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SPACETRACK REPORT NO. 3

PROJECT SPACE TRACK

MODELS FOR PROPAGATION OF
NORAD ELEMENT SETS



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SPACETRACK REPORT NO. 3

MODELS FOR PROPAGATION OF
NORAD ELEMENT SETS

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General perturbations element sets generated by NORAD can be used to predict position and velocity of Earth-orbiting objects. To do this one must be careful to use a prediction method which is compatible with the way in which the elements were generated. Equations for five compatible models are given here along with corresponding FORTRAN IV computer code. With this information a user will be able to make satellite predictions which are completely compatible with NORAD predictions.

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1. INTRODUCTION

NORAD maintains general perturbations element sets on all resident space objects. These element sets are periodically refined so as to maintain a reasonable prediction capability on all space objects. In turn, these element sets are provided to users. The purpose of this report is to provide the user with a means of propagating these element sets in time to obtain a position and velocity of the space object.

The most important point to be noted is that not just any prediction model will suffice. The NORAD element sets are "mean" values obtained by removing periodic variations in a particular way. In order to obtain good predictions, these periodic variations must be reconstructed (by the prediction model) in exactly the same way they were removed by NORAD. Hence, inputting NORAD element sets into a different model (even though the model may be more accurate or even a numerical integrator) will result in degraded predictions. The NORAD element sets must be used with one of the models described in this report in order to retain maximum prediction accuracy.

All space objects are classified by NORAD as near-Earth (period less than 225 minutes) or deep-space (period greater than or equal 225 minutes). Depending on the period, the NORAD element sets are automatically generated with the near-Earth or deep-space model. The user can then calculate the satellite period and know which prediction model to use.

2. THE PROPAGATION MODELS

Five mathematical models for prediction of satellite position and velocity are available. The first of these, SGP, was developed by Hilton & Kuhlman (1966) and is used for near-Earth satellites. This model uses a simplification of the work of Kozai (1959) for its gravitational model and it takes the drag effect on mean motion as linear in time. This assumption dictates a quadratic variation of mean anomaly with time. The drag effect on eccentricity is modeled in such a way that perigee height remains constant.

The second model, SGP4, was developed by Ken Cranford in 1970 (see Lane and Hoots 1979) and is used for near-Earth satellites. This model was obtained by simplification of the more extensive analytical theory of Lane and Cranford (1969) which uses the solution of Brouwer (1959) for its gravitational model and a power density function for its atmospheric model (see Lane, et al 1962).

The next model, SDP4, is an extension of SGP4 to be used for deep-space satellites. The deep-space equations were developed by Hujasak (1979) and model the gravitational effects of the moon and sun as well as certain sectoral and tesseral Earth harmonics which are of particular importance for half-day and one-day period orbits.

The SGP8 model (see Hoots 1980) is used for near-Earth

satellites and is obtained by simplification of an extensive analytical theory of Hoots (to appear) which uses the same gravitational and atmospheric models as Lane and Cranford did but integrates the differential equations in a much different manner.

Finally, the SDP8 model is an extension of SGP8 to be used for deep-space satellites. The deep-space effects are modeled in SDP8 with the same equations used in SDP4.

3. COMPATIBILITY WITH NORAD ELEMENT SETS

The NORAD element sets are currently generated with either SGP4 or SDP4 depending on whether the satellite is near-Earth or deep-space. For element sets sent to external users, the value of mean motion is altered slightly and a pseudo-drag term ($\dot{n}/2$) is generated. These changes allow an SGP user to make compatible predictions in the following manner. If the satellite is near-Earth, then the pseudo-drag term used in SGP simulates the drag effect of the SGP4 model. If the satellite is deep-space, then the pseudo-drag term used in SGP simulates the deep-space secular effects of SDP4.

For SGP4 and SDP4 users, the mean motion is first recovered from its altered form and the drag effect is obtained from the SGP4 drag term (B^*) with the pseudo-drag term being ignored. The value of the mean motion can be used to determine whether the satellite is near-Earth or deep-space (and hence whether SGP4 or SDP4 was used to generate the element set). From

this information the user can decide whether to use SGP4 or SDP4 for propagation and hence be assured of agreement with NORAD predictions.

The SGP8 and SDP8 models have the same gravitational and atmospheric models as SGP4 and SDP4, although the form of the solution equations is quite different. Additionally, SGP8 and SDP8 use a ballistic coefficient (B term) in the drag equations rather than the B* drag term. However, compatible predictions can be made with NORAD element sets by first calculating a B term from the SGP4 B* drag term.

At the present time consideration is being given to replacing SGP4 and SDP4 by SGP8 and SDP8 as the NORAD satellite models. In such a case the new NORAD element sets would still give compatible predictions for SGP, SGP4, and SDP4 users and, for SGP8 and SDP8 users, would give agreement with NORAD predictions.

4. GENERAL PROGRAM DESCRIPTION

The five ephemeris packages cited in section two have each been programmed in FORTRAN IV as stand-alone subroutines. They each access the two function subroutines ACTAN and FMOD2P and the deep-space equations access the function subroutine THETAG. The function subroutine ACTAN is a two argument (quadrant preserving) arctangent subroutine which has been specifically designed to return the angle within the range of 0 to 2π . The function subroutine FMOD2P takes an angle and returns the modulo

by 2π of that angle. The function subroutine THETAG calculates the epoch time in days since 1950 Jan 0.0 UTC, stores this in common, and returns the right ascension of Greenwich at epoch.

One additional subroutine DEEP is accessed by SDP4 and SDP8 to obtain the deep-space perturbations to be added to the main equations of motion.

The main program DRIVER reads the input NORAD 2-line element set in either G-card internal format or T-card transmission format and calls the appropriate ephemeris package as specified by the user. The DRIVER converts the elements to the units of radians and minutes before calling the appropriate subroutine. The ephemeris package returns position and velocity in units of Earth radii and minutes. These are converted by the DRIVER to kilometers and seconds for printout.

All physical constants are contained in the constants common C1 and can be changed through the data statements in the DRIVER. The one exception is the physical constants used only in DEEP which are set in the data statements in DEEP.

In the following sections the equations and program listing are given for each ephemeris model. Every effort has been made to maintain a strict parallel structure between the equations and the computer code.

5. THE SGP MODEL

The NORAD mean element sets can be used for prediction with SGP. All symbols not defined below are defined in the list of

symbols in section twelve. Predictions are made by first calculating the constants

$$a_1 = \left(\frac{k_e}{n_o}\right)^{2/3}$$

$$\delta_1 = \frac{3}{4} J_2 \frac{a_E^2}{a_1^2} \frac{(3 \cos^2 i_o - 1)}{(1 - e_o^2)^{3/2}}$$

$$a_o = a_1 [1 - \frac{1}{3} \delta_1 + \delta_1^2 + \frac{134}{81} \delta_1^3]$$

$$p_o = a_o (1 - e_o^2)$$

$$q_o = a_o (1 - e_o)$$

$$L_o = M_o + \omega_o + \Omega_o$$

$$\frac{d\Omega}{dt} = - \frac{3}{2} J_2 \frac{a_E^2}{p_o^2} n_o \cos i_o$$

$$\frac{d\omega}{dt} = \frac{3}{4} J_2 \frac{a_E^2}{p_o^2} n_o (5 \cos^2 i_o - 1) .$$

The secular effects of atmospheric drag and gravitation are included through the equations

$$a = a_o \left\{ \frac{n_o}{n_o + 2(\frac{\dot{n}_o}{2})(t - t_o) + 3(\frac{\ddot{n}_o}{6})(t - t_o)^2} \right\}^{2/3}$$

$$e = \begin{cases} 1 - \frac{q_0}{a}, & \text{for } a > q_0 \\ 10^{-6}, & \text{for } a \leq q_0 \end{cases}$$

$$p = a(1-e^2)$$

$$\Omega_{s_0} = \Omega_0 + \frac{d\Omega}{dt} (t - t_0)$$

$$\omega_{s_0} = \omega_0 + \frac{d\omega}{dt} (t - t_0)$$

$$L_s = L_0 + (n_0 + \frac{d\omega}{dt} + \frac{d\Omega}{dt}) (t - t_0) + \frac{\dot{n}_0}{2} (t - t_0)^2 + \frac{\ddot{n}_0}{6} (t - t_0)^3$$

where $(t - t_0)$ is time since epoch.

Long-period periodics are included through the equations

$$a_{yNSL} = e \sin \omega_{s_0} - \frac{1}{2} \frac{J_3}{J_2} \frac{a_E}{p} \sin i_0$$

$$L = L_s - \frac{1}{4} \frac{J_3}{J_2} \frac{a_E}{p} a_{xNSL} \sin i_0 \left[\frac{3 + 5 \cos i_0}{1 + \cos i_0} \right]$$

where

$$a_{xNSL} = e \cos \omega_{s_0}$$

Solve Kepler's equation for $E + \omega$ (by iteration to the desired accuracy), where

$$(E + \omega)_i + 1 = (E + \omega)_i + \Delta(E + \omega)_i$$

with

$$\Delta(E + \omega)_i = \frac{U + a_{yNSL} \cos(E + \omega)_i + a_{xNSL} \sin(E + \omega)_i - (E + \omega)}{a_{yNSL} \sin(E + \omega)_i - a_{xNSL} \cos(E + \omega)_i + 1}$$

$$U = L - \Omega s_o$$

and

$$(E + \omega)_1 = U.$$

Then calculate the intermediate (partially osculating) quantities

$$e \cos E = a_{xNSL} \cos(E + \omega) + a_{yNSL} \sin(E + \omega)$$

$$e \sin E = a_{xNSL} \sin(E + \omega) - a_{yNSL} \cos(E + \omega)$$

$$e_L^2 = (a_{xNSL})^2 + (a_{yNSL})^2$$

$$p_L = a(1 - e_L^2)$$

$$r = a(1 - e \cos E)$$

$$\dot{r} = k_e \sqrt{\frac{a}{r}} e \sin E$$

$$\dot{rv} = k_e \sqrt{\frac{p_L}{r}}$$

$$\sin u = \frac{a}{r} [\sin(E + \omega) - a_{yNSL} - a_{xNSL} \frac{e \sin E}{1 + \sqrt{1 - e_L^2}}]$$

$$\cos u = \frac{a}{r} [\cos(E + \omega) - a_{xNSL} + a_{yNSL} \frac{e \sin E}{1 + \sqrt{1 - e_L^2}}]$$

$$u = \tan^{-1} \left(\frac{\sin u}{\cos u} \right) .$$

Short-period perturbations are now included by

$$r_k = r + \frac{1}{4} J_2 \frac{a_E^2}{p_L^2} \sin^2 i_o \cos 2u$$

$$u_k = u - \frac{1}{8} J_2 \frac{a_E^2}{p_L^2} (7 \cos^2 i_o - 1) \sin 2u$$

$$\Omega_k = \Omega_{s_o} + \frac{3}{4} J_2 \frac{a_E^2}{p_L^2} \cos i_o \sin 2u$$

$$i_k = i_o + \frac{3}{4} J_2 \frac{a_E^2}{p_L^2} \sin i_o \cos i_o \cos 2u.$$

Then unit orientation vectors are calculated by

$$\underline{U} = \underline{M} \sin u_k + \underline{N} \cos u_k$$

$$\underline{V} = \underline{M} \cos u_k - \underline{N} \sin u_k$$

where

$$\underline{M} = \begin{cases} M_x = -\sin \Omega_k \cos i_k \\ M_y = \cos \Omega_k \cos i_k \\ M_z = \sin i_k \end{cases}$$

$$\underline{N} = \begin{cases} N_x = \cos \Omega_k \\ N_y = \sin \Omega_k \\ N_z = 0 \end{cases}$$

Then position and velocity are given by

$$\underline{r} = \underline{r}_k U$$

and

$$\dot{\underline{r}} = \dot{\underline{r}} U + (\underline{r} \dot{v}) V.$$

A FORTRAN IV computer code listing of the subroutine SGP
is given below.

31 OCT 83

```

1      *      SGP
2      SUBROUTINE SGP(IFLAG,TSINCE)
3      COMMON/E1/XMO,XNODEO,OMEGA0,E0,XINCL,XNO,XNDT20,XNDD60,BSTAR,
4      1      X,Y,Z,XDOT,YDOT,ZDOT,EPOCH,DS50
5      COMMON/C1/CK2,CK4,E6A,QOMS2T,S,TOTHRD,
6      1      XJ3,XKE,XKMPER,XMNPDA,AE
7      DOUBLE PRECISION EPOCH,DS50
8
9      IF(IFLAG.EQ.0) GO TO 19
10
11     *      INITIALIZATION
12
13     C1= CK2*1.5
14     C2= CK2/4.0
15     C3= CK2/2.0
16     C4= XJ3*AE**3/(4.0*CK2)
17     COSIO=COS(XINCL)
18     SINIO=SIN(XINCL)
19     A1=(XKE/XNO)**TOTHRD
20     D1= C1/A1/A1*(3.*COSIO*COSIO-1.)/(1.-E0*E0)**1.5
21     A0=A1*(1.-1./3.*D1-D1*D1-134./81.*D1*D1*D1)
22     P0=A0*(1.-E0*E0)
23     Q0=A0*(1.-E0)
24     XLO=XMO+OMEGA0*XNODEO
25     D10= C3 *SINIO*SINIO
26     D20= C2 *(7.*COSIO*COSIO-1.)
27     D30=C1*COSIO
28     D40=D30*SINIO
29     P02NO=XNO/(P0*P0)
30     OMGDT=C1*P02NO*(5.*COSIO*COSIO-1.)
31     XNODOT=-2.*D30*P02NO
32     C5=.5*C4*SINIO*(3.+5.*COSIO)/(1.+COSIO)
33     C6=C4*SINIO
34     IFLAG=0
35
36     *      UPDATE FOR SECULAR GRAVITY AND ATMOSPHERIC DRAG
37
38     19 A=XNO+(2.*XNDT20+3.*XNDD60*TSINCE)*TSINCE
39     A=A0*(XNO/A)**TOTHRD
40     E=E6A
41     IF(A.GT.Q0) E=1.-Q0/A
42     P=A*(1.-E*E)
43     XNODES= XNODEO+XNODOT*TSINCE
44     OMGAS= OMEGA0+OMGDT*TSINCE
45     XLS=FMOD2P(XLO+(XNO+OMGAS+XNODOT+(XNDT20+XNDD60*TSINCE)*
46     1 TSINCE)*TSINCE)
47
48     *      LONG PERIOD PERIODICS
49
50     AXNSL=E*COS(OMGAS)
51     AYNSL=E*SIN(OMGAS)-C6/P
52     XL=FMOD2P(XLS-C5/P*AXNSL)
53
54     *      SOLVE KEPLERS EQUATION
55
56     U=FMOD2P(XL-XNODES)

```

```

57      ITEM3=0
58      E01=U
59      TEM5=1.
60      20 SINE01=SIN(E01)
61      COSE01=COS(E01)
62      IF(ABS(TEM5).LT.E6A) GO TO 30
63      IF(ITEM3.GE.10) GO TO 30
64      ITEM3=ITEM3+1
65      TEM5=1.-COSE01*AXNSL-SINE01*AYNSL
66      TEM5=(U-AYNSL*COSE01+AXNSL*SINE01-E01)/TEM5
67      TEM2=ABS(TEM5)
68      IF(TEM2.GT.1.) TEM5=TEM2/TEM5
69      E01=E01+TEM5
70      GO TO 20
71
72      *      SHORT PERIOD PRELIMINARY QUANTITIES
73
74      30 ECOSINE=AXNSL*COSE01+AYNSL*SINE01
75      ESINE=AXNSL*SINE01-AYNSL*COSE01
76      EL2=AXNSL*AXNSL+AYNSL*AYNSL
77      PL=A*(1.-EL2)
78      PL2=PL*PL
79      R=A*(1.-ECOSINE)
80      RDOT=XKE*SQRT(A)/R*ESINE
81      RVDOT=XKE*SQRT(PL)/R
82      TEMP=ESINE/(1.+SQRT(1.-EL2))
83      SINU=A/R*(SINE01-AYNSL-AXNSL*TEMP)
84      COSU=A/R*(COSE01-AXNSL+AYNSL*TEMP)
85      SU=ACTAN(SINU,COSU)
86
87      *      UPDATE FOR SHORT PERIODICS
88
89      SIN2U=(COSU+COSU)*SINU
90      COS2U=1.-2.*SINU*SINU
91      RK=R+D10/PL*COS2U
92      UK=SU-D20/PL2*SIN2U
93      XNODEK=XNODES+D30*SIN2U/PL2
94      XINCK =XINCL+D40/PL2*COS2U
95
96      *      ORIENTATION VECTORS
97
98      SINUK=SIN(UK)
99      COSUK=COS(UK)
100     SINNOK=SIN(XNODEK)
101     COSNOK=COS(XNODEK)
102     SINIK=SIN(XINCK)
103     COSIK=COS(XINCK)
104     XMX=-SINNOK*COSIK
105     XMY=COSNOK*COSIK
106     UX=XMX*SINUK+COSNOK*COSUK
107     UY=XMY*SINUK+SINNOK*COSUK
108     UZ=SINIK*SINUK
109     VX=XMX*COSUK-COSNOK*SINUK
110     VY=XMY*COSUK-SINNOK*SINUK
111     VZ=SINIK*COSUK
112

```

113 * POSITION AND VELOCITY
114
115 X=RK★UX
116 Y=RK★UY
117 Z=RK★UZ
118 XDOT=RDOT★UX
119 YDOT=RDOT★UY
120 ZDOT=RDOT★UZ
121 XDOT=RVDOT★VX+XDOT
122 YDOT=RVDOT★VY+YDOT
123 ZDOT=RVDOT★VZ+ZDOT
124
125 RETURN
126 END

