NGA.STND.0036_1.0.0_WGS84

NATIONAL GEOSPATIAL-INTELLIGENCE AGENCY (NGA) STANDARDIZATION DOCUMENT

DEPARTMENT OF DEFENSE

WORLD GEODETIC SYSTEM 1984

Its Definition and Relationships with Local Geodetic Systems

2014-07-08

Version 1.0.0

OFFICE OF GEOMATICS

SUBJECT TERMS

Datums, Datum Shifts, Datum Transformations, Datum Transformation Multiple Regression Equations, Defense Mapping Agency (DMA), Earth Gravitational Constant, Earth Gravitational Model (EGM), EGM2008, Earth Orientation Parameters (EOP), Ellipsoids, Ellipsoid Heights, Ellipsoidal Gravity Formula, Gauss Coefficients, Geodesy, Geodetic, Geodetic Heights, Geodetic Systems, Geoids, Geoid Heights, Geoid Undulations, Global Positioning System (GPS), Gravitation, Gravitational Coefficients, Gravitational Model, Gravitational Potential, Gravity, Gravity Formula, Gravity Potential, Local Datums, Local Geodetic Datums, Magnetic Coefficients, Magnetic Declination, Magnetics, Molodensky Datum Transformation Formulas, National Geospatial-Intelligence Agency (NGA), National Imagery and Mapping Agency (NIMA), Orthometric Heights, Regional Datums, Reference Frames, Reference Systems, World Geodetic System (WGS), World Geodetic System 1984 (WGS 84), World Magnetic Model (WMM).

NGA DEFINITION

Department of Defense Directive 5105.60, National Geospatial-Intelligence Agency (NGA) July 29, 2009 directs that:

NGA shall support US national security objectives by providing, timely, relevant, and accurate geospatial intelligence (GEOINT) to the Department of Defense (DoD), the Intelligence Community (IC), and other US Government (USG) departments and agencies; conducting other intelligence-related activities essential for US national security; providing GEOINT for safety of navigation information; preparing and distributing maps, charts, books, and geodetic products; designing, developing, operating, and maintaining systems related to the processing and dissemination of GEOINT; and providing GEOINT in support of the combat objectives of the Armed Forces of the United States.

It is DoD policy per *Department of Defense INSTRUCTION NUMBER 5000.56, Programming Geospatial-Intelligence (GEOINT), Geospatial Information and Services (GI&S), and Geodesy Requirements for Developing Systems Jul, 9, 2010* that:

a. All acquisition programs relying on GEOINT data shall be designed to use standard GEOINT data whose content, formats, and standards conform to those established by the National System for Geospatial Intelligence (NSG) through community government processes.

b. All acquisition programs shall make maximum use of standard military data and digital databases provided by the National Geospatial-Intelligence Agency (NGA) and the NSG.

c. DoD Components shall consider the availability of NGA resources and the impact of the use of unique GEOINT data, products, or services may have on the cost, schedule, or performance of system development or upgrades when unique GEOINT data, products, or services are required in the conduct of the system's operational mission profile.

PREFACE

This NGA Standard defines the Department of Defense (DoD) World Geodetic System 1984 (WGS 84). Significant changes incorporated in this standard include:

- Refined realization of the reference frame
- Description of Earth Orientation Parameters (EOP)
- Discussion of International Earth Rotation and Reference Systems Service (IERS) standards
- Development of a refined Earth Gravitational Model (EGM) and geoid
- Description of a World Magnetic Model (WMM)
- Updated list of datum transformations

Users requiring additional information, clarification, or an electronic version of this document should contact:

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WGS 84 is comprised of a coherent set of models and parameters. Organizations are advised NOT to make a substitution for any of the WGS 84 related parameters or equations. Such a substitution may lead to degraded WGS 84 products, interoperability problems and may have other adverse effects.

EXECUTIVE SUMMARY

This edition of the Department of Defense World Geodetic System 1984 (WGS 84) NGA Standard, NGA.STND.0036_1.0.0_WGS84 (formerly known as TR8350.2), reflects significant improvements and changes to WGS 84 since the 3rd edition was published in 1997. Today, the WGS 84 reflects a family of models, parameters, and a new reference frame update.

This standard will address the following subject areas with the WGS 84:

- Coordinate Systems
- The use of Global Positioning System (GPS) in the development of the WGS 84 Reference Frame
- Ellipsoid and its defining parameters
- Ellipsoidal Gravity formula
- Earth Gravitational Model 2008 (EGM2008)
- EGM2008 Geoid Model
- The World Magnetic Model (WMM)
- WGS 84 relationships with other Geodetic Systems
- Accuracy of WGS 84 and its models
- Implementation Guidelines

For those first time readers, the data, models, and information contained in this standard will address issues and analysis related to the following topical areas:

- Gravity Field determination and related issues
    - Components of the Gravity Field
    - Level Surfaces and Plumb Lines
    - Spherical Harmonic Expansion of the Gravitational Potential
- Geodetic Reference Systems and other Reference Frames
    - Geocentric Coordinate Systems and Polar Motion
    - Astronomic Coordinates and the Global Cartesian Coordinate System
    - Reference Surfaces
    - Normal Gravity Field
    - Satellite Observation and Global Navigation Satellite Systems (GNSS)
- Position, Navigation, and Timing (PNT) activities
    - WMM
    - EGM2008
    - Reference Frame
    - GPS
    - Earth Orientation Parameters (EOP)
    - Use of IERS models and constants
- Mapping and Charting
    - Horizontal Datum
    - Vertical Datum

- Geodetic, GIS Data, and Other High-Accuracy Applications:
  - Reference Frame
  - Coordinates
  - Earth Gravitational Model (EGM)
  - WGS 84 Geoid
  - Temporal considerations

NGA.STND.0036 provides a description and sufficient details regarding the global World Geodetic System's global reference frame to allow users of this information to implement this reference frame and its associated geophysical models and constants.

Today's modern GNSS's, such as GPS, have made it possible to establish a truly global geocentric reference system which can be quickly adapted for precise geodetic positioning, especially over long distances. It is possible to determine distortions and mis-orientations of classical geodetic networks around the world. The entire WGS is being incorporated into a Standards-based environment through the NGA Geospatial Intelligence Standards Working Group (GWG). The result will be that the WGS 84 and its future instantiations will become part of the DoD Information Standards Registry (DISR) thereby increasing interoperability and reducing acquisition costs.

Readers will note NGA is adopting multiple recommendations outlined in the International Earth Rotation and Reference Systems Service (IERS) Technical Note No. 36, IERS Conventions 2010 (hereafter referred to as IERS TN 36). This NGA Standard supplements IERS TN 36 as DoD guidance for implementation, particularly where WGS 84 differs from IERS TN 36. A new realization of the WGS 84 reference frame tied to IERS TN 36 and the International Terrestrial Reference Frame (ITRF) 2008, was established in 2013.

In mid-2015, a new edition of this Standard, incorporating an updated WGS 84 reference frame and an updated WMM 2015 will be written and propagated to the community. This anticipated edition will begin a five-year update cycle of the Standard and its related models as required. The 2020 update may be a rigorous review of the WGS which may result in refining parameters for the WGS.

TABLE OF CONTENTS

1. INTRODUCTION

The National Geospatial-Intelligence Agency (NGA) supports a large number and variety of products and users, which makes it imperative that these products all be related to a common worldwide geodetic reference system. This ensures interoperability in relating information from one product to another, supports increasingly stringent accuracy requirements, and supports military and humanitarian activities worldwide. The refined World Geodetic System 1984 (WGS 84) represents NGA's best geodetic model of the Earth using data, techniques and technology available through 2013.

The definition of the World Geodetic System has evolved within NGA and its predecessor agencies from the initial WGS 60 through subsequent improvements embodied in WGS 66, WGS 72, and WGS 84. The refinement described in this standard has been possible due to improved scientific models and additional global data. These data include those from precise and accurate geodetic positioning, new observations of land gravity data, the availability of extensive altimetry data, and gravity data from the GRACE satellite mission.

Using these data, an improved Earth Gravitational Model 2008 (EGM2008) and its associated geoid were developed, and the World Magnetic Model 2010 (WMM2010) was produced. EGM2008 was developed by NGA with contracted support. WMM2010 was developed jointly by the US National Oceanographic and Atmospheric Administration's National Geophysical Data Center and the British Geological Survey.

Commensurate with these modeling enhancements, significant improvements in the current realization of the WGS 84 reference frame have also been achieved through the continued improvement of the NAVSTAR Global Positioning System (GPS). WGS 84 is realized by the coordinates assigned to the GPS tracking stations used in the calculation of precise GPS orbits at NGA. NGA currently utilizes the six globally dispersed Air Force operational GPS tracking stations augmented by eleven tracking stations operated by NGA. The coordinates of these tracking stations have been determined to an absolute accuracy of ± 1 cm (1σ). WGS 84 coordinates are determined primarily through GPS positioning.

The WGS 84 represents the best global geodetic reference system for the Earth available at this time for practical applications of mapping, charting, geopositioning, and navigation. This standard includes the definition of the coordinate system, fundamental and derived constants, the EGM2008, the ellipsoidal (normal) gravity model, a description of the associated WMM, and a current list of local datum transformations. NGA recommendations regarding the implementation of WGS 84 are given in Chapter Ten of this standard.

2. WGS 84 COORDINATE SYSTEM

2.1 Definition

The WGS 84 Coordinate System is a Conventional Terrestrial Reference System (CTRS). The definition of this coordinate system follows the criteria outlined in International Earth Rotation and Reference Systems Service (IERS) Technical Note No. 36, IERS Conventions 2010 (IERS TN 36) [1]. These criteria are repeated below:

- It is geocentric, the center of mass being defined for the whole Earth including oceans and atmosphere
- Its scale is that of the local Earth frame, in the meaning of a relativistic theory of gravitation
- Its orientation was initially given by the Bureau International de l'Heure (BIH) orientation of 1984.0
- Its time evolution in orientation will create no residual global rotation with regards to the crust

The WGS 84 Coordinate System is a right-handed, Earth-fixed orthogonal coordinate system and is graphically depicted in Figure 2.1.

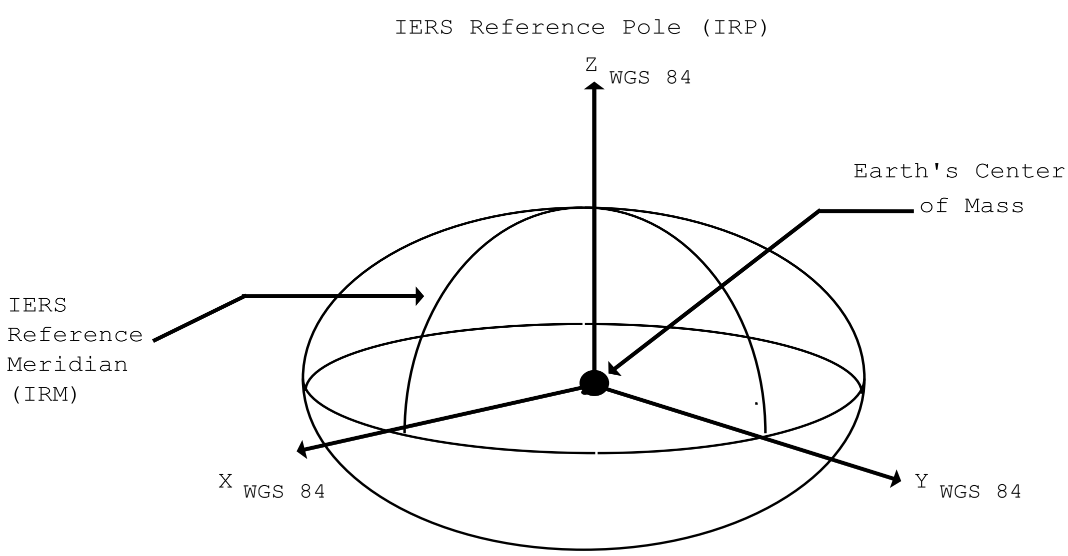


Figure 2.1 The WGS 84 Coordinate System Definition

In Figure 2.1, the origin and axes are defined as follows:

Origin = Earth's center of mass

Z-Axis = The direction of the IERS Reference Pole (IRP). This direction corresponds to the direction of the BIH Conventional Terrestrial Pole (CTP) (epoch 1984.0) with an uncertainty of 0.005″ [1]

X-Axis = Intersection of the IERS Reference Meridian (IRM) and the plane passing through the origin and normal to the Z-axis. The IRM is coincident with the BIH Zero Meridian (epoch 1984.0) with an uncertainty of 0.005″ [1]

Y-Axis = Completes a right-handed, Earth-Centered Earth-Fixed (ECEF) orthogonal coordinate system

The WGS 84 Coordinate System origin also serves as the geometric center of the WGS 84 Ellipsoid, and the Z-axis serves as the rotational axis of this ellipsoid of revolution.

The definition of the WGS 84 CTRS has not changed in any fundamental way since its original implementation. This CTRS continues to be defined as a right-handed, orthogonal, and Earth-fixed coordinate system which is intended to be as closely coincident as possible with the CTRS defined by the IERS or, prior to 1988, its predecessor, the Bureau International de l'Heure (BIH).

The Naval Surface Warfare Center Dahlgren Division (NSWCDD) generates the software which NGA uses for precise GPS orbits and WGS 84 coordinate solutions. To maintain WGS 84 to the highest level of accuracy and integrity, NGA and NSWCDD assess international scientific standards to determine their applicability to WGS 84. WGS 84 (G1762) adheres to IERS TN 36 with limited exceptions as identified in this document. Note that IERS TN 36 often gives multiple approaches for implementation of models. Specifically, WGS 84 (G1762) will not include pole corrections and will not include librations in UT1 and polar motion defined in IERS TN 36 Section 5.5. Previous iterations of WGS 84 (*i.e.* G1150) are consistent with IERS Conventions 1996 Technical Note No. 21 [2]. As new standards are adopted, NGA will review them to determine appropriate implementation. WGS 84 may not conform strictly to all conventions, but relevant differences will be documented.

2.2 Realization

It is important to understand the definition of a coordinate system and the practical realization of a reference frame. Section 2.1 contains the definition of the WGS 84 Coordinate System. To achieve a practical realization of a global geodetic reference frame, a set of station coordinates must be established. A consistent set of station coordinates infers the location of an origin, the orientation of an orthogonal set of Cartesian axes, and a scale. In modern terms, a globally distributed set of consistent station coordinates on the surface of the Earth represents a realization of an ECEF Terrestrial Reference Frame (TRF).

The original WGS 84 reference frame was established in 1987 using the Navy Navigation Satellite System (also called TRANSIT) [3]. The main objective in the original effort was to align, as closely as possible, the origin, scale and orientation of the WGS 84 frame with the BIH Terrestrial System (BTS) frame at an epoch of 1984.0. This development is given in DMA TR 8350.2, Editions 1 and 2. Initial uncertainties, in 1987, were 1-2 meters with respect to the BTS.

Since that time, NGA has used the combined network of the US Air Force (USAF) and NGA GPS satellite tracking stations to improve the WGS 84 reference frame. The WGS 84 reference frame and the GPS Monitor Stations are inherently intertwined. WGS 84 is the reference frame adopted for the operation of GPS, and all users obtain WGS 84 coordinates when they use the GPS Broadcast Navigation Messages. NGA generates a set of 'precise' ephemerides which adopt the WGS 84, and these ephemerides are used for precise positioning. This includes calculation of the WGS 84 positions of the permanent DoD GPS Monitor Stations for both the US Air Force (USAF) and NGA (Figure 2.2) and, therefore, the WGS 84 origin. For highest accuracy applications such as precise positioning applications, temporal effects should be considered as described in Section 2.2.4.

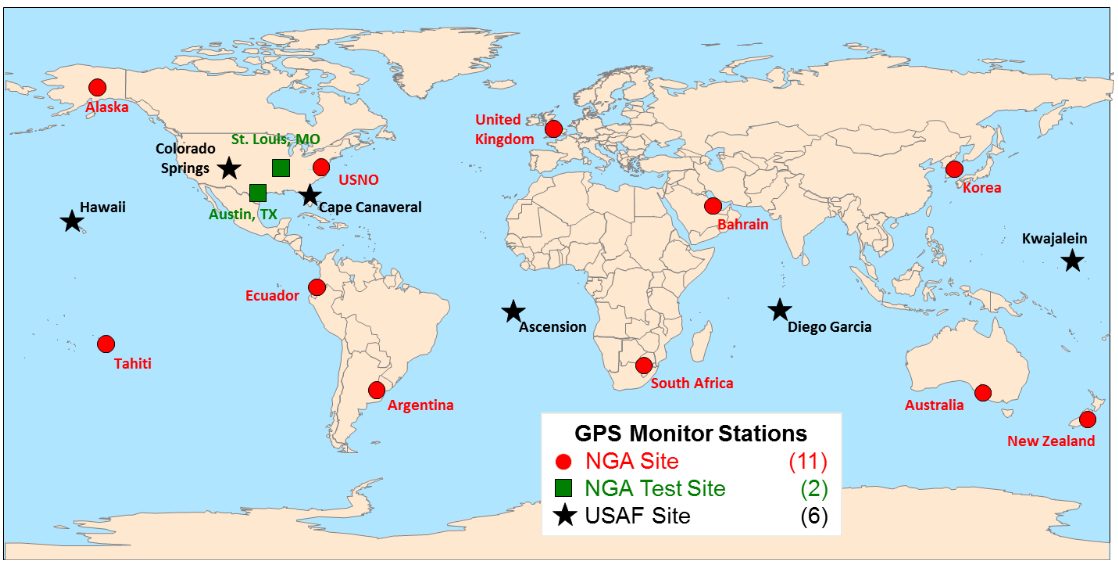


Figure 2.2 WGS 84 (G1762) Reference Frame Stations

NGA is committed to maintaining the highest possible accuracy and stability for the WGS 84 Reference Frame while remaining both practical and accessible. To do so, NGA has generated several realizations of WGS 84 incorporating improved data and advances in geophysical modeling to adequately represent the motion of the Earth's surface. These realizations have not affected the fundamental definition of the WGS 84 reference system; but are necessary for high-accuracy, high-integrity applications. The current realization of the WGS 84 Reference Frame is designated as WGS 84 (G1762). The GPS Operational Control Segment (OCS) implemented WGS 84 (G1762) on 16 Oct 2013 with NGA implementation on the same date.

WGS 84 (G1762) is the sixth update to the realization of the WGS 84 Reference Frame. The previous realizations were designated WGS 84 (G1674) [5], WGS 84 (G1150) [4], WGS 84 (G873), WGS 84 (G730) and WGS 84. The "G" indicates that GPS measurements were used to obtain the coordinates. The number following the "G" indicates the GPS week number during which the coordinates were approved for implementation by NGA. The original TRANSIT realization of WGS 84 has no such designation. Detailed documentation of the previous iterations can be found in the National Imagery and Mapping Agency (NIMA) TR8350.2, Edition 3, (3 Jan 2000) with its Addendum (2002). Table 2.1 shows the name, implementation date by the GPS OCS, epoch, and the overall absolute accuracy of each realization.

Table 2.1 WGS 84 Station Coordinate Updates

Name	Implementation date		Epoch	Accuracy
	GPS Broadcast Orbits	NGA Precise Ephemeris		
WGS 84	1987	1 Jan 1987		1-2 meters
WGS 84 (G730)	29 Jun 1994	2 Jan 1994	1994.0	10 cm/component rms
WGS 84 (G873)	29 Jan 1997	29 Sep 1996	1997.0	5 cm/component rms
WGS 84 (G1150)	20 Jan 2002	20 Jan 2002	2001.0	1cm/component rms
WGS 84 (G1674)	8 Feb 2012	7 May 2012	2005.0	<1cm/component rms
WGS 84 (G1762)	16 Oct 2013	16 Oct 2013	2005.0	<1cm/component rms

2.2.1 WGS 84 (G1762) Methodology

NGA updated the WGS 84 reference frame coordinates in July 2013, to incorporate international conventions in [1] for constants and methodology along with alignment to the International Terrestrial Reference Frame 2008 (ITRF2008). NGA contributes its GPS monitor station tracking data to the International GNSS Service (IGS). NGA used GPS observations from the USAF and NGA GPS monitor stations with IGS stations to estimate updated coordinates for the USAF and NGA stations (Figure 2.2).

- Data were collected from all sites for the period 11 May – 26 May, 2013.
- A subset of IGS stations was selected as control points in the reference frame solution. IGb08 is a realization of ITRF2008 generated by IGS for their products. The IGb08 coordinates, at epoch 2005.0, of the IGS stations were held fixed while the USAF and NGA stations coordinates were adjusted. According to IGS, the difference between reference station coordinates in IGb08 and ITRF2008 are due to antenna calibration changes. Transformation parameters between IGb08 and ITRF2008 are considered to be zero. Reference http://acc.igs.org/reprocess2.html for more information on IGS products, including reference frames.
- The IGS station positions were determined using their velocities and propagating that forward to the data collection timeframe.
- The WGS 84 coordinates are at epoch 2005.0.

- Meteorological data were utilized for all stations. Data from nearby sites or default values were used when meteorological data were not collected at the GPS station.

NGA adopted the velocities generated for the IGb08 coordinates of the NGA stations. For USAF stations, NGA adopted the velocities of nearby IGS sites. The velocities of the USAF and NGA stations used in this realization are given in Table 2.2. Site information for IGS stations may be obtained through the IGS website currently at http://igscb.jpl.nasa.gov.

2.2.2 Results

WGS 84 (G1762) improved the overall accuracy of WGS 84 and will reduce future discrepancies between WGS 84 and ITRF by improving consistency since, in general, both have adopted IERS Conventions 2010 methods and models. Previous WGS 84 realizations have shown consistency with ITRF on the order of 1-2 centimeters at their original implementation date. Over time, small changes in updated models and methods cause differences between the two reference frames to grow. Improvements in methods, models, and data, along with geophysical phenomena such as earthquakes, are the fundamental reasons for periodic updates to the WGS 84 reference frame. Each realization of WGS 84 has improved its accuracy (Table 2.1) and has increased in precision. Table 2.2 gives the positions of the USAF and NGA GPS monitor stations in Cartesian coordinates with station velocities, and Table 2.3 gives the geodetic coordinates.

Table 2.2 WGS 84 (G1762) Cartesian Coordinates* and Velocities for Epoch 2005.0

Station Location	NGA Station Number	X (m)	Y (m)	Z (m)	$\dot{X}$ (m/yr)	$\dot{Y}$ (m/yr)	$\dot{Z}$ (m/yr)
Air Force Stations							
Colorado Springs	85128	-1248599.695	-4819441.002	3976490.117	-0.0146	0.0009	-0.0049
Ascension	85129	6118523.866	-1572350.772	-876463.909	-0.0002	-0.0057	0.0110
Diego Garcia	85130	1916196.855	6029998.797	-801737.183	-0.0448	0.0176	0.0331
Kwajalein	85131	-6160884.028	1339852.169	960843.154	0.0201	0.0663	0.0295
Hawaii	85132	-5511980.264	-2200246.752	2329481.004	-0.0098	0.0628	0.0320
Cape Canaveral	85143	918988.062	-5534552.894	3023721.362	-0.0126	0.0016	0.0011
NGA Stations							
Australia	85402	-3939182.512	3467072.917	-3613217.139	-0.0409	0.0030	0.0485
Argentina	85403	2745499.034	-4483636.563	-3599054.496	0.0045	-0.0079	0.0085
England	85404	4011440.890	-63375.739	4941877.084	-0.0127	0.0168	0.0101
Bahrain	85405	3633910.105	4425277.147	2799862.517	-0.0324	0.0096	0.0270
Ecuador	85406	1272867.304	-6252772.044	-23801.759	0.0067	0.0013	0.0108
US Naval Observatory	85407	1112158.852	-4842855.557	3985497.029	-0.0150	-0.0001	0.0024
Alaska	85410	-2296304.083	-1484805.898	5743078.376	-0.0222	-0.0068	-0.0086
New Zealand	85411	-4749991.001	520984.518	-4210604.147	-0.0219	0.0127	0.0205
South Africa	85412	5066232.068	2719227.028	-2754392.632	-0.0012	0.0197	0.0168
South Korea	85413	-3067863.250	4067640.938	3824295.770	-0.0263	-0.0091	-0.0094
Tahiti	85414	-5246403.943	-3077285.338	-1913839.292	-0.0422	0.0515	0.0327

Notes: * Coordinates are at the Antenna Reference Points.
Reference http://earth-info.nga.mil/GandG/sathtml/ for current values

Table 2.3 WGS 84 (G1762) Geodetic Coordinates* for Epoch 2005.0

Station Location	NGA Station Number	Latitude North (Decimal Degrees)	Longitude East (Decimal Degrees)	Ellipsoid Height (m)
Air Force Stations				
Colorado Springs	85128	38.80293817	255.47540411	1911.778
Ascension	85129	-7.95132931	345.58786964	106.281
Diego Garcia	85130	-7.26984216	72.37092367	-64.371
Kwajalein	85131	8.72250188	167.73052378	39.652
Hawaii	85132	21.56149239	201.76066695	425.789
Cape Canaveral	85143	28.48373823	279.42769502	-24.083
NGA Stations				
Australia	85402	-34.72897999	138.64736789	34.955
Argentina	85403	-34.57370124	301.48070046	48.665
England	85404	51.11761208	359.09487379	139.647
Bahrain	85405	26.20914139	50.60814586	-14.770
Ecuador	85406	-0.21515709	281.50639195	2922.453
US Naval Observatory	85407	38.92056511	282.93368418	59.003
Alaska	85410	64.68789166	212.88698460	177.236
New Zealand	85411	-41.57619313	173.74075198	147.227
South Africa	85412	-25.74634537	28.22403818	1416.334
South Korea	85413	37.07756761	127.02403352	51.755
Tahiti	85414	-17.57702921	210.39381226	99.836

Notes: * Coordinates are at the Antenna Reference Point.
Reference http://earth-info.nga.mil/GandG/sathtml/ for current values.

The differences between WGS 84 (G1762) and WGS 84 (G1674) coordinates (Table 2.4) were computed. These differences result from changes in methodology, including the adoption of several IERS TN 36 [1] recommendations, and antenna movement (in some cases due to site relocation). Since both sets of coordinates are at the same epoch, there was no need to remove the effects of plate tectonic motion using the station velocities.

The G1762 realization marks the first time the WGS 84 Reference Frame (RF) is tied to the site's Antenna Reference Point (ARP) located at the base of the ground plane which supports the antenna's electronics. All previous realizations of the WGS 84 RF were tied to the antenna's electrical phase center so as to conform to the GPS Control Segment's expectations. However, modern standards for reporting reference frame anchor points are for the ARP. This difference, coupled with NGA's adaptation of elevation angle dependent antenna phase center calibration tables, is the cause of the relatively large discrepancies seen in Table 2.4, almost all in the vertical direction.

Table 2.4 WGS 84 (G1674) Minus WGS 84 (G1762) for Epoch 2005.0

Station Location	NGA Station Number	ΔX (m)	ΔY (m)	ΔZ (m)	Total (m)
Air Force Stations					
Colorado Springs	85128	0.018	-0.031	0.017	0.040
Ascension	85129	0.044	0.017	0.000	0.047
Diego Garcia	85130	-0.033	0.025	0.032	0.053
Kwajalein	85131	-0.011	-0.004	-0.002	0.012
Hawaii	85132	-0.003	-0.047	-0.007	0.047
Cape Canaveral	85143	0.011	-0.039	0.016	0.043
NGA Stations					
Australia	85402	-0.018	0.048	-0.036	0.062
Argentina	85403	0.061	-0.032	-0.039	0.079
England	85404	0.007	-0.002	0.038	0.039
Bahrain	85405	-0.015	0.012	0.020	0.027
Ecuador	85406	0.022	-0.091	-0.008	0.094
US Naval Observatory	85407	0.012	-0.050	0.037	0.064
Alaska	85410	0.010	-0.017	0.003	0.021
New Zealand	85411	-0.038	-0.016	-0.064	0.077
South Africa	85412	0.054	0.019	-0.027	0.063
South Korea	85413	-0.026	0.058	0.046	0.079
Tahiti	85414	-0.056	-0.006	-0.016	0.059

2.2.3 Comparisons to Other Reference Frames

NGA and its predecessor organizations have ensured that WGS 84 is consistent with the most recent ITRF realization. The purpose of this alignment is to adhere to international standards and pursue the highest possible level of practical global reference frame accuracy. The ITRF incorporates multiple methods to realize their series of reference systems such as Satellite Laser Ranging (SLR) and Very-Long-Baseline Interferometry (VLBI) that NGA does not include. Constraining the WGS 84 reference frame to align with ITRF as closely as possible allows the WGS 84 reference frame to take advantage of those methods without directly incorporating them into the coordinate determination software. This alignment is necessary for interoperability with other Global Navigation Satellite Systems (GNSS). While GPS is currently the most widely used GNSS, there are other systems operating or being developed. The US bi-lateral agreement with the European Union states that WGS 84 and ITRF will be closely aligned to support interoperability between GPS and Galileo.

