Lecture 15: Symbolic Execution

17-355/17-655/17-819: Program Analysis

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* Course materials developed with Claire Le Goues

Recall: VCGen from Axiomatic Semantics

$$\{P\}$$
$$x := e_1$$
$$x := e_2$$
$$\{Q\}$$

What is the Proof Obligation for backwards reasoning?

What if we just went forwards?

$$\{P\}$$
$$x := e_1$$
$$x := e_2$$
$$\{Q\}$$

Generate “fresh” math variables for every mutable program variable

Proof Obligation:

$$\forall x_n : ([x_0/x]P \wedge x_1 = [x_0/x]e_1 \wedge x_2 = ([x_1/x]e_2)) \Rightarrow [x_2/x]Q$$

What if we just went forwards?

$$\{P\}$$
$$x := e_1$$
$$x := e_2$$
$$\{Q\}$$

$$\{x > 0\}$$
$$x := x * 2$$
$$x := x + 1$$
$$\{x > 1\}$$

Proof Obligation:

$$\forall x_n : ([x_0/x]P \land x_1 = [x_0/x]e_1 \land x_2 = ([x_1/x]e_2)) \Rightarrow [x_2/x]Q$$

$$\forall x_0, x_1, x_2 \in \mathbb{Z} : (x_0 > 0 \land x_1 = x_0 * 2 \land x_2 = x_1 + 1) \Rightarrow x_2 > 1$$

Dealing with conditional paths

```
$\{true\}$
if $(x > 0)$ :
        $y := -x$
else :
        $y := x$
$\{y \geqslant 0\}$
```

Dynamic Symbolic Execution:

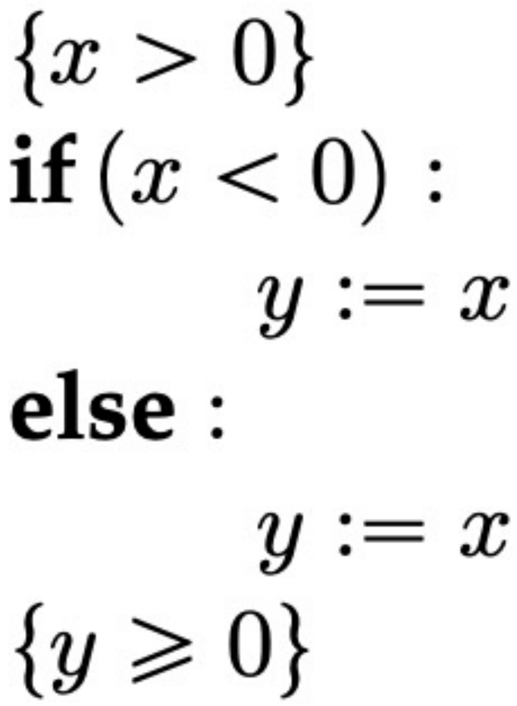
$$\forall x_0, y_0 \in \mathbb{Z} : (x_0 < 0 \wedge y_0 = -x_0) \Rightarrow y_0 \geqslant 0$$

$$\forall x_0, y_0 \in \mathbb{Z} : (x_0 \geqslant 0 \wedge y_0 = x_0) \Rightarrow y_0 \geqslant 0$$

Static Symbolic Execution:

$$\forall x_0, y_0 \in \mathbb{Z} : ((x_0 < 0 \Rightarrow y_0 = -x_0) \vee (x_0 \geqslant 0 \Rightarrow y_0 = x_0)) \Rightarrow y_0 \geqslant 0$$

Dealing with conditional paths



```
$\{x > 0\}$
if $(x < 0)$ :
        $y := x$
else :
        $y := x$
$\{y \geqslant 0\}$
```

Exercise: Generate the VC for this program. Is it true?

