

Operational Semantics

Operational semantics provides a way of understanding what a program means by mimicking, at a high level, the operation of a computer executing the program. Operational semantics falls under two broad classes: *big-step* operational semantics, which specifies the entire operation of a given expression or statement; and *small-step* operational semantics, which specifies the operation of the program one step at a time. Both are powerful tools for verifying the correctness and other desired properties of programs.

Exercises

1. Use the big-step operational semantics rules for the WHILE language to write a well-formed derivation with $\langle y := 3; \text{if } y > 1 \text{ then } z := y \text{ else } z := 2, E \rangle \Downarrow E[y \mapsto 3; z \mapsto 3]$ as its conclusion. Make sure to indicate which rule you used to prove each premise or conclusion.

2. For homework 2, you will be partially proving that if a statement terminates, then the big- and small-step semantics for WHILE will obtain equivalent results; i.e.,

$$\forall S \in \text{Stmt}. \forall E, E' \in \text{Var} \mapsto \mathbb{Z}. \langle S, E \rangle \rightarrow^* \langle \text{skip}, E' \rangle \iff \langle S, E \rangle \Downarrow E'$$

You will prove this by induction on the structure of derivations for each direction of \iff .

For your homework proof, you are only required to show

- The base case(s).
- The inductive case for let using the semantics developed in question 1 of the homework.
- Two more representative inductive cases.

You may assume that this property holds for arithmetic and boolean expressions, i.e., you may assume the following hold:

$$\forall a \in \text{AExp}. \forall n \in \mathbb{Z}. \langle a, E \rangle \rightarrow_a^* n \iff \langle a, E \rangle \Downarrow_a n \quad (1)$$

$$\forall P \in \text{BExp}. \forall b \in \{\text{true}, \text{false}\}. \langle P, E \rangle \rightarrow_b^* b \iff \langle P, E \rangle \Downarrow_b b \quad (2)$$

You may also assume the small-step if congruence of $\langle S, E \rangle \rightarrow^* \langle S', E' \rangle$:

$$\frac{\langle P, E \rangle \rightarrow_b^* P'}{\langle \text{if } P \text{ then } S_1 \text{ else } S_2, E \rangle \rightarrow^* \langle \text{if } P' \text{ then } S_1 \text{ else } S_2, E \rangle} \quad (3)$$

For this exercise, you will prove the following representative inductive case:

$$\forall S \in \text{Stmt}. \forall E, E' \in \text{Var} \mapsto \mathbb{Z}. \langle \text{if } P \text{ then } S_1 \text{ else } S_2, E \rangle \Downarrow E' \iff \langle \text{if } P \text{ then } S_1 \text{ else } S_2, E \rangle \rightarrow^* \langle \text{skip}, E' \rangle$$