6. THE SGP4 MODEL

The NORAD mean element sets can be used for prediction with SGP4. All symbols not defined below are defined in the list of symbols in section twelve. The original mean motion (n_o'') and semimajor axis (a_o'') are first recovered from the input elements by the equations

$$a_1 = \left(\frac{k_e}{n_o} \right)^{2/3}$$

$$\delta_1 = \frac{3}{2} \frac{k_2}{a_1^2} \frac{(3 \cos^2 i_o - 1)}{(1 - e_o^2)^{3/2}}$$

$$a_o = a_1 \left(1 - \frac{1}{3} \delta_1 - \delta_1^2 - \frac{134}{81} \delta_1^3 \right)$$

$$\delta_o = \frac{3}{2} \frac{k_2}{a_o^2} \frac{(3 \cos^2 i_o - 1)}{(1 - e_o^2)^{3/2}}$$

$$n_o'' = \frac{n_o}{1 + \delta_o}$$

$$a_o'' = \frac{a_o}{1 - \delta_o}$$

For perigee between 98 kilometers and 156 kilometers, the value of the constant s used in SGP4 is changed to

$$s^* = a_o'' (1 - e_o) - s + a_E$$

For perigee below 98 kilometers, the value of s is changed to

$$s^* = 20/XKMPER + a_E.$$

If the value of s is changed, then the value of $(q_o - s)^4$ must be replaced by

$$(q_o - s^*)^4 = \left[[(q_o - s)^4]^{1/4} + s - s^* \right]^4.$$

Then calculate the constants (using the appropriate values of s and $(q_o - s)^4$)

$$\theta = \cos i_o$$

$$\xi = \frac{1}{a''_o - s}$$

$$\beta_o = (1 - e_o^2)^{1/2}$$

$$\eta = a''_o e_o \xi$$

$$C_2 = (q_o - s)^4 \xi^4 n''_o (1 - \eta^2)^{-7/2} [a''_o (1 + \frac{3}{2} \eta^2 + 4e_o \eta + e_o \eta^3) + \frac{3}{2} \frac{k_2 \xi}{(1 - \eta^2)} (-\frac{1}{2} + \frac{3}{2} \theta^2) (8 + 24\eta^2 + 3\eta^4)]$$

$$C_1 = B^* C_2$$

$$C_3 = \frac{(q_o - s)^4 \xi^5 A_{3,0} n''_o a_E \sin i_o}{k_2 e_o}$$

$$C_4 = 2n''_o (q_o - s)^4 \xi^4 a''_o \beta_o^2 (1 - \eta^2)^{-7/2} \left([2\eta (1 + e_o \eta)$$

$$+ \frac{1}{2} e_o + \frac{1}{2} \eta^3] - \frac{2k_2 \xi}{a''_o (1 - \eta^2)} [3 (1 - 3\theta^2) (1$$

$$+ \frac{3}{2} \eta^2 - 2e_o \eta - \frac{1}{2} e_o \eta^3] + \frac{3}{4} (1 - \theta^2) (2\eta^2 - e_o \eta$$

$$- e_o \eta^3) \cos 2\omega_o \right)$$

$$C_5 = 2 (q_o - s)^4 \xi^4 a_o'' \beta_o^2 (1 - n^2)^{-7/2} [1 + \frac{11}{4}n (n + e_o) \\ + e_o n^3]$$

$$D_2 = 4 a_o'' \xi C_1^2$$

$$D_3 = \frac{4}{3} a_o'' \xi^2 (17 a_o'' + s) C_1^3$$

$$D_4 = \frac{2}{3} a_o'' \xi^3 (221 a_o'' + 31 s) C_1^4.$$

The secular effects of atmospheric drag and gravitation are included through the equations

$$M_{DF} = M_o + [1 + \frac{3k_2(-1 + 3\theta^2)}{2a_o''^2 \beta_o^3} + \frac{3k_2^2(13 - 78\theta^2 + 137\theta^4)}{16a_o''^4 \beta_o^7}] n_o'' (t - t_o)$$

$$\omega_{DF} = \omega_o + [- \frac{3k_2(1 - 5\theta^2)}{2a_o''^2 \beta_o^4} + \frac{3k_2^2(7 - 114\theta^2 + 395\theta^4)}{16a_o''^4 \beta_o^8} \\ + \frac{5k_4(3 - 36\theta^2 + 49\theta^4)}{4a_o''^4 \beta_o^8}] n_o'' (t - t_o)$$

$$\Omega_{DF} = \Omega_o + [- \frac{3k_2 \theta}{a_o''^2 \beta_o^4} + \frac{3k_2^2(4\theta - 19\theta^3)}{2a_o''^4 \beta_o^8} + \frac{5k_4 \theta(3 - 7\theta^2)}{2a_o''^4 \beta_o^8}] n_o'' (t - t_o)$$

$$\delta\omega = B^* C_3 (\cos \omega_o) (t - t_o)$$

$$\delta M = -\frac{2}{3} (q_o - s)^4 B^* \xi^4 \frac{a_E}{e_o n} [(1 + n \cos M_{DF})^3 - (1 + n \cos M_o)^3]$$

$$M_p = M_{DF} + \delta\omega + \delta M$$

$$\omega = \omega_{DF} - \delta\omega - \delta M$$

$$\Omega = \Omega_{DF} - \frac{21}{2} \frac{n'' k_2 \theta}{a_o''^2 \beta_o^2} C_1 (t - t_o)^2$$

$$e = e_o - B^* C_4 (t - t_o) - B^* C_5 (\sin M_p - \sin M_o)$$

$$a = a_o'' [1 - C_1 (t - t_o) - D_2 (t - t_o)^2 - D_3 (t - t_o)^3$$

$$- D_4 (t - t_o)^4]^2$$

$$\begin{aligned} L = M_p + \omega + \Omega + n_o'' & [\frac{3}{2} C_1 (t - t_o)^2 + (D_2 + 2C_1^2) (t - t_o)^3 \\ & + \frac{1}{4} (3D_3 + 12C_1 D_2 + 10C_1^3) (t - t_o)^4 + \frac{1}{5} (3D_4 + 12C_1 D_3 \\ & + 6D_2^2 + 30C_1^2 D_2 + 15C_1^4) (t - t_o)^5] \end{aligned}$$

$$\beta = \sqrt{(1 - e^2)}$$

$$n = k_e / a^{3/2}$$

where $(t - t_o)$ is time since epoch. It should be noted that when epoch perigee height is less than 220 kilometers, the equations for a and L are truncated after the C_1 term, and the terms involving C_5 , $\delta\omega$, and δM are dropped.

Add the long-period periodic terms

$$a_{xN} = e \cos \omega$$

$$L_L = \frac{A_{3,0} \sin i_o}{8k_2 a \beta^2} (e \cos \omega) \left(\frac{3}{1} + \frac{5\theta}{\theta} \right)$$

$$a_{yNL} = \frac{A_{3,0} \sin i_0}{4k_2 a \beta^2}$$

$$L_T = L + L_L$$

$$a_{yN} = e \sin \omega + a_{yNL}.$$

Solve Kepler's equation for $(E + \omega)$ by defining

$$U = L_T - \Omega$$

and using the iteration equation

$$(E + \omega)_{i+1} = (E + \omega)_i + \Delta(E + \omega)_i$$

with

$$\Delta(E + \omega)_i = \frac{U - a_{yN} \cos (E + \omega)_i + a_{xN} \sin (E + \omega)_i - (E + \omega)_i}{-a_{yN} \sin (E + \omega)_i - a_{xN} \cos (E + \omega)_i + 1}$$

and

$$(E + \omega)_1 = U.$$

The following equations are used to calculate preliminary quantities needed for short-period periodics.

$$e \cos E = a_{xN} \cos (E + \omega) + a_{yN} \sin (E + \omega)$$

$$e \sin E = a_{xN} \sin (E + \omega) - a_{yN} \cos (E + \omega)$$

$$e_L = (a_{xN}^2 + a_{yN}^2)^{1/2}$$

$$p_L = a (1 - e_L^2)$$

$$r = a (1 - e \cos E)$$

$$\dot{r} = k_e \sqrt{\frac{a}{r}} e \sin E$$

$$\dot{r}_f = k_e \sqrt{\frac{p_L}{r}}$$

$$\cos u = \frac{a}{r} [\cos(E + \omega) - a_{xN} + \frac{a_{yN} (e \sin E)}{1 + \sqrt{1 - e_L^2}}]$$

$$\sin u = \frac{a}{r} [\sin(E + \omega) - a_{yN} - \frac{a_{xN} (e \sin E)}{1 + \sqrt{1 - e_L^2}}]$$

$$u = \tan^{-1} \left(\frac{\sin u}{\cos u} \right)$$

$$\Delta r = \frac{k_2}{2p_L} (1 - \theta^2) \cos 2u$$

$$\Delta u = - \frac{k_2}{4p_L^2} (7\theta^2 - 1) \sin 2u$$

$$\Delta \Omega = \frac{3k_2 \theta}{2p_L^2} \sin 2u$$

$$\Delta i = \frac{3k_2 \theta}{2p_L^2} \sin i_o \cos 2u$$

$$\Delta r = - \frac{k_2 n}{p_L} (1 - \theta^2) \sin 2u$$

$$\Delta \dot{r}_{rf} = \frac{k_2 n}{p_L} [(1 - \theta^2) \cos 2u - \frac{3}{2} (1 - 3\theta^2)]$$

The short-period periodics are added to give the osculating quantities

$$r_k = r [1 - \frac{3}{2} k_2 \frac{\sqrt{1 - e_L^2}}{p_L^2} (3\theta^2 - 1)] + \Delta r$$

$$u_k = u + \Delta u$$

$$\Omega_k = \Omega + \Delta \Omega$$

$$i_k = i_o + \Delta i$$

$$\dot{r}_k = \dot{r} + \Delta \dot{r}$$

$$\dot{rf}_k = \dot{rf} + \Delta \dot{rf} .$$

Then unit orientation vectors are calculated by

$$\underline{U} = \underline{M} \sin u_k + \underline{N} \cos u_k$$

$$\underline{V} = \underline{M} \cos u_k - \underline{N} \sin u_k$$

where

$$\underline{M} = \left\{ \begin{array}{l} M_x = - \sin \Omega_k \cos i_k \\ M_y = \cos \Omega_k \cos i_k \\ M_z = \sin i_k \end{array} \right\}$$

$$\underline{N} = \begin{Bmatrix} N_x = \cos \Omega_k \\ N_y = \sin \Omega_k \\ N_z = 0 \end{Bmatrix}.$$

Then position and velocity are given by

$$\underline{r} = \underline{r}_k \underline{U}$$

and

$$\dot{\underline{r}} = \dot{\underline{r}}_k \underline{U} + (\underline{r} \dot{\underline{f}})_k \underline{V}.$$

A FORTRAN IV computer code listing of the subroutine SGP4 is given below. These equations contain all currently anticipated changes to the SCC operational program. These changes are scheduled for implementation in March, 1981.

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```
1      *      SGP4
2      SUBROUTINE SGP4(IFLAG,TSINCE)
3      COMMON/E1/XNO,XNODEQ,OMEGA0,E0,XINCL,XNO,XNDT20,
4      1      XNDD60,BSTAR,X,Y,Z,XDOT,YDOT,ZDOT,EPOCH,DS50
5      COMMON/C1/CK2,CK4,E6A,QOMS2T,S,TOTHRD
6      1      XJ3,XKE,XKMPER,XMNPDA,AE
7      DOUBLE PRECISION EPOCH,DS50
8
9      IF (IFLAG .EQ. 0) GO TO 100
10
11     *      RECOVER ORIGINAL MEAN MOTION (XNODP) AND SEMIMAJOR AXIS (AODP)
12     *      FROM INPUT ELEMENTS
13
14     A1=(XKE/XNO)**TOTHRD
15     COSIO=COS(XINCL)
16     THETA2=COSIO*COSIO
17     X3THM1=3.*THETA2-1.
18
19     EOSQ=E0*E0
20     BETA02=1.-EOSQ
21     BETA0=SQRT(BETA02)
22     DEL1=1.5*CK2*X3THM1/(A1*A1*BETA0*BETA02)
23     AO=A1*(1.-DEL1*(.5*TOTHRD+DEL1*(1.+134./31.*DEL1)))
24     DELO=1.5*CK2*X3THM1/(AO*AO*BETA0*BETA02)
25     XNODP=XNO/(1.+DELO)
26     AODP=AO/(1.-DELO)
27
28     *      INITIALIZATION
29
30     *      FOR PERIGEE LESS THAN 220 KILOMETERS, THE ISIMP FLAG IS SET AND
31     *      THE EQUATIONS ARE TRUNCATED TO LINEAR VARIATION IN SQRT A AND
32     *      QUADRATIC VARIATION IN MEAN ANOMALY. ALSO, THE C3 TERM, THE
33     *      DELTA OMEGA TERM, AND THE DELTA M TERM ARE DROPPED.
34
35     ISIMP=0
36     IF((AODP*(1.-E0)/AE) .LT. (220./XKMPER+AE)) ISIMP=1
37
38     *      FOR PERIGEE BELOW 156 KM, THE VALUES OF
39     *      S AND QOMS2T ARE ALTERED
40
41     S4=S
42     QOMS24=QOMS2T
43     PERIGE=(AODP*(1.-E0)-AE)*XKMPER
44     IF(PERIGE .GE. 156.) GO TO 10
45     S4=PERIGE-78.
46     IF(PERIGE .GT. 98.) GO TO 9
47     S4=20.
48     9 QOMS24=((120.-S4)*AE/XKMPER)**4
49     S4=S4/XKMPER+AE
50     10 PINVSQ=1./(AODP*AODP*BETA02*BETA02)
51     TSI=1./(AODP-S4)
52     ETA=AODP*E0*TSI
53     ETASQ=ETA*ETA
54     EETA=E0*ETA
55     PSISQ=ABS(1.-ETASQ)
56     COEF=QOMS24*TSI**4
57     COEF1=COEF/PSISQ**3.5
```

```

57      C2=COEF1*XNODP*(AODP*(1.+1.5*ETASQ+EETA*(4.+ETASQ))+.75*
58          CK2*TSI/PSISQ*X3THM1*(8.+3.*ETASQ*(8.+FTASQ)))
59      C1=BSTAR*C2
60      SINIO=SIN(XINCL)
61      A3CVK2=-XJ3/CK2*AE**3
62      C3=COEF*TSI*A30VK2*XNODP*AE*SINIO/EO
63      X1MTH2=1.-THETA2
64      C4=2.*XNODP*COEF1*AODP*BETA02*(ETA*
65          (2.+5*ETASQ)+EO*(.5+2.*ETASQ)-2.*CK2*TSI/
66          (AODP*PSISQ)*(-3.*X3THM1*(1.-2.*EETA+ETASQ*
67          (1.5-.5*EETA))+.75*X1MTH2*(2.*ETASQ-EETA*
68          (1.+ETASQ)))*COS(2.*OMEGAO)))
69      C5=2.*COEF1*AODP*BETA02*(1.+2.75*(ETASQ+EETA)+EFTA*ETASQ)
70      THETA4=THETA2*THETA2
71      TEMP1=3.*CK2*PINVSQ*XNODP
72      TEMP2=TEMP1*CK2*PINVSQ
73      TEMP3=1.25*CK4*PINVSQ*PINVSQ*XNODP
74      XMDOT=XNODP+.5*TEMP1*BETA0*X3THM1+.0625*TEMP2*BFTAO*
75          (13.-78.*THETA2+137.*THETA4)
76      X1M5TH=1.-5.*THETA2
77      OMGDOT=-.5*TEMP1*X1M5TH+.0625*TEMP2*(7.-114.*THFTA2+
78          395.*THETA4)+TEMP3*(3.-36.*THETA2+49.*THETA4)
79      XHDOT1=-TEMP1*COSIO
80      XNODOT=XHDOT1+(.5*TFMP2*(4.-19.*THETA2)+2.*TEMP3*(3.-
81          7.*THETA2))*COSIO
82      OMGCDF=BSTAR*C3*COS(OMEGAO)
83      XMCOF=-TOTHRD*COEF*RSTAR*AE/EETA
84      XNODCF=3.5*BETA02*XHDOT1*C1
85      T2COF=1.5*C1
86      XLCDF=.125*A30VK2*SINIO*(3.+5.*COSIO)/(1.+COSIO)
87      AYCOF=.25*A30VK2*SINIO
88      DELMO=(1.+ETA*COS(XMO))**3
89      SINMO=SIN(XMO)
90      X7THM1=7.*THETA2-1.
91      IF(ISIMP.EQ.1) GO TO 90
92      C1SQ=C1*C1
93      D2=4.*AODP*TSI*C1SQ
94      TEMP=D2*TSI*C1/3.
95      D3=(17.*AODP+S4)*TEMP
96      D4=.5*TEMP*AODP*TSI*(221.*AODP+31.*S4)*C1
97      T3COF=D2+2.*C1SQ
98      T4COF=.25*(3.*D3+C1*(12.*D2+10.*C1SQ))
99      T5COF=.2*(3.*D4+12.*C1*D3+6.*D2*D2+15.*C1SQ*(
100          2.*D2+C1SQ))
101     90 IFLAG=0
102
103 *      UPDATE FOR SECULAR GRAVITY AND ATMOSPHERIC DRAG
104
105 100 XMDF=XMO+XMDOT*TSINCE
106      OMGADF=OMEGAO+OMGDOT*TSINCE
107      XNODEDF=XNODE0+XNODOT*TSINCE
108      OMEGA=OMGADF
109      XMP=XMDF
110      TSQ=TSINCE*TSINCE
111      XNODE=XNODEDF+XNODCF*TSQ
112      TEMPA=1.-C1*TSINCE

```

```

113      TEMPE=BSTAR*C4*TSINCE
114      TEMPL=T2COF*TSQ
115      IF(ISIMP.EQ.1) GO TO 110
116      DELOMG=OMGCOF*TSINCE
117      DELM=XMCOF*((1.+ETA*COS(XMDF))**3-DELM0)
118      TEMP=DELOMG+DELM
119      XMP=XMDF+TEMP
120      OMEGA=OMGADF-TEMP
121      TCUBE=TSQ*TSINCE
122      TFOUR=TSINCE*TCUBE
123      TEMPA=TEMPA-D2*TSQ-D3*TCUBE-D4*TFOUR
124      TEMPE=TEMPE+BSTAR*C5*(SIN(XMP)-SIN10)
125      TEMPL=TEMPL+T3COF*TCUBE+
126      1   -TFOUR*(T4COF+TSINCE*T5COF)
127      110 A=AODP*TEMPA**2
128      E=E0-TEMPE
129      XL=XMP+OMEGA+XNODE+XNODP*TEMPL
130      BETA=SQRT(1.-E*E)
131      XN=XKE/A**1.5
132
133      *      LONG PERIOD PERIODICS
134
135      AXN=E*COS(OMEGA)
136      TEMP=1./(A*BETA*BETA)
137      XLL=TEMP*XLCOF*AXN
138      AYNL=TEMP*AYCOF
139      XLT=XL+XLL
140      AYN=E*SIN(OMEGA)+AYNL
141
142      *      SOLVE KEPLERS EQUATION
143
144      CAPU=FMOD2P(XLT-XNODE)
145      TEMP2=CAPU
146      DO 130 I=1,10
147      SINEPW=SIN(TEMP2)
148      COSEPW=COS(TEMP2)
149      TEMP3=AXN*SINEPW
150      TEMP4=AYN*COSEPW
151      TEMP5=AXN*COSEPW
152      TEMP6=AYN*SINEPW
153      EPW=(CAPU-TEMP4+TEMP3-TEMP2)/(1.-TEMPS-TEMP6)+TEMP2
154      IF(ABS(EPW-TEMP2).LE.E6A) GO TO 140
155      130 TEMP2=EPW
156
157      *      SHORT PERIOD PRELIMINARY QUANTITIES
158
159      140 ECOS=TEMPS+TEMP6
160      ESINE=TEMP3-TEMP4
161      ELSQ=AXN*AXN+AYN*AYN
162      TEMP=1.-ELSQ
163      PL=A*TEMP
164      R=A*(1.-ECOS)
165      TEMP1=1./R
166      RDOT=XKE*SQRT(A)*ESINE*TEMP1
167      RFDOT=XKE*SQRT(PL)*TEMP1
168      TEMP2=A*TEMP1

```

