PROBLEM SET 2

16822 GEOMETRY-BASED METHODS IN VISION (FALL 2022)
https://piazza.com/cmu/fall2022/16822

OUT: Sep. 20, 2022
DUE: Sep. 27, 2022 11:59 PM
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START HERE: Instructions

- **Collaboration policy:** All are encouraged to work together BUT you must do your own work (code and write up). If you work with someone, please include their name in your write up and cite any code that has been discussed. If we find highly identical write-ups or code without proper accreditation of collaborators, we will take action according to university policies, i.e. you will likely fail the course. See the Academic Integrity Section detailed in the initial lecture for more information.
- **Late Submission Policy:** There are **no** late days for Problem Set submissions.
- **Submitting your work:**
  - We will be using Gradescope (`https://gradescope.com/`) to submit the Problem Sets. Please use the provided template. Submissions can be written in LaTeX. Regrade requests can be made, however this gives the TA the opportunity to regrade your entire paper, meaning if additional mistakes are found then points will be deducted. Each derivation/proof should be completed on a separate page. For short answer questions you **should** include your work in your solution.
- **Materials:** The data that you will need in order to complete this assignment is posted along with the writeup and template on Piazza.

For multiple choice or select all that apply questions, replace `\choice` with `\CorrectChoice` to obtain a shaded box/circle, and don't change anything else.

Instructions for Specific Problem Types

For "Select One" questions, please fill in the appropriate bubble completely:

Select One: Who taught this course?

- ● Shubham Tulsiani
- ○ Deepak Pathak
- ○ Fernando De la Torre
- ○ Deva Ramanan

For "Select all that apply" questions, please fill in all appropriate squares completely:

Select all that apply: Which are scientists?

- ■ Stephen Hawking
- ■ Albert Einstein
- ■ Isaac Newton
- □ None of the above

For questions where you must fill in a blank, please make sure your final answer is fully included in the given space. You may cross out answers or parts of answers, but the final answer must still be within the given space.

Fill in the blank: What is the course number?

$\boxed{\text{16-822}}$

1 2D Projective Geometry [18 pts]

1. (a) **[2 pts]** Calculate the line passing through 2 given points: (1) $\mathbf{p}_1 = [3, 4, 1]^T, \mathbf{p}_2 = [4, 3, 0]^T$, (2) $\mathbf{p}_1 = [3, 4, 2022]^T, \mathbf{p}_2 = [3, 4, -1967]^T$.

   (b) **[2 pts]** Calculate the intersection point between 2 given lines: (1) $\mathbf{l}_1 = [3, 4, 1]^T, \mathbf{l}_2 = [0, 0, 1]^T$, (2) $\mathbf{l}_1 = [3, 4, 1]^T, \mathbf{l}_2 = [3, 4, 2]^T$.

a)

1) $p_1 = [3,4,1]^T$, $p_2 = [4,3,0]^T$

Line passing through p_1, p_2 given by

$\ell = p_1 \times p_2$

$= (3\hat{i} + 4\hat{j} + \hat{k}) \times (4\hat{i} + 3\hat{j} + 0\hat{k})$

$\ell = -3\hat{i} + 4\hat{j} - 7\hat{k}$

$\therefore \ell = [-3, 4, -7]^T$

a)

2) $p_1 = [3, 4, 2022]$, $p_2 = [3, 4, -1967]$

line passing through p_1, p_2 given by

$\ell = p_1 \times p_2$

$= (3\hat{i} + 4\hat{j} + 2022\hat{k}) \times (3\hat{i} + 4\hat{j} - 1967\hat{k})$

$= -4(2022+1967)\hat{i} + 3(2022+1967)\hat{j}$

$$\therefore \ell = \begin{bmatrix} -4(2022+1967) \\ 3(2022+1967) \\ 0 \end{bmatrix} \equiv \begin{bmatrix} -4 \\ 3 \\ 0 \end{bmatrix}$$

2. **[3 pts]** Suppose a conic in 2D projective space is given by $\mathbf{C} = \mathbf{l}\mathbf{m}^T + \mathbf{m}\mathbf{l}^T$, where $\mathbf{l}$ and $\mathbf{m}$ are 2 lines. Show that a point belongs to $\mathbf{C}$ if and only if it is on $\mathbf{m}$ or $\mathbf{l}$.