Formalizing DSE with Guards and Symbolic Formulas

$$\Sigma \in Var \rightarrow a_s$$

$$\begin{array}{rcl} g & ::= & \text{true} \\ & | & \text{false} \\ & | & \text{not } g \\ & | & g_1 \ op_b \ g_2 \\ & | & a_{s1} \ op_r \ a_{s2} \end{array}$$

$$\begin{array}{rcl} a_s & ::= & \alpha \\ & | & n \\ & | & a_{s1} \ op_a \ a_{s2} \end{array}$$

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Symbolic Evaluation of Expressions

$$\frac{}{\langle \Sigma, n \rangle \Downarrow n}\ \textit{big-int}$$

$$\frac{}{\langle \Sigma, x \rangle \Downarrow \Sigma(x)}\ \textit{big-var}$$

$$\frac{\langle \Sigma, a_1 \rangle \Downarrow a_{s1} \quad \langle \Sigma, a_2 \rangle \Downarrow a_{s2}}{\langle \Sigma, a_1 + a_2 \rangle \Downarrow a_{s1} + a_{s2}}\ \textit{big-add}$$

Symbolic Execution of Statements (DSE)

$$\frac{}{\langle g, \Sigma, \texttt{skip} \rangle \Downarrow \langle g, \Sigma \rangle} \; \textit{big-skip}$$

$$\frac{\langle g, \Sigma, s_1 \rangle \Downarrow \langle g', \Sigma' \rangle \quad \langle g', \Sigma', s_2 \rangle \Downarrow \langle g'', \Sigma'' \rangle}{\langle g, \Sigma, s_1; s_2 \rangle \Downarrow \langle g'', \Sigma'' \rangle} \; \textit{big-seq}$$

$$\frac{\langle \Sigma, a \rangle \Downarrow a_s}{\langle g, \Sigma, x := a \rangle \Downarrow \langle g, \Sigma[x \mapsto a_s] \rangle} \; \textit{big-assign}$$

Symbolic Execution with Branching (DSE)

$$\frac{\langle \Sigma, b \rangle \Downarrow g' \quad g \wedge g' \mathtt{SAT} \quad \langle g \wedge g', \Sigma, s_1 \rangle \Downarrow \langle g'', \Sigma' \rangle}{\langle g, \Sigma, \mathtt{if}\ b\ \mathtt{then}\ s_1\ \mathtt{else}\ s_2, \rangle \Downarrow \langle g'', \Sigma' \rangle}\ \textit{big-iftrue}$$

$$\frac{\langle \Sigma, b \rangle \Downarrow g' \quad g \wedge \neg g' \mathtt{SAT} \quad \langle g \wedge \neg g', \Sigma, s_2 \rangle \Downarrow \langle g'', \Sigma' \rangle}{\langle g, \Sigma, \mathtt{if}\ b\ \mathtt{then}\ s_1\ \mathtt{else}\ s_2, \rangle \Downarrow \langle g'', \Sigma' \rangle}\ \textit{big-iffalse}$$

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Symbolic Execution of Loops

Q. What's wrong here?

$$\frac{\langle \Sigma, b\rangle \Downarrow g' \quad g \wedge g'\,\mathtt{SAT} \quad \langle g \wedge g', \Sigma, \mathtt{while}\ b\ \mathtt{do}\ s\rangle \Downarrow \langle g'', \Sigma'\rangle}{\langle g, \Sigma, \mathtt{while}\ b\ \mathtt{do}\ s,\rangle \Downarrow \langle g'', \Sigma'\rangle}\ \textit{big-whiletrue}$$

$$\frac{\langle \Sigma, b\rangle \Downarrow g' \quad g \wedge \neg g'\,\mathtt{SAT}}{\langle g, \Sigma, \mathtt{while}\ b\ \mathtt{do}\ s,\rangle \Downarrow \langle g \wedge \neg g', \Sigma\rangle}\ \textit{big-whilefalse}$$