```

69      BETAL=SQRT(TEMP)
70      TEMP3=1./ (1.+BETAL)
71      COSU=TEMP2*(COSEPN-AXN+AYN*ESINE*TEMP3)
72      SINU=TEMP2*(SINEPN-AYN-AXN*ESINE*TEMP3)
73      U=ACTAN(SINU,COSU)
74      SIN2U=2.*SINU*COSU
75      COS2U=2.*COSU*COSU-1.
76      TEMP=1./PL
77      TEMP1=CK2*TEMP
78      TEMP2=TEMP1*TEMP
79
80      *      UPDATE FOR SHORT PERIODICS
81
82      RK=R*(1.-1.5*TEMP2*BETAL*X3THM1)+.5*TEMP1*X1MTH2*COS2U
83      UK=U-.25*TEMP2*X7THM1*SIN2U
84      XNODEK=XNODE+1.5*TEMP2*COSIO*SIN2U
85      XINCK=XINCL+1.5*TEMP2*COSIO*SINIO*COS2U
86      RDOTK=RDOT-XN*TEMP1*X1MTH2*SIN2U
87      RFDOIK=RFDOT+XN*TEMP1*(X1MTH2*COS2U+1.5*X3THM1)
88
89      *      ORIENTATION VECTORS
90
91      SINUK=SIN(UK)
92      COSUK=COS(UK)
93      SINIK=SIN(XINCK)
94      COSIK=COS(XINCK)
95      SINNOK=SIN(XNODEK)
96      COSNOK=COS(XNODEK)
97      XMX=-SINNOK*COSIK
98      XMY=COSNOK*COSIK
99      UX=XMX*SINUK+COSNOK*COSUK
200     UY=XMY*SINUK+SINNOK*COSUK
201     UZ=SINIK*SINUK
202     VX=XMX*COSUK-COSNOK*SINUK
203     VY=XMY*COSUK-SINNOK*SINUK
204     VZ=SINIK*COSUK
205
206      *      POSITION AND VELOCITY
207
208      X=RK*UX
209      Y=RK*UY
210      Z=RK*UZ
211      XDOT=RDOTK*UX+RFDOIK*VX
212      YDOT=RDOTK*UY+RFDOIK*VY
213      ZDOT=RDOTK*UZ+RFDOIK*VZ
214
215      RETURN
216      END

```

7. THE SDP4 MODEL

The NORAD mean element sets can be used for prediction with SDP4. All symbols not defined below are defined in the list of symbols in section twelve. The original mean motion (n_o'') and semimajor axis (a_o'') are first recovered from the input elements by the equations

$$a_1 = \left(\frac{k_e}{n_o} \right)^{2/3}$$

$$\delta_1 = \frac{3}{2} \frac{k_2}{a_1^2} \frac{(3 \cos^2 i_o - 1)}{(1 - e_o^2)^{3/2}}$$

$$a_o = a_1 \left(1 - \frac{1}{3} \delta_1 - \delta_1^2 - \frac{134}{81} \delta_1^3 \right)$$

$$\delta_o = \frac{3}{2} \frac{k_2}{a_o^2} \frac{(3 \cos^2 i_o - 1)}{(1 - e_o^2)^{3/2}}$$

$$n_o'' = \frac{n_o}{1 + \delta_o}$$

$$a_o'' = \frac{a_o}{1 - \delta_o} .$$

For perigee between 98 kilometers and 156 kilometers, the value of the constant s used in SDP4 is changed to

$$s^* = a_o'' (1 - e_o) - s + a_E .$$

For perigee below 98 kilometers, the value of s is changed to
 $s^* = 20/XKMPER + a_E .$

If the value of s is changed, then the value of $(q_o - s)^4$ must be replaced by

$$(q_o - s^*)^4 = \left[[(q_o - s)^4]^{1/4} + s - s^* \right]^4.$$

Then calculate the constants (using the appropriate values of s and $(q_o - s)^4$)

$$\theta = \cos i_o$$

$$\xi = \frac{1}{a''_o - s}$$

$$\beta_o = (1 - e_o^2)^{1/2}$$

$$\eta = a''_o e_o \xi$$

$$C_2 = (q_o - s)^4 \xi^4 n''_o (1 - \eta^2)^{-7/2} \left[a''_o \left(1 + \frac{3}{2} \eta^2 + 4e_o \eta \right. \right. \\ \left. \left. + e_o \eta^3 \right) + \frac{3}{2} \frac{k_2 \xi}{(1 - \eta^2)} \left(-\frac{1}{2} + \frac{3}{2} \theta^2 \right) (8 + 24\eta^2 + 3\eta^4) \right]$$

$$C_1 = B^* C_2$$

$$C_4 = 2n''_o (q_o - s)^4 \xi^4 a''_o \beta_o^2 (1 - \eta^2)^{-7/2} \left([2\eta (1 + e_o \eta) \right. \\ \left. + \frac{1}{2} e_o + \frac{1}{2} \eta^3] - \frac{2k_2 \xi}{a''_o (1 - \eta^2)} [3 (1 - 3\theta^2) (1 + \frac{3}{2} \eta^2 \right. \\ \left. - 2e_o \eta - \frac{1}{2} e_o \eta^3) + \frac{3}{4} (1 - \theta^2) (2\eta^2 - e_o \eta - e_o \eta^3) \cos 2\omega_o] \right)$$

$$M = \left[1 + \frac{3k_2 (-1 + 3\theta^2)}{2a''_o^2 \beta_o^3} + \frac{3k_2^2 (13 - 78\theta^2 + 137\theta^4)}{16a''_o^4 \beta_o^7} \right] n''_o$$

$$\omega = \left[-\frac{3k_2(1 - 5\theta^2)}{2a''_o^2 \beta_o^4} + \frac{3k_2^2(7 - 114\theta^2 + 395\theta^4)}{16a''_o^4 \beta_o^8} \right.$$

$$\left. + \frac{5k_4(3 - 36\theta^2 + 49\theta^4)}{4a''_o^4 \beta_o^4} \right] n''_o$$

$$\Omega_1 = -\frac{3k_2\theta}{a''_o^2 \beta_o^4} n''_o$$

$$\Omega = \Omega_1 + \left[\frac{3k_2^2(4\theta - 19\theta^3)}{2a''_o^4 \beta_o^8} + \frac{5k_4\theta(3 - 7\theta^2)}{2a''_o^4 \beta_o^8} \right] n''_o$$

At this point SDP4 calls the initialization section of DEEP which calculates all initialized quantities needed for the deep-space perturbations (see section ten).

The secular effects of gravity are included by

$$M_{DF} = M_o + \dot{M}(t - t_o)$$

$$\omega_{DF} = \omega_o + \dot{\omega}(t - t_o)$$

$$\Omega_{DF} = \Omega_o + \dot{\Omega}(t - t_o)$$

where $(t - t_o)$ is time since epoch. The secular effect of drag on longitude of ascending node is included by

$$\Omega = \Omega_{DF} - \frac{21}{2} \frac{n''_o k_2^2 \theta}{a''_o^2 \beta_o^2} c_1 (t - t_o)^2.$$

Next, SDP4 calls the secular section of DEEP which adds the deep-space secular effects and long-period resonance effects to

the six classical orbital elements (see section ten).

The secular effects of drag are included in the remaining elements by

$$a = a_{DS} [1 - C_1 (t - t_o)]^2$$

$$e = e_{DS} - B*C_4 (t - t_o)$$

$$L = M_{DS} + \omega_{DS} + \Omega_{DS} + n''_o [\frac{3}{2} C_1 (t - t_o)^2]$$

where a_{DS} , e_{DS} , M_{DS} , ω_{DS} , and Ω_{DS} , are the values of n_o , e_o , M_{DF} , ω_{DF} , and Ω after deep-space secular and resonance perturbations have been applied.

Here SDP4 calls the periodics section of DEEP which adds the deep-space lunar and solar periodics to the orbital elements (see section ten). From this point on, it will be assumed that n , e , I , ω , Ω , and M are the mean motion, eccentricity, inclination, argument of perigee, longitude of ascending node, and mean anomaly after lunar-solar periodics have been added.

Add the long-period periodic terms

$$a_{xN} = e \cos \omega$$

$$\beta = \sqrt{(1 - e^2)}$$

$$L_L = \frac{A_{3,0} \sin i_o}{8k_2 a \beta^2} (e \cos \omega) (\frac{3 + 5\theta}{1 + \theta})$$

$$a_{yNL} = \frac{A_{3,0} \sin i_o}{4k_2 a \beta^2}$$

$$\mathbf{L}_T = \mathbf{L} + \mathbf{L}_L$$

$$a_{yN} = e \sin \omega + a_{yNL} .$$

Solve Kepler's equation for $(E + \omega)$ by defining

$$U = \mathbf{L}_T - \Omega$$

and using the iteration equation

$$(E + \omega)_{i+1} = (E + \omega)_i + \Delta(E + \omega)_i$$

with

$$\Delta(E + \omega)_i = \frac{U - a_{yN} \cos(E + \omega)_i + a_{xN} \sin(E + \omega)_i - (E + \omega)_i}{-a_{yN} \sin(E + \omega)_i - a_{xN} \cos(E + \omega)_i + 1}$$

and

$$(E + \omega)_1 = U.$$

The following equations are used to calculate preliminary quantities needed for the short-period periodics.

$$e \cos E = a_{xN} \cos(E + \omega) + a_{yN} \sin(E + \omega)$$

$$e \sin E = a_{xN} \sin(E + \omega) - a_{yN} \cos(E + \omega)$$

$$e_L = (a_{xN}^2 + a_{yN}^2)^{1/2}$$

$$p_L = a(1 - e_L^2)$$

$$r = a(1 - e \cos E)$$

$$\dot{r} = k_e \sqrt{\frac{a}{r}} e \sin E$$

$$\dot{rf} = k_e \sqrt{\frac{p_L}{r}}$$

$$\cos u = \frac{a}{r} [\cos(E + \omega) - a_{xN} + \frac{a_{yN} (e \sin E)}{\sqrt{1 - e_L^2}}]$$

$$\sin u = \frac{a}{r} [\sin(E + \omega) - a_{yN} - \frac{a_{xN} (e \sin E)}{\sqrt{1 - e_L^2}}]$$

$$u = \tan^{-1} \left(\frac{\sin u}{\cos u} \right)$$

$$\Delta r = \frac{k_2}{2p_L} (1 - \theta^2) \cos 2u$$

$$\Delta u = - \frac{k_2}{4p_L^2} (7\theta^2 - 1) \sin 2u$$

$$\Delta \Omega = \frac{3k_2 \theta}{2p_L^2} \sin 2u$$

$$\Delta i = \frac{3k_2 \theta}{2p_L^2} \sin i_o \cos 2u$$

$$\Delta \dot{r} = - \frac{k_2 n}{p_L} (1 - \theta^2) \sin 2u$$

$$\Delta \dot{rf} = \frac{k_2 n}{p_L} [(1 - \theta^2) \cos 2u - \frac{3}{2} (1 - 3\theta^2)]$$

The short-period periodics are added to give the osculating quantities

$$r_k = r [1 - \frac{3}{2} k_2 \frac{\sqrt{1 - e_L^2}}{p_L^2} (3\theta^2 - 1)] + \Delta r$$

$$u_k = u + \Delta u$$

$$\Omega_k = \Omega + \Delta\Omega$$

$$i_k = I + \Delta i$$

$$\dot{r}_k = \dot{r} + \Delta \dot{r}$$

$$\dot{rf}_k = \dot{rf} + \Delta \dot{rf}.$$

Then unit orientation vectors are calculated by

$$\underline{U} = \underline{M} \sin u_k + \underline{N} \cos u_k$$

$$\underline{V} = \underline{M} \cos u_k - \underline{N} \sin u_k$$

where

$$\underline{M} = \left\{ \begin{array}{l} M_x = -\sin \Omega_k \cos i_k \\ M_y = \cos \Omega_k \cos i_k \\ M_z = \sin i_k \end{array} \right\}$$

$$\underline{N} = \left\{ \begin{array}{l} N_x = \cos \Omega_k \\ N_y = \sin \Omega_k \\ N_z = 0 \end{array} \right\} .$$

Position and velocity are given by

$$\underline{r} = \underline{r}_k \underline{U}$$

and

$$\dot{\underline{r}} = \dot{\underline{r}}_k \underline{U} + (\underline{r} \underline{f})_k \underline{V}.$$

A FORTRAN IV computer code listing of the subroutine SDP4 is given below. These equations contain all currently anticipated changes to the SCC operational program. These changes are scheduled for implementation in March, 1981.

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```

1      *      SDP4
2      SUBROUTINE SDP4(IFLAG,TSINCE)
3      COMMON/E1/XMO,XNODEO,OMEGA0,E0,XINCL,XNO,XNDT20,
4      1           XNDD60,BSTAR,X,Y,Z,XDOT,YDOT,ZDOT,EPOCH,DS50
5      COMMON/C1/CK2,CK4,E6A,QOMS2T,S,TOTHRD,
6      1           XJ3,XKE,XKMPER,XMNPDA,AE
7      DOUBLE PRECISION EPOCH,DS50
8
9      IF (IFLAG .EQ. 0) GO TO 100
10
11     *      RECOVER ORIGINAL MEAN MOTION (XNODP) AND SEMIMAJOR AXIS (AODP)
12     *      FROM INPUT ELEMENTS
13
14     A1=(XKE/XNO)**TOTHRD
15     COSIO=COS(XINCL)
16     THETA2=COSIO*COSIO
17     X3THM1=3.*THETA2-1.
18     EOSQ=E0*E0
19     BETA02=1.-EOSQ
20     BETA0=SQRT(BETA02)
21     DEL1=1.5*CK2*X3THM1/(A1*A1*BETA0*BETA02)
22     AO=A1*(1.-DEL1*(.5*TOTHRD+DEL1*(1.+134./81.*DEL1)))
23     DELO=1.5*CK2*X3THM1/(AO*AO*BETA0*BETA02)
24     XNODP=XNO/(1.+DELO)
25     AODP=AO/(1.-DELO)
26
27     *      INITIALIZATION
28
29     *      FOR PERIGEE BELOW 156 KM, THE VALUES OF
30     *      S AND QOMS2T ARE ALTERED
31
32     S4=S
33     QOMS24=QOMS2T
34     PERIGE=(AODP*(1.-E0)-AE)*XKMPER
35     IF(PERIGE .GE. 156.) GO TO 10
36     S4=PERIGE-78.
37     IF(PERIGE .GT. 98.) GO TO 9
38     S4=20.
39     9 QOMS24=((120.-S4)*AE/XKMPER)**4
40     S4=S4/XKMPER+AE
41     10 PINVSQ=1./(AODP*AODP*BETA02*BETA02)
42     SING=SIN(OMEGA0)
43     COSG=COS(OMEGA0)
44     TSI=1./(AODP-S4)
45     ETA=AODP*E0*TSI
46     ETASQ=ETA*ETA
47     EETA=E0*ETA
48     PSISQ=ABS(1.-ETASQ)
49     COEF=QOMS24*TSI**4
50     COEFT=COEF/PSISQ**3.5
51     C2=COEFT1*XNODP*(AODP*(1.+1.5*ETASQ+EETA*(4.+ETASQ))+.75*
52     1           CK2*TSI/PSISQ*X3THM1*(8.+3.*ETASQ*(8.+ETASQ)))
53     C1=BSTAR*C2
54     SINIO=SIN(XINCL)
55     A30VK2=-XJ3/CK2*AE**3
56     XTMTH2=1.-THETA2

```

```

57      C4=2.*XNODP*COEF1*AODP*BETA02*(ETA*
58      1     (2.+.5*ETASQ)+E0*(.5+2.*ETASQ)-2.*CK2*TSI/
59      2     (AODP*PSISQ)*(-3.*X3THM1*(1.-2.*EETA+ETASQ*
60      3     (1.5-.5*EETA))+.75*X1MTH2*(2.*ETASQ-EETA*
61      4     (1.+ETASQ))*COS(2.*OMEGA0)))
62      THETA4=THETA2*THETA2
63      TEMP1=3.*CK2*PINVSQ*XNODP
64      TEMP2=TEMP1*CK2*PINVSQ
65      TEMP3=1.25*CK4*PINVSQ*PINVSQ*XNODP
66      XMDOT=XNODP+.5*TEMP1*BETA0*X3THM1+.0625*TEMP2*BETA0*
67      1     (13.-78.*THETA2+137.*THETA4)
68      X1M5TH=1.-5.*THETA2
69      OMGDOT=-.5*TEMP1*X1M5TH+.0625*TEMP2*(7.-114.*THETA2+
70      1     395.*THETA4)+TEMP3*(3.-36.*THETA2+49.*THETA4)
71      XHDOT1=-TEMP1*COSIO
72      XNODOT=XHDOT1+(.5*TEMP2*(4.-19.*THETA2)+2.*TEMP3*(3.-
73      1     7.*THETA2))*COSIO
74      XNODCF=3.5*BETA02*XHDOT1*C1*
75      T2COF=1.5*C1
76      XLCOF=.125*A30VK2*SINIO*(3.+5.*COSIO)/(1.+COSIO)
77      AYCOF=.25*A30VK2*SINIO
78      X7THM1=7.*THETA2-1.
79      90 IFLAG=0
80      CALL DPINIT(EOSQ,SINIO,COSIO,BETA0,AODP,THETA2,
81      1     SING,COSG,BETA02,XMDOT,OMGDOT,XNODOT,XNODP)
82
83      *      UPDATE FOR SECULAR GRAVITY AND ATMOSPHERIC DRAG
84
85      100 XMDF=XMO+XMDOT*TSINCE
86      OMGADE=OMEGA0+OMGDOT*TSINCE
87      XNODEDF=XNODE0+XNODOT*TSINCE
88      TSQ=TSINCE*TSINCE
89      XNODE=XNODEDF+XNODCF*TSQ
90      TEMPA=1.-C1*TSINCE
91      TEMPE=BSTAR*C4*TSINCE
92      TEMPL=T2COF*TSQ
93      XN=XNODP
94      CALL DPSEC(XMDF,OMGADE,XNODE,EM,XINC,XN,TSINCE)
95      A=(XKE/XN)**TOTHRD*TEMPE**2
96      E=EM-TEMPE
97      XMAM=XMDF+XNODP*TEMPL
98      CALL DPPER(E,XINC,OMGADE,XNODE,XMAM)
99      XL=XMAM+OMGADE+XNODE
100     BETA=SQRT(1.-E**2)
101     XN=XKE/A**1.5
102
103     *      LONG PERIOD PERIODICS
104
105     AXN=E*COS(OMGADE)
106     TEMP=1./ (A*BETA*BETA)
107     XLL=TEMP*XLCOF*AXN
108     AYNL=TEMP*AYCOF
109     XLT=XL+XLL
110     AYN=E*SIN(OMGADE)+AYNL
111
112     *      SOLVE KEPLERS EQUATION

```

```

113
114      CAPU=FMOD2P(XLT-XNODE)
115      TEMP2=CAPU
116      DO 130 I=1,10
117      SINEPW=SIN(TEMP2)
118      COSEPW=COS(TEMP2)
119      TEMP3=AXN*SINEPW
120      TEMP4=AYN*COSEPW
121      TEMP5=AXN*COSEPW
122      TEMP6=AYN*SINEPW
123      EPW=(CAPU-TEMP4+TEMP3-TEMP2)/(1.-TEMP5-TEMP6)+TEMP2
124      IF(ABS(EPW-TEMP2) .LE. E6A) GO TO 140
125      130 TEMP2=EPW

```

126
127 * SHORT PERIOD PRELIMINARY QUANTITIES

```

128
129      140 ECOS=TEMP5+TEMP6
130      ESINE=TEMP3-TEMP4
131      ELSQ=AXN*AXN+AYN*AYN
132      TEMP=1.-ELSQ
133      PL=A*TEMP
134      R=A*(1.-ECOS)
135      TEMP1=1./R
136      RDOT=XKE*SQRT(A)*ESINE*TEMP1
137      RFdot=XKE*SQRT(PL)*TEMP1
138      TEMP2=A*TEMP1
139      BETAL=SQRT(TEMP)
140      TEMP3=1./(1.+BETAL)
141      COSU=TEMP2*(COSEPW-AXN+AYN*ESINE*TEMP3)
142      SINU=TEMP2*(SINEPW-AYN-AXN*ESINE*TEMP3)
143      U=ACTAN(SINU,COSU)
144      SIN2U=2.*SINU*COSU
145      COS2U=2.*COSU*COSU-1.
146      TEMP=1./PL
147      TEMP1=CK2*TEMP
148      TEMP2=TEMP1*TEMP
149

```

150 * UPDATE FOR SHORT PERIODICS

```

151
152      RK=R*(1.-1.5*TEMP2*BETAL*X3THM1)+.5*TEMP1*X1MTH2*COS2U
153      UK=U-.25*TEMP2*X7THM1*SIN2U
154      XNODEK=XNODE+1.5*TEMP2*COSIO*SIN2U
155      XINC=XINC+1.5*TEMP2*COSIO*SINIO*COS2U
156      RDOTK=RDOT-XN*TEMPT*XTMTH2*SIN2U
157      RFdotK=RFdot+XN*TEMP1*(X1MTH2*COS2U+1.5*X3THM1)
158

```

159 * ORIENTATION VECTORS

```

160
161      SINUK=SIN(UK)
162      COSUK=COS(UK)
163      SINIK=SIN(XINC)
164      COSIK=COS(XINC)
165      SINNOK=SIN(XNODEK)
166      COSNOK=COS(XNODEK)
167      XMX=-SINNOK*COSIK
168      XMY=COSNOK*COSIK

```

169 UX=XMX★SINUUK+COSNOK★COSUK
170 UY=XMY★SINUUK+SINNOK★COSUK
171 UZ=SINIK★SINUUK
172 VX=XMX★COSUK-COSNOK★SINUUK
173 VY=XMY★COSUK-SINNOK★SINUUK
174 VZ=SINIK★COSUK
175

176 * POSITION AND VELOCITY
177

178 X=RK★UX
179 Y=RK★UY
180 Z=RK★UZ
181 XDOT=RDOTK★UX+RF DOTK★VX
182 YDOT=RDOTK★UY+RF DOTK★VY
183 ZDOT=RDOTK★UZ+RF DOTK★VZ
184
185 RETURN
186 END

8. THE SGP8 MODEL

The NORAD mean element sets can be used for prediction with SGP8. All symbols not defined below are defined in the list of symbols in section twelve. The original mean motion (n''_o) and semimajor axis (a''_o) are first recovered from the input elements by the equations

$$a_1 = \left(\frac{k_e}{n_o}\right)^{2/3}$$

$$\delta_1 = \frac{3}{2} \frac{k_2}{a_1^2} \frac{(3 \cos^2 i_o - 1)}{(1 - e_o^2)^{3/2}}$$

$$a_o = a_1 \left(1 - \frac{1}{3} \delta_1 - \delta_1^2 - \frac{134}{81} \delta_1^3\right)$$

$$\delta_o = \frac{3}{2} \frac{k_2}{a_o^2} \frac{(3 \cos^2 i_o - 1)}{(1 - e_o^2)^{3/2}}$$

$$n''_o = \frac{n_o}{1 + \delta_o}$$

$$a''_o = \frac{a_o}{1 - \delta_o}$$

The ballistic coefficient (B term) is then calculated from the B* drag term by

$$B = 2B*/\rho_o$$

where

$$\rho_o = (2.461 \times 10^{-5}) \text{ XKMPER kg/m}^2 / \text{Earth radii}$$

is a reference value of atmospheric density.

