = G_{7-6}G_{7-8}, \tag{B81}$$

$$g_7 = P_c - G_{7-9}. \tag{B82}$$

Constraint function $g_8$:

$$G_{8-1} = x_1^2, \tag{B83}$$

$$G_{8-2} = (1 + S_1)x_2, \tag{B84}$$

$$G_{8-3} = G_{8-1}G_{8-2}, \tag{B85}$$

$$G_{8-4} = x_3x_4, \tag{B86}$$

$$G_{8-5} = S_2 G_{8-4}, \tag{B87}$$

$$G_{8-6} = x_2 + L_c, \tag{B88}$$

$$G_{8-7} = G_{8-5} G_{8-6}, \tag{B89}$$

$$G_{8-8} = G_{8-3} + G_{8-7}, \tag{B90}$$

$$g_8 = G_{8-8} - 3.5. \tag{B91}$$

The code lists defined by Eqs. (B24) - (B91) are used in the PITSA software. Notably, each of these code lists is not unique, and a different code list could be generated if necessary. Since subscripts and hyphens cannot be included in the symbolic names of variables in Python, $G_{i\text{-}j}$ in Eqs. (B24) - (B91) is defined as G*i*_*j* in the PITSA software, where $i \in \{1, 2, \ldots, 8\}$, $j \in \{1, 2, \ldots, 22\}$. For example, $G_{1\text{-}1}$ is defined as G1_1 in the PITSA software.

## References

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