Consider pt. Conic C s.t. $x^T C x = 0$ & $C = \ell m^T + m\ell^T$

$\therefore x^T(\ell m^T + m\ell^T)x = 0$

$x^T \ell m^T x + x^T m \ell^T x = 0$ —— 1

Case 1: Pt. lies on ℓ

$\therefore x^T\ell = \ell^T x = 0$

$\therefore$ For such a pt. x, eqn 1 becomes

$0(m^T x) + x^T m(0)$

$= 0$

$\therefore$ Pt. lies on Conic C as well — 2

Case 2: Pt. lies on m

$\therefore m^T x = x^T m = 0$

For such a pt. x, eqn 1 becomes

$\underbrace{x^T\ell(0) + (0)\ell^T x}_{=0}$

$\therefore$ Pt. lies on Conic C as well — 3

Case 3: Pt. doesnt lie on m & ℓ

$\therefore x^T m, m^T x, \ell^T x, x^T \ell \neq 0$

$\therefore$ Eqn 1, $x^T \ell m^T x + x^T m \ell^T x \neq 0$

$\therefore$ Pt. doesnt lie on conic C — 4

From conclusions 2, 3, 4

Pt. x lies on conic C iff it lies on m or ℓ

3. **[4 pts]** Given a transformation $\mathbf{H} = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$,

   (a) transform a point $\mathbf{p} = [3, 4, 1]^T$,

   (b) transform a line $\mathbf{l} = [-4, 3, 0]$

   (c) transform a conic $\mathbf{C} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$

   (d) does this transformation leaves the circular points at infinity unchanged? Explain the reason without calculation.

1) b) 1) $l_1 = [3,4,1]^T$, $l_2 = [0,0,1]^T$

Point of intersection given by

$$p = l_1 \times l_2$$
$$= (3\hat{i} + 4\hat{j} + \hat{k}) \times (\hat{k})$$
$$= (-3\hat{j} + 4\hat{i})$$
$$\therefore p = [4, -3, 0]^T$$

b) 2) $l_1 = [3,4,1]^T$, $l_2 = [3,4,2]^T$

Point of intersection given by

$$p = l_1 \times l_2$$
$$= (3\hat{i} + 4\hat{j} + \hat{k}) \times (3\hat{i} + 4\hat{j} + 2\hat{k})$$
$$= (4\hat{i} - 3\hat{j})$$
$$p = [4, -3, 0]^T$$

3)

a) $p' = Hp$

$$= \begin{bmatrix} 3 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 3 \\ 4 \\ 1 \end{bmatrix}$$

$$p' = \begin{bmatrix} 9 \\ 8 \\ 1 \end{bmatrix}$$

b) $l' = H^{-T} l$

$$= \frac{1}{6} \begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 6 \end{bmatrix} \begin{bmatrix} -4 \\ 3 \\ 0 \end{bmatrix}$$

$$l' = \frac{1}{6} \begin{bmatrix} -8 \\ 9 \\ 0 \end{bmatrix} = \begin{bmatrix} -1.33 \\ 1.5 \\ 0 \end{bmatrix}$$

c) $C' = H^{-T} C H^{-1}$

$$= \frac{1}{36} \begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 6 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix} \begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 6 \end{bmatrix}$$

$$= \frac{1}{36} \begin{bmatrix} 4 & 0 & 0 \\ 0 & 18 & 0 \\ 0 & 0 & 108 \end{bmatrix}$$

$$\therefore C' = \begin{bmatrix} 0.111 & 0 & 0 \\ 0 & 0.5 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

d) No, this transformation doesnt leave the circular points unchanged.
Circular points remain unchanged iff H is a similarity.
A similarity transformation looks like $H_s = \begin{bmatrix} s\cos\theta & -s\sin\theta & t_x \\ s\sin\theta & s\cos\theta & t_y \\ 0 & 0 & 1 \end{bmatrix}$

However, in this case $s\cos\theta = 3$ and $s\cos\theta = 2$
which is not possible. $\therefore$ H is not a similarity

4. **[4 pts]** Are these 2D projective transformations?