Then calculate the constants

$$\beta^2 = 1 - e^2$$

$$\theta = \cos i$$

$$\dot{M}_1 = -\frac{3}{2} \frac{n''k_2}{a''^2 \beta^3} (1 - 3\theta^2)$$

$$\dot{\omega}_1 = -\frac{3}{2} \frac{n''k_2}{a''^2 \beta^4} (1 - 5\theta^2)$$

$$\dot{\Omega}_1 = -3 \frac{n''k_2}{a''^2 \beta^4} \theta$$

$$\dot{M}_2 = \frac{3}{16} \frac{n''k_2^2}{a''^4 \beta^7} (13 - 78\theta^2 + 137\theta^4)$$

$$\dot{\omega}_2 = \frac{3}{16} \frac{n''k_2^2}{a''^4 \beta^8} (7 - 114\theta^2 + 395\theta^4)$$

$$+ \frac{5}{4} \frac{n''k_4}{a''^4 \beta^8} (3 - 36\theta^2 + 49\theta^4)$$

$$\dot{\Omega}_2 = \frac{3}{2} \frac{n''k_2^2}{a''^4 \beta^8} \theta (4 - 19\theta^2)$$

$$+ \frac{5}{2} \frac{n''k_4}{a''^4 \beta^8} \theta (3 - 7\theta^2)$$

$$\dot{\lambda} = n'' + \dot{M}_1 + \dot{M}_2$$

$$\dot{\omega} = \dot{\omega}_1 + \dot{\omega}_2$$

$$\dot{\Omega} = \dot{\Omega}_1 + \dot{\Omega}_2$$

$$\xi = \frac{1}{a''\beta^2 - s}$$

$$\eta = es\xi$$

$$\psi = \sqrt{1-\eta^2}$$

$$\alpha^2 = 1 + e^2$$

$$C_0 = \frac{1}{2}B \rho_0 (q_0 + s)^4 n'' a'' \xi^4 \alpha^{-1} \psi^{-7}$$

$$C_1 = \frac{3}{2} n'' \alpha^4 C_0$$

$$D_1 = \xi \psi^{-2} / a'' \beta^2$$

$$D_2 = 12 + 36\eta^2 + \frac{9}{2}\eta^4$$

$$D_3 = 15\eta^2 + \frac{5}{2}\eta^4$$

$$D_4 = 5\eta + \frac{15}{4}\eta^3$$

$$D_5 = \xi \psi^{-2}$$

$$B_1 = -k_2 (1 - 3\theta^2)$$

$$B_2 = -k_2 (1 - \theta^2)$$

$$B_3 = \frac{A_{3,0}}{k_2} \sin i$$

$$C_2 = D_1 D_3 B_2$$

$$C_3 = D_4 D_5 B_3$$

$$\dot{n}_o = C_1 (2 + 3\eta^2 + 20e\eta + 5e\eta^3 + \frac{17}{2} e^2$$

$$+ 34e^2\eta^2 + D_1 D_2 B_1 + C_2 \cos 2\omega \\ + C_3 \sin \omega)$$

$$C_4 = D_1 D_7 B_2$$

$$C_5 = D_5 D_8 B_3$$

$$D_6 = 30\eta + \frac{45}{2}\eta^3$$

$$D_7 = 5\eta + \frac{25}{2}\eta^3$$

$$D_8 = 1 + \frac{27}{4}\eta^2 + \eta^4$$

$$\dot{e}_o = - C_o (4\eta + \eta^3 + 5e + 15e\eta^2 \\ + \frac{31}{2} e^2\eta + 7e^2\eta^3 + D_1 D_6 B_1 \\ + C_4 \cos 2\omega + C_5 \sin \omega)$$

$$\dot{\alpha}/\alpha = e \dot{e} \alpha^{-2}$$

$$C_6 = \frac{1}{3} \frac{\dot{n}}{n''}$$

$$\dot{\xi}/\xi = 2a''\xi (C_6 \beta^2 + ee)$$

$$\dot{\eta} = (\dot{e} + e \dot{\xi}/\xi) s\xi$$

$$\dot{\psi}/\psi = - \eta \eta \dot{\psi}^{-2}$$

$$\dot{C}_0/C_0 = C_6 + 4 \dot{\xi}/\xi - \dot{\alpha}/\alpha - 7 \dot{\psi}/\psi$$

$$\dot{C}_1/C_1 = \dot{n}/n'' + 4 \dot{\alpha}/\alpha + \dot{C}_0/C_0$$

$$D_9 = 6\eta + 20e + 15en^2 + 68e^2\eta$$

$$D_{10} = 20\eta + 5\eta^3 + 17e + 68en^2$$

$$D_{11} = 72\eta + 18\eta^3$$

$$D_{12} = 30\eta + 10\eta^3$$

$$D_{13} = 5 + \frac{45}{4}\eta^2$$

$$D_{14} = \dot{\xi}/\xi - 2 \dot{\psi}/\psi$$

$$D_{15} = 2 (C_6 + ee \beta^{-2})$$

$$\dot{D}_1 = D_1 (D_{14} + D_{15})$$

$$\dot{D}_2 = \eta D_{11}$$

$$\dot{D}_3 = \eta D_{12}$$

$$\dot{D}_4 = \eta D_{13}$$

$$D_5 = D_5 D_{14}$$

$$\dot{C}_2 = B_2 (D_1 D_3 + D_1 \dot{D}_3)$$

$$\dot{C}_3 = B_3 (D_5 D_4 + D_5 \dot{D}_4)$$

$$\ddot{\omega} = -\frac{3}{2} \frac{n'' k_2}{a''^2 \beta^4} (1 - 5\theta^2)$$

$$D_{16} = D_9 \dot{n} + D_{10} \dot{e} + B_1 (D_1 D_2 + D_1 \dot{D}_2)$$

$$+ C_2 \cos 2\omega + C_3 \sin \omega$$

$$+ \dot{\omega} (C_3 \cos \omega - 2 C_2 \sin 2\omega)$$

$$\ddot{n}_o = \dot{n} \dot{C}_1 / C_1 + C_1 D_{16}$$

$$\ddot{e}_o = \dot{e} \dot{C}_o / C_o - C_o \{ (4 + 3n^2 + 30en$$

$$+ \frac{31}{2} e^2 + 21e^2 n^2) \dot{n} + (5 + 15n^2$$

$$+ 31en + 14en^3) \dot{e} + B_1 [D_1 D_6$$

$$+ D_1 \dot{n} (30 + \frac{135}{2} n^2)] + B_2 [D_1 D_7$$

$$+ D_1 \dot{n} (5 + \frac{75}{2} n^2)] \cos 2\omega$$

$$+ B_3 [D_5 D_8 + D_5 n \dot{n} (\frac{27}{2}$$

$$+ 4\eta^2)] \sin \omega + \dot{\omega} (C_5 \cos \omega - 2 C_4 \sin 2\omega)$$

$$D_{17} = \ddot{n}/n'' - (\dot{n}/n'')^2$$

$$\ddot{\xi}/\xi = 2 (\dot{\xi}/\xi - C_6) \dot{\xi}/\xi + 2a''\xi (\frac{1}{3} D_{17}\beta^2 - 2 C_6 \dot{e}\dot{e} + \dot{e}^2 + \ddot{e}\ddot{e})$$

$$\ddot{\eta} = (\ddot{e} + 2\dot{e} \dot{\xi}/\xi) s\xi + \eta \ddot{\xi}/\xi$$

$$D_{18} = \ddot{\xi}/\xi - (\dot{\xi}/\xi)^2$$

$$D_{19} = - (\dot{\psi}/\psi)^2 (1 + \eta^{-2}) - \eta \ddot{\psi} \psi^{-2}$$

$$\ddot{D}_1 = \dot{D}_1 (D_{14} + D_{15}) + D_1 (D_{18} - 2D_{19})$$

$$+ \frac{2}{3} D_{17} + 2 \alpha^2 \dot{e}^2 \beta^{-4} + 2 \ddot{e}\ddot{e} \beta^{-2})$$

$$\ddot{n}_o = \dot{n} [\frac{4}{3} D_{17} + 3 \dot{e}^2 \alpha^{-2} + 3 \ddot{e}\ddot{e} \alpha^{-2}$$

$$- 6 (\dot{\alpha}/\alpha)^2 + 4 D_{18} - 7 D_{19}]$$

$$+ \ddot{n} \dot{c}_1/c_1 + c_1 \{ D_{16} \dot{c}_1/c_1$$

$$+ D_9 \ddot{\eta} + D_{10} \ddot{e} + \dot{\eta}^2 (6 + 30e\eta$$

$$+ 68e^2) + \dot{\eta}\dot{e} (40 + 30\eta^2 + 272e\eta)$$

$$\begin{aligned}
& + \dot{e}^2 (17 + 68 \dot{n}^2) + B_1 [\ddot{D}_1 D_2 \\
& + 2\dot{D}_1 \dot{D}_2 + D_1 (\ddot{n} D_{11} + \dot{n}^2 (72 \\
& + 54 \dot{n}^2))] + B_2 [\ddot{D}_1 D_3 + 2\dot{D}_1 \dot{D}_3 \\
& + D_1 (\ddot{n} D_{12} + \dot{n}^2 (30 \\
& + 30 \dot{n}^2))] \cos 2\omega + B_3 [(\dot{D}_5 D_{14} \\
& + D_5 (D_{18} - 2D_{19})) D_4 + 2\dot{D}_4 \dot{D}_5 \\
& + D_5 (\ddot{n} D_{13} + \frac{45}{2} n \dot{n}^2)] \sin \omega \\
& + \dot{\omega} [(7 C_6 + 4 e \dot{e} \beta^{-2}) (C_3 \cos \omega \\
& - 2 C_2 \sin 2\omega) + 2 C_3 \cos \omega \\
& - 4 C_2 \sin 2\omega - \dot{\omega} (C_3 \sin \omega \\
& + 4 C_2 \cos 2\omega)] \}
\end{aligned}$$

$$p = \frac{2\dot{n}_o^2 - \dot{n}_o \ddot{n}_o}{\dot{n}_o^2 - \dot{n}_o \ddot{n}_o}$$

$$\gamma = - \frac{\ddot{n}_o}{\dot{n}_o} \frac{1}{(p-2)}$$

$$n_D = \frac{\dot{n}_o}{p\gamma}$$

$$q = 1 - \frac{\dot{e}_o}{\dot{e}_o \gamma}$$

$$e_D = \frac{\dot{e}_o}{q\gamma}$$

where all quantities are epoch values.

The secular effects of atmospheric drag and gravitation are included by

$$n = n_o'' + n_D [1 - (1 - \gamma(t - t_o))^p]$$

$$e = e_o + e_D [1 - (1 - \gamma(t - t_o))^q]$$

$$\omega = \omega_o + \dot{\omega}_1 [(t - t_o) + \frac{7}{3} \frac{1}{n_o''} z_1] + \dot{\omega}_2 (t - t_o)$$

$$\Omega = \Omega_o'' + \dot{\Omega}_1 [(t - t_o) + \frac{7}{3} \frac{1}{n_o''} z_1] + \dot{\Omega}_2 (t - t_o)$$

$$M = M_o + n_o'' (t - t_o) + z_1 + \dot{M}_1 [(t - t_o) + \frac{7}{3} \frac{1}{n_o''} z_1] \\ + \dot{M}_2 (t - t_o)$$

where

$$z_1 = \frac{\dot{n}_o}{p\gamma} \{(t - t_o) + \frac{1}{\gamma(p+1)} [(1 - \gamma(t - t_o))^{p+1} - 1]\}$$

If drag is very small ($\frac{\dot{n}_o}{n_o''}$ less than $1.5 \times 10^{-6}/\text{min}$) then the secular equations for n, e , and z_1 should be replaced by

$$n = n_o'' + \dot{n} (t - t_o)$$

$$e = e_0'' + \dot{e} (t - t_0)$$

$$z_1 = \frac{1}{2} \dot{n}_0 (t - t_0)^2$$

where $(t - t_0)$ is time since epoch and where

$$\dot{e} = -\frac{2}{3} \frac{\dot{n}_0}{n_0''} (1 - e_0'').$$

Solve Kepler's equation for E by using the iteration equation

$$E_{i+1} = E_i + \Delta E_i$$

with

$$\Delta E_i = \frac{M + e \sin E_i - E_i}{1 - e \cos E_i}$$

and

$$E_1 = M + e \sin M + \frac{1}{2} e^2 \sin 2M.$$

The following equations are used to calculate preliminary quantities needed for the short-period periodics.

$$a = \left(\frac{k_e}{n}\right)^{2/3}$$

$$\beta = (1 - e^2)^{1/2}$$

$$\sin f = \frac{\beta \sin E}{1 - e \cos E}$$

$$\cos f = \frac{\cos E - e}{1 - e \cos E}$$

$$u = f + \omega$$

$$r'' = \frac{a\beta^2}{1 + e \cos f}$$

$$\dot{r}'' = \frac{n a e}{\beta} \sin f$$

$$(rf)'' = \frac{n a^2 \beta}{r}$$

$$\delta r = \frac{1}{2} \frac{k_2}{a\beta^2} [(1 - \theta^2) \cos 2u$$

$$+ 3(1 - 3\theta^2)] - \frac{1}{4} \frac{A_{3,0}}{k_2} \sin i_o \sin u$$

$$\delta \dot{r} = -n \left(\frac{a}{r}\right)^2 \left[\frac{k_2}{a\beta^2} (1 - \theta^2) \sin 2u + \frac{1}{4} \frac{A_{3,0}}{k_2} \sin i_o \cos u\right]$$

$$\delta I = \theta \left[\frac{3}{2} \frac{k_2}{a^2 \beta^4} \sin i_o \cos 2u - \frac{1}{4} \frac{A_{3,0}}{k_2 a \beta^2} e \sin \omega \right]$$

$$\delta (rf) = -n \left(\frac{a}{r}\right)^2 \delta r + na \left(\frac{a}{r}\right) \frac{\sin i_o}{\theta} \delta I$$

$$\delta u = \frac{1}{2} \frac{k_2}{a^2 \beta^4} \left[\frac{1}{2} (1 - 7\theta^2) \sin 2u - 3(1\right.$$

$$\left. - 5\theta^2) (f - M + e \sin f) \right]$$

$$- \frac{1}{4} \frac{A_{3,0}}{k_2 a \beta^2} [\sin i_o \cos u (2 + e \cos f)$$

$$+ \frac{1}{2} \frac{\theta^2}{\sin i_o / 2 \cos i_o / 2} e \cos \omega]$$

$$\delta \lambda = \frac{1}{2} \frac{k_2}{a^2 \beta^4} \left[\frac{1}{2} (1 + 6\theta - 7\theta^2) \sin 2u \right.$$

$$\left. - 3(1 + 2\theta - 5\theta^2) (f - M + e \sin f) \right]$$

$$+ \frac{1}{4} \frac{A_{3,0}}{k_2 a^\beta^2} \sin i_o [\frac{e\theta}{1+\theta} \cos \omega$$

$$- (2 + e \cos f) \cos u]$$

The short-period periodics are added to give the osculating quantities

$$r = r'' + \delta r$$

$$\dot{r} = \dot{r}'' + \delta \dot{r}$$

$$\ddot{rf} = (\dot{rf})'' + \delta(\dot{rf})$$

$$y_4 = \sin i_o/2 \sin u + \cos u \sin i_o/2 \delta u$$

$$+ \frac{1}{2} \sin u \cos i_o/2 \delta I$$

$$y_5 = \sin i_o/2 \cos u - \sin u \sin i_o/2 \delta u$$

$$+ \frac{1}{2} \cos u \cos i_o/2 \delta I$$

$$\lambda = u + \Omega + \delta \lambda .$$

Unit orientation vectors are calculated by

$$U_x = 2y_4 (y_5 \sin \lambda - y_4 \cos \lambda) + \cos \lambda$$

$$U_y = -2y_4 (y_5 \cos \lambda + y_4 \sin \lambda) + \sin \lambda$$

$$U_z = 2y_4 \cos I/2$$

$$V_x = 2y_5 (y_5 \sin \lambda - y_4 \cos \lambda) - \sin \lambda$$

$$V_y = -2y_5 (y_5 \cos \lambda + y_4 \sin \lambda) + \cos \lambda$$

$$V_z = 2y_5 \cos I/2$$

where

$$\cos I/2 = \sqrt{1 - y_4^2 - y_5^2}$$

Position and velocity are given by

$$\underline{r} = r \underline{U}$$

$$\dot{\underline{r}} = \dot{r} \underline{U} + r \dot{f} \underline{V}$$

A FORTRAN IV computer code listing of the subroutine SGP8 is given below.