Symbolic Execution of Loops

Bounded exploration (*k*-limited)

$$\frac{k > 0 \quad \langle \Sigma, b\rangle \Downarrow g' \quad g \wedge g' \mathrm{SAT} \quad \langle g \wedge g', \Sigma, \texttt{\textbf{while}}_{\texttt{k}-1}\ b\ \textbf{do}\ s\rangle \Downarrow \langle g'', \Sigma'\rangle}{\langle g, \Sigma, \texttt{\textbf{while}}_{\texttt{k}}\ b\ \textbf{do}\ s,\rangle \Downarrow \langle g'', \Sigma'\rangle}\ \textit{big-whiletrue}$$

$$\frac{\langle \Sigma, b\rangle \Downarrow g' \quad g \wedge \neg g' \mathrm{SAT}}{\langle g, \Sigma, \texttt{\textbf{while}}_{\texttt{k}}\ b\ \textbf{do}\ s,\rangle \Downarrow \langle g \wedge \neg g', \Sigma\rangle}\ \textit{big-whilefalse}$$

Q. What are the implications?

Symbolic Execution with Loops

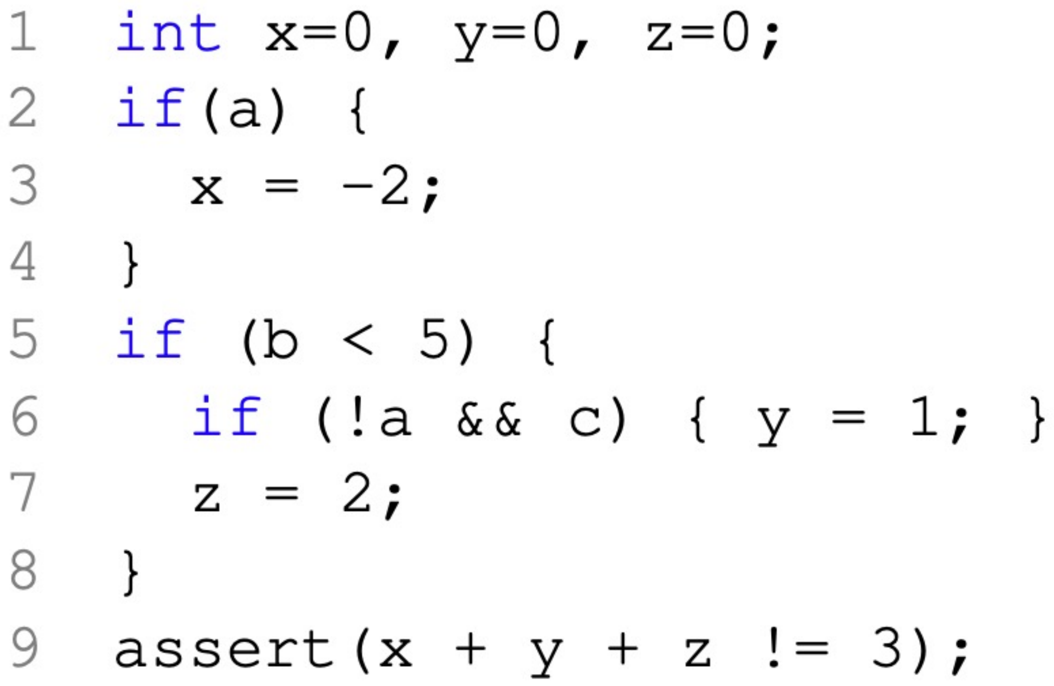
- Loop invariants can be used if given
  - Often works better with SSE
- But we can choose to explore only partial set of paths
  - K-bounded loops (often: k < 3)
  - "Unsound" for verification
  - Sound but "Incomplete" for bug finding when used with DSE
    - DSE formulas for a given path can be solved to find a witness = test input

Digression: Soundness and Completeness

- Soundness = “Doesn’t lie” or “all claims are true”
- Completeness = “All truths are claimed”
- For Verification (claim is “*program is correct*”)
  - Soundness: Reasoning along all possible paths (over-approximation)
- For Bug-Finding (claim is “*a bug exists*”)
  - Soundness: Reasoning along feasible paths only (under-approximation)

- Soundness & Completeness is impossible in general (Rice’s theorem)
  - Most systems are sound but incomplete (e.g. can’t prove all programs, or can’t find all bugs)

