

Homework 6:

Axiomatic Semantics and Hoare-style Verification

17-355/17-665/17-819: Program Analysis *Fall 2025*
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Due: Thursday, 10/30/2025 at 11:59 pm EST

100 points total

Assignment Objectives:

- Generate verification conditions and show how they are used in program verification.
- Write new Hoare Rules/axiomatic semantics for a particular language.
- Reason about soundness/completeness of a system for axiomatic semantics.

Handin Instructions. Submit your assignment through Gradescope link by the due date. When submitting, indicate which pages of the PDF correspond to each homework question. Typesetting is not required, but is strongly suggested; you may submit photos or scans of handwritten answers, but they must be clear and legible. Consult homework 2 for pointers to useful latex packages.

We are *not* making correct handin worth 5 points explicitly, but we reserve the right to *remove* 5 points if the homework is not submitted correctly.

Question 1, VCGen, (24 points). Consider the following rules for VCGen. We saw the first two in class; consider the third one as a proposed rule for `let`:

$$\begin{aligned} VCGen(S_1; S_2, Q) &= VCGen(S_1, VCGen(S_2, Q)) \\ VCGen(x := e, Q) &= [e/x]Q \\ VCGen(\text{let } x = e \text{ in } S, Q) &= [e/x]VCGen(S, Q) \end{aligned}$$

The rule for `let` is incorrect.

- Explain why, briefly, in English prose,
- Give a correct rule for `let`.

Question 2, Let rule soundness, (18 points). Recall that we aim to compute a verification condition VC using a verification condition generator procedure $VCGen(S, Q)$ such that our final proof obligation $VC(\{P\}S\{Q\})$ is $P \Rightarrow VCGen(S, Q)$. Given $\{P\}S\{Q\}$, we desire that $P \Rightarrow VCGen(S, Q) \Rightarrow wp(S, Q)$. We say that our VCGen rules are *sound* iff $VC(\{P\}S\{Q\}) \Rightarrow \models \{P\}S\{Q\}$. Demonstrate the unsoundness of the buggy `let` rule above by giving/showing the following six things:

- (a) a statement S and
- (b) a post-condition Q and
- (c) a state E , all such that
- (d) $E \models VCGen(S, Q)$ and
- (e) $\langle S, E \rangle \Downarrow E'$ but
- (f) $E' \not\models Q$

Question 3, Do-while, (10 points). Write a sound and complete Hoare rule for `do S while b` . The statement has the standard semantics (i.e., S is executed at least once, before b is tested). You do not need to formally prove soundness/completeness, but make sure the rule makes sense.

Question 4, Loop proof obligations, (24 points). Consider the following program:

```
{N > 0 }
i := 0;
while (i < N) do
    i := i + 1
{i=N}
```

1. What is a suitable loop invariant for proving this post-condition holds?
2. Write out the verification condition for this program (don't forget to use the precondition!). You don't have to solve/prove the verification condition, just write it out neatly; it should be formatted so that it's especially clear which parts correspond to the three parts of the proof obligations for the loop:
 - Invariant is initially true:
 - Invariant is preserved by the loop body:
 - Invariant and exit condition imply postcondition:

Hint: you can get full credit on the second part even if your loop invariant (answer to the first part) is incorrect.

Question 5, Soundness/Completeness, (24 points). Consider the following three Hoare rules:

$$\frac{\vdash \{X\} S \{b \Rightarrow X \wedge \neg b \Rightarrow Y\}}{\vdash \{b \Rightarrow X \wedge \neg b \Rightarrow Y\} \text{ while } b \text{ do } S \{Y\}} \text{ rule1} \quad \frac{\vdash \{X \wedge b\} S \{X\}}{\vdash \{X\} \text{ while } b \text{ do } S \{X\}} \text{ rule2}$$

$$\frac{\vdash \{X\} S \{X\}}{\{X\} \text{ while } b \text{ do } S \{X \wedge \neg b\}} \text{ rule3}$$

Recall that a system of axiomatic semantics is *sound* if everything we can prove is also true: if $\vdash \{P\} S \{Q\}$ then $\models \{P\} S \{Q\}$. A system of axiomatic semantics is *complete* if we can prove all true things: if $\models \{P\} S \{Q\}$ then $\vdash \{P\} S \{Q\}$. All three of the rules above are sound, but none are complete.

For two of the identified incomplete rules, give/show example P , Q , E , S , and E' such that $\langle S, E \rangle \Downarrow E'$ and both $E \models P$ and $E' \models Q$, but it is not possible given the rule to prove $\vdash \{P\} S \{Q\}$. Be clear about which rules you have chosen.

(Educational) note: An incomplete system cannot prove all possible properties or handle all possible programs. Incompleteness in an axiomatic semantics or type system is typically not as dire as unsoundness. Many research results that claim to work for the C language, for example, are actually incomplete because they fail to address, say, setjmp/longjmp or bitfields. (Many of them are also unsound because they do not correctly model various language features like unsafe casts, pointer arithmetic, or integer overflow.)

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