1 * SGP8

14 NOV 80

2 SUBROUTINE SGP8(IFLAG,TSINCE)
 3 COMMON/E1/XMO,XNODE0,OMEGA0,E0,XINCL,XNO,XHDT20
 4 1 XNDD60,BSTAR,X,Y,Z,XDOT,YDOT,ZDOT,EPUCR,DS50
 5 COMMON/C1/CK2,CK4,E6A,T0THRD,S,T0THRD,
 6 1 XJ3,XKE,XKMPER,XMNPDA,AE
 7 DOUBLE PRECISION EPOCH,DS50
 8 DATA RHO/.15696615/

9 IF (IFLAG .EQ. 0) GO TO 100

10 * RECOVER ORIGINAL MEAN MOTION (XNODP) AND SEMIMAJOR AXIS (AODP)
 11 * FROM INPUT ELEMENTS ----- CALCULATE BALLISTIC COEFFICIENT
 12 * (B TERM) FROM INPUT B* DRAG TERM

13 A1=(XKE/XNO)**T0THRD

14 COSI=COS(XINCL)

15 THETA2=COSI*COSI

16 TTHMUN=3.*THETA2-1.

17 EOSQ=E0*E0

18 BETA02=1.-EOSQ

19 BETA0=SQRT(BETA02)

20 DEL1=1.5*CK2*TTHMUN/(A1*A1*BETA0*BETA02)

21 AO=A1*(1.-DEL1*(.5*T0THRD+DEL1*(1.+134./31.*DEL1)))

22 DEL0=1.5*CK2*TTHMUN/(AO*AO*BETA0*BETA02)

23 AODP=AO/(1.-DEL0)

24 XNODP=XNO/(1.+DEL0)

25 B=2.*BSTAR/RHO

26 * INITIALIZATION

27 ISIMP=0

28 P0=AODP*BETA02

29 P0M2=1./(P0*P0)

30 SINI=SIN(XINCL)

31 SING=SIN(OMEGA0)

32 COSG=COS(OMEGA0)

33 TEMP=.5*XINCL

34 SINI02=SIN(TEMP)

35 COSI02=COS(TEMP)

36 THETA4=THETA2**2

37 UNM5TH=1.-5.*THETA2

38 UNMTH2=1.-THETA2

39 A3COF=-XJ3/CK2*AE**3

40 PARDT1=3.*CK2*P0M2*XNODP

41 PARDT2=PARDT1*CK2*P0M2

42 PARDT4=1.25*CK4*P0M2*P0M2*XNODP

43 XMDT1=.5*PARDT1*BETA0*TTHMUN

44 XGDT1=-.5*PARDT1*UNM5TH

45 XHDT1=-PARDT1*COSI

46 XLLDOT=XNODP+XMDT1+

47 2 .0625*PARDT2*BETA0*(13.-73.*THETA2+137.*THETA4)

48 0MGDT=XGDT1+

49 1 .0625*PARDT2*(7.-114.*THETA2+395.*THETA4)+PARDT4*(3.-30.*

50 2 THETA2+49.*THETA4)

51 XNODOT=XHDT1+

```

57      1      (.5★PARDT2★(4.-19.★THETA2)+2.★PARDT4★(3.-7.★THETA2))★COSI
58      TSI=1./ (PO-S)
59      ETA=E0★S★TSI
60      ETA2=ETA**2
61      PSIM2=A3S(1./ (1.-ETA2))
62      ALPHA2=1.+EOSQ
63      EETA=E0★ETA
64      COS2G=2.★COSG**2-1.
65      D5=TSI★PSIM2
66      D1=D5/PO
67      D2=12.+ETA2★(36.+4.5★ETA2)
68      D3=ETA2★(15.+2.5★ETA2)
69      D4=EITA★(5.+3.75★ETA2)
70      B1=CK2★TTHMUN
71      B2=CK2★UNMTH2
72      B3=A3COF★SINI
73      C0=.5★B★RH0★Q0MS2I★XNODP★AODP★TSI**4★PSI-12★★3.5/SQRT(ALPHA2)
74      C1=1.5★XNODP★ALPHA2**2★C0
75      C4=D1★D3★B2
76      C5=D5★D4★B3
77      XNDT=C1★(
78      1  (2.+ETA2★(3.+34.★EOSQ)+5.★EETA★(4.+ETA2)+8.5★EOSQ) +
79      1  D1★D2★B1+ C4★COS2G+C5★SING)
80      XNDTN=XNDT/XNODP
81
82      *      IF DRAG IS VERY SMALL, THE ISIMP FLAG IS SET AND THE
83      *      EQUATIONS ARE TRUNCATED TO LINEAR VARIATION IN MEAN
84      *      MOTION AND QUADRATIC VARIATION IN MEAN ANOMALY
85
86      IF(ABS(XNDTN★XMNPDA) .LT. 2.16E-3) GO TO 50
87      D6=ETA★(30.+22.5★ETA2)
88      D7=ETA★(5.+12.5★ETA2)
89      D8=1.+ETA2★(6.75+ETA2)
90      C8=D1★D7★B2
91      C9=D5★D8★B3
92      EDOT=-C0★(
93      1  ETA★(4.+ETA2+EOSQ★(15.5+7.★ETA2))+E0★(5.+15.★ETA2)+_
94      1  D1★D6★B1 +
95      1  C8★COS2G+C9★SING)
96      D20=.5★TOTHRD★XNDTN
97      ALDTAL=E0★EDOT/ALPHA2
98      TSDDTS=2.★AODP★TSI★(D20★BETA02+E0★EDOT)
99      ETDT=(EDOT+E0★TSDDTS)★TSI★S
100     PSDTPS=-ETA★ETDT★PSIM2
101     SIN2G=2.★SING★COSG
102     CODTC0=D20+4.★TSDDTS-ALDTAL-7.★PSDTPS
103     C1DTC1=XNDTN+4.★ALDTAL+CODTC0
104     D9=ETA★(6.+68.★EOSQ)+E0★(20.+15.★ETA2)
105     D10=5.★ETA★(4.+ETA2)+E0★(17.+68.★ETA2)
106     D11=ETA★(72.+18.★ETA2)
107     D12=ETA★(30.+10.★ETA2)
108     D13=5.+11.25★ETA2
109     D14=TSDDTS-2.★PSDTPS
110     D15=2.★(D20+E0★EDOT/BETA02)
111     D1DT=D1★(D14+D15)
112     D2DT=ETDT★D11

```

```

113 D3DT=ETDT★D12
114 D4DT=ETDT★D13
115 D5DT=D5★D14
116 C4DT=B2★(D1DT★D3+D1★D3DT)
117 C5DT=B3★(D5DT★D4+D5★D4DT)
118 D16=
119   1 D9★ETDT+D10★EDOT +
120   1 B1★(D1DT★D2+D1★D2DT) +
121   1 C4DT★COS2G+C5DT★SING+XGDT1★(C5★COSG-2★C4★SIN2G)
122 XNDDT=C1DTC1★XNDT+C1★D16
123 EDDOT=C0DTC0★EDOT-C0★(
124   1 (4.+3.*ETA2+30.*EETA+E0SQ*(15.5+21.*ETA2))*ETDT+(5.+15.*ETA2
125   . +EETA*(31.+14.*ETA2))*EDOT +
126   1 B1★(D1DT★D6+D1★ETDT*(30.+67.5*ETA2)) +
127   1 B2★(D1DT★D7+D1★ETDT*(5.+37.5*ETA2))*COS2G+
128   1 B3★(D5DT★D8+D5★ETDT★ETA*(13.5+4.*ETA2))*SING+XGDT1★(C9★
129   . COSG-2★C8★SIN2G))
130 D25=EDOT★★2
131 D17=XNDDT/XNODP-XNDTN★★2
132 TSDDTS=2.*TSDDTS★(TSDDTS-D20)+AODP★TSI★(TOTHRD★BETA02★D17-4.*D20★
133   . E0★EDOT+2.*D25+E0★EDDOT))
134 ETDDT=(EDDOT+2.*EDOT★TSDDTS)*TSI★S+TSDDTS★ETA
135 D18=TSDDTS-TSDDTS★★2
136 D19=-PSDTPS★★2/ETA2-ETA★ETDDT★PSIM2-PSDTPS★★2
137 D23=ETDT★ETDT
138 D1DDT=D1DT★(D14+D15)+D1★(D13-2.*D19+TOTHRD★D17+2.*ALPHA2★D25
139   . /BETA02+E0★EDDOT)/BETA02)
140 XNTRDT=XNDT★(2.*TOTHRD★D17+3.*★
141   1 (D25+E0★EDDOT)/ALPHA2-6.*ALDTAL★★2 +
142   1 4.*D18-7.*D19 ) +
143   1 C1DTC1★XNDT+C1★(C1DTC1★D16+
144   1 D9★ETDDT+D10★EDDOT+D23★(6.+30.*EETA+68.*E0SQ) +
145   1 ETDT★EDOT★(40.+30.*★
146   . ETA2+272.*EETA)+D25★(17.+68.*ETA2) +
147   1 B1★(D1DDT★D2+2.*D1DT★D2DT+D1★(ETDDT★D11+D23★(72.+54.*ETA2))) +
148   1 B2★(D1DDT★D3+2.*D1DT★D3DT+D1★(ETDDT★D12+D23★(30.+30.*ETA2))) +
149   1 COS2G+
150   1 B3★((D5DT★D14+D5★(D18-2.*D19)) *
151   1 D4+2.*D4DT★D5DT+D5★(ETDDT★D13+22.5★ETA★D23)) *SING+XGDT1★
152   1 ((7.*D20+4.*E0★EDOT/BETA02)*
153   . (C5★COSG-2★C4★SIN2G))
154   . +((2.*C5DT★COSG-4.*C4DT★SIN2G)-XGDT1★(C5★SING+4.*C4★COS2G)))
155   .
156 TMNDDT=XNDDT★1.E9
157 TEMP=TMNDDT★★2-XNDT★1.E18★XNTRDT
158 PP=(TEMP+TMNDDT★★2)/TEMP
159 GAMMA=-XNTRDT/(XNDDT★(PP-2.))
160 XND=XNDT/(PP★GAMMA)
161 QQ=1.-EDDOT/(EDOT★GAMMA)
162 ED=EDOT/(QQ★GAMMA)
163 OVGPP=1./ (GAMMA*(PP+1.))
164 GO TO 70
165 50 ISIMP=1
166 EDDOT=-TOTHRD★XNDTN*(1.-E0)
167 70 IFLAG=0
168

```

169 * UPDATE FOR SECULAR GRAVITY AND ATMOSPHERIC DRAG

170.
 171 100 XMAM=FMOD2P(XMO+XLL DOT TSINCE)
 172 OMGASM=OMEGA0+OMGDT TSINCE
 173 XNODES=XNODEQ+XNODE DOT TSINCE
 174 IF(CISIMP.EQ.1) GO TO 105
 175 TEMP=1.-GAMMA TSINCE
 176 TEMP1=TEMP**PP
 177 XN=XNODP+XND*(1.-TEMP1)
 178 EM=EO+ED*(1.-TEMP**QQ)
 179 Z1=XND*(TSINCE+0VGPP*(TEMP*TEMP1-1.))
 180 GO TO 108
 181 105 XN=XNODP+XNDT TSINCE
 182 EM=EO+EDOT TSINCE
 183 Z1=.5*XNDT TSINCE TSINCE
 184 108 Z7=3.5*TOTH RD*Z1/XNODP
 185 XMAM=FMOD2P(XMAM+Z1+Z7*XMDT1)
 186 OMGASM=OMGASM+Z7*XGDT1
 187 XNODES=XNODES+Z7*XHDT1

188 * SOLVE KEPLERS EQUATION

189 190 ZC2=XMAM+EM*SIN(XMAM)*(1.+EM*COS(XMAM))

191 DO 130 I=1,10

192 SINE=SIN(ZC2)

193 COSE=COS(ZC2)

194 ZC5=1./(1.-EM*COSE)

195 CAPE=(XMAM+EM*SINE-ZC2)*

196 1 ZC5+ZC2

197 IF(ARS(CAPE-ZC2).LE.E6A) GO TO 140

198 130 ZC2=CAPE

200 * SHORT PERIOD PRELIMINARY QUANTITIES

201 202 140 AM=(XKE/XN)**TOTH RD

203 BETA2M=1.-EM*EM

204 SINOS=SIN(OMGASM)

205 COSOS=COS(OMGASM)

206 AXNM=EM*COSOS

207 AYNM=EM*SINOS

208 PM=AM*BETA2M

209 G1=1./PM

210 G2=.5*CK2*G1

211 G3=G2*G1

212 BETA=SQRT(BETA2M)

213 G4=.25*A3COF*SINI

214 G5=.25*A3COF*G1

215 SNF=BETA*SINE*ZCS

216 CSF=(COSE-EM)*ZCS

217 FM=ACTAN(SNF,CSF)

218 SNFG=SNF*COSOS-CSF*SINOS

219 CSFG=CSF*COSOS-SNF*SINOS

220 SN2F2G=2.*SNFG*CSFG

221 CS2F2G=2.*CSFG**2-1.

222 ECOSF=EM*CSF

223 G10=FM-XMAM+EM*SNF

```

225 RM=PM/(1.+ECOSE)
226 AOVr=AM/RM
227 G13=XN*AOVR
228 G14=-G13*AOVR
229 DR=G2*(UNMTH2*CS2F2G-3.*ITHMUN)-G4*SNFG
230 DIWC=3.*G3*SINI*CS2F2G-G5*AYNM
231 DI=DIWC*COSI
232
233 * UPDATE FOR SHORT PERIOD PERIODICS
234
235 SNI2DU=SINI02*
236 1 G3*(.5*(1.-7.*THETA2)*SN2F2G-3.*UNMTH*G10)-G5*SINI*CSFG*(2.+
237 2 ECOSF))-5*G5*THETA2*AXNM/COSI02
238 XLAMB=FM+OMGASM+XNODES+G3*(.5*(1.+6.*COSI-7.*THETA2)*SN2F2G-3.*+
239 1 (UNMTH+2.*COSI)*G10)+G5*SINI*(COSI*AXNM/(1.+COSI)-(2.-
240 2 +ECOSF)*CSFG)
241 Y4=SINI02*SNEG+CSFG*SNI2DU-.5*SNEG*COSI02*DI
242 Y5=SINI02*CSFG-SNFG*SNI2DU+.5*CSFG*COSI02*DI
243 R=RM+DR
244 RDOT=XN*AM*EM*SNF/BETA+G14*(2.*G2*UNMTH2*SN2F2G+G4*CSFG)
245 RVDOT=XN*AM**2*BETA/RM+
246 1 G14*DR+AM*G13*SINI*DIWC
247
248 * ORIENTATION VECTORS
249
250 SNLAMB=SIN(XLAMB)
251 CSLAMB=COS(XLAMB)
252 TEMP=2.* (Y5*SNLAMB-Y4*CSLAMB)
253 UX=Y4*TEMP+CSLAMB
254 VX=Y5*TEMP-SNLAMB
255 TEMP=2.* (Y5*CSLAMB+Y4*SNLAMB)
256 UY=-Y4*TEMP+SNLAMB
257 VY=-Y5*TEMP+CSLAMB
258 TEMP=2.*SQRT(1.-Y4*Y4-Y5*Y5)
259 UZ=Y4*TEMP
260 VZ=Y5*TEMP
261
262 * POSITION AND VELOCITY
263
264 X=R*UX
265 Y=R*UY
266 Z=R*UZ
267 XDOT=RDOT*UX+RVDOT*VX
268 YDOT=RDOT*UY+RVDOT*VY
269 ZDOT=RDOT*UZ+RVDOT*VZ
270
271 RETURN
272 END

```

9. THE SDP8 MODEL

The NORAD mean element sets can be used for prediction with SDP8. All symbols not defined below are defined in the list of symbols in section twelve. The original mean motion (n_o'') and semimajor axis (a_o'') are first recovered from the input elements by the equations

$$a_1 = \left(\frac{k_e}{n_o}\right)^{2/3}$$

$$\delta_1 = \frac{3}{2} \frac{k_2}{a_1^2} \frac{(3 \cos^2 i_o - 1)}{(1 - e_o^2)^{3/2}}$$

$$a_o = a_1 \left(1 - \frac{1}{3} \delta_1 - \delta_1^2 - \frac{134}{81} \delta_1^3\right)$$

$$\delta_o = \frac{3}{2} \frac{k_2}{a_o^2} \frac{(3 \cos^2 i_o - 1)}{(1 - e_o^2)^{3/2}}$$

$$n_o'' = \frac{n_o}{1 + \delta_o}$$

$$a_o'' = \frac{a_o}{1 - \delta_o}$$

The ballistic coefficient (B term) is then calculated from the B* drag term by

$$B = 2B^*/\rho_o$$

where

$$\rho_o = (2.461 \times 10^{-5}) \text{ XKMPER kg/m}^2 / \text{Earth radii}$$

is a reference value of atmospheric density.

Then calculate the constants

$$\beta^2 = 1 - e^2$$

$$\theta = \cos i$$

$$\dot{M}_1 = -\frac{3}{2} \frac{n''k_2}{a''^2 \beta^3} (1 - 3\theta^2)$$

$$\dot{\omega}_1 = -\frac{3}{2} \frac{n''k_2}{a''^2 \beta^4} (1 - 5\theta^2)$$

$$\dot{\Omega}_1 = -3 \frac{n''k_2}{a''^2 \beta^4} \theta$$

$$\dot{M}_2 = \frac{3}{16} \frac{n''k_2^2}{a''^4 \beta^7} (13 - 78\theta^2 + 137\theta^4)$$

$$\dot{\omega}_2 = \frac{3}{16} \frac{n''k_2^2}{a''^4 \beta^8} (7 - 114\theta^2 + 395\theta^4)$$

$$+ \frac{5}{4} \frac{n''k_4}{a''^4 \beta^8} (3 - 36\theta^2 + 49\theta^4)$$

$$\dot{\Omega}_2 = \frac{3}{2} \frac{n''k_2^2}{a''^4 \beta^8} \theta (4 - 19\theta^2) + \frac{5}{2} \frac{n''k_4}{a''^4 \beta^8} \theta (3 - 7\theta^2)$$

$$\dot{\lambda} = n_o'' + \dot{M}_1 + \dot{M}_2$$

$$\dot{\omega} = \dot{\omega}_1 + \dot{\omega}_2$$

$$\dot{\Omega} = \dot{\Omega}_1 + \dot{\Omega}_2$$

$$\xi = \frac{1}{a'' \beta^2 - s}$$

$$\eta = es\xi$$

$$\psi = \sqrt{1-\eta^2}$$

$$\alpha^2 = 1+e^2$$

$$C_0 = \frac{1}{2}B \rho_0 (q_0 - s)^4 n'' a'' \xi^4 \alpha^{-1} \psi^{-7}$$

$$C_1 = \frac{3}{2} n'' \alpha^4 C_0$$

$$D_1 = \xi \psi^{-2} / a'' \beta^2$$

$$D_2 = 12 + 36\eta^2 + \frac{9}{2}\eta^4$$

$$D_3 = 15\eta^2 + \frac{5}{2}\eta^4$$

$$D_4 = 5\eta + \frac{15}{4}\eta^3$$

$$D_5 = \xi \psi^{-2}$$

$$B_1 = -k_2 (1 - 3\theta^2)$$

$$B_2 = -k_2 (1 - \theta^2)$$

$$B_3 = \frac{A_3, 0}{k_2} \sin i$$

$$C_2 = D_1 D_3 B_2$$

$$C_3 = D_4 D_5 B_3$$

$$\dot{n}_o = C_1 (2 + 3n^2 + 20en + 5en^3 + \frac{17}{2}e^2$$

$$+ 34e^2 n^2 + D_1 D_2 B_1 + C_2 \cos 2\omega + C_3 \sin \omega)$$

$$\dot{e}_o = -\frac{2}{3} \frac{\dot{n}}{n^{\pi}} (1 - e)$$

where all quantities are epoch values.

At this point SDP8 calls the initialization section of DEEP which calculates all initialized quantities needed for the deep-space perturbations (see section ten).

The secular effect of gravity is included in mean anomaly by

$$M_{DF} = M_o + \dot{\lambda} (t - t_o)$$

and the secular effects of gravity and atmospheric drag are included in argument of perigee and longitude of ascending node by

$$\omega = \omega_o + \dot{\omega} (t - t_o) + \dot{\omega}_1 z_7$$

$$\Omega = \Omega_o + \dot{\Omega} (t - t_o) + \dot{\Omega}_1 z_7$$

where

$$z_7 = \frac{7}{3} z_1 / n_o''$$

with

$$z_1 = \frac{1}{2} \dot{n}_o (t - t_o)^2$$

Next, SDP8 calls the secular section of DEEP which adds the deep-space secular effects and long-period resonance effects to the six classical orbital elements (see section ten).

The secular effects of drag are included in the remaining elements by

$$n = n_{DS} + \dot{n}_o (t - t_o)$$

$$e = e_{DS} + \dot{e}_o (t - t_o)$$

$$M = M_{DS} + z_1 + M_1 z_7$$

where n_{DS} , e_{DS} , M_{DS} are the values of n_o , e_o , M_{DF} after deep-space secular and resonance perturbations have been applied.

Here, SDP8 calls the periodics section of DEEP which adds the deep-space lunar and solar periodics to the orbital elements (see section ten). From this point on, it will be assumed that n , e , I , ω , Ω , and M are the mean motion, eccentricity, inclination, argument of perigee, longitude of ascending node, and mean anomaly after lunar-solar periodics have been added.