The most recent update of WGS 84 (G1762) aligns WGS 84 with ITRF with an accuracy better than one cm-per-component, resulting in an overall difference of less than one cm. Specific comparisons with other reference systems are given in Chapter 7. Transformations between WGS 84 (G1762) and earlier versions of WGS 84 and IGb08 are given in Table 2.5. The comparison of WGS 84 (G1762) and WGS 84 (G1674) shows that a bias was introduced into WGS 84 (G1674) which is now corrected in WGS 84 (G1762). This bias is also indicated by a resultant zero rotation when WGS 84 (G1762) is compared to WGS 84 (G1150). As shown in Table 2.5, the transformation between WGS 84 (G1762) and IGb08 is zero by statistical analysis. Mean differences are 1-2 millimeters and significantly less than the error in the conversion, making the transformation parameters effectively zero. This is expected since the IGb08 coordinates and velocities were adopted for NGA stations with corrections only for movement of the stations or antennas. Because of the relationship of IGb08 to ITRF2008, it is equivalent to state that the transformation parameters between WGS 84 (G1762) and ITRF2008 are also zero.

Table 2.5 Comparison to WGS 84 (G1762)

Reference Frame (reference frame epoch)	**Δx (mm) (sigma)**	**Δy (mm) (sigma)**	**Δz (mm) (sigma)**	**D (ppb) (sigma)**	**Rx (mas) (sigma)**	**Ry (mas) (sigma)**	**Rz (mas) (sigma)**
WGS 84 (G1674)# (2005.0)	-4 (5.2)	3 (5.2)	4 (5.2)	-6.9 (0.82)	0.27 (0.215)	-0.27 (0.212)	0.38 (0.196)
WGS 84 (G1150)# (2001.0)	-6 (4.7)	5 (4.7)	20 (4.7)	-4.5 (0.74)	0 *	0 *	0 *
ITRF2008* (2005.0)	0 *	0 *	0 *	0 *	0 *	0 *	0 *

Notes: The parameters are defined from the listed reference frame to WGS 84 (G1762) at epoch 2005.0.

The sign convention for the rotations Rx, Ry, and Rz is what NGA uses in its orbit comparison programs and is opposite to that of IERS Conventions 2010.

*Mean differences are 1-2 millimeters and significantly less than the error in the conversion, thus they are effectively zero.

where Δx, Δy, Δz, D, R_x, R_y, and R_z describe the transformation in the equation

$$\left(\begin{bmatrix} 1 & R_z & -R_y \\ -R_z & 1 & R_x \\ R_y & -R_x & 1 \end{bmatrix} + D\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\right)\begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_{WGS\ 84} + \begin{bmatrix} \Delta x \\ \Delta y \\ \Delta z \end{bmatrix} = \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_{ITRF\,2008}$$

where *X*, *Y*, and *Z* represent the Cartesian position in the reference frame.

2.2.4 Temporal Effects

In an ECEF reference frame, geophysical phenomena such as plate tectonics, earthquakes, subsidence and tides, cause changes to station positions.

Each of these temporal changes must be quantified and station coordinates updated in order to maintain the accuracy of the reference frame (NGA provides updated station coordinates for inclusion in the GPS broadcast ephemeris process when necessary). To perform this process, the timeframe (epoch) of the station coordinates and their velocities must be known (Tables 2.1 and 2.2). By applying these values, the user can determine the instantaneous center of the Earth at the stated accuracy of the reference frame. Neglecting these effects adds error to the GPS orbit determination process. Users must determine if these temporal effects should be accounted for, or are negligible to their system or application requirements.

The precision of the current realization of the WGS 84 Reference Frame (G1762) is better than one centimeter overall (<1 cm per component). Maintaining that one-centimeter accuracy poses special challenges as discussed below in detail.

2.2.4.1 Plate Tectonic Motion

To maintain centimeter-level accuracy in a CTRF, a given set of station positions represented at a particular epoch must be updated for the effects of plate tectonic motion. Every point on the Earth, at a particular time or epoch, experiences relative motion with respect to other points on the Earth. This motion, known as plate tectonic motion, has been observed to be as much as 7 cm/year at some DoD GPS tracking stations. One way to handle motions is to estimate velocity parameters along with the station position parameters. This is recommended for applications that require the highest level of accuracy. For most DoD applications, however, this approach is not practical since the observation period is not of a sufficient length to provide significant velocity estimates, nor do the geodetic surveying algorithms in common use provide station velocities. Instead, if accuracy requirements warrant it, DoD practitioners must decide the best method to account for plate tectonic motion. This can be done using a plate motion model instead of known velocities to account for these translational effects. A map of the sixteen major tectonic plates is given in Figure 2.3 [6].

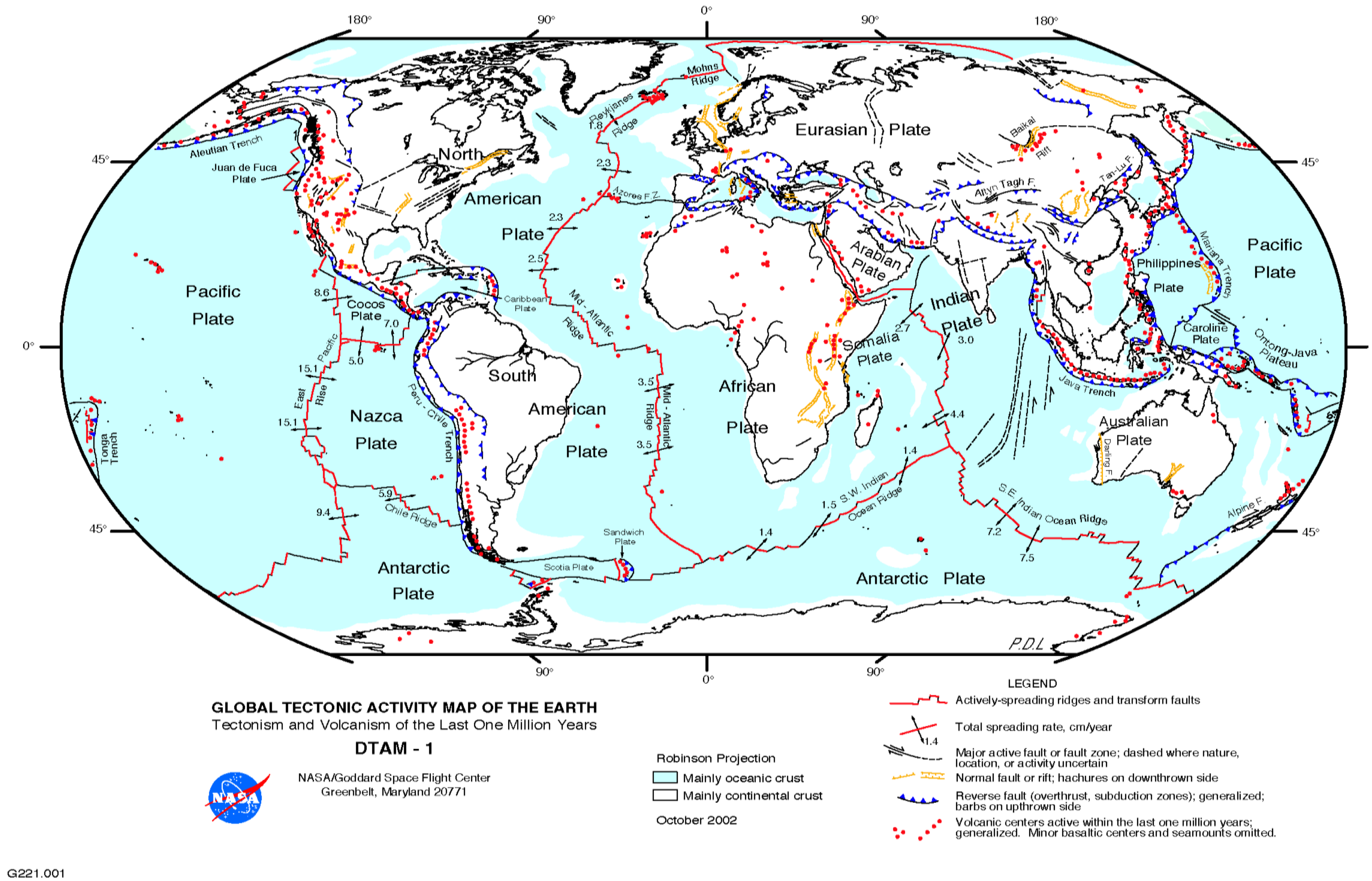


Figure 2.3 Major Plate Boundaries

NGA uses stations velocities to account for plate tectonic motion in both the development of G1762 and in precise GPS orbit calculations. At the beginning of each year, NGA supplies the USAF with station coordinates of both the USAF and NGA sites. The coordinates are adjusted to an epoch at the half year mark for plate tectonic motion for use in GPS orbit computation. For example, on 25 Jan 2013, the USAF began using coordinates for the DoD GPS reference sites with an epoch of 2013.5. This results in an annual step-wise update to the WGS 84 frame used to generate the GPS broadcast message.

The amount of time elapsed between the epoch of a station's coordinates and the time of interest will be a dominant factor in deciding whether application of station velocities or a plate motion model is warranted. For example, a station on a plate that moves at a rate of 5 cm/year may not require station velocity/plate motion model correction if the epoch of the coordinates is less than a year old. If, however, these same coordinates are used over a 5-year period, 25 cm of horizontal displacement will have accumulated in that time and application of a correction may be advisable, depending on the accuracy requirements of the geodetic survey, application, or system.

2.2.4.2 Sudden Displacement of Stations

Earthquakes and other episodic movements of the Earth's crust may cause sudden displacement of GPS monitor stations. These sudden displacements must be incorporated in station coordinate references for high accuracy positioning and navigation. In November 2002, NGA's Alaska station had an approximately 8 cm displacement caused by a 7.9 magnitude earthquake. In March 2011, the 9.8 magnitude earthquake that struck Japan moved NGA's station in South Korea approximately 4 cm. When sudden movements like these occur, a re-determination of the monitor station's coordinates is necessary for the station to maintain its stated accuracy. New coordinates are calculated and moved to the epoch of the current reference frame to maintain consistency. Station displacement varies depending upon the earthquake's magnitude and distance from the station. In each of the cases above, the steady-state velocity of the station was assumed to be unchanged due to the earthquake. Depending on the accuracy required by the user, this may not be a valid assumption. Small changes in station velocities have been documented immediately following these major events. Users with sub-centimeter requirements cannot ignore this issue.

Equipment changes or moves can also cause sudden monitor station displacement. In 2009 and 2010, new antennas were installed at USAF and NGA monitor stations resulting in each site being displaced by small amounts. The actual movements were minimal in comparison to plate tectonic motion. Users who are not concerned with better than decimeter-level accuracy are not affected.

When any displacement of NGA or USAF stations occur that would warrant permanent changes that impact station coordinates, NGA provides direct notification to the USAF so that the new coordinates may be utilized in their GPS orbit determination process;

furthermore, NGA also notifies GPS users via http://earth-info.nga.mil/GandG/sathtml/ which should be consulted for current authoritative values.

2.2.4.3 Earth Tide Effects

Earth tide phenomena are an additional source of temporal and permanent displacements of a station's coordinates. These displacements can be modeled to some degree depending on the desired accuracy and spatial extent. In the most demanding applications (centimeter-level or better accuracy), the user must address these changes. The IERS TN 36 outlines procedures for handling these displacements. The results of following these conventions lead to station coordinates in a 'zero-tide' system. In practice, however, the coordinates are typically represented in a 'tide-free' system. The NGA GPS precise ephemeris estimation process uses a tide-free model as described in [1], Section 7.1.1. In this 'tide-free' system, both the temporal and permanent displacements are removed from a station's coordinates.

Note that many practical geodetic surveying algorithms are not equipped to rigorously account for tidal effects. Often, these effects are completely ignored or allowed to 'average-out'. This approach may be adequate if the data collection period is long enough since the majority of the displacement is diurnal and semi-diurnal in nature. Coordinates determined from GPS differential (baseline) processing would typically contain whatever tidal components are present in the coordinates of the fixed (known) end of the baseline. If decimeter level or better absolute accuracy is required, careful consideration must be given to these station displacements since the peak absolute, instantaneous effect can be as large as 42 cm [7]. In the most demanding applications, a rigorous model such as that outlined in [1] should be applied.

2.3 Earth Orientation Parameters

Objects on the Earth are measured in an earth-fixed reference system or CTRS. Satellite orbit determination is performed in an inertial reference system. Earth Orientation Parameters (EOP) are required for transformation between the CTRS and the inertial reference system.

Since satellite equations of motion are appropriately handled in an inertial coordinate system, the concept of a Conventional Celestial Reference System (CCRS) (alternately known as a Conventional Inertial System (CIS)) is employed in most DoD orbit determination operations. The CCRS is based on a kinematic definition, making the axes' directions fixed with respect to distant matter in the universe. In other words, the reference frame is defined by the very precise coordinates of extragalactic objects, mostly quasars [8]. This is known as the International Celestial Reference System (ICRS) with the J2000.0 Earth-Centered Inertial (ECI) reference frame being a practical realization for orbit determination applications. Two equivalent procedures for the coordinate transformation from the ITRS to the Geocentric Celestial Reference System (GCRS) expressed in Eq. (2-1) are presented in IERS TN 36 and the earlier IERS Conventions [2]. According to the International Astronomical Union (IAU), these procedures differ in the origin adopted for the right ascension coordinate measured along the equator of the Celestial Intermediate Pole (CIP) (*i.e.* the equinox or the celestial intermediate origin (CIO)). In the case of the former, the reference frame is called "equinox-based" and, in the case of the latter, "CIO-based". Each procedure is based on a specific representation of the transformation matrix components $Q(t)$ and $R(t)$ of Eq. (2-1), which depends on the corresponding origin on the CIP equator, while the representation of the transformation matrix component $W(t)$ is common to both techniques. NGA has implemented the Celestial Intermediate Origin (CIO) in its orbit determination process. Since a detailed definition of these concepts is beyond the scope of this document, the reader is referred to Appendix A and [1] for in-depth discussion. Appendix A discusses the transformation between the CIO and GCRS.

Traditionally, the mathematical relationship between the ICRS and a CTRS (in this case, the WGS 84 Coordinate System) at the date (t) is expressed as

$$ICRS = [Q(t)][R(t)][W(t)]\ CTRS \qquad (2\text{-}1)$$

where the matrices Q(t), R(t), and W(t) are the transformational matrices representing the effects of nutation and precession, Earth rotation, and polar motion, respectively. Precession/nutation (coordinates of CIP in the ICRS or celestial pole offsets), Earth rotation, and polar motion parameters are known as Earth Orientation Parameters (EOP).

The specific formulations for the generation of matrices Q(t), R(t), and W(t) can be found in the references cited above and appendix A. With adoption of new precession-nutation models, IAU 2006/2000 Precession-Nutation Model [1], the coordinates of the CIP in the GCRS, X and Y, can be modeled at the 0.2 milli arc-second (mas) level. With such high levels of precision, observations and predictions of the coordinates X and Y or celestial pole offsets (i.e. Q matrix elements) are not necessary for near real-time orbit determination applications. Therefore, the discussion of EOPs and EOP predictions presented here will only refer to Earth rotation and polar motion (i.e. R and W matrix elements).

Note that for near-real-time orbit determination applications, EOPs must be predicted values. Because it is difficult to model Earth rotation and polar motion with high accuracy over an extended period of time, predictions are performed daily. Within the DoD, NGA and the USNO supply these daily predictions. When the model is evaluated at a specific time, polar motion and Earth rotation predictions can be computed. The polar motion (x_p and y_p) predictions represent directional offsets from the IERS Reference Pole (IRP) in the direction of 0° and 270° longitude, respectively. The Earth rotation predictions, more specifically, UT1-UTC predictions, represent the difference between the actual rotational time scale, UT1, and the uniform time scale, UTC (Coordinated Universal Time). For consistency with NGA's usage of the EOP parameters, the user needs to restore the zonal tides as presented in [1, Section 8.1], apply the diurnal and semi-diurnal tide corrections to polar motion and UT1-UTC using the "pmut1_oceans" routine contained in [1, Section 8.2, alternate software 'interp.f'] and not apply the libration effects [1, Section 5.5.1]. Note the interp.f software contains higher precision tables than those published in [1, Tables 8.2 & 8.3]. Position error due to unaccounted EOPs (combined effect of polar motion and Earth rotation variability) could exceed one meter per day on the surface of the Earth. In the future, these predictions may be performed more than once per day. The NGA EOP prediction models are given in equation (2-2). The NGA EOP prediction models are documented and the quantities A, B, C_n, D_n, E, F, G_n, H_n, P_n, Q_n, I, J, K_n, L_n, R_n, T_a, T_b, and T are defined on http://earth-info.nga.mil/GandG/sathtml/.

$$
\begin{aligned}
x_p &= A + B(T - T_a) + \sum_{n=1}^{2} C_n \cdot \sin\left(\frac{2\pi(T - T_a)}{P_n}\right) + \sum_{n=1}^{2} D_n \cdot \cos\left(\frac{2\pi(T - T_a)}{P_n}\right) \\
y_p &= E + F(T - T_a) + \sum_{n=1}^{2} G_n \cdot \sin\left(\frac{2\pi(T - T_a)}{Q_n}\right) + \sum_{n=1}^{2} H_n \cdot \cos\left(\frac{2\pi(T - T_a)}{Q_n}\right) \qquad (2\text{-}2) \\
UT1 - UTC &= I + J(T - T_b) + \underbrace{\sum_{n=3}^{4} K_n \cdot \sin\left(\frac{2\pi(T - T_b)}{R_n}\right) + \sum_{n=3}^{4} L_n \cdot \cos\left(\frac{2\pi(T - T_b)}{R_n}\right)}_{seasonal\ variations}
\end{aligned}
$$

2.3.1 Tidal Variations in the Earth's Rotation

The actual Earth rotation rate (represented by UT1) undergoes periodic variations due to tidal deformation of the polar moment of inertia. These highly predictable periodic variations have a peak-to-peak amplitude of 3 milliseconds and can be modeled by using the formulation found in Chapter 8 of [1]. If an orbit determination application requires extreme accuracy and uses tracking data from stations on the Earth, these UT1 variations should be modeled in the orbit estimation process. See Appendix A.

2.4 Summary

WGS 84 (G1762) represents the most recent realization of the WGS 84 Reference Frame. Further improvements and future realizations of the WGS 84 Reference Frame are anticipated. When new stations are added to the permanent DoD GPS tracking network or when existing stations (and/or antennas) are moved or replaced, new station coordinates will be required. NGA will update the WGS 84 Reference Frame as new standards are developed and as new methods and models are implemented. As these changes occur, NGA will take steps to ensure that the highest possible degree of fidelity is maintained and changes are identified to the appropriate organizations. Changes to WGS 84 will be made to ensure that it remains state-of-the-art while considering the general user is not unduly impacted by frequent updates. Users should reference http://earth-info.nga.mil/GandG/sathtml/ for authoritative information particularly for WGS 84 reference frame coordinates and Earth orientation models.

3. WGS 84 ELLIPSOID

3.1 General

Many geodetic applications involve three different surfaces, which should be clearly defined. The first of these is the Earth's topographic surface. This surface includes the familiar landmass topography as well as the ocean bottom topography. In addition to this highly irregular topographic surface, a mathematically manageable reference surface (an ellipsoid) approximating the broad features of the figure and of the gravity field of the Earth is useful both in cartographic and in gravimetric applications. Finally, an equipotential surface of the gravity field of the Earth, called the geoid (Chapter 6), is fundamental to gravimetric applications.

An oblate ellipsoid of revolution (i.e., bi-axial), whose surface is also an equipotential surface of its gravity field, rotating with the same (average) angular velocity as the Earth around the same (average) rotation axis, constitutes a mathematical construct with a suitable reference surface and reference (normal) gravity field. Due to the requirement that its surface is also an equipotential surface of its gravity field, four defining parameters are sufficient to determine completely and uniquely its geometric properties, as well as the dynamic properties of its associated (normal) gravity field, sometimes referred to as the "Somigliana-Pizzetti" normal gravity field. The WGS 84 ellipsoid is defined by the semi-major axis (a) and reciprocal flattening (1/f) of an oblate ellipsoid of revolution, and the Geocentric Gravitational Constant (GM) and angular rotational velocity (ω). The first two parameters (a, 1/f) completely define the geometry of the rotational ellipsoid, while the other two parameters (GM, ω) permit the unique determination of its associated normal gravity field.

3.2 Historical Development

While selecting the WGS 84 Ellipsoid and associated parameters, the original WGS 84 Development Committee decided to closely adhere to the approach used by the International Union of Geodesy and Geophysics (IUGG), when the latter established and adopted the Geodetic Reference System 1980 (GRS 80) [9]. Accordingly, a geocentric ellipsoid of revolution was taken as the form for the WGS 84 Ellipsoid. The parameters selected to originally define the WGS 84 Ellipsoid were the semi-major axis (a), the Earth's gravitational constant (GM), the angular velocity of the Earth (ω), and the normalized second degree zonal gravitational coefficient ($\bar{C}_{2,0}$). These parameters are identical to those of the GRS 80 Ellipsoid with one exception. The form of the coefficient used for the normalized second degree zonal is that of the original WGS 84 Earth Gravitational Model rather than the dynamical form factor of the Earth (J_2) used with GRS 80. J_2 and $\bar{C}_{2,0}$ are related by:

$$\bar{C}_{2,0} = -1 \times \frac{J_2}{\sqrt{5}} \qquad (3\text{-}1)$$

In 1993, two efforts were initiated which resulted in significant refinements to the original WGS 84 defining parameters. The first refinement occurred when DMA recommended, based on a body of empirical evidence, a refined value for the GM parameter [10], [11]. In 1994, this improved GM parameter was recommended for use in all high-accuracy DoD orbit determination applications. The second refinement occurred when the joint NASA/NIMA Earth Gravitational Model 1996 (EGM96) project produced a new estimated dynamic value for the second degree zonal coefficient.

A decision was made to retain the original WGS 84 Ellipsoid semi-major axis and flattening factor of the Earth values (a = 6378137.0 m and 1/f = 298.257223563). For this reason the four defining parameters for WGS 84 were chosen then to be a, f, GM, and ω. Further details regarding this decision are provided in [12]. The reader should also note that the refined GM value is within 1σ of the original (1987) GM value. Additionally, there are now distinct values for the $\bar{C}_{2,0}$ term, one dynamically derived as part of the WGS 84 Earth Gravitational Models and the other geometric, implied by the defining parameters.

3.3 Alignment to International Standards

One of the goals of the WGS 84 Development Committee was to examine the various International Standards to see where WGS 84 could best be aligned with them, where practical, without causing detrimental change for the users of WGS 84. The WGS 84 team used IERS TN 36 as the major source document to update and align certain constants and parameters. The following sections will highlight those constants that are in agreement with International Standards.

3.4 WGS 84 Defining Parameters

3.4.1 Semi-major Axis (a)

The semi-major axis (a) is one of the defining parameters for WGS 84. The original WGS 84 semi-major axis will be retained. This value is the same as that of the GRS 80 Ellipsoid. As stated in [13], the GRS 80, and thus the WGS 84 semi-major axis is based on estimates derived from work performed during 1976-1979 using laser, Doppler and radar altimeter data and techniques. Its adopted value is:

$$a = 6378137.0 \, meters \tag{3-2}$$

Although more recent, improved estimates of this parameter have become available, these new estimates differ from the above value by only a few decimeters. More importantly, the vast majority of practical applications, such as GPS receivers and mapping processes, use the ellipsoid only as a convenient reference surface; they do not require an ellipsoid that best fits the geoid. Retaining the original major-axis value for the WGS 84 Ellipsoid eliminates the need to enact numerous software modifications to GPS receivers and mapping processes and to transform or re-compute coordinates for the large body of accurate geospatial data which have been collected and referenced to the WGS 84 Ellipsoid in the last two and half decades. Highly specialized applications and experiments which require the 'best-fitting' ellipsoid parameters can be handled separately, outside the mainstream of DoD geospatial information generation.

3.4.2 Flattening Factor of the Earth (1/f)

The flattening factor of the Earth (1/f) is one of the defining parameters for WGS 84 and remains the same as previously defined. This term was called 'flattening'. This edition will rename this term to be 'flattening factor of the Earth' to align with other international naming conventions. Its adopted value is:

$$1/f = 298.257223563 \tag{3-3}$$

As discussed in 3.4.1, there are numerous practical reasons for retaining this flattening factor of the Earth value along with the semi-major axis as part of the definition of the WGS 84 Ellipsoid.

The original WGS 84 development effort used the fully-normalized second degree zonal harmonic coefficient value $\bar{C}_{2,0}$ as a defining parameter. In this case, the flattening factor of the Earth value was derived from $\bar{C}_{2,0}$ through an accepted, rigorous expression. This derived flattening factor turned out to be slightly different than the GRS 80 flattening because the $\bar{C}_{2,0}$value was truncated in the normalization process. Although this slight difference has no practical consequences, the flattening factor of the Earth for the WGS 84 Ellipsoid is numerically distinct from the GRS 80 flattening factor of the Earth.

3.4.3 Geocentric Gravitational Constant (GM)

The central term in the Earth's gravitational field (GM) is known with much greater accuracy than either 'G', the universal gravitational constant, or 'M', the mass of the Earth. Significant improvement in the knowledge of GM has occurred since the original WGS 84 development effort and the original WGS 84 value was updated in 1994 and reflected in [12]. The value from TR8350.2 3rd edition will be retained. The term will be renamed to 'Geocentric Gravitational Constant (GM)' to align with other international naming conventions. The value of the WGS 84 GM parameter is:

$$GM = 3.986004418 \times 10^{+14}\, meters^3/seconds^2 \tag{3-4}$$

This value is recommended in [1] and includes the mass of the atmosphere. The GPS Operational Control Segment (OCS) shall use this value in its orbit determination process. See Sec. 3.7.1 for further discussion on the use of GM for GPS equipment. Also see Sec. 3.6.2 for further discussion on the individual 'G' and 'M' terms.

3.4.4 Nominal Mean Angular Velocity of the Earth (ω)

The nominal mean angular velocity of the Earth (ω) is one of the defining parameters for WGS 84. The original WGS 84 nominal mean angular velocity of the Earth from previous TR8350.2 will be retained. The term will be renamed to Nominal Mean Angular Velocity of the Earth to align with other international naming conventions.

$$\omega = 7.292115 \times 10^{-5}\ radians/second \tag{3-5}$$

This value represents a standard Earth rotating with a constant angular velocity, and is recommended in [1]. Note that the actual angular velocity of the Earth fluctuates with time. Some geodetic applications that require angular velocity do not need to consider these fluctuations.

Table 3.1 WGS 84 Defining Parameters

Parameter	Symbol	Value	Units
Semi-major Axis	a	6378137.0	m
Flattening Factor of the Earth	1/f	298.257223563	
Geocentric Gravitational Constant	GM	$3.986004418 \times 10^{+14}$	m^3 / s^2
Nominal Mean Angular Velocity of the Earth	ω	7.292115×10^{-05}	rads / s

3.5 Special WGS 84 Parameters

3.5.1 WGS 84 EGM2008 Dynamic Second Degree Zonal and Sectorial Harmonics

The following values are the dynamic second degree zonal and sectorial harmonics directly from the EGM2008 spherical harmonics data file. They should not be confused with the purely geometric second degree zonal and sectorial harmonics that can be derived from the ellipsoid parameters. To add to this, multiple EGMs now exist within WGS 84. In an effort to help reduce ambiguity and clearly identify which term is being used, the suffix of 'dyn' or 'geo' are being added to the $\bar{C}_{2,0}$ term to represent the EGM dynamic or purely geometric parameter, respectively. The dynamic parameters will include the year of the WGS 84 EGM used. As demonstrated below, the EGM2008 derived dynamic second degree zonal and sectorial harmonics are represented as

$$\bar{C}_{2,0dyn[2008]} = -4.84165143790815 \times 10^{-04} \tag{3-6}$$

$$\bar{C}_{2,2dyn[2008]} = 2.43938357328313 \times 10^{-06} \tag{3-7}$$

These values, along with the dynamical ellipticity, are used in the computation of the Moments of Inertia.