Symbolic Execution: A Generalization of Testing

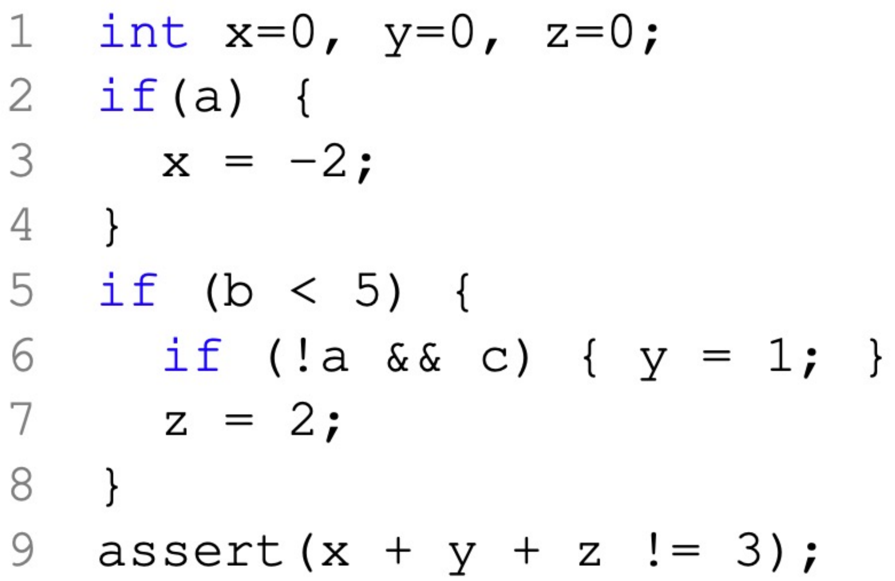


```
int x=0, y=0, z=0;
if(a) {
  x = -2;
}
if (b < 5) {
  if (!a && c) { y = 1; }
  z = 2;
}
assert(x + y + z != 3);
```

What input values of a,b,c will cause the assert to fail?

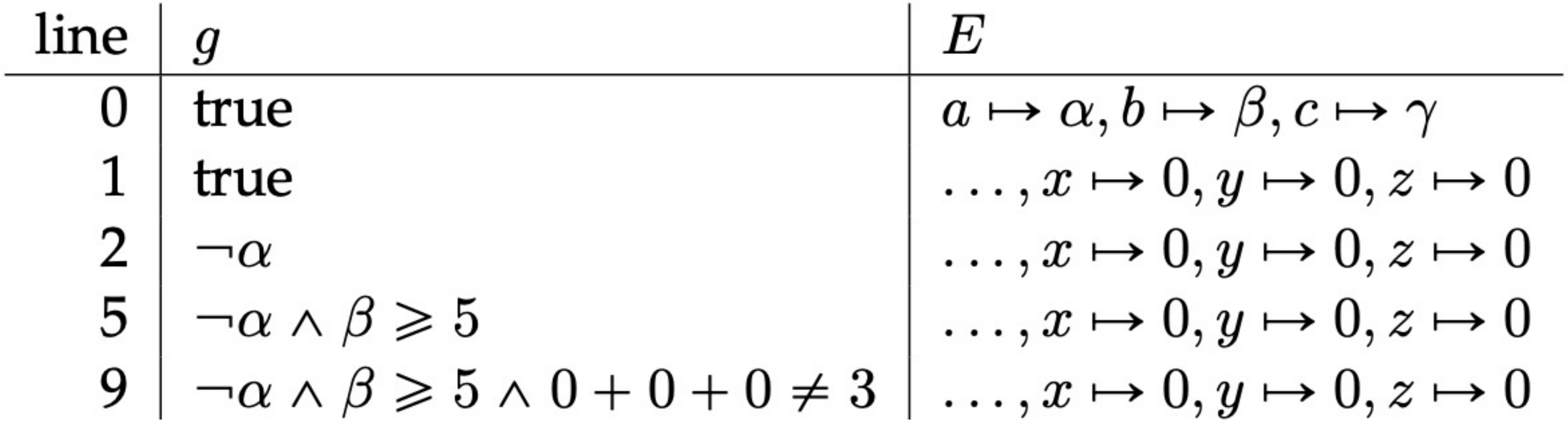
```
1  int x=0, y=0, z=0;
2  if(a) {
3    x = -2;
4  }
5  if (b < 5) {
6    if (!a && c) { y = 1; }
7    z = 2;
8  }
9  assert(x + y + z != 3);
```