(a) Reflection along a line,

(b) Doubling spherical coordinates: $(r, \theta) \rightarrow (2r, 2\theta)$,

(c) A picture hanging on a wall and its image taken by a camera,

(d) Transformation between these 2 world maps.

a) Yes
b) No — no 1-1 mapping
c) Yes
d) No — Collinearity not preserved

5. **[3 pts]** Are these statements true or false?

(a) Given a line $\mathbf{l}$, if both $\mathbf{H}_A$ and $\mathbf{H}_B$ map $\mathbf{l}$ to $[0, 0, 1]^T$, then $\mathbf{H}_A\mathbf{H}_B^{-1}$ is an affine transformation.

(b) Instead of annotating orthogonal lines, if we annotate multiple pairs of lines that form 45 degree angles in the metric space, we can still calculate $\mathbf{C}_\infty^{\star\prime}$.

(c) If we are allowed to annotate pairs of parallel and orthogonal lines, we need at least 5 pairs of them to calculate $\mathbf{C}_\infty^{\star\prime}$.

a) False - The only H that can map any line l to $[0,0,1]^T$ is of the form $H = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ a & b & c \end{bmatrix}$. Such a matrix is singular & not invertible. $\therefore$ H_b^{-1} doesnt exist

b) True - Possible but complicated equations.

c) False - Just 2 pairs of parallel & 2 pairs of orthogonal are enough.

2 3D Projective Geometry [12 pts]

6. **[3 pts]** Show that the Plucker Representation of a 3D line $\mathbf{L} = \mathbf{x}_1\mathbf{x}_2^T - \mathbf{x}_2\mathbf{x}_1^T$ is equivalent to representing the line as $(\tilde{\mathbf{d}}, \tilde{\mathbf{x}} \times \tilde{\mathbf{d}})$, i.e. show they have the same elements up to scale.

   Notations: $\tilde{\mathbf{d}}$ is the unit direction vector along the line, and $\tilde{\mathbf{x}}$ is any point on the line. Note that $\tilde{\mathbf{x}}$ and $\tilde{\mathbf{d}}$ are 3-dim Euclidean coordinates while $\mathbf{x}_1$ and $\mathbf{x}_2$ are 4-dim homogeneous coordinates.

Consider homogeneous pts. $x_1 = [x_1, y_1, z_1, 1]$ & $x_2 = [x_2, y_2, z_2, 1]$

$\therefore L = x_1 x_2^T - x_2 x_1^T$

$$= \begin{bmatrix} x_1x_2 & x_1y_2 & x_1z_2 & x_1 \\ y_1x_2 & y_1y_2 & y_1z_2 & y_1 \\ z_1x_2 & z_1y_2 & z_1z_2 & z_1 \\ x_2 & y_2 & z_2 & 1 \end{bmatrix} - \begin{bmatrix} x_2x_1 & x_2y_1 & x_2z_1 & x_2 \\ y_2x_1 & y_2y_1 & y_2z_1 & y_2 \\ z_2x_1 & z_2y_1 & z_2z_1 & z_2 \\ x_1 & y_1 & z_1 & 1 \end{bmatrix}$$

$$L = \begin{bmatrix} 0 & x_1y_2 - x_2y_1 & x_1z_2 - x_2z_1 & x_1 - x_2 \\ -(x_1y_2 - x_2y_1) & 0 & y_1z_2 - y_2z_1 & y_1 - y_2 \\ -(x_1z_2 - x_2z_1) & -(y_1z_2 - y_2z_1) & 0 & z_1 - z_2 \\ -(x_1 - x_2) & -(y_1 - y_2) & -(z_1 - z_2) & 0 \end{bmatrix}$$

$\therefore$ Plucker coordinates given by

$L = [(x_1y_2 - x_2y_1), (x_1z_2 - x_2z_1), (x_1 - x_2), (y_1z_2 - y_2z_1), (y_1 - y_2), (z_1 - z_2)]^T$ — (1)

contd.