Table 3.2 Special WGS 84 Parameters

Parameter	Symbol	Value
WGS 84 Earth Gravitational Model 2008 Dynamic Second Degree Zonal and Sectorial Harmonics	$\bar{C}_{2,0\,\mathrm{dyn[2008]}}$	$-4.84165143790815 \times 10^{-04}$
	$\bar{C}_{2,2\,\mathrm{dyn[2008]}}$	$2.43938357328313 \times 10^{-06}$

3.6 Other Fundamental Constants

3.6.1 Velocity of Light (in a vacuum) (c)

The accepted value for the velocity of light in a vacuum (c) [1] is:

$$c = 2.99792458 \times 10^{+08} meters/second \qquad (3\text{-}8)$$

3.6.2 Universal Constant of Gravitation (G)

The recently updated and accepted value for the universal constant of gravitation (G) [1] is:

$$G = 6.67428 \times 10^{-11} meters^3/(kilograms \cdot second^2) \qquad (3\text{-}9)$$

It should be noted that the Geocentric Gravitational Constant (GM) is simply the product of the Universal Constant of Gravitation (G) and the Mass of the Earth (M). With the recent update to G, the various International Standards groups agreed to continue to maintain the historical product of G × M and adjust the G and M terms accordingly. The WGS 84 Development Committee has implemented these updated changes. Therefore, it should be noted that the Mass of the Earth, M, has changed as well.

3.6.3 Total Mean Mass of the Atmosphere (with water vapor) (M_A)

Lacking an internationally accepted value for the mass of the atmosphere (with water vapor) (MA), the recommend value [14] is:

$$M_A = 5.1480 \times 10^{+18}\ kilograms \qquad (3\text{-}10)$$

3.6.4 Dynamic Ellipticity (H)

The dynamical ellipticity (H) is necessary to determine the Earth's principal moments of inertia; A, B, and C. In literature H is variously referred to as dynamical ellipticity, mechanical ellipticity, or the precessional constant. It is a factor in the theoretical value of the rate of precession of the equinoxes, which is well known from observation. The accepted value for the dynamic ellipticity (H) [1] is:

$$H = 3.273795 \times 10^{-03} \tag{3-11}$$

Table 3.3 Other Fundamental Constants and Best Accepted Values

Parameter	**Symbol**	**Value**	**Units**
Velocity of Light (in a vacuum)	c	$2.997\ 924\ 58 \times 10^{+08}$	m / s
Universal Constant of Gravitation	G	$6.674\ 28 \times 10^{-11}$	m^3 / kg s^2
Total Mean Mass of the Atmosphere (with water vapor)	M_A	$5.148\ 0 \times 10^{+18}$	kg
Dynamic Ellipticity	H	$3.273\ 795 \times 10^{-03}$	unitless

3.7 Special Applications of the WGS 84 Defining Parameters

3.7.1 Special Considerations for GPS

Based on a recommendation in a DMA letter to the Air Force [11], the refined WGS 84 GM value ($3.986004418 \times 10^{14}\ m^3/s^2$) was implemented in the GPS OCS during the fall of 1994. This improvement removed a 1.3 meter radial bias from the OCS orbit estimates.

The process that generates the predicted broadcast navigation messages in the OCS also uses a GM value to create the quasi-Keplerian elements from the predicted Cartesian state vectors. These broadcast elements are then interpolated by a GPS receiver to obtain the satellite position at a given epoch.

To avoid any loss of accuracy, the GPS receiver's interpolation process must use the same GM value that was used to generate the fitted parameters of the broadcast message. Note that this fitting process is somewhat arbitrary but must be commensurate with the algorithm in the receiver. There are many thousands of GPS receivers in use around the world, and any proposed, coordinated software modifications to these receivers would be a costly, unmanageable endeavor. As a result, Aerospace Corporation [15] suggested that the original WGS 84 GM value be retained in GPS receivers and in the OCS process which fits a set of broadcast elements to the Cartesian vectors. This approach takes advantage of the improved orbit accuracy for both the estimated and predicted states facilitated by the refined GM value and avoids the expense of software modifications to all GPS receivers.

For the above reasons, the GPS interface control document (ICD-GPS-200), which defines the space segment to user segment interface, shall retain the original WGS 84 GM value. The refined WGS 84 GM value shall be used in the OCS orbit estimation process. Most importantly, this approach avoids the introduction of any error to a GPS user.

To reduce previous ambiguity, the original WGS 84 GM value for use in GPS receivers, in the OCS process which fits the broadcast elements to the Cartesian vectors, and by ICD-GPS-200 shall be named as GM_{GPSNAV}.

$$GM_{GPSNAV} = 3.9860050 \times 10^{+14} meters^3/seconds^{02} \quad (3\text{-}12)$$

3.7.2 Special Consideration for the Angular Velocity

Although the angular velocity (ω) is suitable for use with a standard Earth and the WGS 84 Ellipsoid, it is the International Astronomical Union (IAU), or the GRS 67, version of this value (ω′) that was used with the new definition of time [16].

$$\omega'^{\prime} = 7.2921151467 \times 10^{-05}\, radians/second \quad (3\text{-}13)$$

For consistent satellite applications**,** the value of the Earth's angular velocity, (ω′), rather than ω, should be used in the following formula to obtain the angular velocity of the Earth in a precessing reference frame, (ω*).

$$\omega^{*} = \omega^{'} + m \quad (3\text{-}14)$$

In the above equation [16] [14], the precession rate in right ascension (m) is:

$$m = (7.086 \;\times 10^{-12} + \; 4.3 \;\times 10^{-15}\, T_U)\, radians/second \quad (3\text{-}15)$$

where

$$T_U = Julian\ Centuries\ from\ Epoch\ J2000.0$$

$$T_U = d_U/36525$$

$$\begin{aligned} d_U = {} & Number\ of\ days\ of\ Universal\ Time\ (UT) \\ & from\ Julian\ Date\ (JD)\ 2451545.0\ UT1, \\ & taking\ on\ values\ of\ \pm 0.5, \pm 1.5, \pm 2.5 \ldots \end{aligned}$$

$$d_U = JD - 2451545$$

Therefore, the angular velocity of the Earth in a precessing reference frame, for satellite applications, is given by:

$$\omega^* = (7.2921158553 \times 10^{-05} + 4.3 \times 10^{-15} T_U)\ rads/sec \tag{3-16}$$

Note that values for ω, ω′, and ω* have remained unchanged from previous editions of TR8350.2.

Table 3.4 Special WGS 84 Parameters

Parameter	Symbol	Value	Units
Gravitational Constant for GPS Navigation Message	GM_{GPSNAV}	$3.9860050 \times 10^{+14}$	m^3 / s^2
Angular Velocity of the Earth (in a precessing reference frame)	ω*	$7.292\,115\,855\,3 \times 10^{-05} + 4.3 \times 10^{-15}\ T_U$	rads / s

3.8 WGS 84 Derived Geometric and Physical Constants

Many constants associated with the WGS 84 Ellipsoid, other than the four defining parameters (Table 3.1), are needed for geodetic applications. Using the four defining parameters, and supplemented with values from Sec. 3.5 and 3.6, it is possible to derive these associated constants. The more commonly used geometric and physical constants associated with the WGS 84 Ellipsoid are listed in Tables 3.5 and 3.6. The formulas used in the calculation of these constants are primarily from [9], [17], [18] and [19]. Derived constants should retain the listed significant digits if consistency among the precision levels of the various parameters is to be maintained.

The differences between the dynamic and geometric even degree zonal harmonics to degree 20 are used in spherical harmonic expansions to calculate the geoid and other geodetic quantities as described in Chapters 5 and 6 of this standard. The EGM2008 values provided in Table 5.1 should be used in orbit determination applications. A description of the EGM2008 geopotential coefficients can be found in Chapter 5, while details on its development and evaluation are provided in [20].

Table 3.5 WGS 84 Ellipsoid Derived Geometric Constants

Parameter	Symbol	Value	Units
WGS 84 Flattening (reduced)	f	$3.3528106647475 \times 10^{-03}$	
Semi-minor Axis (Polar Radius of the Earth)	b	6356752.314 2	m
First Eccentricity	e	$8.1819190842622 \times 10^{-02}$	
First Eccentricity Squared	e^2	$6.694379990141 \times 10^{-03}$	
Second Eccentricity	e'	$8.2094437949696 \times 10^{-02}$	
Second Eccentricity Squared	e'^2	$6.739496742276 \times 10^{-03}$	
Linear Eccentricity	E	$5.2185400842339 \times 10^{+05}$	m
Polar Radius of Curvature	R_p	6399593.6258	m
Axis Ratio (b/a)	AR	$9.96647189335 \times 10^{-01}$	
Mean Radius of the Three Semi-axes	R_1	6371008.7714	m
Radius of a Sphere of Equal Area	R_2	6371007.1810	m
Radius of a Sphere of Equal Volume	R_3	6371000.7900	m
Second Degree Zonal Harmonic	$\bar{C}_{2,0\,\mathrm{geo}}$	$-4.84166774985 \times 10^{-04}$	
Dynamical Form Factor	$J_{2\mathrm{geo}}$	$1.082629821313 \times 10^{-03}$	

Table 3.6 WGS 84 Derived Physical Constants

Parameter	Symbol	Value	Units
Normal Gravity Potential on the Ellipsoid	U_0	$6.26368517146 \times 10^{+07}$	m^2 / s^2
Normal Gravity at the Equator (on the Ellipsoid)	γ_e	9.7803253359	m / s^2
Normal Gravity at the Pole (on the Ellipsoid)	γ_p	9.8321849379	m / s^2
Mean Value of Normal Gravity	$\overline{\gamma}$	9.7976432223	m / s^2
Somigliana's Formula - Normal Gravity Formula Constant	k	$1.931852652458 \times 10^{-03}$	
Normal Gravity Formula Constant ($\omega^2 a^2 b/GM$)	m	$3.449786506841 \times 10^{-03}$	
Mass of the Earth (including atmosphere)	M	$5.9721864 \times 10^{+24}$	kg
Geocentric Gravitational Constant with Earth's Atmosphere Excluded	GM'	$3.986000982 \times 10^{+14}$	m^3 / s^2
Gravitational Constant of the Earth's Atmosphere	GM_A	$3.4359 \times 10^{+08}$	m^3 / s^2

Table 3.7 WGS 84 Derived Moments of Inertia

Parameter	Symbol	Value	Units
Dynamic Moment of Inertia – A	$A_{dyn[2008]}$	$8.0079215 \times 10^{+37}$	$kg\ m^2$
Dynamic Moment of Inertia – B	$B_{dyn[2008]}$	$8.0080746 \times 10^{+37}$	$kg\ m^2$
Dynamic Moment of Inertia – C	$C_{dyn[2008]}$	$8.0343007 \times 10^{+37}$	$kg\ m^2$
Geometric Moment of Inertia – A	A_{geo}	$8.0467266 \times 10^{+37}$	$kg\ m^2$
Geometric Moment of Inertia – C	C_{geo}	$8.0730294 \times 10^{+37}$	$kg\ m^2$
Geometric Solution for Dynamical Ellipticity	H_{geo}	$3.2581006 \times 10^{-03}$	

4. WGS 84 ELLIPSOIDAL GRAVITY FORMULA

4.1 General

The WGS 84 Ellipsoid is identified as being a geocentric, equipotential ellipsoid of revolution. An equipotential ellipsoid is simply an ellipsoid defined to be an equipotential surface, i.e., a surface on which the value of the gravity potential is the same everywhere. The WGS 84 ellipsoid of revolution is defined as an equipotential surface with a specific theoretical gravity potential (U). The equation is derived in [18]. This theoretical gravity potential can be uniquely determined, independent of the density distribution within the ellipsoid, by using any system of four independent constants as the defining parameters of the ellipsoid. As noted earlier in the case of the WGS 84 Ellipsoid (Chapter 3), these are the semi-major axis (a), the flattening factor of the Earth (f), the nominal mean angular velocity of the Earth (ω), and the geocentric gravitational constant (GM).

4.2 Normal Gravity on the Ellipsoidal Surface

Normal gravity (γ), the magnitude of the gradient of the normal potential function (U), is given on (at) the surface of the ellipsoid (h=0) by the closed formula of Somigliana [18] as

$$\gamma = \gamma_e \frac{1 + k \sin^2 \phi}{\sqrt{1 - e^2 \sin^2 \phi}} \qquad (4\text{-}1)$$

where

$$k = \frac{b\gamma_p}{a\gamma_e} - 1$$

a, b = *semi-major and semi-minor axes of the ellipsoid respectively*

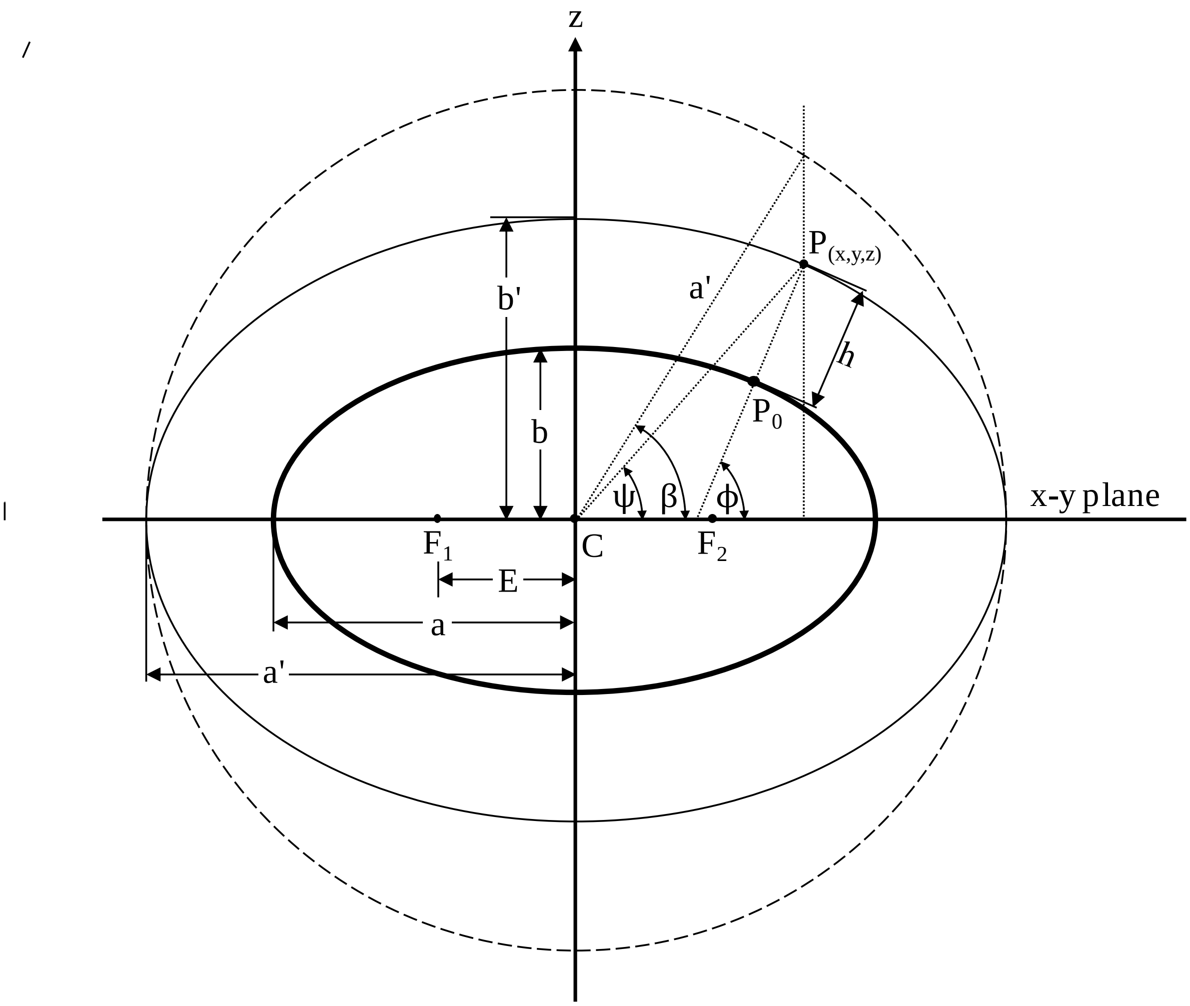
γ_e, γ_p = *normal gravity at the equator and poles, respectively*

e^2 = *first eccentricy squared of the ellipsoid*

ϕ = *geodetic latitude*

This form of the normal gravity equation is the WGS 84 Ellipsoidal Gravity Formula. The equipotential ellipsoid not only serves as the reference for horizontal and vertical surfaces, or geometric figure of the Earth, but also serves as the reference surface for the normal gravity of the Earth.

If the MKS (meter-kilogram-second) unit system is used to evaluate Equation (4-1), or any gravity formula in this Chapter, the gravity unit will be m/s^2 which can be converted to milligals (abbreviated mgal) by the conversion factor, $1\ m/s^2 = 10^5$ mgal.



Reference Ellipsoid
a = semi-major axis
b = semi-minor axis
F_1, F_2 = foci
E = linear eccentricity
(semi focal length)
C = center of ellipsoid

Confocal Ellipsoid through P
a' = semi-major axis
b' = semi-minor axis
F1, F2 = foci
E = linear eccentricity
C = center of ellipsoid
ψ = geocentric latitude of P
β = reduced latitude of P
ϕ = geodetic latitude of P
h = geodetic height

Sphere
a' = radius
C = center of sphere

$$a' = \sqrt{u^2 + E^2}$$

$$b' = u$$

$$E = \sqrt{a^2 - b^2} = \sqrt{a'^2 - b'^2}$$

(λ, ϕ, h) geodetic coordinates: longitude, latitude, and height
(λ, β, u) ellipsoidal coordinates: longitude, reduced latitude, and ellipsoidal parameter
(x, y, z) 3D Cartesian coordinates: aligned with the ellipsoids' axes

Figure 4.1 Ellipsoidal Coordinates

4.3 Normal Gravity Above the Ellipsoid

When the geodetic height (h) is small, the normal gravity above the ellipsoid (γ_h) can be estimated by upward continuing γ at the ellipsoidal surface using a truncated Taylor series expansion as

$$\gamma_h = \gamma + \frac{\partial \gamma}{\partial h} h + \frac{1}{2} \cdot \frac{\partial^2 \gamma}{\partial h^2} h^2 \tag{4-2}$$

where γ and its derivatives are referred to the ellipsoid (h=0).

A frequently used Taylor series expansion for the magnitude of normal gravity above the ellipsoid with a positive direction downward along the geodetic normal to the reference ellipsoid is [18]:

$$\gamma_h = \gamma \left[1 - \frac{2}{a}(1 + f + m - 2f \sin^2 \phi) h + \frac{3}{a^2} h^2 \right] \tag{4-3}$$

where

$$m = \frac{\omega^2 a^2 b}{GM}$$

The derivation of Equation (4-3) can be found in [18].

At moderate and high geodetic heights where Equation (4-3) may yield results with less than desired accuracy, an alternate approach based on formulating normal gravity in the ellipsoidal coordinate system (λ, β, u) is recommended over the Taylor series method. The coordinate u is the semi-minor axis (b'), therefore (u = b'), of an ellipsoid of revolution whose surface passes through the point P in Figure 4.1. This ellipsoid is confocal with the reference ellipsoid and therefore has the same linear eccentricity (E). Its semi-major axis (a′) will reduce to the semi-major axis (a) of the reference ellipsoid when u = b. The β coordinate is known in geodesy as the “reduced latitude” (the definition is seen in Figure 4.1), and λ is the usual geocentric longitude with a value in the open interval 0° - 360°E.

The component that is colinear with the geodetic normal line and directed positively downward (γ_h) of the total normal gravity vector ($\vec{\gamma}_{total}$) for point P in Figure 4.2 can be estimated with sub-microgal precision to geodetic heights of at least 20,000 meters by using the normal gravity components (γ_β, γ_λ, γ_u,) in the ellipsoidal coordinate system:

$$\gamma_h \cong |\vec{\gamma}_{total}| = \sqrt{\gamma_\beta^2 + \gamma_\lambda^2 + \gamma_u^2} \tag{4-4}$$

The normal gravity field from the ellipsoidal representation is symmetrical about the rotation axis, and therefore $\gamma_\lambda = 0$. The radical expression in Equation (4-4) is the true magnitude of the total normal gravity vector $\vec{\gamma}_{total}$ that is perpendicular to the equipotential surface passing through the point P at geodetic height h. The fact that the angular separation (ε) in the inset of Figure 4.2 between the component γ_h and the total normal gravity vector $\vec{\gamma}_{total}$ at the point P is small, even for large geodetic heights, is the basis for using Equation (4-4) to approximate the component γ_h. On the reference ellipsoidal surface where h = 0, $\gamma_\beta = 0$, and u = b, Equation (4-4) is equivalent to Somigliana's Equation (4-1).

The two ellipsoidal components (γ_β, γ_u) of the normal gravity vector ($\vec{\gamma}_{total}$) that are needed in Equation (4-4) are shown in [18] to be functions of the ellipsoidal coordinates (β, u) shown in Figure 4.1. These two components can be computed with unlimited numerical accuracy by the closed expressions:

$$\gamma_u(\beta,u) = -\frac{1}{w}\left[\frac{GM}{u^2+E^2} + \frac{\omega^2 a^2 E}{u^2+E^2}\cdot\frac{q'}{q_0}\left(\frac{1}{2}\sin^2\beta - \frac{1}{6}\right)\right] + \frac{1}{w}\omega^2 u\cos^2\beta \tag{4-5}$$

$$\gamma_\beta(\beta,u) = \frac{1}{w}\cdot\frac{\omega^2 a^2}{\sqrt{u^2+E^2}}\cdot\frac{q}{q_0}\sin\beta\cos\beta - \frac{1}{w}\omega^2\sqrt{u^2+E^2}\sin\beta\cos\beta \tag{4-6}$$

where

$$E = \sqrt{a^2-b^2} \tag{4-7}$$

$$u = \sqrt{\frac{1}{2}(x^2+y^2+z^2-E^2)\left[1+\sqrt{1+\frac{4E^2z^2}{(x^2+y^2+z^2-E^2)^2}}\right]} \tag{4-8}$$

$$\beta = \arctan\left(\frac{z\sqrt{u^2+E^2}}{u\sqrt{x^2+y^2}}\right) \tag{4-9}$$

$$w = \sqrt{\frac{u^2+E^2\sin^2\beta}{u^2+E^2}} \tag{4-10}$$

$$q = \frac{1}{2}\left[\left(1+3\frac{u^2}{E^2}\right)\arctan\left(\frac{E}{u}\right) - 3\frac{u}{E}\right] \tag{4-11}$$

$$q_0 = \frac{1}{2}\left[\left(1+3\frac{b^2}{E^2}\right)\arctan\left(\frac{E}{b}\right) - 3\frac{b}{E}\right] \tag{4-12}$$

$$q' = 3\left(1+\frac{u^2}{E^2}\right)\left[1-\frac{u}{E}\arctan\left(\frac{E}{u}\right)\right] - 1 \tag{4-13}$$

The rectangular coordinates (x, y, z) required in Equations (4-8) and (4-9) can be computed from known geodetic coordinates (λ, φ, h) through the equations:

$$x = (N + h)\cos\phi\cos\lambda$$
$$y = (N + h)\cos\phi\sin\lambda \tag{4-14}$$
$$z = \left[\left(\frac{b^2}{a^2}\right)N + h\right]\sin\phi$$

where

$$N = \frac{a}{\sqrt{1 - e^2\sin^2\phi}} \tag{4-15}$$
$$= \textit{the radius of curvature in the prime vertical}$$

To compute the component γ_h at point P in Figure 4.2 exactly, (account for the angle ε in Figure 4.2 that is being treated as negligible in Equation (4-4)), the ellipsoidal normal gravity components (γ_β, γ_u) are rotated to a spherical coordinate system (λ, ψ, r) resulting in the spherical normal gravity components, (γ_ψ, γ_r). Then, the spherical components are projected onto the geodetic normal line through point P using the angular difference, $\alpha = \varphi - \psi$, between geodetic (φ) and geocentric (ψ) latitudes. The equations to calculate the exact value of γ_h at point P follow:

$$\gamma_h = -\gamma_r\cos\alpha - \gamma_\psi\sin\alpha \tag{4-16}$$

where from [18]:

$$\underset{\substack{Ellipsoid\\System}}{\overset{\vec{\gamma}_E}{\begin{Bmatrix}\gamma_u\\\gamma_\beta\\\gamma_\lambda\end{Bmatrix}}} \xrightarrow{\vec{\gamma}_R = R_1\cdot\vec{\gamma}_E} \underset{\substack{Rectangular\\System}}{\overset{\vec{\gamma}_R}{\begin{Bmatrix}\gamma_x\\\gamma_y\\\gamma_z\end{Bmatrix}}} \xrightarrow{\vec{\gamma}_S = R_2\cdot\vec{\gamma}_R} \underset{\substack{Spherical\\System}}{\overset{\vec{\gamma}_S}{\begin{Bmatrix}\gamma_r\\\gamma_\psi\\\gamma_\lambda\end{Bmatrix}}} \Rightarrow \vec{\gamma}_S = R_2R_1\cdot\vec{\gamma}_E \tag{4-17}$$

$$R_1 = \begin{bmatrix} \frac{u}{w\sqrt{u^2+E^2}}\cos\beta\cos\lambda & -\frac{1}{w}\sin\beta\cos\lambda & -\sin\lambda \\ \frac{u}{w\sqrt{u^2+E^2}}\cos\beta\sin\lambda & -\frac{1}{w}\sin\beta\sin\lambda & \cos\lambda \\ \frac{1}{w}\sin\beta & \frac{u}{w\sqrt{u^2+E^2}}\cos\beta & 0 \end{bmatrix} \tag{4-18}$$

$$R_2 = \begin{bmatrix} \cos\psi\cos\lambda & \cos\psi\sin\lambda & \sin\psi \\ -\sin\psi\cos\lambda & -\sin\psi\sin\lambda & \cos\psi \\ -\sin\lambda & \cos\lambda & 0 \end{bmatrix} \tag{4-19}$$

$$\alpha = \phi - \psi \tag{4-20}$$

The γ_λ component in the two normal gravity vectors, $\vec{\gamma}_E$ and $\vec{\gamma}_S$, in Equation (4-17) is zero since the normal gravity potential is not a function of longitude, λ. The definitions for the other two relevant angles depicted in the inset of Figure 4.2 are:

$$\varepsilon = \theta - \alpha \tag{4-21}$$

$$\theta = \arctan\left(\frac{\gamma_\psi}{\gamma_r}\right) \tag{4-22}$$

$$such\ that\ -\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}$$

The equations listed here for the angles (α, ε, θ) are applicable to both the northern and southern hemispheres. For positive h, each of these angles is zero when point P is directly above one of the poles or lies in the equatorial plane. Elsewhere for h > 0, they have the same sign as the geodetic latitude for point P. For h = 0, the angles α and θ are equal and ε = 0. Numerical results have indicated that the angular separation (ε) between the component γ_h and the total normal gravity vector $\vec{\gamma}_{total}$ satisfies the inequality | ε | < 4 arcseconds for geodetic heights up to 20,000 meters. For completeness the component (γ_ϕ of the total normal gravity vector $\vec{\gamma}_{\text{total}}$ at point P in Figure 4.2 that is orthogonal to γ_h and lies in the meridian plane for point P is given by the expression:

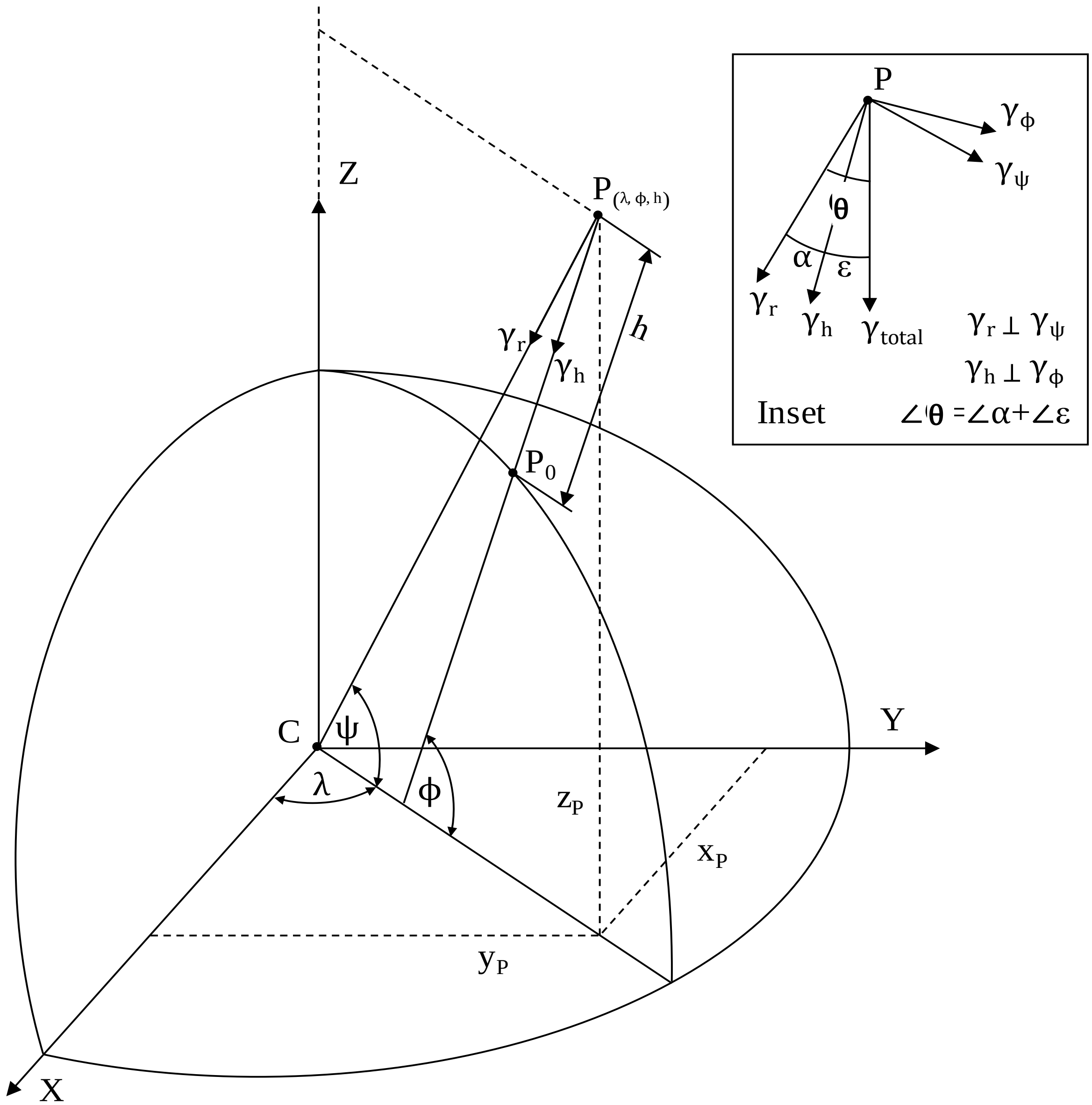
$$\gamma_\phi = -\gamma_r \sin\alpha + \gamma_\psi \cos\alpha \tag{4-23}$$

The component $\gamma_\phi$has a positive sense northward. For geodetic height h = 0, the $\gamma_\phi$component is zero. Numerical tests with whole degree latitudes showed that the magnitude of γ_ϕ remains less than 0.002% of the value of γ_h for geodetic heights up to 20,000 meters. Equations (4-16) and (4-23) provide an alternative way to compute the magnitude | $\vec{\gamma}_{\text{total}}$ | of the total normal gravity vector through the equation:

$$|\vec{\gamma}_{total}| = \sqrt{\gamma_h^2 + \gamma_\phi^2} \tag{4-24}$$

In summary, for near-surface geodetic heights when sub-microgal precision is not necessary, the Taylor series expansion Equation (4-3) for γ_h should suffice. When the intended application for γ_h requires high accuracy, Equation (4-4) will be a close approximation to the exact Equation (4-16) for geodetic heights up to 20,000 meters. Of course, γ_h can be computed using the exact Equation (4-16), but this requires that the computational procedure include the two transformations, R_1 and R_2, that are shown in Equation (4-17). Because the difference in results between Equations (4-4) and (4-16) is less than one microgal (10^{-6} gal) for geodetic heights up to 20,000 meters, the transformation approach would probably be unnecessary in most situations. For applications requiring pure attraction (attraction without centrifugal force) due to the normal gravitational potential (V), the u- and β-vector components of normal gravitation can be computed easily in the ellipsoidal coordinate system by omitting the last term in Equations (4-

5) and (4-6), respectively. These last attraction terms account for the centrifugal force due to the angular velocity (ω) of the reference ellipsoid.



λ = geodetic longitude of P
ψ = geocentric latitude of P
ϕ = geodetic latitude of P
h = geodetic height

C = center of ellipsoid
x_P, y_P, z_P = 3D Cartesian coordinates of P
γ_r = normal gravity component along the geocentric radius
γ_h = normal gravity component along the geodetic normal

(λ, φ, h) geodetic coordinates: longitude, latitude, and height
(λ, β, u) ellipsoidal coordinates: longitude, reduced latitude, and ellipsoidal parameter
(x, y, z) 3D Cartesian coordinates: aligned with the ellipsoids' axes

Figure 4.2 Normal Gravity Components

5. WGS 84 EGM2008 GRAVITATIONAL MODELING

5.1 EGM2008

The form of the WGS 84 EGM2008 gravitational model is a spherical harmonic expansion of the gravitational potential (V). The WGS 84 EGM2008 is complete up to degree (n) and order (m) 2159, and contains additional spherical harmonic coefficients up to degree 2190 and order 2159. EGM2008 contains approximately 4.7 million spherical harmonic coefficients. Table 5.1 lists the low degree portion of EGM2008, up to degree and order 18.

EGM2008 [20] was developed through a least-squares combination of the ITG-Grace03s [21] satellite-only model, with the gravitational information obtained from the analysis of a global set of 5′ x 5′ area-mean values of gravity anomalies. The ITG-Grace03s model was developed by analyzing range-rate data from the GRACE Satellite-to-Satellite Tracking (SST) mission [22], spanning 57 months. No other satellite data were used in the development of ITG-Grace03s. The global set of 5′ x 5′ area-mean values of gravity anomalies was formed by merging terrestrial, airborne, and altimetry-derived gravity data. Over some areas, the available terrestrial data were proprietary and could only be used up to the 15′ x 15′ resolution, which corresponds to harmonic degree 720. To compensate for this restriction, the gravitational signal implied by the Earth’s topography, in the form of Residual Terrain Model effect [23], was used to supplement the spectral bandwidth of the proprietary data. This approach was first tested and verified through comparisons over areas where the full bandwidth of the data is available (see [24] for details). Post-production analysis has shown that EGM2008 incorporated the proprietary gravity data information up to maximum harmonic degree 900 which corresponds approximately to 12 arc-minute resolution. The topographic information that was used within the development of EGM2008 originates from the global Digital Topographic Model (DTM) DTM2006.0 [24]. In DTM2006.0, more than 80% of the total land area of the Earth is covered with data acquired by the Shuttle Radar Topography Mission (SRTM) [25].

EGM2008 was originally developed in ellipsoidal harmonic coefficients, complete to degree and order 2159. The conversion from ellipsoidal to spherical harmonic coefficients preserves the order, but not the degree (see [26] for details). For this reason, EGM2008 in its final, spherical harmonic form, extends beyond degree 2159, and up to degree 2190. The extra coefficients are necessary to preserve the modeling fidelity of the expansion, especially at high latitudes. The user need not be concerned with ellipsoidal harmonic coefficients, which represent an intermediate step towards the development of the EGM2008 spherical harmonic expansion.

The sensitivity of an Earth-orbiting satellite to the gravitational potential is strongly influenced by the satellite’s altitude range and other orbital parameters. For satellites like Starlette, LAGEOS, and the GPS spacecraft (with orbit radii of 7331 km, 12270 km, and 26600 km, respectively) the approximate maximum degree is 90, 20, and 18, respectively. However, DoD programs performing satellite orbit determinations are advised to determine the maximum degree and order that is most appropriate for their particular mission and orbit accuracy requirements and to use the previous estimates only as general guidelines.

The WGS 84 EGM2008 coefficients through degree and order 18 are provided in Table 5.1 in fully-normalized form. Coefficient error estimates are available for the entire model up to degree 2190 and order 2159. In addition, global 5′ x 5′ grids containing the EGM2008 commission error estimates in gravity anomalies, geoid undulations, and in the north-south, east-west (ξ,η) components of the deflection of the vertical, are available at the NGA web site at:

http://earth-info.nga.mil/GandG/wgs84/gravitymod/egm2008/index.html

The gravity anomaly degree variances implied by EGM2008 up to degree and order 2159 are plotted in Figure 5.1.

5.2 Gravity Potential (W)

The Earth's total gravity potential (W) is defined as

$$W = V + \Phi \tag{5-1}$$

where V is the gravitational potential and Φ is the potential due to the Earth's rotation. If ω is the angular velocity of the Earth [Equation (3-5)], then:

$$\Phi = \frac{1}{2}\omega^2(x^2 + y^2) \tag{5-2}$$

where x and y are the geocentric coordinates of a given point in the WGS 84 reference frame (See Figure 2.1).

The gravitational potential function (V) is defined as

$$V = \frac{GM}{r}\left[1 + \sum_{n=2}^{n_{max}} \left(\frac{a}{r}\right)^n \sum_{m=0}^{n} \left(\overline{C}_{nm} \cos m\lambda + \overline{S}_{nm} \sin m\lambda\right) \overline{P}_{nm}(\sin \phi')\right] \tag{5-3}$$

where

$V = gravitational\ potential$

$GM = geocentric\ gravitational\ constant$

$r = distance\ from\ the\ Earth's\ center\ of\ mass$

$a = semi\text{-}major\ axis\ of\ the\ WGS\ 84\ ellipsoid$

$n, m = degree\ and\ order, respectively$

$\phi' = geocentric\ latitude$

$\lambda = geocentric\ longitude$
$\quad = geodetic\ longitude$

$\overline{C}_{nm}, \overline{S}_{nm} = fully\ normalized\ gravitational\ coefficients$

$\overline{P}_{nm}(\sin \phi') = fully\ normailzed\ associated\ Legendre\ function$

$$= \left[\frac{(n-m)!\,(2n+1)k}{(n+m)!}\right]^{\frac{1}{2}} P_{nm}(\sin \phi')$$

$P_{nm}(\sin \phi') = associated\ Legendre\ function$

$$= (\cos \phi')^m \frac{d^m}{d(\sin \phi')^m}[P_n(\sin \phi')]$$

$[P_n(\sin \phi')] = Legendre\ polynomial$

$$= \frac{1}{2^n n!} \frac{d^n}{d(\sin \phi')^n} (\sin^2 \phi' - 1)^n$$

Note:

$$\begin{vmatrix} \overline{C}_{nm} \\ \overline{S}_{nm} \end{vmatrix} = \left[\frac{(n+m)!}{(n-m)!\,(2n+1)k}\right]^{\frac{1}{2}} \begin{vmatrix} C_{nm} \\ S_{nm} \end{vmatrix}$$

where

$C_{nm}, S_{nm} = conventional\ (unnormalized) gravitational\ coefficients$

for $\quad m = 0, k = 1$
$\quad m \neq 0, k = 2$

Convergence of the series (5-3) is guaranteed for r ≥ a [18]. The series can be used with negligible error for points located near or on the Earth's surface. The series should not be used for points located below Earth's surface.

Table 5.1 Earth Gravitational Model 2008
Truncated at n=m=18

Degree and Order		Normalized Gravitational Coefficients	
n	m	$\bar{C}_{nm}$	$\bar{S}_{nm}$
2	0	-0.484165143790815E-03	
2	1	-0.206615509074176E-09	0.138441389137979E-08
2	2	0.243938357328313E-05	-0.140027370385934E-05
3	0	0.957161207093473E-06	
3	1	0.203046201047864E-05	0.248200415856872E-06
3	2	0.904787894809528E-06	-0.619005475177618E-06
3	3	0.721321757121568E-06	0.141434926192941E-05
4	0	0.539965866638991E-06	
4	1	-0.536157389388867E-06	-0.473567346518086E-06
4	2	0.350501623962649E-06	0.662480026275829E-06
4	3	0.990856766672321E-06	-0.200956723567452E-06
4	4	-0.188519633023033E-06	0.308803882149194E-06
5	0	0.686702913736681E-07	
5	1	-0.629211923042529E-07	-0.943698073395769E-07
5	2	0.652078043176164E-06	-0.323353192540522E-06
5	3	-0.451847152328843E-06	-0.214955408306046E-06
5	4	-0.295328761175629E-06	0.498070550102351E-07
5	5	0.174811795496002E-06	-0.669379935180165E-06
6	0	-0.149953927978527E-06	
6	1	-0.759210081892527E-07	0.265122593213647E-07
6	2	0.486488924604690E-07	-0.373789324523752E-06
6	3	0.572451611175653E-07	0.895201130010730E-08
6	4	-0.860237937191611E-07	-0.471425573429095E-06
6	5	-0.267166423703038E-06	-0.536493151500206E-06
6	6	0.947068749756882E-08	-0.237382353351005E-06
7	0	0.905120844521618E-07	
7	1	0.280887555776673E-06	0.951259362869275E-07
7	2	0.330407993702235E-06	0.929969290624092E-07
7	3	0.250458409225729E-06	-0.217118287729610E-06
7	4	-0.274993935591631E-06	-0.124058403514343E-06
7	5	0.164773255934658E-08	0.179281782751438E-07
7	6	-0.358798423464889E-06	0.151798257443669E-06
7	7	0.150746472872675E-08	0.241068767286303E-07
8	0	0.494756003005199E-07	
8	1	0.231607991248329E-07	0.588974540927606E-07
8	2	0.800143604736599E-07	0.652805043667369E-07
8	3	-0.193745381715290E-07	-0.859639339125694E-07
8	4	-0.244360480007096E-06	0.698072508472777E-07
8	5	-0.257011477267991E-07	0.892034891745881E-07
8	6	-0.659648680031408E-07	0.308946730783065E-06

Table 5.1 Earth Gravitational Model 2008
Truncated at n=m=18

Degree and Order		Normalized Gravitational Coefficients	
n	**m**	$\overline{C}_{nm}$	$\overline{S}_{nm}$
8	7	0.672569751771483E-07	0.748686063738231E-07
8	8	-0.124022771917136E-06	0.120551889384997E-06
9	0	0.280180753216300E-07	
9	1	0.142151377236084E-06	0.214004665077510E-07
9	2	0.214144381199757E-07	-0.316984195352417E-07
9	3	-0.160612356882835E-06	-0.742658786809216E-07
9	4	-0.936529556592536E-08	0.199026740710063E-07
9	5	-0.163134050605937E-07	-0.540394840426217E-07
9	6	0.627879491161446E-07	0.222962377434615E-06
9	7	-0.117983924385618E-06	-0.969222126840068E-07
9	8	0.188136188986452E-06	-0.300538974811744E-08
9	9	-0.475568433357652E-07	0.968804214389955E-07
10	0	0.533304381729473E-07	
10	1	0.837623112620412E-07	-0.131092332261065E-06
10	2	-0.939894766092874E-07	-0.512746772537482E-07
10	3	-0.700709997317429E-08	-0.154139929404373E-06
10	4	-0.844715388074630E-07	-0.790255527979406E-07
10	5	-0.492894049964295E-07	-0.506137282060864E-07
10	6	-0.375849022022301E-07	-0.797688616388143E-07
10	7	0.826209286523474E-08	-0.304903703914366E-08
10	8	0.405981624580941E-07	-0.917138622482163E-07
10	9	0.125376631604340E-06	-0.379436584841270E-07
10	10	0.100435991936118E-06	-0.238596204211893E-07
11	0	-0.507683787085927E-07	
11	1	0.156127678638183E-07	-0.271235374123689E-07
11	2	0.201135250154855E-07	-0.990003954905590E-07
11	3	-0.305773531606647E-07	-0.148835345047152E-06
11	4	-0.379499015091407E-07	-0.637669897493018E-07
11	5	0.374192407050580E-07	0.495908160271967E-07
11	6	-0.156429128694775E-08	0.342735099884706E-07
11	7	0.465461661449953E-08	-0.898252194924903E-07
11	8	-0.630174049861897E-08	0.245446551115189E-07
11	9	-0.310727993686101E-07	0.420682585407293E-07
11	10	-0.522444922089646E-07	-0.184216383163730E-07
11	11	0.462340571475799E-07	-0.696711251523700E-07
12	0	0.364361922614572E-07	
12	1	-0.535856270449833E-07	-0.431656037232084E-07
12	2	0.142665936828290E-07	0.310937162901519E-07
12	3	0.396211271409354E-07	0.250622628960907E-07
12	4	-0.677284618097416E-07	0.383823469584472E-08

Table 5.1 Earth Gravitational Model 2008
Truncated at n=m=18

Degree and Order		Normalized Gravitational Coefficients	
n	**m**	$\overline{C}_{nm}$	$\overline{S}_{nm}$
12	5	0.308775410911475E-07	0.759066416791107E-08
12	6	0.313421100991039E-08	0.389801868153392E-07
12	7	-0.190517957483100E-07	0.357268620672699E-07
12	8	-0.258866871220994E-07	0.169362538600173E-07
12	9	0.419147664170774E-07	0.249625636010847E-07
12	10	-0.619955079880774E-08	0.309398171578482E-07
12	11	0.113644952089825E-07	-0.638551119140755E-08
12	12	-0.242377235648074E-08	-0.110993698692881E-07
13	0	0.417293021685027E-07	
13	1	-0.514421009206120E-07	0.386910482386637E-07
13	2	0.553118515702855E-07	-0.626943474947239E-07
13	3	-0.215570388049647E-07	0.976866679032941E-07
13	4	-0.365127902764428E-08	-0.117512717960252E-07
13	5	0.583702330251927E-07	0.672244622794413E-07
13	6	-0.350445484464565E-07	-0.627356859556194E-08
13	7	0.301412465951003E-08	-0.732068659961970E-08
13	8	-0.100532117993105E-07	-0.985787032645980E-08
13	9	0.247702773255876E-07	0.458875609452034E-07
13	10	0.411080488026299E-07	-0.368403909177750E-07
13	11	-0.445213404110823E-07	-0.484141059455725E-08
13	12	-0.313130628228171E-07	0.879376493656904E-07
13	13	-0.612004732532594E-07	0.681501470347338E-07
14	0	-0.226681154094404E-07	
14	1	-0.187724885657433E-07	0.288602024410603E-07
14	2	-0.359186725681205E-07	-0.405327051356456E-08
14	3	0.365140497848863E-07	0.196941950099316E-07
14	4	0.160184144282003E-08	-0.226625156915448E-07
14	5	0.293092637238844E-07	-0.167894170071708E-07
14	6	-0.190674352461174E-07	0.245661933018802E-08
14	7	0.376297554551457E-07	-0.393364671078740E-08
14	8	-0.349417459823694E-07	-0.154475521495720E-07
14	9	0.319517827137725E-07	0.284642263273996E-07
14	10	0.388008374622606E-07	-0.129351349981739E-08
14	11	0.156475715628498E-07	-0.390403676399681E-07
14	12	0.846317130098805E-08	-0.311211374558506E-07
14	13	0.322437165995707E-07	0.451472951879378E-07
14	14	-0.518650713590088E-07	-0.481611072612157E-08
15	0	0.219216154508434E-08	
15	1	0.942942849227573E-08	0.104833279954842E-07
15	2	-0.205302993025519E-07	-0.303007325301024E-07

Table 5.1 Earth Gravitational Model 2008
Truncated at n=m=18

Degree and Order		Normalized Gravitational Coefficients	
n	**m**	$\bar{C}_{nm}$	$\bar{S}_{nm}$
15	3	0.534162906407382E-07	0.176634209489453E-07
15	4	-0.401721760931603E-07	0.681343055884874E-08
15	5	0.122424622638639E-07	0.762075300880907E-08
15	6	0.328522324096078E-07	-0.364691072357445E-07
15	7	0.596540954755518E-07	0.507394600032322E-08
15	8	-0.320875010777956E-07	0.221729246735453E-07
15	9	0.132999993095746E-07	0.379914748780028E-07
15	10	0.102639183602256E-07	0.146906332576870E-07
15	11	-0.130462854422872E-08	0.185210748548004E-07
15	12	-0.324147324255678E-07	0.156080393130757E-07
15	13	-0.283649640930711E-07	-0.457549001836693E-08
15	14	0.519862755176957E-08	-0.243950380180467E-07
15	15	-0.190443752608698E-07	-0.469941074395975E-08
16	0	-0.471037252266068E-08	
16	1	0.261852447892310E-07	0.333423726704204E-07
16	2	-0.245117847209327E-07	0.280314862616323E-07
16	3	-0.339147390918482E-07	-0.213401079775597E-07
16	4	0.408540187833972E-07	0.479877498751224E-07
16	5	-0.121209356470009E-07	-0.344283772175451E-08
16	6	0.138747261713477E-07	-0.355969015666675E-07
16	7	-0.806188863620981E-08	-0.865178660695190E-08
16	8	-0.212044215093991E-07	0.540717302974902E-08
16	9	-0.224151915247857E-07	-0.396686472741101E-07
16	10	-0.118064629182132E-07	0.115374382529194E-07
16	11	0.191118463013820E-07	-0.320054944714364E-08
16	12	0.195631858722835E-07	0.672539764312181E-08
16	13	0.137744095920323E-07	0.104745467774466E-08
16	14	-0.193451561175034E-07	-0.386522367774771E-07
16	15	-0.144196100302905E-07	-0.327760669358925E-07
16	16	-0.382992884797529E-07	0.295860714710389E-08
17	0	0.191875988417387E-07	
17	1	-0.253645204657949E-07	-0.317042482135934E-07
17	2	-0.201017447758198E-07	0.681376058343928E-08
17	3	0.630839562027722E-08	0.508308654394536E-08
17	4	0.647515778433184E-08	0.253354668311939E-07
17	5	-0.162193992995631E-07	0.803001732911605E-08
17	6	-0.117320425248787E-07	-0.294401279670302E-07
17	7	0.249764681793243E-07	-0.438554429095768E-08
17	8	0.390006528364112E-07	0.363439990543582E-08
17	9	0.347901841118582E-08	-0.276427048005195E-07

Table 5.1 Earth Gravitational Model 2008
Truncated at n=m=18

Degree and Order		Normalized Gravitational Coefficients	
n	**m**	$\bar{C}_{nm}$	$\bar{S}_{nm}$
17	10	-0.379589397747057E-08	0.183778246503248E-07
17	11	-0.160174948249694E-07	0.110870581174922E-07
17	12	0.287221517856901E-07	0.204582230384783E-07
17	13	0.165063239919241E-07	0.201356543378344E-07
17	14	-0.142747771121823E-07	0.115742828658331E-07
17	15	0.553783372805876E-08	0.524340656496201E-08
17	16	-0.303992561155655E-07	0.360274437895663E-08
17	17	-0.346971352906772E-07	-0.198743584916705E-07
18	0	0.609862871807421E-08	
18	1	0.720152312518112E-08	-0.392970098282429E-07
18	2	0.147251428316923E-07	0.108333873087810E-07
18	3	-0.504457513185901E-08	-0.584923266822307E-08
18	4	0.546253493560536E-07	-0.786748537792632E-09
18	5	0.597651128344404E-08	0.261398616579252E-07
18	6	0.135775785802306E-07	-0.132446105152748E-07
18	7	0.679347259309753E-08	0.746790172709486E-08
18	8	0.304992690074792E-07	0.434439798188153E-08
18	9	-0.195696822181729E-07	0.361306774690565E-07
18	10	0.521528994919861E-08	-0.424068379064049E-08
18	11	-0.688640849277642E-08	0.211925103597343E-08
18	12	-0.297489841938418E-07	-0.165609165083241E-07
18	13	-0.625379710747338E-08	-0.349391913136191E-07
18	14	-0.829497209425585E-08	-0.128330547008713E-07
18	15	-0.404690219510979E-07	-0.202806589920597E-07
18	16	0.101653114622966E-07	0.650425930141024E-08
18	17	0.348327556578355E-08	0.437638847248534E-08
18	18	0.299062325911737E-08	-0.108590059665012E-07

Table 5.1 Earth Gravitational Model 2008
Truncated at n=m=18

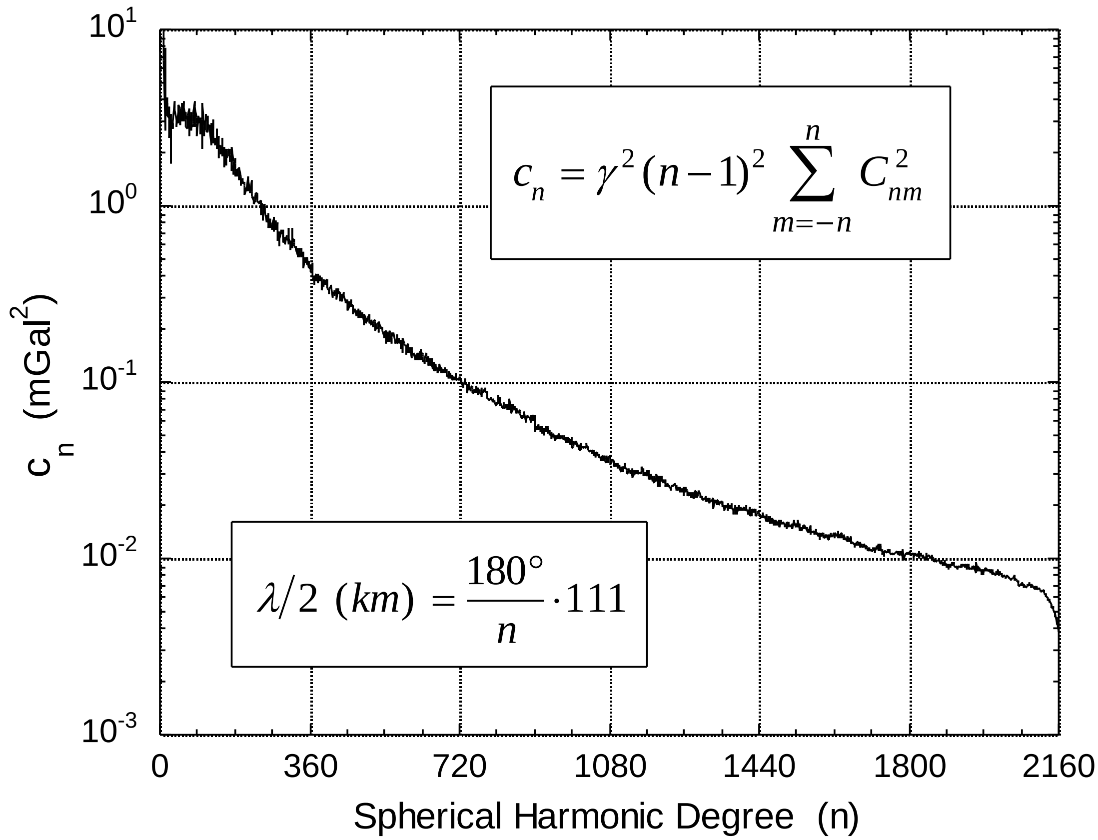


Figure 5.1 Anomaly Degree Variances

6. WGS 84 EGM2008 GEOID

6.1 General

In geodetic applications, three primary reference surfaces for the Earth are used: 1) the Earth's topographic surface, 2) an ellipsoid of revolution, which is a reference surface of purely mathematical nature, and 3) the geoid.

The locus of points (X, Y, Z) at which the gravity potential (W) maintains a constant value, as shown in Equation (6-1), defines an equipotential surface called a geop [18]. There is an infinite number of geops associated with the Earth's gravity field. Each one of these can be defined by changing the numerical value of the constant in Equation (6-1). The geoid is that particular geop that is closely associated with the mean ocean surface.

$$W(X, Y, Z) = constant \tag{6-1}$$

Traditionally, when a geoid model is developed, the value of the constant in Equation (6-1), representing the potential anywhere on this surface, is constrained or assumed to be equal to the normal potential (U_0) on the surface of a 'best-fitting' ellipsoid. Throughout this refinement effort, however, the authors recognize that the WGS 84 ellipsoid no longer represents a true 'best-fitting' ellipsoid. In terms of the geoid, this effect is handled through the application of a zero-degree undulation (N_0). With this approach, the WGS 84 ellipsoid and its associated normal gravity field can be retained, without the introduction of any additional errors by not using the 'best-fitting' ellipsoid.