Solve Kepler's equation for E by using the iteration equation

$$E_{i+1} = E_i + \Delta E_i$$

with

$$\Delta E_i = \frac{M + e \sin E_i - E_i}{1 - e \cos E_i}$$

and

$$E_1 = M + e \sin M + 1/2 e^2 \sin 2M .$$

The following equations are used to calculate preliminary quantities needed for the short-period periodics.

$$a = \left(\frac{k_e}{n}\right)^{2/3}$$

$$\beta = (1 - e^2)^{1/2}$$

$$\sin f = \frac{\beta \sin E}{1 - e \cos E}$$

$$\cos f = \frac{\cos E - e}{1 - e \cos E}$$

$$u = f + \omega$$

$$r'' = \frac{a\beta^2}{1 + e \cos f}$$

$$\dot{r}'' = \frac{nae}{\beta} \sin f$$

$$\ddot{(rf'')} = \frac{na^2\beta}{r}$$

$$\begin{aligned} \delta r &= \frac{1}{2} \frac{k_2}{a\beta^2} [(1 - \theta^2) \cos 2u \\ &\quad + 3(1 - 3\theta^2)] - \frac{1}{4} \frac{A_{3,0}}{k_2} \sin i_o \sin u \end{aligned}$$

$$\delta \dot{r} = -n \left(\frac{a}{r}\right)^2 \left[\frac{k_2}{a\beta^2} (1 - \theta^2) \sin 2u + \frac{1}{4} \frac{A_{3,0}}{k_2} \sin i_o \cos u \right]$$

$$\delta I = \theta \left[\frac{3}{2} \frac{k_2}{a^2 \beta^4} \sin i_o \cos 2u + \frac{1}{4} \frac{A_{3,0}}{k_2 a \beta^2} e \sin \omega \right]$$

$$\delta(rf) = -n \left(\frac{a}{r}\right)^2 \delta r + na \left(\frac{a}{r}\right) \frac{\sin i_o}{\theta} \delta I$$

$$\begin{aligned} \delta u &= \frac{1}{2} \frac{k_2}{a^2 \beta^4} \left[\frac{1}{2} (1 - 7\theta^2) \sin 2u + 3(1 - 5\theta^2) (f - M + e \sin f) \right] \\ &\quad - \frac{1}{4} \frac{A_{3,0}}{k_2 a \beta^2} [\sin i_o \cos u (2 + e \cos f) \right. \end{aligned}$$

$$+ \frac{1}{2} \frac{\theta^2}{\sin i_o / 2 \cos i_o / 2} e \cos \omega]$$

$$\begin{aligned} \delta \lambda &= \frac{1}{2} \frac{k_2}{a^2 \beta^4} \left[\frac{1}{2} (1 + 6\theta - 7\theta^2) \sin 2u - 3(1 + 2\theta - 5\theta^2) (f - M + e \sin f) \right] \\ &\quad - \frac{1}{4} \frac{A_{3,0}}{k_2 a \beta^2} \sin i_o \left[\frac{e\theta}{1 + \theta} \cos \omega \right. \end{aligned}$$

$$- (2 + e \cos f) \cos u]$$

The short-period periodics are added to give the osculating quantities

$$r = r'' + \delta r$$

$$\dot{\underline{r}} = \dot{\underline{r}''} + \delta \dot{\underline{r}}$$

$$\dot{\underline{rf}} = (\dot{\underline{rf}})'' + \delta \dot{\underline{rf}}$$

$$y_4 = \sin I/2 \sin u + \cos i_o/2 \delta u$$

$$+ \frac{1}{2} \sin u \cos i_o/2 \delta I$$

$$y_5 = \sin I/2 \cos u - \sin u \sin i_o/2 \delta u$$

$$+ \frac{1}{2} \cos u \cos i_o/2 \delta I$$

$$\lambda = u + \Omega + \delta \lambda .$$

Unit orientation vectors are calculated by

$$U_x = 2y_4 (y_5 \sin \lambda - y_4 \cos \lambda) + \cos \lambda$$

$$U_y = -2y_4 (y_5 \cos \lambda + y_4 \sin \lambda) + \sin \lambda$$

$$U_z = 2y_4 \cos I/2$$

$$V_x = 2y_5 (y_5 \sin \lambda - y_4 \cos \lambda) - \sin \lambda$$

$$V_y = -2y_5 (y_5 \cos \lambda + y_4 \sin \lambda) + \cos \lambda$$

$$V_z = 2y_5 \cos I/2$$

where

$$\cos I/2 = \sqrt{1 - y_4^2 - y_5^2} .$$

Position and velocity are given by

$$\underline{r} = \underline{rU}$$

$$\dot{\underline{r}} = \dot{\underline{rU}} + \dot{\underline{rfV}} .$$

A FORTRAN IV computer code listing of the subroutine
SDP8 is given below.

14 NOV 80

```
1 * SDP8
2 SUBROUTINE SDP8(IFLAG,TSINCE)
3 COMMON/E1/XMO,XNODEO,OMEGAO,EO,XINCL,XNO,XNDT20,
4 1 XNDD60,BSTAR,X,Y,Z,XDOT,YDOT,ZDOT,EPOCH,DS50
5 COMMON/C1/CK2,CK4,E6A,QOMS2T,S,TOTHRD,
6 1 XJ3,XKE,XKMPER,XMNPDA,AE
7 DOUBLE PRECISION EPOCH,DS50
8 DATA RHO/.15696615/
9
10 IF (IFLAG .EQ. 0) GO TO 100
11
12 * RECOVER ORIGINAL MEAN MOTION (XNODP) AND SEMIMAJOR AXIS (AODP)
13 * FROM INPUT ELEMENTS ----- CALCULATE BALLISTIC COEFFICIENT
14 * (B TERM) FROM INPUT B* DRAG TERM
15
16 A1=(XKE/XNO)**TOTHRD
17 COSI=COS(XINCL)
18 THETA2=COSI*COSI
19 TTHMUN=3.*THETA2-1.
20 EOSQ=EO*EO
21 BETA02=1.-EOSQ
22 BETA0=SQRT(BETA02)
23 DEL1=1.5*CK2*TTHMUN/(A1*A1*BETA0*BETA02)
24 AO=A1*(1.-DEL1*(.5*TOTHRD+DEL1*(1.+134./81.*DELT)))
25 DELO=1.5*CK2*TTHMUN/(AO*AO*BETA0*BETA02)
26 AODP=AO/(1.-DELO)
27 XNODP=XNO/(1.+DELO)
28 B=2.*BSTAR/RHO
29
30 * INITIALIZATION
31
32 P0=AODP*BETA02
33 P0M2=1./(P0*P0)
34 SINI=SIN(XINCL)
35 SING=SIN(OMEGAO)
36 COSG=COS(OMEGAO)
37 TEMP=.5*XINCL
38 SINIO2=SIN(TEMP)
39 COSIU2=COS(TEMP)
40 THETA4=THETA2**2
41 UNM5TH=1.-5.*THETA2
42 UNMTH2=1.-THETA2
43 A3COF=-XJ3/CK2*AE**3
44 PARDT1=3.*CK2*P0M2*XNODP
45 PARDT2=PARDT1*CK2*P0M2
46 PARDT4=1.25*CK4*P0M2*P0M2*XNODP
47 XMDT1=.5*PARDT1*BETA0*TTHMUN
48 XGDT1=-.5*PARDT1*UNM5TH
49 XHDT1=-PARDT1*COSI
50 XLLDOT=XNODP+XMDT1+
51 2 .0625*PARDT2*BETA0*(13.-78.*THETA2+137.*THETA4)
52 OMGDT=XGDT1+
53 1 .0625*PARDT2*(7.-114.*THETA2+395.*THETA4)+PARDT4*(3.-36.*THETA2+49.*THETA4)
54 2 XNODQT=XHDT1+
55 1 (.5*PARDT2*(4.-19.*THETA2)+2.*PARDT4*(3.-7.*THETA2))*COSI
```

```

57      TSI=1./(PO-S)
58      ETA=E0*S*TSI
59      ETA2=ETA**2
60      PSIM2=ABS(1. / (1.-ETA2))
61      ALPHA2=1.+EOSQ
62      EETA=E0*ETA
63      COS2G=2.*COSG**2-1.
64      D5=TSI*PSIM2
65      D1=D5/PO
66      D2=12.+ETA2*(36.+4.5*ETA2)
67      D3=ETA2*(15.+2.5*ETA2)
68      D4=ETA*(5.+3.75*ETA2)
69      B1=CK2*TTHMUN
70      B2=-CK2*UNMTH2
71      B3=A3COF*SINI
72      CO=.5*B*RHO*QOMS2T*XNODP*AODP*TSI**4*PSIM2**3.5/SQRT(ALPHA2)
73      C1=1.5*XNODP*ALPHA2**2*CO
74      C4=D1*D3*B2
75      C5=D5*D4*B3
76      XNDT=C1*
77      1 (2.+ETA2*(3.+34.*EOSQ)+5.*EETA*(4.+ETA2)+8.5*EOSQ) +
78      1 D1*D2*B1+ C4*COS2G+C5*SING)
79      XNDTN=XNDT/XNODP
80      EDOT=-TOTHRD*XNDTN*(1.-EO)
81      IFLAG=0
82      CALL DPINIT(EOSQ,SINI,COSI,BETAO,AODP,THETA2,SING,COSG,
83      1 BETA02,XLDDOT,OMGDT,XNODOT,XNODP)
84
85      * UPDATE FOR SECULAR GRAVITY AND ATMOSPHERIC DRAG
86
87      100 Z1=.5*XNDT*TSINCE*TSINCE
88      Z7=3.5*TOTHRD*Z1/XNODP
89      XMAMDF=XMO+XLDDOT*TSINCE
90      OMGASM=OMEGA0+OMGDT*TSINCE+Z7*XGDT1
91      XNODES=XNODE0+XNODOT*TSINCE+Z7*XHDT1
92      XN=XNODP
93      CALL DPSEC(XMAMDF,OMGASM,XNODES,EM,XINC,XN,TSINCE)
94      XN=XN+XNDT*TSINCE
95      EM=EM+EDOT*TSINCE
96      XMAM=XMAMDF+Z1+Z7*XMDT1
97      CALL DPPER(EM,XINC,OMGASM,XNODES,XMAM)
98      XMAM=FMOD2P(XMAM)
99
100     * SOLVE KEPLERS EQUATION
101
102     ZC2=XMAM+EM*SIN(XMAM)*(1.+EM*COS(XMAM))
103     DO 130 I=1,10
104     SINE=SIN(ZC2)
105     COSE=COS(ZC2)
106     ZC5=1. / (1.-EM*COSE)
107     CAPE=(XMAM+EM*SINE-ZC2)*
108     1 ZC5+ZC2
109     IF(ABS(CAPE-ZC2) .LE. E6A) GO TO 140
110     130 ZC2=CAPE
111
112     * SHORT PERIOD PRELIMINARY QUANTITIES

```

```

113
114      140 AM=(XKE/XN)**TOTHRD
115      BETA2M=1.-EM*EM
116      SINOS=SIN(OMGASM)
117      COSOS=COS(OMGASM)
118      AXNM=EM*COSOS
119      AYNM=EM*SINOS
120      PM=AM*BETA2M
121      G1=1./PM
122      G2=.5*CK2*G1
123      G3=G2*G1
124      BETA=SQRT(BETA2M)
125      G4=.25*A3COF*SINI
126      G5=.25*A3COF*G1
127      SNF=BETA*SINE*ZCS
128      CSF=(COSE-EM)*ZCS
129      FM=ACTAN(SNF,CSF)
130      SNFG=SNF*COSOS+CSF*SINOS
131      CSFG=CSF*COSOS-SNF*SINOS
132      SN2F2G=2.*SNFG*CSFG
133      CS2F2G=2.*CSFG**2-1.
134      ECOSF=EM*CSF
135      G10=FM-XMAM+EM*SNF
136      RM=PM/(1.+ECOSF)
137      AOVR=AM/RM
138      G13=XN*AOVR
139      G14=-G13*AOVR
140      DR=G2*(UNMTH2*CS2F2G-3.*TTHMUN)-G4*SNFG
141      DIWC=3.*G3*SINI*CS2F2G-G5*AYNM
142      DI=DIWC*COSI
143      SINI2=SIN(.5*XINC)
144
145      *      UPDATE FOR SHORT PERIOD PERIODICS
146
147      SNI2DU=SINI02*
148      1  G3*(.5*(1.-7.*THETA2)*SN2F2G-3.*UNM5TH*G10)-G5*SINI*CSFG*(2.+
149      2  ECOSF))-5*G5*THETA2*AXNM/COSI02
150      XLAMB=FM+OMGASM+XNODES+G3*(.5*(1.+6.*COSI-7.*THETA2)*SN2F2G-3.*+
151      1  (UNM5TH+2.*COSI)*G10)+G5*SINI*(COSI*AXNM/(1.+COSI)-(2.-
152      2  +ECOSF)*CSFG)
153      Y4=SINI2*SNFG+CSFG*SNI2DU+.5*SNFG*COSI02*DI
154      Y5=SINI2*CSFG-SNFG*SNI2DU+.5*CSFG*COSI02*DI
155      R=RM+DR
156      RDOT=XN*AM*EM*SNF/BETA+G14*(2.*G2*UNMTH2*SN2F2G+G4*CSFG)
157      RVDOT=XN*AM**2*BETA/RM+
158      1  G14*DR+AM*G13*SINI*DIWC
159
160      *      ORIENTATION VECTORS
161
162      SNLAMB=SIN(XLAMB)
163      CSLAMB=COS(XLAMB)
164      TEMP=2.*(Y5*SNLAMB-Y4*CSLAMB)
165      UX=Y4*TEMP+CSLAMB
166      VX=Y5*TEMP-SNLAMB
167      TEMP=2.*(Y5*CSLAMB+Y4*SNLAMB)
168      UY=-Y4*TEMP+SNLAMB

```

169 VY=-Y5*TEMP+CSLAMB
170 TEMP=2.*SQRT(1.-Y4*Y4-Y5*Y5)
171 UZ=Y4*TEMP
172 VZ=Y5*TEMP
173
174 * POSITION AND VELOCITY
175
176 X=R*UX
177 Y=R*UY
178 Z=R*UZ
179 XDOT=RDOT*UX+RVDOT*VX
180 YDOT=RDOT*UY+RVDOT*VY
181 ZDOT=RDOT*UZ+RVDOT*VZ
182
183 RETURN
184 END

10. THE DEEP-SPACE SUBROUTINE

The two deep-space models, SDP4 and SDP8, both access the subroutine DEEP to obtain the deep-space perturbations of the six classical orbital elements. The perturbation equations are quite extensive and will not be repeated here. Rather, this section will concentrate on a general description of the flow between the main program and the deep-space subroutines. A specific listing of the equations is available in Hujasak (1979) or Hujasak and Hoots (1977).

The first time the deep-space subroutine is accessed is during the initialization portion of SDP4/SDP8 and is via the entry DPINIT. Through this entry, certain constants already calculated in SDP4/SDP8 are passed to the deep-space subroutine which in turn calculates all initialized (time independent) quantities needed for prediction in deep space. Additionally, a determination is made and flags are set concerning whether the orbit is synchronous and whether the orbit experiences resonance effects.

The next access to the deep-space subroutine occurs during the secular update portion of SDP4/SDP8 and is via the entry DPSEC. Through this entry, the current secular values of the "mean" orbital elements are passed to the deep-space subroutine which in turn adds the appropriate deep-space secular and long-period resonance effects to these mean elements.

Code Modification for Subroutine DEEP

1. Following line 6, add the following line

COMMON/C2/DE2RA, PI, PI02, TWOPI, X3PI02

2. Following line 297, add the following 4 lines:

IF(XINC .GE. 0.) GO TO 90

XINC = -XINC

XNODES = XNODES + PI

OMGASM = OMGASM - PI

3. Modify line 298 to become statement label 90.

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1 * DEEP SPACE

2 SUBROUTINE DEEP
 3 COMMON/E1/XMO,XNODE0,OMEGA0,E0,XINCL,XNO,XNDT20,
 4 XNDD60,BSTAR,X,Y,Z,XDOT,YDOT,ZD0T,EPOCH,DS50
 5 COMMON/C1/CK2,CK4,ESA,DS2,T,S,TOTHRD
 6 XJ3,XKE,XKMPER,XMNPDA,AE
 7 DOUBLE PRECISION EPOCH,DS50

8 DOUBLE PRECISION

9 * DAY,PREEP,XNODCE,ATIME,DELT,SAVTSN,STEP2,STEPP
 10 DATA ZNS, C1SS, ZES/
 11 A 1.12459E-5 2.936477E-6 0.1375/
 12 DATA ZNL, C1L, ZEL/
 13 A 1.5835213E-6 4.7268055E-7 0.5490/
 14 DATA ZCOSIS, ZSINIS, ZSINGS/
 15 A 91744862 3.7785415 -0.28038458/
 16 DATA ZCOSGS, ZCOSHs, ZSINHS/
 17 A 1.945905 1.0 0.0/
 18 DATA Q22,Q31,Q33/1.7891679E-6,2.1460743E-5,2.2123015E-7/
 19 DATA G22,G32/5.7636395,0.95240898/
 20 DATA G44,G52/1.3014998,1.0508330/
 21 DATA G54/4.4108898/
 22 DATA ROOT22,ROOT32/1.7891679E-6,3.7393792E-7/
 23 DATA ROOT44,ROOT52/7.3636953E-9,1.1428639E-2/
 24 DATA ROOT54/2.1765803E-9/
 25 DATA THDT/4.3752691E-3/

26 * ENTRANCE FOR DEEP SPACE INITIALIZATION

27 ENTRY DPINIT(EQSQ,SINIQ,COSIQ,RTEQSQ,AQ,COSQ2,SINQAO,COSOMO,
 28 BSQ,XLLDOT,OMGDT,XNODOT,XNODP)
 29 IHGR=THEIAG(EPOCH)
 30 EQ = EO
 31 XNQ = XNODP
 32 AQNV = 1./AO
 33 XQNCL = XINCL
 34 XMAO=XMO
 35 XPIDOT=OMGDT+XNODOT
 36 SINQ = SIN(XNODE0)
 37 COSQ = COS(XNODE0)
 38 OMEGAQ = OMEGA0

41 * INITIALIZE LUNAR SOLAR TERMS

42 5 DAY=DS50+18261.500
 43 IF (DAY.EQ.PREEP) GO TO 10
 44 PREEP = DAY
 45 XNODCE=4.5236020-9.2422029E-4*DAY
 46 STEM=DSIN (XNODCE)
 47 CTEM=DCOS (XNODCE)
 48 ZCOSIL=.91375164-.03563096*CTEM
 49 ZSINIL=SQRT (1.-ZCOSIL*ZCOSIL)
 50 ZSINHL=.089683511*STEM/ZSINIL
 51 ZCOSHs=SQRT (1.-ZSINHL*ZSINHL)
 52 C=4.7199672+.22997150*DAY
 53 GAM=5.8351514+.0019443680*DAY
 54 ZMOL = FMOD2P(C-GAM)

The last access to the deep-space subroutine occurs at the beginning of the osculation portion (periodics application) of SDP4/SDP8 and is via the entry DP PER. Through this entry, the current values of the orbital elements are passed to the deep-space subroutine which in turn adds the appropriate deep-space lunar and solar periodics to the orbital elements.

During initialization the deep-space subroutine calls the function subroutine THETAG to obtain the location of Greenwich at epoch and to convert epoch to minutes since 1950. All physical constants which are unique to the deep-space subroutine are set via data statements in DEEP rather than being passed through a common.