7. **[2 pts]** Suppose $\mathbf{U}$ is a 4×4 matrix. $\mathbf{U}_{4\times4} = [\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3, \mathbf{u}_4]$ and $\mathbf{U}^T\mathbf{U} = \mathbf{I}$.

   (a) Suppose $\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3$ represent 3 points in the 3D space. What is the plane passing through these 3 points?

   (b) Suppose $\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3, \mathbf{u}_4$ represent 4 points in the 3D space. Let $\mathbf{l}_1$ be the line passing through $\mathbf{u}_1, \mathbf{u}_2$, and $\mathbf{l}_2$ be the line passing through $\mathbf{u}_3, \mathbf{u}_4$. Do $\mathbf{l}_1$ and $\mathbf{l}_2$ intersect or not? (Only consider real-number points.)

6) Contd.

Consider $K = \tilde{x}_1 \times \tilde{x}_2 = \begin{bmatrix} x_1 \\ y_1 \\ z_1 \end{bmatrix} \times \begin{bmatrix} x_2 \\ y_2 \\ z_2 \end{bmatrix} = \begin{bmatrix} y_1 z_2 - z_1 y_2 \\ x_2 z_1 - x_1 z_2 \\ x_1 y_2 - x_2 y_1 \end{bmatrix} = \begin{bmatrix} k_1 \\ k_2 \\ k_3 \end{bmatrix}$ —— ②

Consider a unit direction vector along line L

$$\hat{d} = \frac{\tilde{x}_2 - \tilde{x}_1}{\|\tilde{x}_2 - \tilde{x}_1\|} = \frac{1}{\alpha}(\tilde{x}_2 - \tilde{x}_1) \quad \text{where } \alpha = \|\tilde{x}_2 - \tilde{x}_1\|$$

→ ③

$$\therefore \hat{d} = \frac{1}{\alpha}\begin{bmatrix} x_2 - x_1 \\ y_2 - y_1 \\ z_2 - z_1 \end{bmatrix}$$ —— ④

Also, consider ② again

$$K = \tilde{x}_1 \times \tilde{x}_2$$

$$= \tilde{x}_1 \times (\tilde{x}_2 - \tilde{x}_1 + \tilde{x}_1) \qquad \text{Adding \& subtracting } \tilde{x}_1$$

$$= \tilde{x}_1 \times (\tilde{x}_2 - \tilde{x}_1) + \cancel{\tilde{x}_1 \times \tilde{x}_1}^{\,0}$$

From ③

$$K = \alpha (x_1 \times \hat{d})$$

Now, consider any general pt. on the line. Say, $\tilde{x}$

$$\therefore K = \alpha\left[(\tilde{x}_1 - \tilde{x} + \tilde{x}) \times \hat{d}\right] \qquad \text{Adding \& subtracting } \tilde{x}$$

$$= \alpha\left[\cancel{(\tilde{x}_1 - \tilde{x}) \times \hat{d}}^{\,0} + \tilde{x} \times \hat{d}\right]$$

However, $(x_1 - x)$ & $\hat{d}$ are in same direction $\therefore$ cross product 0

$$K = \alpha(\tilde{x} \times \hat{d})$$ —— ⑤

From ② & ⑤

$$K = \alpha(\tilde{x} \times \hat{d}) = \begin{bmatrix} y_1 z_2 - y_2 z_1 \\ x_2 z_1 - x_1 z_2 \\ x_1 y_2 - x_2 y_1 \end{bmatrix} = \alpha \begin{bmatrix} (\tilde{x} \times \hat{d})_1 \\ (\tilde{x} \times \hat{d})_2 \\ (\tilde{x} \times \hat{d})_3 \end{bmatrix}$$ —— ⑥

Rewriting ① using ④ & ⑥

$$L = \begin{bmatrix} \alpha(\tilde{x} \times \hat{d})_3 \\ -\alpha(\tilde{x} \times \hat{d})_2 \\ -\alpha \hat{d}_1 \\ \alpha(\tilde{x} \times \hat{d})_1 \\ -\alpha \hat{d}_2 \\ -\alpha \hat{d}_3 \end{bmatrix} \equiv \begin{bmatrix} (\tilde{x} \times \hat{d})_3 \\ -(\tilde{x} \times \hat{d})_2 \\ -\hat{d}_1 \\ (\tilde{x} \times \hat{d})_1 \\ -\hat{d}_2 \\ -\hat{d}_3 \end{bmatrix} \longrightarrow (7)$$