In common practice, the geoid is expressed at a given point in terms of the distance above (+N) or below (-N) the ellipsoid. For practical reasons, the geoid has been used to serve as a vertical reference surface for Mean Sea Level (MSL) heights. In areas where elevation data are not available from conventional geodetic leveling, an approximation of MSL heights, using orthometric heights, can be obtained from the following equation [18]:

$$H = h - N \tag{6-2}$$

where

$H = orthometric\ height\ (height\ relative\ to\ the\ geoid)$

$h = geodetic\ (ellipsoid)\ height\ (height\ relative\ to\ the\ ellipsoid)$

$N = geoid\ height\ or\ geoid\ undulation$

Utilization of orthometric heights requires knowledge of crustal mass densities. Lack of such information introduces errors to the geoid undulation model. As an alternative, some countries replace orthometric heights with normal heights and geoid undulations with height anomalies [18]. This use of height anomalies eliminates assumptions about the density of masses between the geoid and the ground. Therefore, Equation (6-2) can be re-formulated as

$$h = H + N \approx H^* + \zeta \tag{6-3}$$

where

$H^* = Normal\ height$

$\zeta = height\ anomaly$

The telluroid [38] is a surface defined in such way that the normal potential (U) at every point Q on the telluroid is equal to the actual potential (W) at its corresponding point P on the Earth's surface [18]. Point Q is located on the line normal to the ellipsoid that passes through P. The height anomaly is the distance between point Q on the telluroid and point P on the Earth's surface.

Equation (6-2) illustrates the use of geoid undulations in the determination of orthometric heights (H) from geodetic heights (h) derived using satellite positioning (*e.g.*, from GPS) located on the Earth's physical surface or aboard a vehicle operating near the Earth's surface.

6.2 Formulas, Representations and Analysis

The WGS 84 EGM2008 geoid undulations were computed following the same general procedure as in the case of the EGM96 geoid undulations [18]. The difference is that in the case of EGM2008 [20] the expansion in Equation (6-4) extends to degree and order 2190, while in the case of EGM96, it extended only to degree and order 360. Using the EGM2008 spherical harmonic coefficients in Equation (6-4), height anomalies are first computed with respect to an 'ideal' (best-fitting) mean-Earth ellipsoid, in the tide-free system. According to [28], the best estimates of the parameters of the 'ideal' mean-Earth ellipsoid, in the tide-free system (Love number k = 0.3), are: semi-major axis a = 6378136.58 meters and reciprocal flattening 1/f = 298.257686. The height anomalies are then transformed to geoid undulations using the procedure described in [27] and [29], and a set of correction coefficients complete up to degree and order 2160. Finally, the constant value of -0.41 meters is added to these geoid undulations. This is the zero-degree height anomaly (ζ_z) that accounts for the difference between the 'ideal' mean-Earth ellipsoid in a tide-free system and the WGS 84 ellipsoid. Due to the fact that the height anomaly to geoid undulation conversion terms do not average to zero globally, the -41 cm (ζ_z) value results in a -46.3 cm zero-degree geoid undulation value (N_0) [20].

The zero-degree height anomaly (ζ_z) that was computed when the WGS 84 EGM96 geoid was released was equal to -0.53 meters [27]. The primary reason for the change in the numerical value of ζ_z from the EGM96 to the current best estimate is the discovery by Quan-Zan Zanife (CLS, France) of an error in the Oscillator Drift correction applied to TOPEX altimeter data [30]. The erroneous correction was producing TOPEX Sea Surface Heights biased by 12 to 13 centimeters.

6.2.1 Formulas

The formula for calculating the WGS 84 EGM2008 geoid undulations starts with the calculation of the height anomaly (ζ) [29]:

$$\zeta(\phi, \lambda, r) = \frac{GM}{\gamma(\phi) r}\left[\sum_{n=2}^{N_{max}} \left(\frac{a}{r}\right)^n \sum_{m=0}^{n} \left(\overline{C}_{nm} \cos m\lambda + \overline{S}_{nm} \sin m\lambda\right) \overline{P}_{nm}(\sin \phi')\right] \qquad (6\text{-}4)$$

$where\ (following\ notation\ from\ Equation$ (5-3))

$\gamma(\phi) = normal\ gravity\ at\ point\ P$

$\overline{C}_{nm}, \overline{S}_{nm} = fully\ normalized\ gravitational\ coefficients$
$from\ EGM2008$

Equation (6-4) is evaluated at a point P(r, ϕ', λ) on the surface of the Earth's topography.

In Equation (6-4), the even degree zonal coefficients of degree 2 through 20 are remainders after subtracting from the EGM2008 gravitational potential coefficients, the corresponding coefficients implied by the WGS 84 normal gravitational field.

Table 6.1 WGS 84 Normal Gravitational Potential Coefficients

$\overline{C}_{2,0(normal)}$	-0.484166774985001E-03	$\overline{C}_{12,0(normal)}$	-0.410790141413244E-16
$\overline{C}_{4,0(normal)}$	0.790303733511320E-06	$\overline{C}_{14,0(normal)}$	0.447177357025841E-18
$\overline{C}_{6,0(normal)}$	-0.168724961151417E-08	$\overline{C}_{16,0(normal)}$	-0.346362564744706E-20
$\overline{C}_{8,0(normal)}$	0.346052468394228E-11	$\overline{C}_{18,0(normal)}$	0.241145603218922E-22
$\overline{C}_{10,0(normal)}$	-0.265002225746918E-14	$\overline{C}_{20,0(normal)}$	-0.160243292851218E-24

The relationship between the height anomaly and disturbing potential is given by:

$$\zeta(\phi, \lambda, r) = \frac{T(\phi, \lambda, r)}{\gamma_P} \qquad (6\text{-}5)$$

The geodetic latitude of point P is denoted in the following by φ. Note that φ and ϕ' correspond to the same physical location. To calculate the geoid undulation (N) with respect to the WGS 84 reference ellipsoid, we use the formula [27]:

$$N(\phi,\lambda) = \zeta_Z + \zeta(\phi,\lambda,r) + \frac{\Delta g_{BA}(\phi,\lambda)}{\bar{\gamma}} H(\phi,\lambda) \tag{6-6}$$

where

$\zeta_Z = -0.41\ meters\ (zero\ degree\ height\ anomaly)$

$\Delta g_{BA}(\phi,\lambda) = Bouguer\ gravity\ anomaly\ from\ EGM2008\ and\ the\ Digital\ Topographic\ Model\ 2006\ (DTM2006.0)$

$\bar{\gamma} = average\ value\ of\ normal\ gravity$

$H(\phi,\lambda) = defined\ from\ harmonic\ analysis\ of\ the\ elevations\ of\ the\ DTM2006.0$

The DTM2006.0 is described in [24]. The Bouguer anomaly can be computed from the EGM2008 spherical harmonic coefficients and the coefficients from the harmonic analysis of the DTM2006.0 elevation database using:

$$\Delta g_{BA}(\phi,\lambda) = \Delta g_{FA}(\phi,\lambda) = -0.1119 \times H(\phi,\lambda) \tag{6-7}$$

where

$\Delta g_{FA}(\phi,\lambda) = Free\ air\ gravity\ anomaly\ from\ EGM2008$

The conversion of height anomalies to geoid undulations is expressed in a set of spherical harmonic coefficients complete up to degree and order 2160.

6.2.2 Permanent Tide Systems

In the calculation of geoid undulations from the EGM2008, the second degree zonal harmonic coefficient is given in the tide-free system. The tide-free definition means that any geoid undulations calculated from EGM2008 exist for a tide-free Earth with all (direct and indirect) tidal effects of the Sun and Moon removed. Other geoids to consider are the mean geoid (geoid which would exist in the presence of the Sun and Moon) and the zero-tide geoid (geoid which exists if the permanent direct effects of the Sun and Moon are removed but the indirect effect related to the Earth's elastic deformation is retained). A complete set of equations to convert from one tide system to another can be found in [31].

To calculate the geoid in the zero-tide system use the formula:

$$N_Z = N_n + k(9.9 - 29.6 \sin^2 \varphi) \tag{6-8}$$

where

$N_Z = zero\ tide\ geoid$

$N_n = tide\ free\ geoid$

$k = Love\ Number, = 0.3\ by\ international\ agreement$

6.2.3 Representations and Analysis

The geoid undulations can be depicted as a color map showing the deviations of the geoid from the ellipsoid, the latter being selected as the mathematical figure of the Earth. Figure 6.1 is such a color map, created from a worldwide 2.5′ x 2.5′ grid of the WGS 84 EGM2008 geoid undulations which were calculated using the procedure described in Section 6.2.1. These values exhibit the following statistics:

Mean	-0.46 meters
Standard Deviation	30.59 meters
Minimum	-106.91 meters
Maximum	85.82 meters

The locations of the minimum and maximum undulations are:

Minimum	ϕ = 04.667° N, λ = 078.750° E
Maximum	ϕ = 08.417° S, λ = 147.375° E

Figure 6.2 shows the propagated (commission) error that was estimated for the WGS 84 EGM2008 geoid undulations. These values were computed using the procedure described in [32] and [20], on a global 5′x5′ grid. These error estimates have a global Root Mean Square (RMS) value of approximately ±11 centimeters, but vary considerably in a geographic sense, reflecting the geographically varying accuracy of the gravity data that were available for the development of EGM2008.

6.3 Availability of WGS 84 EGM2008 Data Products

Several products and associated documentation related to the WGS 84 EGM2008 geoid undulations and their error estimates are available from:

http://earth-info.nga.mil/GandG/wgs84/gravitymod/egm2008/egm08_wgs84.html

These products include:

- The EGM2008 spherical harmonic coefficients to degree 2190 and order 2159.

- The correction coefficients necessary to convert height anomalies to geoid undulations, complete up to degree and order 2160.

- A FORTRAN program that can be used to compute the WGS 84 EGM2008 geoid undulations, via harmonic synthesis, using the EGM2008 and the correction coefficient files.

- A 1′ x 1′ WGS 84 EGM2008 geoid undulation grid file calculated using the procedure described in Section 6.2.1, along with software that can be used to interpolate from this grid. Differences in the results using these versus harmonic synthesis do not exceed 1 millimeter.

- A 2.5′ x 2.5′ WGS 84 EGM2008 geoid undulation grid file calculated using the procedure described in Section 6.2.1, along with software that can be used to interpolate from this grid. Differences in the results using these versus harmonic synthesis do not exceed 1 centimeter.

Additional information on the WGS 84 EGM2008 geoid undulations, associated software and data files can be obtained from the location and addresses in the PREFACE.

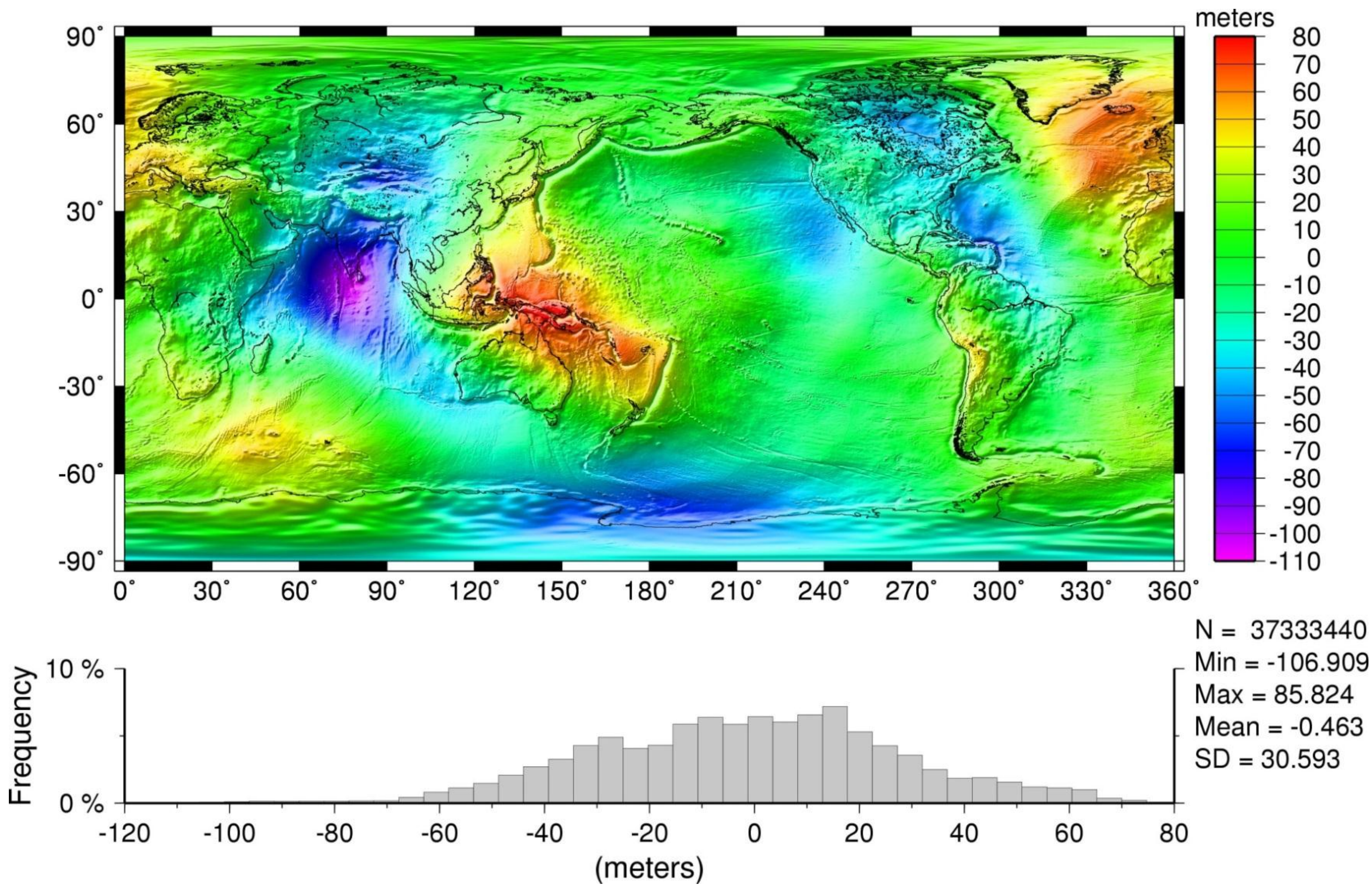


Figure 6.1 EGM2008 Geoid Undulations on a Global 2.5′ x 2.5′ Grid, with respect to the WGS 84 Ellipsoid. Unit is Meter.

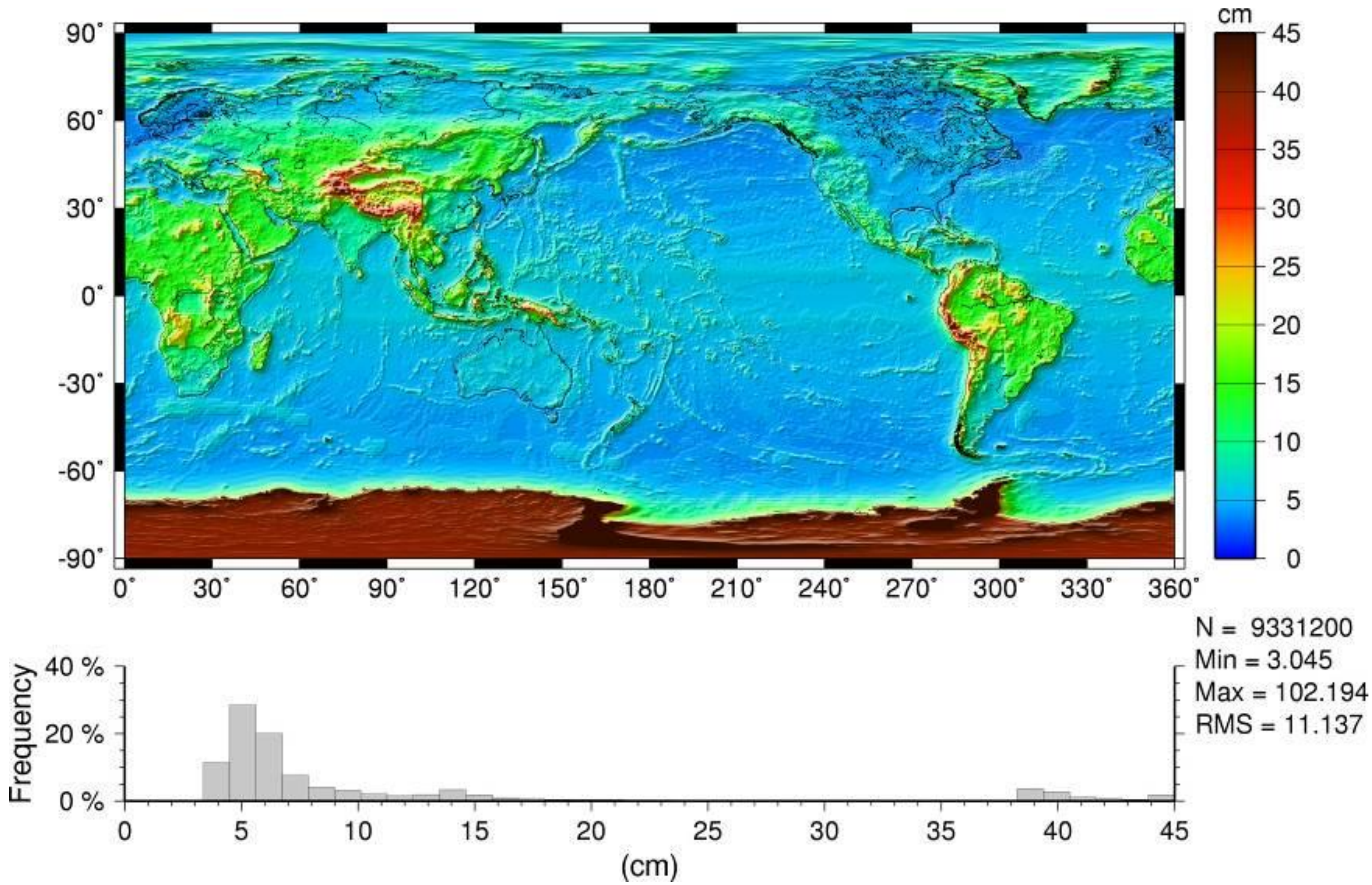


Figure 6.2 Propagated (Commission) Errors of the EGM2008 Geoid Undulations on a Global 5′ x 5′ Grid. Unit is Centimeter.

7. WGS 84 RELATIONSHIPS WITH OTHER GEODETIC SYSTEMS

7.1 General

One of the principal purposes of a world geodetic system is to eliminate the use of local horizontal geodetic datums by providing a globally consistent reference system. Although the number of local horizontal geodetic datums, counting island and astronomic-based datums, continues to significantly decrease, there are still several hundred currently in use. Until a global geodetic datum is accepted, used, and implemented worldwide, a means to relate these traditional geodetic datums to WGS 84 is required. To accomplish the conversion, local geodetic datum and WGS coordinates are both required at one or more sites within the local datum area so that a local geodetic datum to WGS datum shift can be computed using a suitable model. Satellite stations positioned within WGS 84, with known local geodetic datum coordinates, were the basic ingredients in the development of local geodetic datum to WGS 84 datum shifts.

Local horizontal datums were developed in the past to satisfy mapping and navigation requirements for specific regions of the Earth. Geocentric datums of large geographic extent are the Geocentric Datum of Australia (GDA) and the North American Datum of 1983 (NAD 83). In the past couple of decades, development of global geocentric datums has become possible, *e.g.*, WGS 84 and the ITRF.

Applications requiring precise point positions should not rely on coordinates derived from a datum transformation. The most accurate approach for obtaining WGS 84 coordinates is to acquire satellite tracking data at the site of interest and position it directly within WGS 84 using GPS positioning techniques. Direct occupation of the site, however, is not always possible or warranted; and in these cases, a datum transformation can be used to convert coordinates from the local system to WGS 84.

Transforming local datum coordinates to WGS 84 coordinates does not improve the accuracy of the transformed coordinates. Distortions in the local datum will combine with any uncertainties in the datum transformation parameters.

NGA maintains datum transformation/coordinate conversion software, "MSP GeoTrans," suitable for cartographic purposes. MSP GeoTrans is available at http://osis.nga.mil/GandG/geotrans/index.htm .

7.2 Relationship of WGS 84 to the ITRF

WGS 84 has undergone several enhancements since its original definition. The practical realization of the reference frame is determined by a set of globally distributed, permanent GPS tracking stations which are aligned with highly accurate ITRF coordinates.

7.2.1 Agreement With the ITRF

The WGS 84 (G1762) Reference Frame compared to ITRF2008 shows a Root Mean Square (RMS) difference of one centimeter overall. Comparisons between the NGA GPS precise ephemerides, referenced to WGS 84 (G1762), and International GNSS Service (IGS) GPS precise ephemerides, referenced to ITRF2008, validate that the two reference systems are consistent. This indicates that these two reference frames are essentially identical with differences being statistically insignificant for most applications.

7.2.2 Datums Based on the ITRF

Some countries and regions have been converting to datums based on the ITRF: such ITRF-based datums can be considered as identical to WGS 84, *e.g.*, the European Terrestrial Reference Frame 1989 (EUREF89) at mapping and charting scales. Users requiring high accuracies will need to quantify the differences between them.

7.3 Relationship of WGS 84 to the NAD 83

In 1986, a group of institutions representing Canada, Greenland, and the United States of America (USA) adopted the North American Datum of 1983 (NAD 83) as their official spatial reference system for geometric positioning [33]. NAD 83 replaced the North American Datum of 1927 (NAD 27), and the U.S. National Geodetic Survey (NGS) has already expressed its intention to replace NAD 83 around 2022 with a newer geometric reference system. Snay [34] discusses the evolution of NAD 83 in the USA since its adoption in 1986, and Craymer [35] discusses the evolution of NAD 83 in Canada.

NAD 83 uses the Geodetic Reference System 1980 (GRS 80) ellipsoid as its reference ellipsoid with the geometric center of the ellipsoid coincident with the center of mass of the Earth and the origin of the coordinate system. The semi-major axis and flattening factor of the Earth parameters are adopted directly as

$$a = 6378137 \; meters$$

$$1/f = 298.257222101$$

The WGS 84 ellipsoid is, for all practical purposes, identical to the GRS 80 ellipsoid. They both use the same value for the semi-major axis and have the same orientation with respect to the center of mass and the coordinate system origin. However, WGS 84 uses a derived value for the flattening factor that is computed from the normalized second degree zonal harmonic gravitational coefficient ($\bar{C}_{2,0}$). This value was derived from the GRS 80 value for J_2 and truncated to 8 significant digits as

$$\bar{C}_{2,0} = -1 \times \frac{J_2}{\sqrt{5}} \tag{7-1}$$

The resulting WGS 84 value for 1/f is 298.257223563. The difference between the GRS 80 and WGS 84 values for f creates a difference of 0.1 mm in the derived semi-minor axes of the two ellipsoids.

The original NAD 83 reference frame, denoted NAD 83(1986) in the USA, is based on a horizontal adjustment of triangulation and trilateration data with the inclusion of Transit Satellite Doppler data and Very Long Baseline Interferometry (VLBI) data. The global Doppler and VLBI observations were used to align NAD 83(1986) to the BIH Terrestrial System of 1984. Consequently, the origin and orientation of the Earth-centered, Earth-fixed (ECEF) coordinate axes of NAD 83(1986) are identical to that of the original WGS 84 reference frame.

After 1986, both GPS and Satellite Laser Ranging (SLR) measurements allowed geodesists to locate the geocenter with a precision of a few centimeters. In doing so, these technologies revealed that the geocenter adopted for both NAD 83(1986) and the original WGS 84 reference frame is displaced by more than two meters from the true geocenter. Similarly, GPS, SLR, and VLBI revealed that the orientation of the Cartesian axes of NAD 83(1986) and the original WGS 84 is misaligned by over 0.03 arc seconds relative to the orientation associated with current versions of the International Terrestrial Reference Frame (ITRF), and that the scale of NAD 83(1986) and the original WGS 84 differs by approximately 0.0871 parts per million (ppm) from the true definition of a meter.

These discrepancies caused significant concern as the use of highly accurate GPS measurements proliferated. As a result, both NAD 83 and WGS 84 have evolved over the past few decades. In 1994, DoD introduced a new WGS 84 realization, called WGS 84 (G730), whose origin, orientation and scale would be identical to those adopted for the International Terrestrial Reference Frame of 1991 (ITRF91). Hence, from 1994 onward, NAD 83 coordinates and WGS 84 coordinates have systematically differed by more than two meters.

7.3.1 Transforming WGS 84 (G1762) Coordinates to Current NAD 83 Coordinates

Because WGS 84 (G1762) is aligned to ITRF2008, the transformation from WGS 84 (G1762) to each of the current NAD 83 reference frames; namely, NAD 83(2011), NAD 83(PA11) and NAD 83(MA11); is the same as the corresponding transformation from ITRF2008 to each of these three current NAD 83 reference frames. Pearson and Snay [36] present the mathematical procedure for transforming coordinates from ITRF2008 to the three current NAD 83 frames. For the sake of completeness, their procedure is given here in terms of transforming coordinates from WGS 84 (G1762) to the three current NAD 83 frames.

Let x(t)A ,y(t)A, and z(t)A denote the positional coordinates of a location at time t referred to reference frame A in a 3D-ECEF Cartesian coordinate system. These coordinates are expressed as a function of time to reflect the reality of crustal motion. Similarly, let x(t)B, y(t)B, and z(t)B denote the positional coordinates of this same location at time (t) referred to reference frame B also in a 3D-ECEF Cartesian coordinate system. The coordinates in frame A are approximately related to those in frame B via the following equations of a 14-parameter transformation:

$$
\begin{aligned}
x(t)B &= Tx(t) + [1 + s(t)] \cdot x(t)A + wz(t) \cdot y(t)A - wy(t) \cdot z(t)A \\
y(t)B &= Ty(t) - wz(t) \cdot x(t)A + [1 + s(t)] \cdot y(t)A + wx(t) \cdot z(t)A \\
z(t)B &= Tz(z) + wy(t) \cdot x(t)A - wx(t)y(t)A + [1 + s(t)] \cdot z(t)A
\end{aligned} \tag{7-2}
$$

Tx(t), Ty(t), and Tz(t) are translations along the x-, y-, and z-axis, respectively; wx(t), wy(t), and wz(t) are counterclockwise rotations about these same three axes; and s(t) is the differential scale between reference frame A and reference frame B. These approximate equations suffice because the three rotations have relatively small magnitudes. Note: Each of the seven quantities is represented as a function of time because modern geodetic technology has enabled scientists to detect their time-related variations with some degree of accuracy. These time-related variations are assumed to be linear, so that each of the seven quantities may be expressed by an equation of the form:

$$P(t) = P(\tau) + \dot{P}(t - \tau) \tag{7-3}$$

where τ denotes a prespecified time of reference and the two quantities, P(τ) and $\dot{P}$, are constants. Thus, the seven quantities give rise to 14 parameters, but notice that the values of seven of these parameters depend on the value chosen for τ. The following table provides parameter values for τ = 1997.0; i.e., January 1, 1997.

Table 7.1 Parameters for transforming WGS 84 (G1762) coordinates to NAD 83 coordinates.

Parameter	Units	NAD 83(2011)	NAD 83(PA11)	NAD 83(MA11)
T_x(1997.0)	meters	+0.99343	+0.9080	+0.9080
T_y(1997.0)	meters	-1.90331	-2.0161	-2.0161
T_z(1997.0)	meters	-0.52655	-0.5653	-0.5653
w_x(1997.0)	nanoradians	+125.63787	+134.49216	+140.45537
w_y(1997.0)	nanoradians	+45.70072	+65.29956	+50.51759
w_z(1997.0)	nanoradians	+56.23524	+13.14815	+43.28416
s(1997.0)	parts per billion	+1.71504	+1.10	+1.10
$\dot{T}_x$	meters/yr	+0.00079	+0.0001	+0.0001
$\dot{T}_y$	meters/yr	-0.00060	+0.0001	+0.0001
$\dot{T}_z$	meters/yr	-0.00134	-0.0018	-0.0018
$\dot{w}_x$	nanoradians/yr	+0.32322	-1.86168	-0.09696
$\dot{w}_y$	nanoradians/yr	-3.67217	+4.88207	+0.50905
$\dot{w}_z$	nanoradians/yr	-0.24886	-10.59803	-1.68230
$\dot{s}$	parts-per-billion/yr	-0.10201	+0.08	+0.08

As reflected in Equations (7-2) and (7-3), the transformation from WGS 84 (G1762) to each of the current NAD 83 reference frames depends on the variable t (expressed in decimal years). This variable denotes the epoch associated with the coordinates. This epoch often corresponds to the date the coordinates were measured; however, sometimes the coordinates have been converted to some other epoch of particular significance so that coordinates measured at one time may be compared with coordinates measured at other times. Such a conversion requires knowledge of how the coordinates have moved over time. The Horizontal Time-Dependent Positioning (HTDP) utility (www.geodesy.noaa.gov/TOOLS/Htdp/Htdp.shtml) contains numerical models for horizontal crustal motion which enables its users to convert horizontal coordinates from one epoch to another for locations in the USA. HTDP also incorporates the parameters given here to enable its users to transform coordinates between WGS 84 (G1762) and the current NAD 83 reference frames.