A FORTRAN IV computer code listing of the subroutine DEEP is given below. These equations contain all currently anticipated changes to the SCC operational program. These changes are scheduled for implementation in March, 1981.

```

57      ZX=.39785416★STEM/ZSINIL
58      ZY=ZCOSHL★CTEM+0.91744367★ZSINHL★STEM
59      ZX=ACTAN(ZX,ZY)
60      ZX=GAM+ZX-XNODE
61      ZCOSGL=COS(ZX)
62      ZSINGL=SIN(ZX)
63      ZMOS=6.256583700+.01720197700★DAY
64      ZMOS=FMOD2P(ZMOS)
65
66      * DO SOLAR TERMS
67
68      10 LS = 0
69      SAVTSN=1.020
70      ZCOSG=ZCOSGS
71      ZSING=ZSINGS
72      ZCOSI=ZCOSIS
73      ZSINI=ZSINIS
74      ZCOSH=COSQ
75      ZSINH=SINQ
76      CC=C1SS
77      ZN=ZNS
78      ZE=ZES
79      ZMO=ZMOS
80      XNOI=1./XNQ
81      ASSIGN 30 TO LS
82      20 A1=ZCOSG★ZCOSH+ZSING★ZCOSI★ZSINH
83      A3=-ZSING★ZCOSH+ZCOSG★ZCOSI★ZSINH
84      A7=-ZCOSG★ZSINH+ZSING★ZCOSI★ZCOSH
85      A8=ZSING★ZSINI
86      A9=ZSING★ZSINH+ZCOSG★ZCOSI★ZCOSH
87      A10=ZCOSG★ZSINI
88      A2= COSIQ★A7+ SINIQ★A8
89      A4= COSIQ★A9+ SINIQ★A10
90      A5=- SINIQ★A7+ COSIQ★A8
91      A6=- SINIQ★A9+ COSIQ★A10
92      C
93      X1=A1★COSOMO+A2★SINOMO
94      X2=A3★COSOMO+A4★SINOMO
95      X3=-A1★SINOMO+A2★COSOMO
96      X4=-A3★SINOMO+A4★COSOMO
97      X5=A5★SINOMO
98      X6=A6★SINOMO
99      X7=A5★COSOMO
100     X8=A6★COSOMO
101     C
102     Z31=12.*X1★X1-3.*X3★X3
103     Z32=24.*X1★X2-6.*X3★X4
104     Z33=12.*X2★X2-3.*X4★X4
105     Z1=3.* (A1★A1+A2★A2)+Z31★EQSQ
106     Z2=6.* (A1★A3+A2★A4)+Z32★EQSQ
107     Z3=3.* (A3★A3+A4★A4)+Z33★EQSQ
108     Z11=-6.*A1★A5+EQSQ ★(-24.*X1★X7-6.*X3★X5)
109     Z12=-6.* (A1★A6+A3★A5)+EQSQ ★(-24.* (X2★X7+X1★X8)-6.* (X3★X6+X4★X5))
110     Z13=-6.*A3★A6+EQSQ ★(-24.*X2★X8-6.*X4★X6)
111     Z21=6.*A2★A5+EQSQ ★(24.*X1★X5-6.*X3★X7)
112     Z22=6.* (A4★A5+A2★A6)+EQSQ ★(24.* (X2★X5+X1★X6)-6.* (X4★X7+X3★X5))

```

```

113      Z23=6.*A4*A6+FQSQ *(24.*X2*X5-6.*X4*X8)
114      Z1=Z1+Z1+BSQ*Z31
115      Z2=Z2+Z2+BSQ*Z32
116      Z3=Z3+Z3+BSQ*Z33
117      S3=CC*XNOI
118      S2=-.5*S3/RTEQSQ
119      S4=S3*RTEQSQ
120      S1=-15.*EQ*S4
121      S5=X1*X3+X2*X4
122      S6=X2*X3+X1*X4
123      S7=X2*X4-X1*X3
124      SE=S1*ZN*S5
125      SI=S2*ZN*(Z11+Z13)
126      SL=-ZN*S3*(Z1+Z3-14.-6.*EQSQ)
127      SGH=S4*ZN*(Z31+Z33-6.)
128      SH=-ZN*S2*(Z21+Z23)
129      IF(XQNCL.LT.5.2359877E-2) SH=0.0
130      EE2=2.*S1*S6
131      E3=2.*S1*S7
132      XI2=2.*S2*Z12
133      XI3=2.*S2*(Z13-Z11)
134      XL2=-2.*S3*Z2
135      XL3=-2.*S3*(Z3-Z1)
136      XL4=-2.*S3*(-21.-9.*EQSQ)*ZE
137      XGH2=2.*S4*Z32
138      XGH3=2.*S4*(Z33-Z31)
139      XGH4=-18.*S4*ZE
140      XH2=-2.*S2*Z22
141      XH3=-2.*S2*(Z23-Z21)
142      GO TO LS
143
144      * DO LUNAR TERMS
145
146      30 SSE = SE
147      SSI=SI
148      SSL=SL
149      SSH=SH/SINI0
150      SSG=SGH-COSIQ*SSH
151      SE2=EE2
152      SI2=XI2
153      SL2=XL2
154      SGH2=XGH2
155      SH2=XH2
156      SE3=E3
157      SI3=XI3
158      SL3=XL3
159      SGH3=XGH3
160      SH3=XH3
161      SL4=XL4
162      SGH4=XGH4
163      LS=1
164      ZCOSG=ZCOSGL
165      ZSING=ZSINGL
166      ZCOSI=ZCOSIL
167      ZSINI=ZSINIL
168      ZCOSH=ZCOSHL*COSQ+ZSINHL*SING

```

```

69      ZSINH=SINQ★ZCOSHL-COSQ★ZSINHL
70      ZN=ZNL
71      CC=C1L
72      ZE=ZEL
73      ZMO=ZMOL
74      ASSIGN 40 TO LS
75      GO TO 20
76      40 SSE = SSE+SE
77      SSI=SSI+SI
78      SSL=SSL+SL
79      SSG=SSG+SGH-COSIQ/SINIQ★SH
80      SSH=SSH+SH/SINIQ
81
82      *      GEOPOTENTIAL RESONANCE INITIALIZATION FOR 12 HOUR ORBITS
83
84      IRESFL=0
85      ISYNEL=0
86      IF(XNQ.LT.(.0052359877).AND.XNQ.GT.(.0034906585)) GO TO 70
87      IF (XNQ.LT.(3.26E-3) OR XNQ.GT.(9.24E-3)) RETURN
88      IF (EQ.LT.0.5) RETURN
89      IRESFL=1
90      EOC=EQ★EQSQ
91      G201=-.306-(EQ-.64)*.440
92      IF(EQ.GT.(.65)) GO TO 45
93      G211=3.616-13.247★EQ+16.290★EQSQ
94      G310=-19.302+117.390★EQ-228.419★EQSQ+156.591★EOC
95      G322=-18.9068+109.7927★EQ-214.6334★EQSQ+146.5816★EOC
96      G410=-41.122+242.694★EQ-471.094★EQSQ+313.953★EOC
97      G422=-146.407+841.380★EQ-1629.014★EQSQ+1033.435★EOC
98      G520=-532.114+3017.977★EQ-5740★EQSQ+3708.276★EOC
99      GO TO 55
100     45 G211=-72.099+331.819★EQ-508.738★EQSQ+266.724★EOC
101     G310=-346.844+1532.851★EQ-2415.925★EQSQ+1246.113★EOC
102     G322=-342.585+1554.908★EQ-2366.899★EQSQ+1215.972★EOC
103     G410=-1052.797+4758.686★EQ-7193.992★EQSQ+3651.957★EOC
104     G422=-3581.69+16178.11★EQ-24462.77★EQSQ+12422.52★EOC
105     IF(EQ.GT.(.715)) GO TO 50
106     G520=1464.74-4664.75★EQ+3763.64★EQSQ
107     GO TO 55
108     50 G520=-5149.66+29936.92★EQ-54087.36★EQSQ+31324.55★EOC
109     55 IF(EQ.GE.(.72)) GO TO 60
110     G533=-919.2277+4988.61★EQ-9064.77★EQSQ+5542.21★EOC
111     G521 = -822.71072+4568.6173★EQ-8491.4146★EQSQ+5337.524★EOC
112     G532 = -853.666+4690.25★EQ-3624.77★EQSQ+5341.4★EOC
113     GO TO 65
114     60 G533=-37995.78+161616.52★EQ-229838.2★EQSQ+109377.94★EOC
115     G521 = -51752.104+218913.95★EQ-309468.16★EQSQ+146349.42★EOC
116     G532 = -40023.88+170470.39★EQ-242699.48★EQSQ+115605.82★EOC
117     65 SINI2=SINIQ★SINIQ
118     F220=.75*(1.+2.*COSIQ+COSQ2)
119     F221=1.5★SINI2
120     F321=1.875★SINIQ*(1.-2.*COSIQ-3.*COSQ2)
121     F322=-1.875★SINIQ*(1.+2.*COSIQ-3.*COSQ2)
122     F441=35.*SINI2★F220
123     F442=39.3750★SINI2★SINI2
124     F522=9.84375★SINI2*(SINI2*(1.-2.*COSIQ-5.*COSQ2))

```

```

225      1 + .33333333*(-2.+4.*COSIQ+6.*COSQ2)
226      F523 = SINIQ*(4.92187512*SINI2*(-2.-4.*COSIQ+10.*COSQ2)
227      * +6.56250012*(1.+2.*COSIQ-3.*COSQ2))
228      F542 = 29.53125*SINIQ*(2.-8.*COSIQ+COSQ2*(-12.+3.*COSIQ
229      * +10.*COSQ2))
230      F543=29.53125*SINIQ*(-2.-8.*COSIQ+COSQ2*(12.+8.*COSIQ-10.*COSQ2))
231      XNQ2=XNQ*XNQ
232      AINV2=AQN V*AQN V
233      TEMP1 = -3.*XNQ2*AINV2
234      TEMP = TEMP1*ROOT22
235      D2201 = TEMP*F220*G201
236      D2211 = TEMP*F221*G211
237      TEMP1 = TEMP1*AQN V
238      TEMP = TEMP1*ROOT32
239      D3210 = TEMP*F321*G310
240      D3222 = TEMP*F322*G322
241      TEMP1 = TEMP1*AQN V
242      TEMP = 2.*TEMP1*ROOT44
243      D4410 = TEMP*F441*G410
244      D4422 = TEMP*F442*G422
245      TEMP1 = TEMP1*AQN V
246      TEMP = TEMP1*ROOT52
247      D5220 = TEMP*F522*G520
248      D5232 = TEMP*F523*G532
249      TEMP = 2.*TEMP1*ROOT54
250      D5421 = TEMP*F542*G521
251      D5433 = TEMP*F543*G533
252      XLAMO = XMA0+XNODE0+XNODE0-THGR-THGR
253      BFACT = XLLDOT+XNODOT+XNODOT-THDT-THDT
254      BFACT=BFACT+SSL+SSH+SSH
255      GO TO 80
256
257      * SYNCHRONOUS RESONANCE TERMS INITIALIZATION
258
259      70 IRESEL=1
260      ISYNFL=1
261      G200=1.0+EQS Q*(-2.5+.8125*EQSQ)
262      G310=1.0+2.0*EQSQ
263      G300=1.0+EQSQ*(-6.0+6.60937*EQSQ)
264      F220=.75*(1.+COSIQ)*(1.+COSIQ)
265      F311=.9375*SINIQ*SINIQ*(1.+3.*COSIQ)-.75*(1.+COSIQ)
266      F330=1.+COSIQ
267      F330=1.875*F330*F330*F330
268      DEL1=3.*XNQ*XNQ*AQN V*AQN V
269      DEL2=2.*DEL1*F220*G200*Q22
270      DEL3=3.*DEL1*F330*G300*Q33*AQN V
271      DEL1=DEL1*F311*G310*Q31*AQN V
272      FASX2=.13130908
273      FASX4=2.8843198
274      FASX6=.37448087
275      XLAMO=XMA0+XNODE0+OMEGA0-THGR
276      BFACT = XLLDOT+XPIDOT-THDT
277      BFACT=BFACT+SSL+SSG+SSH
278      80 XFACT=BFACT-XNQ
279      C
280      C      INITIALIZE INTEGRATOR

```

```

281      C
282      XLI=XLAM0.
283      XNI=XNQ
284      ATIME=0.00
285      STEPP=720.00
286      STEPN=-720.00
287      STEP2 = 259200.00
288      RETURN
289
290      * ENTRANCE FOR DEEP SPACE SECULAR EFFECTS
291
292      ENTRY DPSEC(XLL,OMGASM,XNODES,EM,XINC,XN,T)
293      XLL=XLL+SSL*T
294      OMGASM=OMGASM+SSG*T
295      XNODES=XNODES+SSH*T
296      EM=EO+SSE*T
297      XINC=XINCL+SSI*T
298      IF(IRESFL .EQ. 0) RETURN
299      100 IF (ATIME.EQ.0.00) GO TO 170
300      IF(T.GE.(0.00).AND.ATIME.LT.(0.00)) GO TO 170
301      IF(T.LT.(0.00).AND.ATIME.GE.(0.00)) GO TO 170
302      105 IF(DABS(T).GE.DABS(ATIME)) GO TO 120
303      DELT=STEPP
304      IF (T.GE.0.00) DELT = STEPN
305      110 ASSIGN 100 TO IRET
306      GO TO 160
307      120 DELT=STEPN
308      IF (T.GT.0.00) DELT = STEPP
309      125 IF (DABS(T-ATIME).LT.STEPP) GO TO 130
310      ASSIGN 125 TO IRET
311      GO TO 160
312      130 FT = T-ATIME
313      ASSIGN 140 TO IRETN
314      GO TO 150
315      140 XN = XNI+XNDOT*FT+XNDDT*FT*FT*0.5
316      XL = XLI+XLDOT*FT+XNDOT*FT*FT*0.5
317      TEMP = -XNODES+THGR+T*THDT
318      XLL = XL-OMGASM+TEMP
319      IF (ISYNFL.EQ.0) XLL = XL+TEMP+TEMP
320      RETURN
321      C
322      C DOT TERMS CALCULATED
323      C
324      150 IF (ISYNFL.EQ.0) GO TO 152
325      XNDOT=DEL1*SIN (XLI-FASX2)+DEL2*SIN (2.*(XLI-FASX4))
326      1     +DEL3*SIN (3.*(XLI-FASX6))
327      XNDDT = DEL1*COS(XLI-FASX2)
328      *     +2.*DEL2*COS(2.*(XLI-FASX4))
329      *     +3.*DEL3*COS(3.*(XLI-FASX6))
330      GO TO 154
331      152 XOMI = OMEGAQ+OMGDT*ATIME
332      X2OMI = XOMI+XOMI
333      X2LI = XLI+XLI
334      XNDOT = D2201*SIN(X2OMI+XLI-G22)
335      *     +D2211*SIN(XLI-G22)
336      *     +D3210*SIN(XOMI+XLI-G32)

```

```

37      *      +D3222*SIN(-XOMI+XLI-G32)
38      *      +D4410*SIN(X2OMI+X2LI-G44)
39      *      +D4422*SIN(X2LI-G44)
40      *      +D5220*SIN(XOMI+XLI-G52)
41      *      +D5232*SIN(-XOMI+XLI-G52)
42      *      +D5421*SIN(XOMI+X2LI-G54)
43      *      +D5433*SIN(-XOMI+X2LI-G54)
44      XNDT = D2201*COS(X2OMI+XLI-G22)
45      *      +D2211*COS(XLI-G22)
46      *      +D3210*COS(XOMI+XLI-G32)
47      *      +D3222*COS(-XOMI+XLI-G32)
48      *      +D5220*COS(XOMI+XLI-G52)
49      *      +D5232*COS(-XOMI+XLI-G52)
50      *      +2.* (D4410*COS(X2OMI+X2LI-G44)
51      *      +D4422*COS(X2LI-G44)
52      *      +D5421*COS(XOMI+X2LI-G54)
53      *      +D5433*COS(-XOMI+X2LI-G54))
54      154 XLDOT=XNI+XFACT
55      XNDT = XNDT*XLDOT
56      GO TO IRETN
57      C
58      C      INTEGRATOR
59      C
60      160 ASSIGN 165 TO IRETN
61      GO TO 150
62      165 XLI = XLI+XLDOT*DELT+XNDOT*STEP2
63      XNI = XNI+XNDOT*DELT+XNDT*STEP2
64      ATIME=ATIME+DELT
65      GO TO IRET
66      C
67      C      EPOCH RESTART
68      C
69      170 IF (T.GE.0.00)      GO TO 175
70      DELT=STEPR
71      GO TO 180
72      175 DELT = STEPP
73      180 ATIME = 0.00
74      XNI=XNQ
75      XLI=XLAMO
76      GO TO 125
77      C
78      C      ENTRANCES FOR LUNAR-SOLAR PERIODICS
79      C
80      C
81      ENTRY DPPER(EM,XINC,OMGASM,XNODES,XLL)
82      SINIS = SIN(XINC)
83      COSIS = COS(XINC)
84      IF (DABS(SAVTSN-T).LT.(30.00))      GO TO 210
85      SAVTSN=T
86      ZM=ZMOS+ZNS*T
87      205 ZF=ZM+2.*ZES*SIN(ZM)
88      SINZF=SIN(ZF)
89      F2=-.5*SINZF*SINZF-.25
90      F3=-.5*SINZF*COS(ZF)
91      SES=SE2*F2+SE3*F3
92      SIS=SI2*F2+SI3*F3

```

```

93      SLS=SL2*F2+SL3*F3+SL4*SINZF
94      SGHS=SGH2*F2+SGH3*F3+SGH4*SINZF
95      SHS=SH2*F2+SH3*F3
96      ZM=ZMOL+ZNL*T
97      ZF=ZM+2.*ZEL*SIN(ZM)
98      SINZF=SIN(ZF)
99      F2=-.5*SINZF*SINZF=.25
00      F3=-.5*SINZF*COS(ZF)
01      SEL=EE2*E2+E3*E3
02      SIL=XI2*F2+XI3*F3
03      SLL=XL2*F2+XL3*F3+XL4*SINZF
04      SGHL=XGH2*F2+XGH3*F3+XGH4*SINZF
05      SHL=XH2*F2+XH3*E3
06      PE=SES+SEL
07      PINC=SIS+SIL
08      PL=SLS+SLL
09      210 PGH=SGHS+SGHL
10      PH=SHS+SHL
11      XINC=XINC+PINC
12      EM=EM+PE
13      IF(XQNCL.LT.(.2)) GO TO 220
14      GO TO 218
15      C
16      C      APPLY PERIODICS DIRECTLY
17      C
18      218 PH=PH/SINIQ
19      PGH=PGH-COSIQ*PH
20      OMGASM=OMGASM+PGH
21      XNODES=XNODES+PH
22      XLL=XLL+PL
23      GO TO 230
24      C
25      C      APPLY PERIODICS WITH LYDDANE MODIFICATION
26      C
27      220 SINOK=SIN(XNODES)
28      COSOK=COS(XNODES)
29      ALFDP=SINIS*SINOK
30      BETDP=SINIS*COSOK
31      DALF=PH*COSOK+PINC*COSIS*SINOK
32      DBET=-PH*SINOK+PINC*COSIS*COSOK
33      ALFDP=ALFDP+DALE
34      BETDP=BETDP+DBET
35      XLS=XLL+OMGASM+COSIS*XNODES
36      DLS=PL+PGH-PINC*XNODES*SINIS
37      XLS=XLS+DLS
38      XNODES=ACTAN(ALFDP,BETDP)
39      XLL=XLL+PL
40      OMGASM=XLS-XLL-COS(XINC)*XNODES
41      230 CONTINUE
42      RETURN
43      END

```

11. DRIVER AND FUNCTION SUBROUTINES

The DRIVER controls the input and output function and the selection of the model. The input consists of a program card which specifies the model to be used and the output times and either a G-card or T-card element set.

The DRIVER reads and converts the input elements to units of radians and minutes. These are communicated to the prediction model through the common E1. Values of the physical and mathematical constants are set and communicated through the commons C1 and C2, respectively.

The program card indicates the mathematical model to be used and the start and stop time of prediction as well as the increment of time for output. These times are in minutes since epoch.

In the interest of efficiency the DRIVER sets a flag (IFLAG) the first time the model is called. This flag tells the model to calculate all initialized (time independent), quantities. After initialization, the model subroutine turns off the flag so that all subsequent calls only access the time dependent part of the model. This mode continues until another input case is encountered.

The DRIVER takes the output from the mathematical model (communicated through the common E1) and converts it to units of kilometers and seconds for printout.

The function subroutine ACTAN is passed the values of sine and cosine in that order and it returns the angle in radians within the range of 0 to 2π . The function subroutine FMOD2P is passed an angle in radians and returns the angle in radians within the range of 0 to 2π . The function subroutine THETAG is passed the epoch time exactly as it appears on the input element cards.* The routine converts this time to days since 1950 Jan 0.0 UTC, stores this in the common E1, and returns the right ascension of Greenwich at epoch (in radians).

FORTRAN IV computer code listings of the routines DRIVER, ACTAN, FMOD2P, and THETAG are given below.

*If only one year digit is given (as on standard G-cards) the program assumes the 80 decade. This may be overridden by putting a 2 digit year in columns 30-31 of the first G-card.