Since (1) can be rewritten entirely in terms of the direction vector $(\hat{d})$ & the normal to the line $(\tilde{x} \times \hat{d})$, the following representations

$L = X_1 X_2^T - X_2 X_1^T$ & $L = (\hat{d}, \tilde{x} \times \hat{d})$ are equivalent to scale.

a) Let the plane be defined by its normal as $\pi = \hat{n} = \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix}$

∴ For each of the pt.s u_1, u_2, u_3 to lie on π,

$u_1^T \pi = 0$, $u_2^T \pi = 0$, $u_3^T \pi = 0$

This is only possible if the normal $\hat{n}$ is perpendicular to each u_1, u_2, u_3

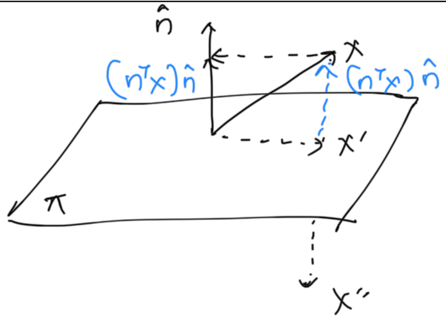
Considering that $U^T U = I$, U is an orthogonal matrix & so, u_1, u_2, u_3, u_4 are all perpendicular to each other.

∴ the only $\hat{n}$ possible that's normal to u_1, u_2, u_3 is u_4

∴ $\pi = u_4$

8. **[4 pts]** (a) Calculate the 3D transformation $\mathbf{H}$ that represents the projection onto a plane $\pi = [\mathbf{n}^T, 0]^T$, where $\mathbf{n} = [a, b, c]^T$ is a unit vector.

(b) Calculate the 3D transformation $\mathbf{H}$ that represents the reflection along a plane $\pi = [\mathbf{n}^T, 0]^T$, where $\mathbf{n} = [a, b, c]^T$ is a unit vector.



Consider $x = \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$

then x' is its projection on plane π & x'' is its reflection along π

a) ∴ Projection $x' = x - (n^T x)\hat{n}$

$$= \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} - (ax + by + cz) \begin{bmatrix} a \\ b \\ c \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} - \begin{bmatrix} a^2x + aby + acz \\ abx + b^2y + bcz \\ acx + bcy + c^2z \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} (1-a^2)x - aby - acz \\ -abx + (1-b^2)y - bcz \\ -acx - bcy + (1-c^2)z \\ 1 \end{bmatrix}$$

7) b) From plucker representation

$$l_1 = u_1u_2^T - u_2u_1^T \quad \& \quad l_2 = u_3u_4^T - u_4u_3^T$$

$\therefore$ Point of intersection can be found by the cross product equivalent of lines l_1 & l_2

$$\therefore p = l_1l_2^T - l_2l_1^T = (u_1u_2^T - u_2u_1^T)(u_3u_4^T - u_4u_3^T)^T - (u_3u_4^T - u_4u_3^T)(u_1u_2^T - u_2u_1^T)^T$$

$$= (u_1u_2^T - u_2u_1^T)(u_4u_3^T - u_3u_4^T) - (u_3u_4^T - u_4u_3^T)\cdot(u_2u_1^T - u_1u_2^T)$$

$$= (u_1u_2^T - u_2u_1^T)\left[\cancel{(u_4u_3^T - u_3u_4^T)} - \cancel{(u_4u_3^T - u_3u_4^T)}\right]$$

$$= 0$$

$\because$ the cross product equivalent is 0, lines l_1 & l_2 do not intersect.