7.4 Three-parameter Geometric Transformations

Local geodetic systems often contain non-linear distortions, poorly known relationships to the geoid, and lack quantifiable absolute accuracy. For most DoD operations and applications involving datum shifts, WGS 84 coordinates will be obtained by a Local Geodetic Datum to WGS 84 Datum Transformation. A shift of an ellipsoid's origin (neglecting changes in scale and misalignment of the axes) satisfies most mapping and charting requirements. A three parameter model can be applied to geocentric Cartesian coordinates; or when used with ellipsoidal change parameters in the Molodensky transformation equations, applied to curvilinear coordinates. These shifts are reversible. The prime meridian was assumed to be Greenwich in the development of these shifts.

The datum shift parameters (ΔX, ΔY, ΔZ) are the distances between the center of a local geodetic system's ellipsoid and the center of the WGS 84 ellipsoid, i.e., the center of the local ellipsoid in WGS 84 coordinates.

Appendix C lists the reference ellipsoid names and parameters (semi-major axis and flattening factor) for local datums currently tied to WGS 84 and used for generating datum transformations.

Appendix D contains horizontal transformation parameters for the geodetic datums/systems which have been generated from satellite ties to the local geodetic control.

Updates to the datum transformation parameters are identified through the use of cycle numbers and issue dates. Cycle numbers have been set to the numerical value of zero for all datum transformations appearing in the August 1993 Insert 1 and the WGS 84 TR8350.2 Second Edition. New datum transformations will carry a cycle number of zero. As updates are made the cycle number will increment by one.

Datum transformation shifts derived from non-satellite information are listed in Appendix E.

7.4.1 Transformation to WGS 84 Cartesian Coordinates

The shift parameters can be applied directly to the local geocentric system as

$$
\begin{aligned}
X_{WGS\,84} &= X_{Local} + \Delta X \\
Y_{WGS\,84} &= Y_{Local} + \Delta Y \\
Z_{WGS\,84} &= Z_{Local} + \Delta Z
\end{aligned}
\qquad (7\text{-}4)
$$

7.4.2 Transformation to WGS 84 Geodetic Coordinates: Three-step Method

Local geodetic coordinates can be transformed to WGS 84 in three steps:

1) Convert the local geodetic coordinates to local geocentric Cartesian coordinates using the local ellipsoid parameters.
2) Shift the local geocentric Cartesian coordinates to WGS 84 geocentric coordinates.
3) Convert the WGS 84 geocentric Cartesian coordinates to WGS 84 geodetic coordinates using the parameters of the WGS 84 ellipsoid.

7.4.3 Transformation to WGS 84 Geodetic Coordinates: Molodensky Shifts

$$
\begin{aligned}
\varphi_{WGS\,84} &= \varphi_{Local} + \Delta\varphi \\
\lambda_{WGS\,84} &= \lambda_{Local} + \Delta\lambda \\
h_{WGS\,84} &= h_{Local} + \Delta h
\end{aligned}
\tag{7-5}
$$

where Δφ, Δλ, Δh are provided by the standard Molodensky transformation formulas [37], [38] as

$$
\begin{aligned}
\Delta\varphi' = \{&-\Delta X \sin\varphi\cos\lambda - \Delta Y \sin\varphi\sin\lambda + \Delta Z\cos\varphi + \Delta a\,(R_N e^2 \sin\varphi\cos\varphi)/a \\
&+ \Delta f[R_M(a/b) + R_N(b/a)]\sin\varphi\cos\varphi\} \cdot \left[(R_M + h)\sin 1''\right]^{-1} \\
\Delta\lambda'' &= [-\Delta X\, sin\,\lambda + \Delta Y\, cos\,\lambda] \cdot [(R_N + h)\, cos\,\varphi\, sin\, 1'']^{-1} \\
\Delta h &= \Delta X\, cos\,\varphi\, cos\,\lambda + \Delta Y\, cos\,\varphi\, sin\,\lambda + \Delta Z\, sin\,\varphi - \Delta a(a/R_N) + \\
&\quad \Delta f(b/a)R_N\, sin^2\,\varphi
\end{aligned}
\tag{7-5}
$$

where

$\varphi, \lambda, h = geodetic\ coordinates\ (old\ ellipsiod)$

$\varphi = geodetic\ latitude.\ The\ angle\ between\ the\ plane\ of\ the\ geodetic\ equator\ and\ the\ ellipsoid\ normal\ at\ a\ point\ (measured\ positive\ north\ from\ the\ geodetic\ equator, negative\ south)$

$\lambda = geodetic\ longitude. The\ angle\ between\ the\ plane\ of\ the\ Zero\ Meridian\ and\ the\ plane\ of\ the\ geodetic\ meridian\ of\ the\ point\ (measured\ in\ the\ plane\ of\ the\ geodetic\ equator, positive\ from\ the\ 0°\ to\ the\ 180°\ E\ and\ negative\ from\ the\ 0°\ to\ the\ 180°\ W)$

$h = N + H$

where

$h = height\ above\ ellipsoid$

$N = geoid\ height$

$H = orthometric\ height\ relative\ to\ the\ geoid$

$\Delta\varphi, \Delta\lambda, \Delta h = corrections\ to\ the\ transformation\ local\ geodetic\ datum\ coordinates\ to\ WGS\ 84\ \varphi, \lambda, h\ values.\ The\ units\ of\ \Delta\varphi\ and\ \Delta\lambda\ are\ arc\ seconds\ (");\ the\ units\ of\ \Delta h\ are\ meters\ (m).$

NOTE: AS “h’s” ARE NOT AVAILABLE FOR LOCAL GEODETIC DATUMS, THE Δh CORRECTION WILL NOT BE APPLICABLE WHEN TRANSFORMING TO WGS 84

$\Delta X, \Delta Y, \Delta Z = shifts\ between\ centers\ of\ the\ local\ geodetic\ datum\ and\ WGS\ 84\ Ellipsoid, corrections\ to\ transformation\ local\ geodetic\ system\text{-}related\ rectangular\ coordinates\ (X, Y, Z)\ to\ WGS\ 84\text{-}\ related\ X, Y, Z\ values$

$a = semi\text{-}major\ axis\ of\ the\ local\ geodetic\ datum\ ellipsoid$

$b = semi\text{-}minor\ axis\ of\ the\ local\ geodetic\ datum\ ellipsoid$

$b/a = 1 - f$

$f = flattening\ factor\ of\ the\ local\ geodetic\ datum\ ellipsoid$

$\Delta a, \Delta f = differences\ between\ the\ semi\text{-}major\ axis\ and\ the\ flattening\ factors\ of\ the\ local\ geodetic\ datum\ ellipsod\ and\ the\ WGS\ 84\ Ellipsoid\ , respectively\ (WGS\ 84\ minus\ local)$

$e = first\ eccentricity$

$e^2 = 2f - f^2$

$$R_N = \frac{a}{\sqrt{1 - e^2 \sin^2 \varphi}}$$
$$= radius\ of\ curvature\ in\ the\ prime\ vertical$$

$$R_M = \frac{a(1 - e^2)}{\sqrt[3]{1 - e^2 \sin^2 \varphi}}$$
$$= radius\ of\ curvature\ in\ the\ meridian$$

NOTE: All Δ-quantities are formed by subtracting local geodetic datum ellipsoid values from WGS 84 Ellipsoid values.

Due to the errors and distortions that affect most local geodetic datums, the use of mean datum shifts (ΔX, ΔY, ΔZ) in the standard Molodensky datum transformation formulas may produce results with a poor quality of “fit”. Improved fit between the local datum and WGS 84 may result only with better and denser ties with local or regional control points.

7.5 Seven-parameter Geometric Transformations

The seven-parameter transformation model assumes the local and WGS 84 Cartesian axes can be related by rotations of the three axes, a change of scale (Δs), and a shift of the origin (ΔX, ΔY, ΔZ).

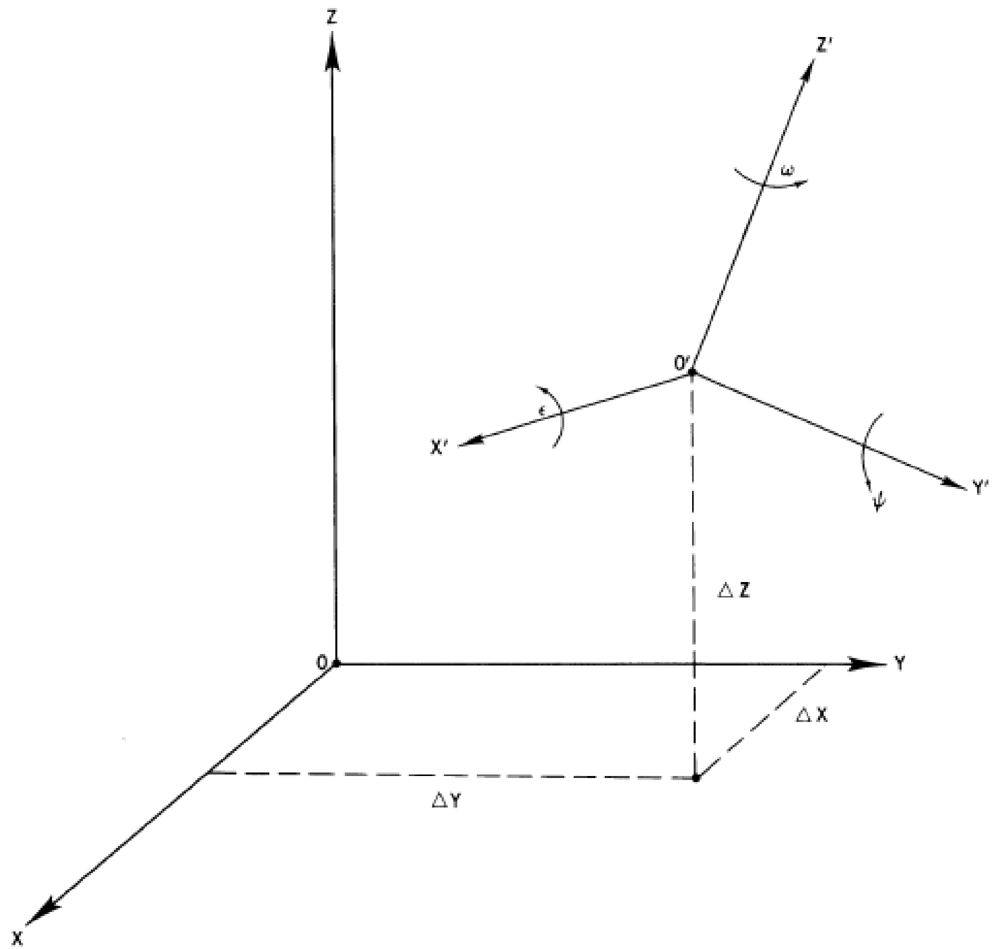


Figure 7.1 Seven-Parameter Transformation

Two seven-parameter transformation models (Coordinate Frame Rotation and Position Vector Transformation) are in widespread use. They work best for geocentric datums that are distortion free, like an ITRF to WGS 84 transformation, or other modern satellite derived datums. They are, however, often used for datums that do not meet these criteria. The models differ by whether rotation is of the axes (frame) or of the position vector. With approximations, these models can be related by changing the signs of the rotation angles. The use of the actual angles instead of their cosines and sines as elements in the rotation matrices (the small angle approximation) has been studied and generally leads to coordinate errors in the millimeter range [39]. If there are no rotations, the models are the same. If there are no rotations or scale change the models degenerate to the three-parameter shifts.

7.5.1 Coordinate Frame Rotation Model

The coordinate frame rotation model, widely used in the United States and Australia, can be expressed as

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_{WGS\,84} = \begin{bmatrix} \Delta X \\ \Delta Y \\ \Delta Z \end{bmatrix} + \begin{bmatrix} 1+\Delta s' & \omega & -\psi \\ -\omega & 1+\Delta s' & \varepsilon \\ \psi & -\varepsilon & 1+\Delta s' \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_{Local} \tag{7-6}$$

where

$\Delta s' = the\ scale\ correction$

$\varepsilon = the\ rotation\ about\ the\ X_{LOCAL}\ axis$

$\psi = the\ rotation\ about\ the\ Y_{LOCAL}\ axis$

$\omega = the\ rotation\ about\ the\ Z_{LOCAL}\ axis$

7.5.2 Position Vector Transformation Model

The position vector transformation model, used by IERS [1] and recommended for use by NATO members, can be expressed as

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_{WGS\,84} = \begin{bmatrix} \Delta X \\ \Delta Y \\ \Delta Z \end{bmatrix} + (1+\Delta S) \begin{bmatrix} 1 & -R_Z & R_Y \\ R_Z & 1 & -R_X \\ -R_Y & R_X & 1 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_{Local} \tag{7-7}$$

where

$\Delta S = the\ scale\ correction$

$R_X = the\ rotation\ about\ the\ X_{LOCAL}\ axis$

$R_Y = the\ rotation\ about\ the\ Y_{LOCAL}\ axis$

$R_Z = the\ rotation\ about\ the\ Z_{LOCAL}\ axis$

7.5.3 Approximate Equivalency of the Models

$$\begin{aligned} \Delta s' &= \Delta S \\ \varepsilon &= -(1+\Delta S)R_X \approx R_X \\ \psi &= -(1+\Delta S)R_Y \approx R_Y \\ \omega &= -(1+\Delta S)R_Z \approx R_Z \end{aligned} \tag{7-8}$$

7.6 Molodensky-Badekas Transformation Model

It may be advantageous to relate a local datum to a global satellite-derived one by the Molodensky-Badekas model; also known as the 7 + 3 model.

In this model, the rotations and scale change take place in the region of interest, not the geocenter, hence the additional three parameters. This is done to alleviate high correlations that may exist among the usual seven parameters when computed at the geocenter.

This model can have two variations of rotations, as was described for the seven parameter models. A drawback to this method is that it is not reversible in the absolute sense. It is described by:

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_{WGS\,84} = (1+\Delta S)\begin{bmatrix} 1 & R_Z & -R_Y \\ -R_Z & 1 & R_X \\ R_Y & -R_X & 1 \end{bmatrix}\begin{bmatrix} X_{Local}-X_0 \\ Y_{Local}-Y_0 \\ Z_{Local}-Z_0 \end{bmatrix} + \begin{bmatrix} X_0 \\ Y_0 \\ Z_0 \end{bmatrix} + \begin{bmatrix} \Delta X \\ \Delta Y \\ \Delta Z \end{bmatrix} \tag{7-9}$$

where

X_0, Y_0, Z_0 *are the local Cartesian coordinates of the point where the rotations and the scale change are to take place*

7.7 Datum Transformation Multiple Regression Equations (MRE)

The development of Local Geodetic Datum to WGS 84 Datum Transformation Multiple Regression Equations [40] was initiated to obtain better fits over continental-size land areas than could be achieved using the standard Molodensky formula with datum shifts (ΔX, ΔY, ΔZ).

For Δφ, the general form of the Multiple Regression Equation is (also see [40]):

$$\Delta\varphi = A_0 + A_1U + A_2V + A_3U^2 + A_4UV + A_5V^2 + \cdots + A_{99}U^9V^9 \tag{7-10}$$

where

$A_0 = constant$

$A_0, A_1, \cdots A_{nn} = coeffiecents\ determined\ in\ the\ development$

$U = k(\varphi - \varphi_m)$
$= normalized\ geodetic\ latitude\ of\ the\ computation\ point$

$V = k(\lambda - \lambda_m)$
$= normalized\ geodetic\ longitude\ of\ the\ computation\ point$

$k = scale\ factor\ and\ degree\text{-}to\text{-}radian\ conversion$

$\varphi, \lambda = local\ geodetic\ latitude\ and\ local\ geodetic\ longitude$
$(in\ degrees), respectively, of\ the\ computation\ point$

$$\varphi_m, \lambda_m = mid\text{-}latitude\ and\ mid\text{-}\ longitude\ values, respectively\ of\ the\ local\ geodetic\ datum\ area\ (in\ degrees)$$

Similar equations are obtained for $\Delta\lambda$ and Δh by replacing $\Delta\varphi$ in the left portion of Equation (7-10) by $\Delta\lambda$ and Δh, respectively.

Local Geodetic Datum to WGS 84 Datum Transformation Multiple Regression Equations for seven major continental-size datums, covering contiguous land areas with large distortion, are provided in Appendix F. The main advantage of MREs lies in modeling of distortion for better fit in geodetic applications. However, caution must be used to ensure that MREs are not extrapolated outside of the area of intended use. Large distortions can be realized in very short distances outside of the area where the stations that were used in the development of the MREs exist.

7.8 WGS 72 to WGS 84

See Appendix G.

7.9 Datums Equivalent to WGS 84 for Mapping and Charting Purposes

See Appendix H for a list of datums that can be considered equivalent to WGS 84 for use at scales of 1:5,000 and larger.

8. ACCURACY OF COORDINATES REFERENCED TO WGS 84

8.1 Discussion

Numerous techniques exist to establish WGS 84 coordinates for a given site or position. The accuracy and precision achieved by these various techniques vary significantly. The most common, currently available techniques are listed below:

- General geodetic solution for station coordinates, orbits, and other parameters of interest

- Direct geodetic point positioning at a stationary, solitary station using a 'geodetic-quality', dual frequency GPS receiver and NGA Precise Ephemerides and Satellite Clock States (note that the effects of Selective Availability (SA) must be removed if SA is not set to zero)

- Same as above but using the Broadcast GPS Ephemerides and Clock States

- GPS differential (baseline) processing from known WGS 84 sites [41]

- GPS Precise Positioning Service (PPS) navigation solutions [42]
  - Instantaneous
  - Mean over some averaging interval

- GPS Standard Positioning Service (SPS) navigation solutions [43]
  - Instantaneous
  - Mean over some averaging interval

- Photogrammetrically-derived coordinates from NGA products

- Map-derived coordinates from digital or paper NGA products

- Coordinate conversion based on transformation from a different geodetic datum to WGS 84

Clearly, the above positioning techniques do not provide WGS 84 coordinates with uniform accuracy and statistical properties. Even within a given technique, accuracy variations can occur due, for example, to the treatment of certain error sources such as the troposphere. Because of these variations and periodic algorithm improvements, full characterization of the accuracy achieved by all the above techniques would be quite challenging and beyond the scope of this document.

In the terminology of Chapter 2, a network of stations obtained from one of these techniques yields a unique realization of the WGS 84 reference frame. Currently, within the DoD, almost all operational geodetic survey requirements can be met with direct geodetic point positioning with GPS. The NGA-developed technique [44] [45] which performs this function has been demonstrated to achieve an accuracy at a single station of:

2012 - present: 5 cm (1σ), in each of the 3 position components (φ, λ, h)

2010 - 2012: 10 cm (1 σ), in each of the 3 position components (φ, λ, h)

1994 - 2010: 30 cm (1 σ), in each of the 3 position components (φ, λ, h)

1989 - 1994: 100 cm (1 σ), in each of the 3 position components (φ, λ, h)

8.2 Techniques for Obtaining WGS 84 Coordinates.

Other techniques based on older, previously established survey coordinates have been used in the past to yield 'WGS 84' coordinates with limited accuracy. Precise positions computed many years ago are no longer precise due to movement of the Earth as discussed in Chapter 2. Knowledge of the inherent accuracy of these positions and the accuracy required of the operation is critical for any usage to be valid for operational requirements. These legacy techniques may be suitable for very limited mapping applications where accuracy is not a concern. Some of these alternate techniques to obtain WGS 84 coordinates are listed below:

- TRANSIT Point Positioning directly in WGS 84 (1 σ = 1-2 m)
- TRANSIT Point Positions transformed from NSWC-9Z2
- GPS differential (baseline) processing from a known (TRANSIT-determined) WGS 84 geodetic point position
- WGS 72 to WGS 84 Coordinate Transformation
- Local Geodetic Datum to WGS 84 Datum Transformation
- Non-WGS 84-based reference systems

Because geospatial information within the DoD often originates from multiple sources and processes, the absolute accuracy of a given WGS 84 position becomes very important when information from these various sources is combined in Geographic Information Systems or 'geospatial databases'. Due to their high fidelity, surveyed WGS 84 geodetic control points can often serve to improve or validate the accuracy of maps, image products, or other geospatial information. Even GPS navigation solutions can serve a similar role, as long as the accuracy of these solutions is well understood.

8.3 Summary

In summary, while WGS 84 provides a common global framework for all geospatial information within the DoD, the accuracy of each 'layer' of information depends largely on the metric fidelity of the process used to collect that information. WGS 84-surveyed control points provide an accuracy level which meets or exceeds all current operational DoD requirements. The user must ensure that the method chosen to obtain WGS 84 coordinates meets their accuracy requirements.

9. THE WORLD MAGNETIC MODEL

9.1 Introduction

At every location on or above the Earth, its magnetic field has a more or less well-known direction which can be used as a reference frame to orient ships, aircraft, satellites, antennas, drilling equipment, and handheld devices. At some places on the globe the horizontal direction of the magnetic field coincides with the direction of geographic north (“true” north), but in general this is not the case. The angular amount by which the horizontal direction of the magnetic field differs from true north is called the magnetic declination, or simply declination (D). This is the correction required to convert between a magnetic bearing and a true bearing. The main utility of the World Magnetic Model (WMM) is to provide the magnetic declination for any desired location on the globe. In addition to the magnetic declination, the WMM also provides the complete geometry of the field from 1 km below the Earth’s surface to 850 km above the surface. The magnetic field extends deep into the Earth and far out into space, but the WMM is not valid there.

The Earth’s magnetism has several sources. All the sources will affect a scientific or navigational instrument, but only some of them are represented in the WMM. The strongest contributor, by far, is the magnetic field produced by the Earth’s liquid-iron outer core, called the “core field”. Magnetic minerals in the crust and upper mantle make a further contribution that can be locally significant. Electric currents induced by the flow of conducting sea water through the ambient magnetic field make a further, albeit weak, contribution to the observed magnetic field. All of these are of “internal” origin. Deliberately excluded from the WMM, by the data selection process and by other means, are the so-called “disturbance fields”. These are contributions arising from electric currents in the upper atmosphere and near-Earth space. Because the “external” magnetic fields so produced are time-varying, there is a further effect. They induce electric currents in the Earth and oceans, producing secondary internal magnetic fields, which are considered part of the disturbance field and are therefore not represented in the WMM.

The mathematical method of the WMM is an expansion of the magnetic potential into spherical harmonic functions up to degree and order 12. The minimum wavelength resolved is $360° / \sqrt{12\ x\ 13} = 28.8°$ in arc-length, corresponding to 3200 km at the Earth’s surface. The WMM is a model of those internal magnetic fields that are not part of the disturbance field and have spatial wavelengths exceeding 30° in arc-length. This includes almost the entire core field and the long-wavelength portion of the crustal and oceanic fields. The term “main field” refers to the portion of the Earth’s magnetic field at epoch 2010.0 that is modeled by the WMM.

The core field changes perceptibly from year to year. This effect, called secular variation (SV), is accounted for in the WMM by a linear SV model. Specifically, a straight line is used as the model of the time-dependence of each coefficient of the spherical harmonic representation of the magnetic potential. Due to unpredictable, non-linear changes in the core field, the values of the WMM coefficients need to be updated every five years. WMM2010, is valid from 2010.0 to 2015.0.

9.2 The Elements of the Magnetic Field

The geomagnetic field vector ($\mathbf{B}_m$) is described by 7 elements. These are the northerly intensity (X), the easterly intensity (Y), the vertical intensity (Z) (positive downwards), and the following quantities derived from X, Y, and Z: the horizontal intensity (H), the total intensity (F), the inclination angle (I), (also called the dip angle and measured from the horizontal plane to the field vector, positive downwards), and the declination angle (D) (also called the magnetic variation and measured clockwise from true north to the horizontal component of the field vector). In the descriptions of X, Y, Z, H, F, I and D above, the vertical direction is perpendicular to the WGS 84 ellipsoid model of the Earth, the horizontal plane is perpendicular to the vertical direction, and the rotational directions clockwise and counter-clockwise are determined by a view from above.

The quantities X, Y, and Z are the sizes of perpendicular vectors that add vectorially to $\mathbf{B}_m$. Conversely, X, Y, and Z can be determined from the quantities F, I, and D (i.e., the quantities that specify the size and direction of $\mathbf{B}_m$).

9.3 Grid Variation

In the polar regions, or near the rotation axis of the Earth, the angle D changes strongly with a change in the longitude of the observer, and is therefore a poor measure of the direction of $\mathbf{B}_m$. For this reason, the WMM technical report and software have defined an auxiliary angle, GV, for the direction of $\mathbf{B}_m$ in the horizontal plane. Its definition is:

$$
\begin{aligned}
&GV = D - \lambda \; for \; \varphi > 55° \\
&GV = D + \lambda \; for \; \varphi < -55° \\
&GV is \; undefined \; otherwise
\end{aligned}
\tag{9-1}
$$

where

$$\lambda = longitude$$

$$\varphi = geodetic \; latitude$$

The angle GV should also be understood as the angle on the plane of the Universal Polar Stereographic (UPS) grid for the appropriate hemisphere at the observer's location measured clockwise from the direction parallel to the UPS Northing axis (y-axis) to the horizontal component of $\mathbf{B}_m$. To emphasize this, the designation GV_{UPS} may be used for the above.

The quantity GV_{UPS} defined above is an example of a more general concept, namely grid variation or grivation. Grivation is the angle on the plane of a chosen grid coordinate system at the observer's location, measured clockwise from the direction parallel to the grid's Northing axis to the horizontal component of $\mathbf{B}_m$. It is useful for local surveys, where location is given by grid coordinates rather than by longitude and latitude. Grivation is dependent on the map projection used to define the grid coordinates. In general it is:

$$GV_{grid} = D - C \tag{9-2}$$

where

$D = magnetic\ declination$

$C = 'convergence\text{-}of\text{-}meridians'(clockwise\ angle\ from\ the\ northward\ meridional\ arc\ to\ the\ grid\ Northing\ direction)$

Large scale military topographic mapping routinely employs the Universal Transverse Mercator (UTM) grid coordinates for the map projection of the sheet, for the definition of a grid to overprint, and for a grivation calculation as defined above. The latter could be notated GV_{UTM}.

9.4 Range of the Magnetic Elements at the Earth's Surface

Table 9.1 shows the expected range of the magnetic field elements, measured in nanoTeslas, and GV at the Earth's surface.

Table 9.1 Ranges of Magnetic Elements and *GV* at the Earth's Surface

Element	Name	Alternative Name	Range at Earth's Surface			Positive Sense
			Min	Max	Unit	
X	North component	Northerly intensity	-17000	42000	nT	North
Y	East component	Easterly intensity	-18000	17000	nT	East
Z	Down component	Vertical intensity	-67000	61000	nT	Down
H	Horizontal intensity		0	42000	nT	
F	Total intensity	Total field	22000	67000	nT	
I	Inclination	Dip	-90	90	Degree	Down
D	Declination	Magnetic variation	-180	180	Degree	East / Clockwise
GV	Grid variation	Grivation	-180	180	Degree	East / Clockwise

9.5 Model Representation

This section describes the representation of the magnetic field in the WMM. All variables in this section adhere to the following measurement conventions: angles are in radians, lengths are in meters, magnetic intensities are in nano-Teslas (nT), and times are in years.

The main magnetic field ($\mathbf{B_m}$) is a potential field and therefore can be written in geocentric spherical coordinates (longitude λ, latitude φ', radius r) as the negative spatial gradient of a scalar potential:

$$\boldsymbol{B}_m(\lambda,\varphi',r,t) = -\nabla V(\lambda,\varphi',r,t) \tag{9-3}$$

This potential can be expanded in terms of spherical harmonics as

$$V(\lambda,\varphi',r,t) = a\left\{\sum_{n=1}^{N}\sum_{m=0}^{n}\left(g_n^m(t)\cos m\lambda + h_n^m(t)\sin m\lambda\right)\left(\frac{a}{r}\right)^{n+1}\breve{P}_n^m \sin\varphi'\right\} \tag{9-4}$$

where:

N = 12 is the degree of the expansion of the WMM

a = (6371200 m), the geomagnetic reference radius

(λ,φ',r) = spherical geocentric longitude, latitude, and radius

$g_n^m(t)$ and $h_n^m(t)$ = the time-dependent Gauss coefficients of degree n and order m describing the Earth's main magnetic field.