3 NOV 80

```

1      *      DRIVER
2
3      *      WGS-72 PHYSICAL AND GEOPOTENTIAL CONSTANTS
4      *      CK2=.5*XJ2*AE**2      CK4=-.375*XJ4*AE**4
5
6      DOUBLE PRECISION EPOCH,DS50
7      COMMON/E1/XMO,XNODEO,OMEGA0,E0,XINCL,XNO,XNDT20,XNDD60,BSTAR,
8      1      X,Y,Z,XDOT,YDOT,ZDOT,EPOCH,DS50
9      COMMON/C1/CK2,CK4,E6A,QOMS2T,S,TOTHRD,
10     1      XJ3,XKE,XKMPER,XMNPDA,AE
11     COMMON/C2/DE2RA,PI,PIO2,TWOP1,X3PI02
12     DATA IHG/1HG/
13     DATA DE2RA,E6A,PI,PIO2,Q0,S0,TOTHRD,TWOP1,X3PI02,XJ2,XJ3,
14     1      XJ4,XKE,XKMPER,XMNPDA,AE/.174532925E-1,1.E-6,
15     2      3.14159265,1.57079633,120.0,78.0,.66666667,
16     4      6.2831853,4.71238898,1.082616E-3,-.253881E-5,
17     5      -1.65597E-6,.743669161E-1,6378.135,1440.,1./
18     DIMENSION ISET(5)
19     CHARACTER ABUF*80(2)
20     DATA (ISET(I),I=1,5)/3HSGP,4HSGP4,4HSOP4,4HSGP8,4HSOP8/
21
22     *      SELECT EPHemeris TYPE AND OUTPUT TIMES
23
24     CK2=.5*XJ2*AE**2
25     CK4=-.375*XJ4*AE**4
26     QOMS2T=((Q0-S0)*AE/XKMPER)**4
27     S=AE*(1.+S0/XKMPER)
28     2 READ (5,700) IEPT, TS, TF, DELT
29     IF(IEPT.LE.0) STOP
30     IDEEP=0
31
32     *      READ IN MEAN ELEMENTS FROM 2 CARD T(TRANS) OR G(INTERN) FORMAT
33
34     READ (5,706) ABUF
35     DECODE(ABUF(1),707) ITYPE
36     IF(ITYPE.EQ.IHG) GO TO 5
37     DECODE (ABUF,702) EPOCH,XNDT20,XNDD60,IEXP,BSTAR,IBEXP,XINCL,
38     1      XNODEO,E0,OMEGA0,XMO,XNO
39     GO TO 7
40     5 DECODE(ABUF,701) EPOCH,XMO,XNODEO,OMEGA0,E0,XINCL,XNO,XNDT20,
41     1      XNDD60,IEXP,BSTAR,IBEXP
42     7 IF(XNO.LE.0.) STOP
43     WRITE(6,704) ABUF,ISET(IEPT)
44     IF(IEPT.GT.5) GO TO 900
45     XNDD60=XNDD60*(10.**IEXP)
46     XNODEO=XNODEO*DE2RA
47     OMEGA0=OMEGA0*DE2RA
48     XMO=XMO*DE2RA
49     XINCL=XINCL*DE2RA
50     TEMP=TWOP1/XMNPDA/XMNPDA
51     XNO=XNO*TEMP*XNPDA
52     XNDT20=XNDT20*TEMP
53     XNDD60=XNDD60*TEMP/XMNPDA
54
55     *      INPUT CHECK FOR PERIOD VS EPHemeris SELECTED
56     *      PERIOD GE 225 MINUTES IS DEEP SPACE

```

```

57
58      A1=(XKE/XNO) ** TOTHRD
59      TEMP=1.5*CK2*(3.*COS(XINCL)**2-1.)/(1.-E0*E0)**1.5
60      DEL1=TEMP/(A1*A1)
61      A0=A1*(1.-DEL1*(.5*TOTHRD+DEL1*(1.+134./81.*DEL1)))
62      DELO=TEMP/(A0*A0)
63      XNODP=XNO/(1.+DELO)
64      IF((TWOP1/XNO/XMNPDA).GE..15625) IDEEP=1
65
66      BSTAR=BSTAR*(10.**IBEXP)/AE
67      TSINCE=TS
68      IFLAG=1
69      IF(IDEEP.EQ.1.AND.(IEPT.EQ.1.OR.IEPT.EQ.2
70      .OR.IEPT.EQ.4)) GO TO 800
71      9 IF(IDEEP.EQ.0.AND.(IEPT.EQ.3.OR.IEPT.EQ.5))
72      1 GO TO 850
73      10 GO TO (21,22,23,24,25), IEPT
74      21 CALL SGP(IFLAG,TSINCE)
75      GO TO 60
76      22 CALL SGP4(IFLAG,TSINCE)
77      GO TO 60
78      23 CALL SDP4(IFLAG,TSINCE)
79      GO TO 60
80      24 CALL SGP8(IFLAG,TSINCE)
81      GO TO 60
82      25 CALL SDP8(IFLAG,TSINCE)
83      60 X=X*XKMPER/AE
84      Y=Y*XKMPER/AE
85      Z=Z*XKMPER/AE
86      XDOT=XDOT*XKMPER/AE*XNPDA/86400.
87      YDOT=YDOT*XKMPER/AE*XNPDA/86400.
88      ZDOT=ZDOT*XKMPER/AE*XNPDA/86400.
89      WRITE(6,705) TSINCE,X,Y,Z,XDOT,YDOT,ZDOT
90      TSINCE=TSINCE+DELT
91      IF(ABS(TSINCE).GT.ABS(TF)) GO TO 2
92      GO TO 10
93      700 FORMAT(I1,3F10.0)
94      701 FORMAT(29X,D14.8,1X,3F8.4,/,6X,F8.7,F8.4,1X,2F11.9,1X,F5.5,I2,
95      1        4X,F8.7,I2)
96      702 FORMAT(18X,D14.8,1X,F10.8,2(1X,F6.5,I2),/7X,2(1X,F8.4),1X,
97      1        F7.7,2(1X,F8.4),1X,F11.8)
98      703 FORMAT(79X,A1)
99      704 FORMAT(1H1,A80,/,1X,A80,/,1X,A4,7H TSINCE,
100     1 14X,1HX,16X,1HY,16X,1HZ,14X,
101     1 4HxDot,13X,4HYDot,13X,4HZDot,/)
102     705 FORMAT(7F17.8)
103     706 FORMAT(A80)
104     707 FORMAT(79X,A1)
105     930 FORMAT("SHOULD USE DEEP SPACE EPHemeris")
106     940 FORMAT("SHOULD USE NEAR EARTH EPHemeris")
107     950 FORMAT("EPHEMERIS NUMBER",I2," NOT LEGAL, WILL SKIP THIS CASE")
108     800 WRITE(6,930)
109     GO TO 9
110     850 WRITE(6,940)
111     GO TO 10
112     900 WRITE(6,950) IEPT

```

113

GO TO 2
END

```
1      FUNCTION ACTAN(SINX,COSX)
2      COMMON/C2/DE2RA,PI,PI02,TWOPI,X3PI02
3      ACTAN=0.
4      IF (COSX.EQ.0. ) GO TO 5
5      IF (COSX.GT.0. ) GO TO 1
6      ACTAN=PI
7      GO TO 7
8      1 IF (SINX.EQ.0. ) GO TO 8
9      IF (SINX.GT.0. ) GO TO 7
10     ACTAN=TWOPI
11     GO TO 7
12     5 IF (SINX.EQ.0. ) GO TO 8
13     IF (SINX.GT.0. ) GO TO 6
14     ACTAN=X3PI02
15     GO TO 8
16     6 ACTAN=PI02
17     GO TO 8
18     7 TEMP=SINX/COSX
19     ACTAN=ACTAN+ATAN(TEMP)
20     8 RETURN
21     END
```

```
1 FUNCTION FMOD2P(X)
2 COMMON/C2/DE2RA,PI,PI02,TW0PI,X3PI02
3 FMOD2P=X.
4 I=FMOD2P/TW0PI
5 FMOD2P=FMOD2P-I*TW0PI
6 IF(FMOD2P.LT.0.) FMOD2P=FMOD2P+TW0PI
7 RETURN
8 END
```

```
1      FUNCTION THETAG(EP)
2      COMMON /ET/XMO,XNODEO,OMEGA0,E0,XINCL,XNO,XNDT20,XNDD60,BSTAR,
3      1 X,Y,Z,XDOT,YDOT,ZDOT,EPOCH,DS50
4      DOUBLE PRECISION EPOCH,D,THETA,TWOP1,YR,TEMP,EP,DS50
5      TWOP1=6.28318530717959D0
6      YR=(EP+2.D-7)*1.D-3
7      JY=YR
8      YR=JY
9      D=EP-YR*1.D3
10     IF(JY.LT.10) JY=JY+80
11     N=(JY-69)/4
12     IF(JY.LT.70) N=(JY-72)/4
13     DS50=7305.D0 + 365.D0*(JY-70) + N + D
14     THETA=1.72944494D0 + 6.3003880987D0*DS50
15     TEMP=THETA/TWOP1
16     I=TEMP
17     TEMP=I
18     THETAG=THETA-TEMP*TWOP1
19     IF(THETAG.LT.0.D0) THETAG=THETAG+TWOP1
20     RETURN
21     END
```

12. USERS GUIDE, CONSTANTS AND SYMBOLS

The first input card is the program card. The format is as follows:

| <u>Column</u> | <u>Format</u> | <u>Description</u> |
|---------------|---------------|--------------------------------------------------------------------------------------|
| 1 | I1 | Ephemeris program desired 1 = SGP 2 = SGP4 3 = SDP4 4 = SGP8 5 = SDP8 |
| 2-11 | F 10.0 | Prediction start time |
| 12-21 | F 10.0 | Prediction stop time |
| 22-31 | F 10.0 | Time increment |

All times are in minutes since epoch and can be positive or negative. The second and third input cards consist of either a 2-card transmission or 2-card G type element set. Either type can be used with the only condition being that the two cards must be in the correct order. For reference a format sheet for the T-card and G-card element sets follows this section.

The values of the physical and mathematical constants used in the program are given below.

| <u>Variable name</u> | <u>Definition</u> | <u>Value</u> |
|----------------------|--------------------------|--------------|
| CK2 | $\frac{1}{2} J_2 a_E^2$ | 5.413080E-4 |
| CK4 | $-\frac{3}{8} J_4 a_E^4$ | .62098875E-6 |
| E6A | 10^{-6} | 1.0 E-6 |

| | | |
|--------|--------------------------------|---------------|
| QOMS2T | $(q_0 - s)^4 (er)^4$ | 1.88027916E-9 |
| S | $s (er)$ | 1.01222928 |
| TOTHRD | $2/3$ | .66666667 |
| XJ3 | J_3 | -.253881E-5 |
| XKE | $k_e (\frac{er}{min})^{3/2}$ | .743669161E-1 |
| XKMPER | kilometers/Earth radii | 6378.135 |
| XMNPDA | time units/day | 1440.0 |
| AE | distance units/ Earth radii | 1.0 |
| DE2RA | radians/degree | .174532925E-1 |
| PI | π | 3.14159265 |
| PI02 | $\pi/2$ | 1.57079633 |
| TWOP1 | 2π | 6.2831853 |
| X3PI02 | $3\pi/2$ | 4.71238898 |

where er = Earth radii. Except for the deep-space models, all ephemeris models are independent of units. Thus, units input or output as well as physical constants can be changed by making the appropriate changes in only the DRIVER program.

Following is a list of symbols commonly used in this report.

n_o = the SGP type "mean" mean motion at epoch
 e_o = the "mean" eccentricity at epoch
 i_o = the "mean" inclination at epoch
 M_o = the "mean" mean anomaly at epoch
 ω_o = the "mean" argument of perigee at epoch
 Ω_o = the "mean" longitude of ascending node at epoch
 \dot{n}_o = the time rate of change of "mean" mean motion at epoch
 \ddot{n}_o = the second time rate of change of "mean" mean motion at epoch
 B^* = the SGP4 type drag coefficient
 k_e = \sqrt{GM} where G is Newton's universal gravitational constant and M is the mass of the Earth
 a_E = the equatorial radius of the Earth
 J_2 = the second gravitational zonal harmonic of the Earth
 J_3 = the third gravitational zonal harmonic of the Earth
 J_4 = the fourth gravitational zonal harmonic of the Earth
 $(t - t_o)$ = time since epoch
 $k_2 = \frac{1}{2} J_2 a_E^2$

$$k_4 = -\frac{3}{8} J_4 a_E^4$$

$$A_{3,0} = -J_3 a_E^3$$

q_0 = parameter for the SGP4/SGP8 density function

s = parameter for the SGP4/SGP8 density function

$B = \frac{1}{2} C_D \frac{A}{m}$, the ballistic coefficient for SGP8

where C_D is a dimensionless drag coefficient
and A is the average cross-sectional area of
the satellite of mass m

SDC/SCC TWO/THREE CARD ELEMENT SET - INTERNAL FORMAT

SDC/SEC TWO CARB ELEMENT SET - TRANSMISSION FORMAT

ELEMENTI SEI FORMAIS

13. SAMPLE TEST CASES

For reference a sample test case is given for each of the five models.* The input used was standard T-cards and the output is given at 360 minute intervals in units of kilometers and seconds.

When implemented on a given computer, the accuracies with which the test cases are duplicated will be dominated by the accuracy of the epoch mean motion. If, after reading and converting, the epoch mean motion has an error $\Delta n = j \times 10^{-k}$ radians/time, then the predicted positions at time t may differ from the test cases by numbers on the order of

$$\Delta r = \Delta n(t - t_0) (6,378.135) \text{ kilometers}$$

*The test cases were generated on a machine with 8 digits of accuracy. After a one day prediction, the test cases have only 5 to 6 digits of accuracy.

1 88888U 80275.98708465 .00073094 13844-3 66816-4 0 8
2 88888 72.8435 115.9689 0086731 52.6988 110.5714 16.05824518 105

| SGP | TSINCE | X | Y | Z |
|---------------|--------|---------------|----------------|---------------|
| 0. | | 2328.96594238 | -5995.21600342 | 1719.97894287 |
| 360.00000000 | | 2456.00610352 | -6071.94232177 | 1222.95977784 |
| 720.00000000 | | 2567.39477539 | -6112.49725342 | 713.97710419 |
| 1080.00000000 | | 2663.03179932 | -6115.37414551 | 195.73919105 |
| 1440.00000000 | | 2742.85470581 | -6079.13580322 | -328.86091614 |

| XDOT | YDOT | ZDOT |
|------------|-------------|-------------|
| 2.91110113 | -0.98164053 | -7.09049922 |
| 2.67852119 | -0.44705850 | -7.22800565 |
| 2.43952477 | 0.09884824 | -7.31889641 |
| 2.19531813 | 0.65333930 | -7.36169147 |
| 1.94707947 | 1.21346101 | -7.35499924 |

| | | | | | | | |
|----------|----------------|-----------|---------|---------|----------|-------------|-----|
| 1 88888U | 80275.98708465 | .00073094 | 13844-3 | 66816-4 | 0 | 8 | |
| 2 88888 | 72.8435 | 115.9689 | 0086731 | 52.6988 | 110.5714 | 16.05824518 | 105 |

| SGP4 TSINCE | X | Y | Z |
|---------------|---------------|----------------|---------------|
| 0. | 2328.97048951 | -5995.22076416 | 1719.97067261 |
| 360.00000000 | 2456.10705566 | -6071.93853760 | 1222.89727783 |
| 720.00000000 | 2567.56195068 | -6112.50384522 | 713.96397400 |
| 1080.00000000 | 2663.09078980 | -6115.48229980 | 196.39640427 |
| 1440.00000000 | 2742.55133057 | -6079.67144775 | -326.38095856 |

| XDOT | YDOT | ZDOT |
|------------|-------------|-------------|
| 2.91207230 | -0.98341546 | -7.09081703 |
| 2.67938992 | -0.44829041 | -7.22879231 |
| 2.44024599 | 0.09810869 | -7.31995916 |
| 2.19611958 | 0.65241995 | -7.36282432 |
| 1.94850229 | 1.21106251 | -7.35619372 |

| | | | | |
|----------|----------------|-----------|---------|----------------------------|
| 1 11801U | 80230.29629788 | .01431103 | 00000-0 | 14311-1 |
| 2 11801 | 46.7916 | 230.4354 | 7318036 | 47.4722 10.4117 2.28537848 |

SDF4 TSINCE

X

Y

Z

| | | | |
|---------------|----------------|----------------|-----------------|
| 0. | 7473.37066650 | 428.95261765 | 5828.74786377 |
| 360.00000000 | -3305.22537232 | 32410.86328125 | -24697.17675781 |
| 720.00000000 | 14271.28759766 | 24110.46411133 | -4725.76837158 |
| 1080.00000000 | -9990.05883789 | 22717.35522461 | -23616.89062501 |
| 1440.00000000 | 9787.86975097 | 33753.34667969 | -15030.81176753 |

XDOT

YDOT

ZDOT

| | | |
|-------------|-------------|-------------|
| 5.10715413 | 6.44468284 | -0.18613096 |
| -1.30113538 | -1.15131518 | -0.28333528 |
| -0.32050445 | 2.67984074 | -2.08405289 |
| -1.01667246 | -2.29026759 | 0.72892364 |
| -1.09425066 | 0.92358845 | -1.52230928 |

| | | | | | | | |
|----------|----------------|-----------|---------|---------|----------|-------------|-----|
| 1 88888U | 80275.98708465 | .00073094 | 13844-3 | 66816-4 | 0 | 8 | |
| 2 88888 | 72.8435 | 115.9689 | 0086731 | 52.6988 | 110.5714 | 16.05824518 | 105 |

| SGP8 TSINCE | X | Y | Z |
|---------------|---------------|----------------|---------------|
| 0. | 2328.87265015 | -5995.21289063 | 1720.04884338 |
| 360.00000000 | 2456.04577637 | -6071.90490722 | 1222.84086609 |
| 720.00000000 | 2567.68383789 | -6112.40881348 | 713.29282379 |
| 1080.00000000 | 2663.49508667 | -6115.18182373 | 194.62816810 |
| 1440.00000000 | 2743.29238892 | -6078.90783691 | -329.73434067 |

| XDOT | YDOT | ZDOT |
|------------|-------------|-------------|
| 2.91210661 | -0.98353850 | -7.09081554 |
| 2.67936245 | -0.44820847 | -7.22888553 |
| 2.43992555 | 0.09893919 | -7.32018769 |
| 2.19525236 | 0.65453661 | -7.36308974 |
| 1.94680957 | 1.21500109 | -7.35625595 |

| | | | | | | |
|----------|----------------|-----------|---------|---------|---------|------------|
| 1 11801U | 80230.29629788 | .01431103 | 00000-0 | 14311-1 | | |
| 2 11801 | 46.7916 | 230.4354 | 7318036 | 47.4722 | 10.4117 | 2628537848 |

| SDP8 TSINCE | X | Y | Z |
|---------------|-----------------|----------------|-----------------|
| 0. | 7469.47631836 | 415.99390792 | 5829.64318848 |
| 360.00000000 | -3337.38992310 | 32351.39086914 | -24658.63037109 |
| 720.00000000 | 14226.54333496 | 24236.08740234 | -4856.19744873 |
| 1080.00000000 | -10151.59838867 | 22223.69848633 | -23392.39770508 |
| 1440.00000000 | 9420.08203125 | 33847.21875000 | -15391.06469727 |

| XDOT | YDOT | ZDOT |
|-------------|-------------|-------------|
| 5.11402285 | 6.44403201 | -0.18296110 |
| -1.30200730 | -1.15603013 | -0.28164955 |
| -0.33951668 | 2.65315416 | -2.08114153 |
| -1.00112480 | -2.33532837 | 0.76987664 |
| -1.11986055 | 0.85410149 | -1.49506933 |

14. SAMPLE IMPLEMENTATION

These FORTRAN IV routines have been implemented on a Honeywell-6000 series computer. This machine has a processing speed in the 1MIPS range and a 36 bit floating point word providing 8 significant figures of accuracy in single precision. The information in the following table is provided to allow a comparison of the relative size and speed of the different models*.

| <u>Model</u> | <u>core used (words)</u> | <u>CPU time per call</u> | <u>(milliseconds)</u> |
|--------------|------------------------------|--------------------------|-----------------------|
| | | <u>Initialize</u> | <u>Continue</u> |
| SGP | 541 | .8 | 2.7 |
| SGP4 | 1,041 | 1.9 | 2.5 |
| SDP4 | 3,095 | 5.1 | 3.6 |
| SGP8 | 1,601 | 1.8 | 2.2 |
| SDP8 | 3,149 | 5.4 | 3.2 |

* The timing results are for the test cases in section thirteen with a one day prediction. Times may vary slightly with orbital characteristics and, for deep-space satellites, with prediction interval.

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