8)

a) Rewriting it as $x' = Hx$

$$x' = \begin{bmatrix} (1-a^2) & -ab & -ac & 0 \\ -ab & (1-b^2) & -bc & 0 \\ -ac & -bc & (1-c^2) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

$\therefore$ Transformation matrix to get projection onto plane π

$$H = \begin{bmatrix} (1-a^2) & -ab & -ac & 0 \\ -ab & (1-b^2) & -bc & 0 \\ -ac & -bc & (1-c^2) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

b) Reflection along plane π

$$x'' = x - 2(n^Tx)\hat{n}$$

$$= \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} - 2(ax+by+cz)\begin{bmatrix} a \\ b \\ c \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} (1-2a^2)x - 2aby - 2acz \\ -2abx + (1-2b^2)y - 2bcz \\ -2acx - 2bcy + (1-2c^2)z \\ 0 \end{bmatrix}$$

Rewriting it as $x'' = Hx$

$$x'' = \begin{bmatrix} (1-2a^2) & -2ab & -2ac & 0 \\ -2ab & (1-2b^2) & -2bc & 0 \\ -2ac & -2bc & (1-2c^2) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

$\therefore$ Transformation matrix to get reflection along plane π

$$H = \begin{bmatrix} (1-2a^2) & -2ab & -2ac & 0 \\ -2ab & (1-2b^2) & -2bc & 0 \\ -2ac & -2bc & (1-2c^2) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

9. **[3 pts]** In the lecture, we introduced an algorithm to compute homography between images from 4 pairs of point correspondences. Design an algorithm that instead uses pairs of line correspondences. Write the constraints provided by each correspondence, and how to compute the **H** that satisfies these.

Finding the equations

$l' = H^{-T} l$

$\therefore \ l = H^T l'$

$$\begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} h_1 & h_4 & h_7 \\ h_2 & h_5 & h_8 \\ h_3 & h_6 & h_9 \end{bmatrix} \begin{bmatrix} a' \\ b' \\ c' \end{bmatrix}$$

$$\therefore \ a = a'h_1 + b'h_4 + c'h_7$$
$$b = a'h_2 + b'h_5 + c'h_8$$
$$c = a'h_3 + b'h_6 + c'h_9$$

$$\therefore \ \frac{a}{c} = \frac{a'h_1 + b'h_4 + c'h_7}{a'h_3 + b'h_6 + c'h_9}$$

$$\therefore \ aa'h_3 + ab'h_6 + ac'h_9 - ca'h_1 - cb'h_4 - cc'h_7 = 0 \quad \text{— } 1$$

IIIrly

$$\frac{b}{c} = \frac{a'h_2 + b'h_5 + c'h_8}{a'h_3 + b'h_6 + c'h_9}$$

$$\therefore \ ba'h_3 + bb'h_6 + bc'h_9 - ca'h_2 - cb'h_5 - cc'h_8 = 0 \quad \text{— } 2$$

Combining 1 & 2

$$\begin{bmatrix} -ca' & 0 & aa' & -cb' & 0 & ab' & -cc' & 0 & ac' \\ 0 & -ca' & ba' & 0 & -cb' & bb' & 0 & -cc' & bc' \end{bmatrix} \begin{bmatrix} h_1 \\ h_2 \\ h_3 \\ h_4 \\ h_5 \\ h_6 \\ h_7 \\ h_8 \\ h_9 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$Ah = 0$

$\therefore$ 1 pair of corresponding lines gives 2 equations.

Also, H has 8 degrees of freedom since there are 8 ratios & we dont care about scale

$\therefore$ Algorithm

- Find 4 pairs of line correspondences & generate a 8×9 matrix A.
- Use SVD to solve for its nullspace & get h.
- However, add a constraint $||h|| = 1$ to avoid a trivial solution

Collaboration Questions Please answer the following:

1. Did you receive any help whatsoever from anyone in solving this assignment?

   ○ Yes

   ● No

   - If you answered 'Yes', give full details:
   - (e.g. "Jane Doe explained to me what is asked in Question 3.4")

2. Did you give any help whatsoever to anyone in solving this assignment?

   ○ Yes

   ● No

   - If you answered 'Yes', give full details:
   - (e.g. "I pointed Joe Smith to section 2.3 since he didn't know how to proceed with Question 2")

3. Did you find or come across code that implements any part of this assignment ?

   ○ Yes

   ● No

   - If you answered 'Yes', give full details: No
   - (book & page, URL & location within the page, etc.).