$\breve{P}_n^m(\mu)$ are the Schmidt semi-normalized associated Legendre functions defined as

$$\begin{aligned}\breve{P}_n^m(\mu) &= \sqrt{2\frac{(n-m)!}{(n+m)!}}\,P_{n,m}(\mu)\ \ if\ \ m>0\\ \breve{P}_n^m(\mu) &= P_{n,m}(\mu)\ \ if\ \ m=0\end{aligned} \tag{9-5}$$

Here, we use the definition of $P_{n,m}(\mu)$ commonly used in geodesy and geomagnetism [18].

WMM2010 comprises two sets of Gauss coefficients to degree and order $N=12$. One set provides a spherical-harmonic main field model for 2010.0 in units of nT, the other set provides a predictive secular variation model for the period 2010.0 to 2015.0 in units of nT/year. Further details on the computation of magnetic field elements can be found in [46].

9.6 The WMM2010 Coefficients

The model coefficients, also referred to as Gauss coefficients, are listed in Table 9.2. These coefficients can be used to compute values for the field elements and their annual rates of change at any location near the surface of the Earth and at any date between 2010.0 and 2015.0.

Table 9.2 Final Coefficients for WMM2010

Units are nT for the main field, and nT per year for the secular variation. The index *n* is the degree and *m* is the order. Since $h_n^m(t_0)$ and $\dot{h}_n^m(t_0)$ are not defined for *m*=0, the corresponding fields are left blank.

n	m	$g_n^m(t_0)$	$h_n^m(t_0)$	$\dot{g}_n^m(t_0)$	$\dot{h}_n^m(t_0)$
1	0	-29496.6		11.6	
1	1	-1586.3	4944.4	16.5	-25.9
2	0	-2396.6		-12.1	
2	1	3026.1	-2707.7	-4.4	-22.5
2	2	1668.6	-576.1	1.9	-11.8
3	0	1340.1		0.4	
3	1	-2326.2	-160.2	-4.1	7.3
3	2	1231.9	251.9	-2.9	-3.9
3	3	634.0	-536.6	-7.7	-2.6
4	0	912.6		-1.8	
4	1	808.9	286.4	2.3	1.1
4	2	166.7	-211.2	-8.7	2.7
4	3	-357.1	164.3	4.6	3.9
4	4	89.4	-309.1	-2.1	-0.8
5	0	-230.9		-1.0	
5	1	357.2	44.6	0.6	0.4
5	2	200.3	188.9	-1.8	1.8
5	3	-141.1	-118.2	-1.0	1.2
5	4	-163.0	0.0	0.9	4.0
5	5	-7.8	100.9	1.0	-0.6
6	0	72.8		-0.2	
6	1	68.6	-20.8	-0.2	-0.2
6	2	76.0	44.1	-0.1	-2.1
6	3	-141.4	61.5	2.0	-0.4
6	4	-22.8	-66.3	-1.7	-0.6

n	m	$g_n^m(t_0)$	$h_n^m(t_0)$	$\dot{g}_n^m(t_0)$	$\dot{h}_n^m(t_0)$
6	5	13.2	3.1	-0.3	0.5
6	6	-77.9	55.0	1.7	0.9
7	0	80.5		0.1	
7	1	-75.1	-57.9	-0.1	0.7
7	2	-4.7	-21.1	-0.6	0.3
7	3	45.3	6.5	1.3	-0.1
7	4	13.9	24.9	0.4	-0.1
7	5	10.4	7.0	0.3	-0.8
7	6	1.7	-27.7	-0.7	-0.3
7	7	4.9	-3.3	0.6	0.3
8	0	24.4		-0.1	
8	1	8.1	11.0	0.1	-0.1
8	2	-14.5	-20.0	-0.6	0.2
8	3	-5.6	11.9	0.2	0.4
8	4	-19.3	-17.4	-0.2	0.4
8	5	11.5	16.7	0.3	0.1
8	6	10.9	7.0	0.3	-0.1
8	7	-14.1	-10.8	-0.6	0.4
8	8	-3.7	1.7	0.2	0.3
9	0	5.4		0.0	
9	1	9.4	-20.5	-0.1	0.0
9	2	3.4	11.5	0.0	-0.2
9	3	-5.2	12.8	0.3	0.0
9	4	3.1	-7.2	-0.4	-0.1
9	5	-12.4	-7.4	-0.3	0.1
9	6	-0.7	8.0	0.1	0.0
9	7	8.4	2.1	-0.1	-0.2
9	8	-8.5	-6.1	-0.4	0.3
9	9	-10.1	7.0	-0.2	0.2
10	0	-2.0		0.0	
10	1	-6.3	2.8	0.0	0.1
10	2	0.9	-0.1	-0.1	-0.1
10	3	-1.1	4.7	0.2	0.0
10	4	-0.2	4.4	0.0	-0.1
10	5	2.5	-7.2	-0.1	-0.1

n	m	$g_n^m(t_0)$	$h_n^m(t_0)$	$\dot{g}_n^m(t_0)$	$\dot{h}_n^m(t_0)$
10	6	-0.3	-1.0	-0.2	0.0
10	7	2.2	-3.9	0.0	-0.1
10	8	3.1	-2.0	-0.1	-0.2
10	9	-1.0	-2.0	-0.2	0.0
10	10	-2.8	-8.3	-0.2	-0.1
11	0	3.0		0.0	
11	1	-1.5	0.2	0.0	0.0
11	2	-2.1	1.7	0.0	0.1
11	3	1.7	-0.6	0.1	0.0
11	4	-0.5	-1.8	0.0	0.1
11	5	0.5	0.9	0.0	0.0
11	6	-0.8	-0.4	0.0	0.1
11	7	0.4	-2.5	0.0	0.0
11	8	1.8	-1.3	0.0	-0.1
11	9	0.1	-2.1	0.0	-0.1
11	10	0.7	-1.9	-0.1	0.0
11	11	3.8	-1.8	0.0	-0.1
12	0	-2.2		0.0	
12	1	-0.2	-0.9	0.0	0.0
12	2	0.3	0.3	0.1	0.0
12	3	1.0	2.1	0.1	0.0
12	4	-0.6	-2.5	-0.1	0.0
12	5	0.9	0.5	0.0	0.0
12	6	-0.1	0.6	0.0	0.1
12	7	0.5	0.0	0.0	0.0
12	8	-0.4	0.1	0.0	0.0
12	9	-0.4	0.3	0.0	0.0
12	10	0.2	-0.9	0.0	0.0
12	11	-0.8	-0.2	-0.1	0.0
12	12	0.0	0.9	0.1	0.0

9.7 Magnetic Poles and Geomagnetic Coordinate Systems

There are different ways of defining magnetic poles. The most common understanding is that they are the positions on the Earth's surface where the geomagnetic field is vertical, that is, perpendicular to the ellipsoid. These positions are called dip poles, and the north and south dip poles do not have to be (and are not now) antipodal.

Other definitions originate from models of the geomagnetic field. The WMM representation of this field includes a magnetic dipole at the center of the Earth. This dipole defines an axis that intersects the Earth's surface at two antipodal points, known as geomagnetic poles. The geomagnetic poles, otherwise known as the dipole poles, can be computed from the first three Gauss coefficients of the WMM. Based on the WMM2010 coefficients for 2010.0, the geomagnetic north pole is at 72.21°W longitude and 80.02°N geocentric latitude (80.08°N geodetic latitude); and the geomagnetic south pole is at 107.79°E longitude and 80.02°S geocentric latitude (80.08°S geodetic latitude). The axis of the dipole is currently inclined at 9.98° to the Earth's rotation axis. The same dipole is the basis for the simple geomagnetic coordinate system of geomagnetic latitude and longitude. The geomagnetic equator is at geomagnetic latitude 0°.

10. IMPLEMENTATION GUIDELINES

10.1 Introduction

WGS 84 represents the current state-of-the-art NSG operational reference system which is a realization of the ITRF 2008 based on the IERS Conventions (2010) as outlined in IERS TN 36. This new realization is titled WGS 84 (G1762). Since G1762 is a realization of the IERS reference frame, it incorporates the most current and modern analysis and data including SLR, LLR and VLBI systems.

WGS 84 (G1762) supports the most stringent accuracy requirements for geodetic positioning, navigation and mapping within the NSG. WGS 84 (G1762) accuracy is less than 1cm in each axis of the WGS 84 ECEF coordinate system.

It is the policy of NGA to continually improve components of the WGS 84 system to maintain it as a state-of-the-art NSG operational World Geodetic System. This will lead to improvements in the definition and realization of the system as

- The basic tracking stations which are used are updated, repositioned or their number is increased
- Additional data make it possible to improve the accuracy of individual local datum relationships to WGS 84
- Additional surface gravity data become available in various areas of the world to improve the gravity and geoid models
- Five year updates of the World Magnetic Model

The current definition of WGS 84 recognizes the continually changing physical Earth, shifting of internal masses and continental plate motions and accommodates this time dependence in its definition through reference station velocities. For the high accuracy geodetic positioning requirements, this requires a time epoch to be defined for all station coordinates since they are in continual motion. As mentioned previously in this standard, the coordinates of the fixed GPS tracking stations WGS 84 (G1762) adheres to IERS TN 36 which is an improvement to previous WGS 84 reference frames.

As a consequence of the inherent high accuracy and continual refinement of the definition and realization of WGS 84, considerable care should be taken in the implementation of this system into existing and future weapons systems and geospatial information systems. This Chapter will address some of the considerations that should be made prior to the implementation of WGS 84 to ensure that the full benefits of WGS 84 are realized and are consistent with the operational product accuracy and interoperability requirements of the users.

10.2 Updates to WGS 84

It is important that systems implement the WGS 84 information relative to the gravity field, geoid, and datum transformations in a manner that will allow future update of specific portions of the data. These data will change with future refinements of WGS 84, as improved information becomes available to NGA, and this may necessitate updates to existing implementations as future operational accuracy requirements become known. For example, implementations of the geoid undulation values as a grid in a lookup table may facilitate easier future updates than implementation as spherical harmonic coefficients**. NGA recommends that NSG systems be designed to allow for updates to the WGS 84 system, data, parameters, and algorithms.**

10.3 WGS 84 Reference Frame and Coordinate Systems

NGA is committed to maintaining the highest possible accuracy and stability for the WGS 84 Reference Frame. To do so, NGA has generated several realizations of WGS 84, incorporating improved data and advances in geophysical modeling to define the motion of the earth's surface. These realizations have not affected the fundamental definition of the WGS 84 reference system but the improvements may be necessary for high-accuracy, high-integrity applications.

As described in Chapter 2, the WGS 84 Coordinate System is a right-handed, orthogonal and Earth-fixed coordinate system which is intended to be as closely coincident as possible with the CTRS defined by the IERS or, prior to 1988, its predecessor, the Bureau International de l'Heure (BIH). The current realization of the WGS 84 Reference Frame is designated as WGS 84 (G1762). The GPS Operational Control Segment (OCS) implemented WGS 84 (G1762) on 16 Oct 2013.

The WGS 84 Reference Frame and Coordinate System are delivered via the GPS. The most accurate approach for obtaining WGS 84 coordinates is to acquire satellite tracking data at the site of interest and position it directly in WGS 84 using GPS positioning techniques. For precise surveying applications, it is recommended that coordinates be maintained with an epoch assigned to each coordinate determination along with an indication of the fixed station GPS coordinate set used for the realization, such as WGS 84 (G1762).

10.4 Ellipsoid and Its Defining Parameters

The WGS 84 is comprised of a coherent set of parameters. **NGA recommends that the NSG implement the WGS 84 parameters and equations presented in this document (and supporting documentation) without substitution or deletion.** Substitution or deletions may lead to degraded WGS 84 products, interoperability problems, and may have other adverse effects.

10.5 The World Magnetic Model (WMM)

The main utility of the World Magnetic Model (WMM) is to provide magnetic declination for any desired location on the globe. In addition to the magnetic declination, the WMM also provides the complete geometry of the field from 1 km below the Earth's surface to 850 km above the surface. The magnetic field extends deep into the Earth and far out into space, but the WMM is not valid there. **NGA recommends that the NSG implement the World**

Magnetic Model as described in, The US/UK World Magnetic Model for 2010-2015, NOAA Technical Report NESDIS/NGDC.

10.6 WGS 84 Relationships with other Geodetic Systems

Before satellite geodetic techniques became available, the local horizontal datum was defined independently of the local vertical datum. NGA has developed datum transformations to convert over 120 local horizontal datums to WGS 84. This generally entails making survey ties between a number of local geodetic control points and their corresponding geodetic positions derived from satellite observations. NGA did this for many years with Doppler observations from TRANSIT satellites and continues to do it now with GPS. Aside from the countless maps and charts which are still based on these classical local datums, numerous land records, property boundaries and other geographic information in many countries are referenced to local datums.

Modern maps, navigation systems and geodetic applications require a single accessible, global, 3-dimensional reference frame. It is important for global operations and interoperability that NSG systems implement and operate as much as possible on WGS 84. **NGA recommends that all NSG systems operate in the 3-dimensional WGS 84 datum.** Universal use of the WGS 84 datum and positional reference system will reduce confusion regarding which datum system is being used and increase interoperability.

There is currently no world vertical system defined to unify and tie together local vertical systems. Generally the vertical datum is defined by a series of tide gauges in the area or by approximating mean sea level by the geoid, leading to numerous realizations of mean sea level. Unfortunately, the local geodetic coordinates on these datums are of limited use for modern survey, navigation and mapping operations. **NGA recommends that all heights be reported as WGS 84 ellipsoidal heights (also known as geodetic heights). If a MSL height is required, NGA recommends that orthometric heights computed from WGS 84 Earth Gravitational Model 2008, or the current model, be used to compute these MSL heights.**

10.7 The Earth Gravitational Model EGM2008 and Geoid Model

Nearly every system and component of WGS 84 has nearly ubiquitous and universal implementation throughout the NSG, except for the Earth gravitational model and geoids. The purpose of the geoid is to perform a datum transformation between ellipsoid heights and orthometric heights (or Mean Sea Level). Numerous and incompatible implementations stem from many sources, including data file size and a general lack of understanding of the true nature of the geoid’s transformation relationship between ellipsoid height and orthometric heights (or Mean Sea Level). Past implementation guidance for geoids recommended that the application determine their overall accuracy and then decimate the geoid grid to their accuracy, in essence, editing and changing the transformation. **For those systems that require a Mean Sea Level height, NGA recommends that orthometric heights computed from the WGS 84 Earth Gravitational Model 2008 be used to compute these MSL heights.**

An Earth orbiting satellite's sensitivity to the geopotential is strongly influenced by the satellite's altitude range and other orbital parameters. NSG programs performing satellite orbit determination are advised to determine the maximum degree and order most appropriate for their particular mission and orbit accuracy requirements. **The EGM2008 through degree and order 70 is recommended for high accuracy satellite orbit determination and prediction purposes.**

10.8 Positioning, Navigation and Targeting (PN&T) Applications

Positioning, Navigation and Targeting is concerned with location and moving in real-time or after the fact on land, air and sea platforms. Accuracy requirements may vary from the centimeter level for precise geodetic applications, such as aerial photogrammetry, to meters for combined integrated GPS/Inertial Navigation Systems, to tens of meters for the SPS GPS user. PN&T applications are also characterized by large numbers of systems in the field, *e.g.* more than 100,000 GPS military receivers, with legacy systems having different implementations of WGS 84 than the newer systems. Implementation and update of WGS 84 in these platforms are costly and almost impossible to accomplish simultaneously. Therefore, a careful analysis must be done for each implementation to determine the essential elements of the refined definition of WGS 84 that need to be implemented.

It should be noted that most users depend on the ECEF coordinate system but for some they must depend on the Earth Centered Inertial (ECI) reference frame. This inertial reference frame is a stationary reference frame from which Newton's equations of motion hold true. The ECI coordinates are then transformed into ECEF coordinates using Earth Orientation Parameters (EOP). They are the parameters which provide the rotation of ECI to ECEF Reference frame as function of time, see definition for EOP. These EOP values and models are available at http://earth-info.nga.mil/GandG/sathtml/index.html and can be downloaded via FTP.

10.9 Geospatial Information Applications

Geospatial databases contain information from various sources as thematic layers from which imagery, imagery intelligence and geospatial information can be derived or extracted. Since the information represented in the various layers has multiple uses and supports applications of different accuracies, it is important that the accuracy of the basic information be retained. Therefore, the refined WGS 84 should be implemented in geospatial systems to maintain the inherent accuracy of the basic data sources.

10.10 Cartographic Applications

The definition and realization of WGS 84 as defined in TR8350.2, Second Edition, satisfies the NSG's mapping and charting requirements to better than 1 meter. Therefore, the improvement and refinements presented in this document for the WGS 84 reference frame, coordinate system, Earth gravitational model and datum transformations, is more than adequate for cartographic and mapping applications.

The current national horizontal map accuracy standard indicates that well defined points should be located with accuracy of better than 1/30 of an inch (0.85 mm) at a 90 percent confidence level on maps with scales greater than 1:20,000. This translates to 8.5 m on a 1:10,000 scale map which is easily met by WGS 84, assuming, of course, that the mapping products are on WGS 84 and not a local datum. If the maps or charts are on a local datum, then the application of appropriate datum transformation is necessary to preserve interoperability with other geospatial information. Depending on the local datum, the accuracy of the datum transformations can vary from 1 meter to over 25 meters in each Cartesian component.

The vertical accuracy of geospatial information and resulting map products depends on how the elevations were compiled. If the elevations are based on first order geodetic leveling, the control heights are very accurate, probably good to centimeters with respect to ‘local mean sea level’. A height bias in the local mean sea level would be the major potential error source. If no leveling data are available for vertical control, elevations are estimated from height above the WGS 84 Ellipsoid and a geoid height derived from the WGS 84 Geoid model. For mapping processes which use imagery, the orthometric heights (height above the geoid) are substituted for elevations above mean sea level. For these products the Earth Gravitational Model 2008 (EGM2008), which provides a geoid with an accuracy of < 25cm worldwide, should be implemented.

10.11 Summary

Users need to implement the refinements of the WGS 84 (NGA.STND.0036) into application systems in a planned and well thought out manner. An analysis into what aspects of these refinements are required for specific applications should be performed. This will ensure that the applications have indeed implemented WGS84 in the most effective manner.

A new realization of the WGS 84 reference frame, tied to the IERS TN 36 and International Terrestrial Reference Frame (ITRF) 2010, will be established in 2014. In Mid-2015, a new edition of NGA.STND.0036 incorporating this 2014 WGS 84 reference frame and an updated WMM 2015 will be written and propagated. This 5th edition will begin a five-year update cycle of the Standard and its related models as required. The 2020 update may be a rigorous review of the WGS which may result in refining parameters for the WGS.

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CHANGES to Appendices: New and Updated information

- Appendix A Transformation of Geocentric Celestial Reference Frame (GCRS, Epoch J2000.0) Coordinates to WGS 84 Terrestrial Reference Frame (TRS, ECEF) Coordinates.

- Appendix B Calculation and Validation of the World Geodetic System 1984 Fundamental Parameters, Derived Constants, and Miscellaneous Principal Values.

- Appendix C.1 Ellipsoids.

  - Clarke 1880 (IGN), CG
  - War Office 1924, WO

- Appendix D.1 Datums and Ellipsoids (Through Satellite Ties).

  - Accra (Ghana, Africa)
  - Circuit (Zimbabwe, Africa)
  - DCS-3 (Astro 1955) (St Lucia, Atlantic Ocean)
  - Fiji 1956 (Fiji Island, Pacific Ocean)
  - Gambia (Gambia, Africa)
  - Lisbon (Portugal, Europe)
  - Observatorio Campos Rodrigues 1907 (Mozambique, Africa)
  - South East Island (Seychelles, Indian Ocean)
  - Tete 1960 (Mozambique, Africa)
  - Timbalai 1968 (Brunei & E. Malaysia, Asia)
  - Yof Astro 1967 (Datum 200)(Senegal, Africa)

- Appendix D.2 AFRICA.

  - Accra (Ghana) New
  - Arc 1950 (Zimbabwe) Updated
  - Arc 1960 (Malawi) New
  - Ayabella Lighthouse (Djibouti) Updated
  - Circuit (Zimbabwe) New
  - Gambia (Gambia) New
  - Observatorio Campos Rodrigues 1907 (Mozambique) New
  - Tete 1960 (Mozambique) New
  - Yof Astro 1967 (Datum 200)(Senegal) New

- Appendix D.3 ASIA.

  - Oman (Oman) Updated
  - Timbalai 1968 (Brunei & E. Malaysia) New

- Appendix D.4 AUSTRALIA.

    - Australian Geodetic Datum 1966 Updated

- Appendix D.5 EUROPE.

    - Hjorsey 1955 (Iceland) Updated
    - S-42 (Pulkovo) (Estonia) New

- Appendix D.7 SOUTH AMERICA

    - Provisional South American 1956 (Chile) New regions defined
    - South American 1969 (Chile) New regions defined

- Appendix D.8 ATLANTIC OCEAN.

    - DCS-3 (Astro 1955) (St Lucia, Windward Islands)
    - Hjorsey 1955 (Iceland) Updated

- Appendix D.9 INDIAN OCEAN.

    - South East Island (Seychelles) New
    - Tananarive Observatory 1925 (Madagascar) <u>Updated: Moved from Non-Satellite</u>

- Appendix D.10 PACIFIC OCEAN.

    - Fiji 1956 (Fiji) New
    - Midway Astro 1951 (Midway Island) Updated 2003
    - Viti Levu 1916 (Viti Levu Island, Fiji Island) Updated

- Appendix E.1 Datums and Ellipsoids (Through Non-Satellite Ties).

    - Aden (Aden, Yemen) New
    - Beijing 1954 (China) New
    - Bekaa Valley 1920 (Lebanon) New
    - Conakry 1905 (Guinea) New
    - Indian (Sri Lanka) New
    - Mayotte Combani (Mayotte Island, Comoros) New
    - Ocotepeque (Costa Rica) New
    - New Triangulation of France (France) New
    - St Pierre et Miquelon 1950 (Miquelon and St Pierre Islands, North Atlantic) New
    - Tananarive Observatory 1925 (Madagascar) <u>Updated & Moved</u> to Satellite section

APPENDIX A

TRANSFORMATION OF GEOCENTRIC CELESTIAL REFERENCE FRAME (GCRS, EPOCH J2000.0) COORDINATES TO WGS 84 TERRESTRIAL REFERENCE FRAME (TRS, ECEF) COORDINATES

APPENDIX A

Transformation of Geocentric Celestial Reference Frame (GCRS, EPOCH J2000.0) Coordinates to WGS 84 Terrestrial Reference Frame (TRS, ECEF) Coordinates

1. General

This appendix provides a summary of the formulation used by NGA for transformation between the Earth-Center Inertial (ECI) system and the Earth-Centered Earth-Fixed coordinate system (ECEF) and is a summary of information contained in the IERS Technical Note 36 [1]. Technical Note 36 provides detailed information on the description of time systems, coordinate frames, and various other models. Technical Note 36 also describes the ECI-ECEF transformation for both the equinox-based approach and the CIO-based approach. The following description focuses on the CIO-based approach. It is noted the description of any ECI-ECEF transformation cannot be exact and is limited to a prescribed accuracy. Also, portions of the transformation are described by various series expressions, some of which entail dozens of terms. Instead of publishing the tables that describe the series expressions, references are made to the necessary electronic files that contain the appropriate data. Also, some hyperlinks are provided to IERS websites for the files; however, the reader is reminded that not all hyperlinks exist forever.

This appendix has several purposes. These are:

- To provide the mathematical relationship between the Conventional Geocentric Celestial Reference System (GCRS, an Earth-Centered Inertial System, Radio Wavelength Based, Epoch J2000.0), the Celestial Intermediate Origin (CIO), the Terrestrial Intermediate Origin (TIO) and a Conventional Terrestrial Reference System (TRS) as they relate to the World Geodetic System 1984 (WGS 84). The CIO-based expressions for the IAU 2006/2000A precession-nutation model have too many terms for complete representation here; however, auxiliary files provide the complete set of necessary terms, software provided by the IERS can be used to verify implementation of these models;
- To provide specific guidance to implement the coordinate transformation presented in the International Earth Rotation and Reference Systems Service (IERS) 2010 Conventions Technical Note 36 (TN36) [1]. This guidance is required to ensure a consistent coordinate transformation implementation across the various DoD programs. Guidance is provided on the application of the NGA-generated, Earth Orientation Parameter Predictions (EOPP), in this coordinate transformation procedure. Note that changes are planned for the EOPP that are consistent with TN36; however, these changes will not be

implemented until further coordination with all DoD components is complete. Until that time, users should reference [12]; and

- To discuss and provide the numerical data needed to transform position and velocity components from the Geocentric Celestial Reference Frame (CRF), the realization of the GCRS, or Earth-Centered-Inertial (ECI) Coordinate System to the WGS 84 Terrestrial Reference Frame (TRF), a realization of an Earth-Centered-Earth-Fixed (ECEF) TRS.

The CRF (ECI) to WGS 84 TRF (ECEF) transformation makes use of the International Astronomical Union (IAU) 2006 precession theory, the IAU2000 nutation theory, the definition of universal time, and the CIO-based transformation adopted by the IAU and as documented in [1].

Definition of terms and background

Throughout this appendix the following definitions and conventions apply. The definitions are taken largely from [1].

Barycentric Celestial Reference System (BCRS) - a system of space-time coordinates within the framework of General Relativity with metric tensor specified by the IAU centered at the solar system barycenter.

Celestial Intermediate Origin (CIO) - origin for right ascension on the intermediate equator in the Celestial Intermediate Reference System. It is the non-rotating origin in the GCRS that is recommended by the IAU.

Celestial Intermediate Pole (CIP) - geocentric equatorial pole defined as the intermediate pole in the transformation from the GCRS to the International Terrestrial Reference System (ITRS), separating nutation from polar motion. Its GCRS location results from (i) the terms of precession-nutation formulation with periods greater than 2 days, (ii) the retrograde diurnal part of polar motion including the free core nutation (FCN), and (iii) the frame bias. Its ITRS location results from (i) the part of polar motion which is outside the retrograde diurnal band in the ITRS and (ii) the motion in the ITRS corresponding to nutations with periods less than 2 days. The motion of the CIP is realized by the IAU precession-nutation plus time-dependent corrections provided by the IERS (celestial pole offsets).

Celestial Intermediate Reference System (CIRS) - geocentric reference system related to the GCRS by a time-dependent rotation taking into account precession-nutation. It is defined by the intermediate equator (of the CIP) and CIO on a specific date. It is similar to the system based on the true equator and equinox of date, but the equatorial origin is at the CIO.

Coordinated Universal Time (UTC) - measure of time that is the basis of all civil timekeeping

Earth Rotation Angle (ERA) - angle measured along the intermediate equator of the Celestial Intermediate Pole (CIP) between the Terrestrial Intermediate Origin (TIO) and the Celestial Intermediate Origin (CIO), positively in the retrograde direction. It is related to UT1 (see below) by a conventionally adopted expression in which ERA is a linear function of UT1.

ecliptic - the plane perpendicular to the mean heliocentric orbital angular momentum vector of the Earth- Moon barycenter.

epoch - a fixed date used to reckon time for expressing time varying quantities. It is often expressed in the system of Julian date, marked by the prefix J (*e.g.* J2000.0), with the Julian year of 365.25 days as unit. The term is also used to designate the date and time of an observation, *e.g.* "epoch of observation," which can also be expressed by "date of observation".

equation of the origins (EO) - angular distance between the CIO and the equinox along the intermediate equator; it is the CIO right ascension of the equinox; alternatively the difference between the Earth rotation angle and Greenwich apparent sidereal time (ERA−GAST).

equinox - either of the two points at which the ecliptic intersects the celestial equator; also the time at which the Sun passes through either of these intersection points; *i.e.*, when the apparent longitude of the Sun is 0° or 180°.

geocenter - center of mass of the Earth including the atmosphere and oceans

Geocentric Celestial Reference System (GCRS) - a system of geocentric space-time coordinates within the framework of General Relativity with metric tensor specified by the IAU. The GCRS is defined such that the transformation between BCRS and GCRS spatial coordinates contains no rotation component, so that GCRS is kinematically non-rotating with respect to BCRS. The spatial orientation of the GCRS is derived from that of the BCRS, that is, unless otherwise stated, by the orientation of the ICRS.