line	g	E
0	true	$a \mapsto \alpha, b \mapsto \beta, c \mapsto \gamma$
1	true	$\ldots, x \mapsto 0, y \mapsto 0, z \mapsto 0$
2	$\neg\alpha$	$\ldots, x \mapsto 0, y \mapsto 0, z \mapsto 0$
5	$\neg\alpha \wedge \beta \geqslant 5$	$\ldots, x \mapsto 0, y \mapsto 0, z \mapsto 0$
9	$\neg\alpha \wedge \beta \geqslant 5 \wedge 0 + 0 + 0 \neq 3$	$\ldots, x \mapsto 0, y \mapsto 0, z \mapsto 0$



```
1  int x=0, y=0, z=0;
2  if(a) {
3    x = -2;
4  }
5  if (b < 5) {
6    if (!a && c) { y = 1; }
7    z = 2;
8  }
9  assert(x + y + z != 3);
```

Exercise: Generate path constraints for another path.



line	g	E
0	true	$a \mapsto \alpha, b \mapsto \beta, c \mapsto \gamma$
1	true	$\ldots, x \mapsto 0, y \mapsto 0, z \mapsto 0$
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5	$\neg\alpha \wedge \beta \geqslant 5$	$\ldots, x \mapsto 0, y \mapsto 0, z \mapsto 0$
9	$\neg\alpha \wedge \beta \geqslant 5 \wedge 0 + 0 + 0 \neq 3$	$\ldots, x \mapsto 0, y \mapsto 0, z \mapsto 0$

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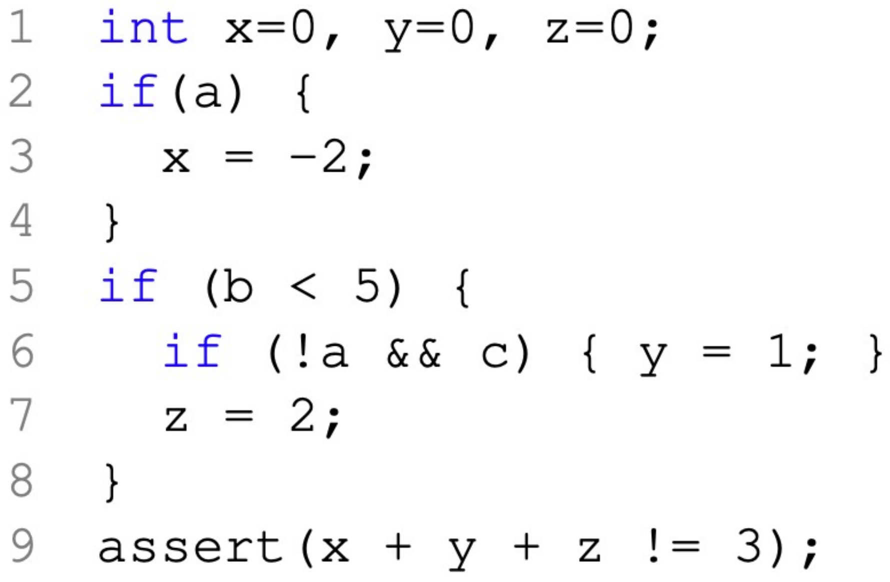
1  int x=0, y=0, z=0;
2  if(a) {
3    x = -2;
4  }
5  if (b < 5) {
6    if (!a && c) { y = 1; }
7    z = 2;
8  }
9  assert(x + y + z != 3);
```

Exercise: Generate path constraints for another path (e.g. one that executes line 6).

line	g	E

Symbolic Execution Tree

Exercise: How many feasible paths are in the program?



```
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