Geocentric Terrestrial Reference System (GTRS) - a system of geocentric space-time coordinates within the framework of General Relativity, co-rotating with the Earth, and related to the GCRS by a spatial rotation that takes into account the Earth orientation parameters.

Greenwich Mean Sidereal Time (GMST) - Greenwich hour angle of the mean equinox defined by a conventional relationship to Earth rotation angle or equivalently to UT1.

Greenwich Apparent Sidereal Time (GAST) - the hour angle of the true equinox from the Terrestrial Intermediate Origin (TIO) meridian (Greenwich or International meridian).

International Atomic Time (TAI) - a practical realization of terrestrial time (TT) with a fixed shift from the latter due to historical reasons (see TT); it is a continuous time scale, now calculated at the Bureau International des Poids et Mesures (BIPM), using data from over three hundred atomic clocks in over fifty national laboratories in accordance with the definition of the SI second.

International Celestial Reference Frame (ICRF) - a set of extragalactic objects whose adopted directions realize the ICRS axes. It is also the name of the catalog of directions whose 212 defining sources is currently the most accurate realization of the ICRS.

International Celestial Reference System (ICRS) - the idealized barycentric coordinate system to which celestial positions are referred. It is kinematically non-rotating with respect to the ensemble of distant extragalactic objects. It has no intrinsic orientation but was aligned close to the mean equator and dynamical equinox of J2000.0

for continuity with previous fundamental reference systems. Its orientation is independent of epoch, ecliptic or equator and is realized by a list of adopted coordinates of extragalactic sources (ICRF).

International Terrestrial Reference Frame (ITRF) - a realization of ITRS, through the realization of its origin, orientation axes and scale, and their time evolution.

International Terrestrial Reference System (ITRS) - the specific GTRS for which the orientation is operationally maintained in continuity with past international agreements. It has no residual rotation with regard to the Earth's surface, and the geocenter is understood as the center of mass of the whole Earth system, including oceans and atmosphere. For continuity with previous terrestrial reference systems, the first alignment was close to the mean equator of 1900 and the Greenwich meridian.

J2000.0 - defined in the framework of General Relativity by IAU as being the epoch at the geocenter and at the date 2000 January 1.5 TT = Julian Date 245 1545.0 TT. Note that this event has different dates in different time scales.

mean pole - the direction on the celestial sphere towards which the Earth's axis points at a particular epoch, with the oscillations due to precession-nutation removed.

modified Julian date (MJD) - The Modified Julian Date or Day (MJD) is defined as $MJD = JD - 2400000.5$, where JD is the Julian Day, a continuous count of days beginning at 12 noon 1 January -4712 (4713BC). [2]

non-rotating origin (NRO) - in the context of the GCRS or the ITRS, the point on the intermediate equator such that its instantaneous motion with respect to the system (GCRS or ITRS as appropriate) has no component along the intermediate equator (*i.e.* its instantaneous motion is perpendicular to the intermediate equator). It is called the CIO and TIO in the GCRS and ITRS, respectively.

polar motion - the motion of the Earth's rotation axis with respect to the ITRS. The main components are the Chandlerian free motion, with a period of approximately 435 days, and an annual motion. It also includes sub-daily variations caused by ocean tides and periodic motions driven by gravitational torques with periods less than two days. Sub-daily variations are not included in the values distributed by the IERS, and are therefore to be added, after interpolation to the date of interest, using a model provided by the IERS Conventions[1].

precession-nutation - the ensemble of effects of external torques on the motion in space of the rotation axis of a freely rotating body, or alternatively, the forced motion of the pole of rotation due to those external torques. In the case of the Earth, a practical definition consistent with the IAU 2000 resolutions is that precession-nutation is the motion of the CIP in the GCRS, including FCN and other corrections to the standard models: precession is the secular part of this motion plus the term of 26000-year period and nutation is that part of the CIP motion not classed as precession.

Terrestrial Intermediate Origin (TIO) - non-rotating origin of longitude recommended by the IAU in the ITRS. The TIO was originally set at the ITRF origin of longitude and throughout 1900-2100 stays within 0.1 mas of the ITRF zero meridian.

Terrestrial Intermediate Reference System (TIRS) - a geocentric reference system defined by the intermediate equator of the CIP and the TIO. It is related to the ITRS by polar motion and the TIO locater, s'. It is related to the Celestial Intermediate Reference System by a rotation of ERA around the CIP, which defines the common z-axis of the two systems.

terrestrial reference frame (TRF) - realization of the Terrestrial Reference System (TRS), through the realization of its origin, orientation axes and scale, and their time evolution.

terrestrial reference system (TRS) - a Terrestrial Reference System (TRS) is a spatial reference system co-rotating with the Earth in its diurnal motion in space.

Terrestrial Time (TT) - a coordinate time whose mean rate is close to the mean rate of the proper time of an observer located on the rotating geoid. At 1977 January 1.0 TAI exactly, the value of TT was 1977 January 1.0003725 exactly. TT may be used as the independent time argument for geocentric ephemerides. An accurate realization of TT is *TT*(*TAI*)=TAI+32.184 seconds. In the past TT was called terrestrial dynamical time (TDT).

UT1 - angle of the Earth's rotation about the CIP axis defined by its conventional linear relation to the Earth rotation angle (ERA). It is related to Greenwich apparent sidereal time through the ERA (see equation of the origins). It is determined by observations, and can be obtained from UTC by using the quantity UT1−UTC, which is provided by the IERS.

(UT1−UTC) - difference between the UT1 parameter derived from observation and the time scale UTC, the latter being currently defined as UTC = TAI + n, where n is an integer number of seconds, such that $|UT1 - UTC| < 0.9$ seconds.

In the following, all coordinate systems are right-handed and orthogonal and positive rotations are counterclockwise. Specifically, the notation R_1, R_2, and R_3 indicates rotations about the x, y, and z axes of the reference system, respectively. That is:

$$R_1(\theta)=\begin{bmatrix}1 & 0 & 0\\ 0 & \cos\theta & \sin\theta\\ 0 & -\sin\theta & \cos\theta\end{bmatrix};$$

$$R_2(\theta)=\begin{bmatrix}\cos\theta & 0 & -\sin\theta\\ 0 & 1 & 0\\ \sin\theta & 0 & \cos\theta\end{bmatrix};$$

and

$$R_3(\theta)=\begin{bmatrix}\cos\theta & \sin\theta & 0\\ -\sin\theta & \cos\theta & 0\\ 0 & 0 & 1\end{bmatrix}. \tag{A-1}$$

2. Time and Epochs

There are various systems based on the diurnal rotation of the Earth that are of interest here, including sidereal time, Julian date, terrestrial time, solar or universal time. Sidereal time is realized by observations of the Earth's rotation angle with respect to celestial objects, particularly extragalactic radio sources (such as quasars and active galactic nuclei). In these observations, consideration must be given to the effects of polar motion. Mean solar time is associated with a mean fictitious sun that moves along the celestial equator with a uniform sidereal motion approximately commensurate with the mean rate of the annual motion of the true sun along the ecliptic. If the hour angle of the mean sun is referred to the BIH-defined Zero Meridian, the resulting time is called universal time (UT). Universal and sidereal time, like other rotation based times, are affected by the irregularities in the rotation of the Earth. These irregularities take the form of polar motion (variations in the position of the Earth's rotation axis with respect to the earth's crust), and variations in the Earth's angular rotation rate.

The time scale distributed by most broadcast time services is coordinated universal time (UTC). It is based on the redefined coordinated universal time, which differs from international atomic time (TAI), another time scale, by an integral number of seconds. The TAI time scale is the most precisely determined time scale available, and results from the analyses by the BIH of data from the atomic time standards of many supporting countries. Another time scale associated with TAI is terrestrial time (TT); TT = TAI+32.184 seconds.

Universal time (UTC), is maintained to within 0.90 second of UT1 through the introduction of one second time steps (called leap seconds) when necessary. Universal and sidereal time are equivalent time systems since UT1 is formally defined by an equation which relates it to mean sidereal time (MST).

On any given date, a star's or source's computed apparent position, upon which its contribution to the UT1 determination depends, is a function not only of its' catalog position and proper motion, but also of the adopted theory of precession, astronomical nutation, aberration, etc. Therefore, a change from one celestial inertial system (CIS) to another and changes in the values of astronomical constants have complex and subtle effects on the value of UT1. For the current GCRS, Greenwich mean sidereal time, which is related to the equinox, can be expressed as a function of the Earth rotation angle (ERA) which is a function of UT1. These relationships are developed in Section 3.3.

The unit of time, T, in the formulas for precession and astronomical nutation is the Julian Century of 36525 days. Since the Julian Day begins at noon, the time interval in the GCRS-to-TRS transformation to be discussed later is Julian Centuries measured from 2000 January 1.5.

The time parameter *t*, used in all of the following expressions, is defined by

$$t = (\text{TT - 2000 January1d 12h TT}) \text{ in days} / 36525 \qquad \text{(A-2)}$$

where *t* is in centuries. Note that 2000 January 1.5 TT = Julian Date 2451545.0 TT.

3. Description of the GCRF (ECI) to WGS 84 TRF (ECEF) Transformation Matrices and Equations

3.1 General

The IAU 2000 and IAU 2006 resolutions presented in TN36 [1], defined the transformation between the CRS, the CIO, the TIO, and the ECEF TRS as realized within the WGS 84 at the date *t* of the observation as

$$[\text{GCRS}] = Q(t)R(t)W(t)\,[\text{TRS}] \qquad \text{(A-3)}$$

where $Q(t)$, $R(t)$, and $W(t)$ are the transformation matrices describing the motion of the celestial pole in the CRF, the rotation of the Earth around the axis associated with the pole and the motion of the pole with respect to the TRF, known as polar motion, respectively.

Similarly, the reverse transform from TRS to GCRS can be defined as

$$[\text{TRS}] = [Q(t)R(t)W(t)]^{\text{T}}[\text{GCRS}] = W(t)^{\text{T}}\,R(t)^{\text{T}}\,Q(t)^{\text{T}}\,[\text{GCRS}] \qquad \text{(A-4)}$$

In the following sub-sections the three matrices that make up this transform will be presented.

3.2 Precession and Nutation, *Q(t)*

In the CIO-based transformation, the effects of precession and nutation are accounted for within a single matrix transform, the $Q(t)$ transform that accounts for the CIO-based motion of the Celestial Intermediate Pole (CIP) in the GCRS, and combines the former two-step process of computing precession and nutation into a single expression:

$$Q(t) = R_3(-E)\cdot R_2(-d)\cdot R_3(E)\cdot R_3(s)$$

or the transpose

$$Q(t)^T = R_3(-s)\cdot R_3(-E)\cdot R_2(d)\cdot R_3(E) \qquad \text{(A-5)}$$

where *E* and *d* are related to the coordinates of the CIP in the GCRS (see Figure 1).

The precession/nutation matrix $Q(t)$ can be written as

$$Q(t) = \begin{bmatrix} 1-aX^2 & -aXY & X \\ -aXY & 1-aY^2 & Y \\ -X & -Y & 1-a\left(X^2+Y^2\right) \end{bmatrix} \cdot R_3(s) \qquad \text{(A-6)}$$

where

$$a = \frac{1}{1+\cos(d)} \cong \tfrac{1}{2} + \tfrac{1}{8}\left(X^2 + Y^2\right) \qquad \text{(A-6a)}$$

The approximation has an accuracy of $1\mu as$, and X and Y are the “coordinates” of the CIP in the CRS that are provided by the conventional IAU 2000A models, which is based on geophysical and astronomical theory.

The quantities E and d are defined with respect to the X, Y, and Z GCRS coordinates as

$$X = \sin(d)\cos(E),$$

$$Y = \sin(d)\sin(E),$$

and

$$Z = \cos(d). \qquad \text{(A-7)}$$

s specifies the position of the Celestial Intermediate Origin on the equator of the Celestial Intermediate Pole and can be expressed as a function of X and Y,

$$s(t) = -\int_{to}^{t} \frac{X\dot{Y} - Y\dot{X}}{1+Z}\, dt - K_o \qquad \text{(A-8)}$$

where $K_o = \sigma_o N_o - \sum_o N_o =$ constant and σ_o and Σ_o are positions of the CIO at J2000.0 and the x-origin of GCRF respectively, and N_o is the ascending node of the equator at J2000.0 in the equator of the GCRS.

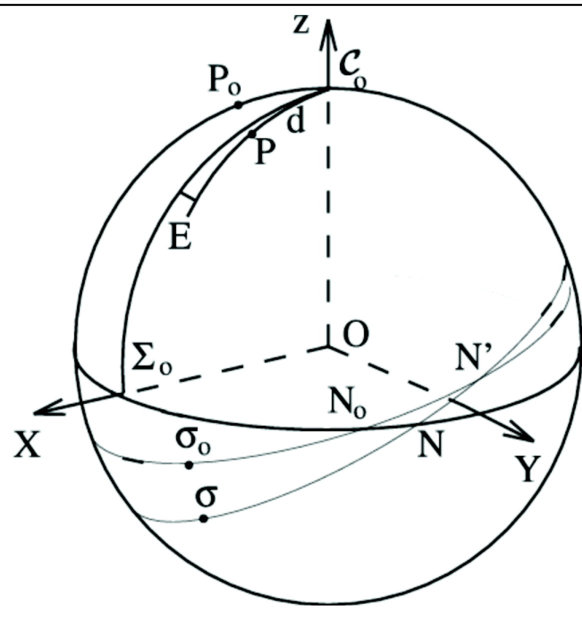

Figure 1. The Geocentric Celestial Reference System, GCRS (pole C_o, X-axis origin Σ_0) and Celestial Intermediate Reference System, CIRS (CIP which is point P, CIP equator, CIO σ). CIP has GCRS coordinates that are defined by X and Y which are functions of E and d (see text definitions). The CIO is represented by point σ and due to the kinematical definition of the non-rotating origin, the instantaneous motion of σ is perpendicular to the CIP equator. P_o and σ_o are the CIP and CIO at J2000.0 [52].

This formulation models the motion of the pole due to lunisolar and planetary motions. The parameters X and Y are given as function of time using the IAU 2006 precession and IAU 2000A nutation (IAU 2006/2000A) models that are valid at the one μas level that corresponds to the pole and equinox at J2000.0 with respect to the GCRS [1, Section 5.5.4].

$$
\begin{aligned}
X = &-0.016617'' + 2004.191898''\, t - 0.4297829''\, t^2 \\
&- 0.19861834''\, t^3 + 0.000007578''\, t^4 + 0.0000059285''\, t^5 \\
&+ \Sigma_i\, [(a_s,0)_i \sin(\text{ARGUMENT}) + (a_c,0)_i \cos(\text{ARGUMENT})] \\
&+ \Sigma_i\, [(a_s,1)_i\, t \sin(\text{ARGUMENT}) + (a_c,1)_i\, t \cos(\text{ARGUMENT})] \\
&+ \Sigma_i\, [(a_s,2)_i\, t^2 \sin(\text{ARGUMENT}) + (a_c,2)_i\, t^2 \cos(\text{ARGUMENT})] + \ldots
\end{aligned}
$$

and

$$
\begin{aligned}
Y = &-0.006951'' - 0.025896''\, t - 22.4072747''\, t^2 \\
&+ 0.00190059''\, t^3 + 0.001112526''\, t^4 + 0.0000001358''\, t^5 \\
&+ \Sigma_i\, [(b_c,0)_i \cos(\text{ARGUMENT}) + (b_s,0)_i \sin(\text{ARGUMENT})] \\
&+ \Sigma_i\, [(b_c,1)_i\, t \cos(\text{ARGUMENT}) + (b_s,1)_i\, t \sin(\text{ARGUMENT})] \\
&+ \Sigma_i\, [(b_c,2)_i\, t^2 \cos(\text{ARGUMENT}) + (b_s,2)_i\, t^2 \sin(\text{ARGUMENT})] + \ldots
\end{aligned}
\tag{A-9}
$$

where t is from equation (A-3) and ARGUMENT represents functions of the fundamental parameters of the nutation theory whose expressions are for the lunisolar and the planetary arguments, a_s, a_c and b_s, b_c as given in [1, Section 5.5.4, Tables 5.2a and 5.2 b], respectively.

Each of the lunisolar terms in the nutation series is characterized by a set of five integers N_j which determines the argument for the term as a linear combination of the five Fundamental Arguments F_j, namely the Delaunay variables(l, l', F, D, Ω):

$$\text{ARGUMENT} = \sum_{j=1}^{5} N_j F_j\ , \tag{A-10}$$

where the values (N_1,...,N_5) of the multipliers characterize the term and the F_j are functions of time as given in [1, Section 5.7.2, Eq. 5.43].

$$
\begin{aligned}
&F_1 \equiv l = \text{Mean Anomaly of the Moon} \\
&= 134.96340251^\circ + 1717915923.2178''\,\text{t} + 31.8792''\,\text{t}^2 + 0.051635''\,\text{t}^3 - 0.0002447''\,\text{t}^4
\end{aligned}
$$

$$F_2 \equiv l' = \text{Mean Anomaly of the Sun}$$
$$= 357.52910918^\circ + 129596581.0481''\mathrm{t} - 0.5532''\mathrm{t}^2 + 0.000136''\mathrm{t}^3 - 0.00001149''\mathrm{t}^4$$

$$F_3 \equiv F = L - \Omega$$
$$= 93.27209062^\circ + 1739527262.8478''\mathrm{t} - 12.7512''\mathrm{t}^2 - 0.001037''\mathrm{t}^3 + 0.00000417''\mathrm{t}^4$$

$$F_4 \equiv D = \text{Mean Elongation of the Moon from the Sun}$$
$$= 297.85019547^\circ + 1602961601.2090''\mathrm{t} - 6.3706''\mathrm{t}^2 + 0.006593''\mathrm{t}^3 - 0.00003169''\mathrm{t}^4$$

$$F_5 \equiv \Omega = \text{Mean Longitude of the Ascending Node of the Moon}$$
$$= 125.04455501^\circ - 6962890.5431''\mathrm{t} + 7.4722''\mathrm{t}^2 + 0.007702''\mathrm{t}^3 - 0.00005939''\mathrm{t}^4 \tag{A-11}$$

where L is the Mean Longitude of the Moon.

The mean longitudes of the planets used in the arguments for the planetary nutation ($F_6,\ldots,F_{14}$) for determination of Celestial Poles X and Y are essentially those provided by Souchay *et al.* [59] and *Petit and Luzum* [1, Section 5.7.2, Eq. 5.43], based on theories and constants of VSOP82 [60] and ELP 2000 [61] and developments of Simon *et al.* [58, Tables 5.8.1-5.8.8]. Their developments are given in Eq. A-12 in radians with t in Julian centuries [1, Section 5.7.3, Eq. 5.44].

The general precession, F_{14}, is from Kinoshita and Souchay [62].

$$F_6 \equiv L_{Me} = 4.402608842 + 2608.7903141574\ t$$

$$F_7 \equiv L_{Ve} = 3.176146697 + 1021.3285546211\ t$$

$$F_8 \equiv L_E = 1.753470314 + 628.3075849991\ t$$

$$F_9 \equiv L_{Ma} = 6.203480913 + 334.0612426700\ t \tag{A-12}$$

$$F_{10} \equiv L_J = 0.599546497 + 52.9690962641\ t$$

$$F_{11} \equiv L_{Sa} = 0.874016757 + 21.3299104960\ t$$

$$F_{12} \equiv L_U = 5.481293872 + 7.4781598567\ t$$

$$F_{13} \equiv L_{Ne} = 5.311886287 + 3.8133035638\ t$$

$$F_{14} \equiv p_A = 0.02438175\ t + 0.00000538691\ t^2$$

The F_j are functions of time, and the angular frequency of the nutation described by the term is given by

$$\omega \equiv d(\mathrm{ARGUMENT})/dt.$$

The angular frequency is positive for most terms, and negative for some terms. The planetary nutation terms differ from the lunisolar nutation terms only in the fact that ARGUMENT $= \sum_{j=6}^{14} N_j F_j$, F_6 to F_{13}, as noted in Tables 5.2a-5.2b and 5.3b of [1], with the mean longitudes of the planets including the Earth (L_{Me}, L_{Ve}, L_E, L_{Ma}, L_J, L_{Sa}, L_U, l_{Ne}) and F'_{14} is the general precession in longitude p_A.

The numerical values of the coefficients of the polynomial part of X and Y are derived from the development as a function of time of the precession in longitude and obliquity and pole offset at J2000.0 and the amplitudes $(a_c,j)_i$, $(a_s,j)_i$, $(b_c,j)_i$, and $(b_s,j)_i$ for j = 0, 1, 2, ... are derived from the amplitudes of the precession and nutation series. The amplitudes $(a_s,0)_i$, $(b_c,0)_i$ of the sine and cosine terms in X and Y respectively are equal to the amplitudes $A_i \times sin\,(\epsilon_0)$ and B_i of the series for nutation in longitude [$\times\, sin\,(\epsilon_0)$] and obliquity, except for a few terms in each coordinate X and Y which contain a contribution from crossed-nutation effects. The coordinates X and Y contain Poisson terms in t sin, t cos, t^2 sin, t^2 cos, ... which originate from crossed terms between precession and nutation.

The numerical development of the quantity $s(t)$ (*i.e.* the CIO locator) appearing in the prior equations, compatible with the IAU 2006/2000A precession-nutation model as well as the corresponding celestial offset at J2000.0, has been derived in a way similar to that based on the IERS Conventions 2003 [48]. The results from the above expression for $s(t)$ [Eq. A-8] using the developments of X and Y as functions of time [49] has the following form [1, Section 5.5.6]:

$$
\begin{aligned}
s(t) = {} & \frac{-XY}{2} \\
& \left.\begin{aligned} & + \; 94.00\text{e-6}'' \; + \; 3808.65\text{e-6}'' t \; - \; 122.68\text{e-6}'' t^2 \\ & - \; 72574.11\text{e-6}'' t^3 \; + \; 27.98\text{e-6}'' t^4 \; + \; 15.62\text{e-6}'' t^5 \end{aligned}\right\} \textit{polynomial component} \\
& \left.+ \sum_{n=1}^{33} \left(c_{s[0,n]} \sin\left(\sum_{m=1}^{8} N_{0,n,m} \cdot F_m \right) + c_{c[0,n]} \cos\left(\sum_{m=1}^{8} N_{0,n,m} \cdot F_m \right) \right) \right\} \textit{periodic component for } t^0 \\
& \left.+ t \cdot \sum_{n=1}^{3} \left(c_{s[1,n]} \sin\left(\sum_{m=1}^{8} N_{1,n,m} \cdot F_m \right) + c_{c[1,n]} \cos\left(\sum_{m=1}^{8} N_{1,n,m} \cdot F_m \right) \right) \right\} \textit{periodic component for } t^1 \\
& \left.+ t^2 \cdot \sum_{n=1}^{25} \left(c_{s[2,n]} \sin\left(\sum_{m=1}^{8} N_{2,n,m} \cdot F_m \right) + c_{c[2,n]} \cos\left(\sum_{m=1}^{8} N_{2,n,m} \cdot F_m \right) \right) \right\} \textit{periodic component for } t^2 \\
& \left.+ t^3 \cdot \sum_{n=1}^{4} \left(c_{s[3,n]} \sin\left(\sum_{m=1}^{8} N_{3,n,m} \cdot F_m \right) + c_{c[3,n]} \cos\left(\sum_{m=1}^{8} N_{3,n,m} \cdot F_m \right) \right) \right\} \textit{periodic component for } t^3 \\
& \left.+ t^4 \cdot \left(c_{s[4,1]} \sin\left(\sum_{m=1}^{8} N_{4,n,m} \cdot F_m \right) + c_{c[4,1]} \cos\left(\sum_{m=1}^{8} N_{4,n,m} \cdot F_m \right) \right) \right\} \textit{periodic component for } t^4
\end{aligned}
\tag{A-13}
$$

The constant term for s, which was previously chosen so that $s(J2000.0) = 0$, was subsequently fitted [49] to ensure continuity of $UT1$ at the date of change (1 January 2003) consistent with the Earth rotation angle (*ERA*) relationship and the current VLBI procedure for estimating $UT1$.

The series of amplitudes (C_k) of sin and cosine terms for $s+XY/2$ with all terms larger than 0.1 μas is available on the IERS Conventions Center website:

http://62.161.69.131/iers/conv2010/conv2010_c5.html

in Table 5.2d [1, Section 5.5.6].

Very Long Baseline Interferometry (VLBI) observations show the precession-nutation theory to be accurate at the ±0.2 μas level. If more accurate values are needed, corrections called celestial pole offsets, can be added to the quantities X and Y. These corrections can be either those produced by an IERS Free Core Nutation model or IERS VLBI observation of the celestial pole offsets. See TN 36 [1] for details on these relationships and models. Note, NGA does not supply the celestial pole offsets, and hence these terms should not be included in the precession-nutation model implementation to be consistent with NGA's usage.

3.3 Earth Rotation, *R(t)*

The CIO based transformation matrix resulting from the rotation of the Earth around the axis of the CIP can be expressed as

$$R(t) = R_3(-ERA(t)) \tag{A-14}$$

or

$$R^T(t) = R_3(ERA(t)) \tag{A-15}$$

where $ERA(t)$ is the Earth Rotation angle between the CIO and the Terrestrial Intermediate Origin (TIO) at the date t on the equator of the CIP, which provides a rigorous definition of the sidereal rotation of the Earth. $ERA(t)$ is defined as [1, Section 5.5.3]

$$ERA(T_u) = 2\pi(UT1 \text{ Julian day fraction} + 0.7790572732640 + 0.00273781191135448T_u). \tag{A-16}$$

where

$$T_u = (\text{Julian } UT1 \text{ date - } 2451545.0)$$

and $UT1$ *Julian day fraction* is the fractional part of a Julian day with

$$UT1 = UTC + (UT1\text{-}UTC)$$

The Earth orientation parameter $UT1$-UTC must be observed. The observed and predicted values of $UT1$-UTC are supplied in the NGA-generated EOPP bulletin as follows (see Section 2.3 for details):

$$UT1 - UTC = I + J\left(MJD - T_b\right) + \underbrace{\sum_{n=3}^{4} K_n \cdot \sin\left(\frac{2\pi\left(MJD - T_b\right)}{R_n}\right) + \sum_{n=3}^{4} L_n \cdot \cos\left(\frac{2\pi\left(MJD - T_b\right)}{R_n}\right)}_{\text{seasonal variations}}. \tag{A-17}$$

Because the Earth's rotation angle is variable, the parameter *UT1* that characterizes that angle must be observed in the form of the Earth orientation parameter $UT1$-UTC. Note: to use the ($UT1$-UTC) parameter, users need to apply zonal, diurnal and semi-diurnal tides as described in [1, Chapter 8]. The restoration of zonal tides should be consistent with the NGA's usage based on [1, Table 8.1]. Similarly, the application of the diurnal and semi-diurnal ocean tides should be consistent with NGA's usage described in [1, Section 8.2].

3.4 Polar Motion, *W(t)*

The transformation for the polar motion can be expressed as

$$W(t) = R_3(-s')\ \ R_2(x_p)\ \ R_1(y_p) \tag{A-18}$$

or its transpose

$$W^T(t) = R_1(-y_p)\ \ R_2(-x_p)\ \ R_3(s')$$

where x_p and y_p being the "pole coordinates" of the CIP in the TRS and s' being the "Terrestrial Intermediate Origin (TIO) locator" that provides the position of the TIO on the equator of the CIP corresponding to the kinematical definition of the "non-rotating" origin (NRO) in the TRS when the CIP is moving with respect to the TRS due to polar motion. The parameter s' is a function of the polar motion and its rate of change over time.

$$s'(t) = \frac{1}{2}\int_{t_0}^{t}\left(x_p\dot{y}_p - y_p\dot{x}_p\right)dt$$

It can be written in terms of the average amplitude of the Chandlerian and Annual wobbles, a_C and a_a, respectively, as

$$s'(t) \cong -0.0015\left(\frac{a_C^2}{1.2} + a_a^2\right)t \tag{A-19}$$

Using current mean amplitudes (Lambert and Bizouard, 2002),

$$s'(t) \cong -47t\ \mu as. \tag{A-20}$$

The x_p and y_p polar coordinates in (A-18) must be observed. The observed and predicted values of the polar coordinates x_p and y_p are supplied in NGA-generated EOPP bulletin as follows:

$$\begin{aligned} x_p &= A + B(T - T_a) + \sum_{n=1}^{2} C_n \cdot \sin\left(\frac{2\pi(T - T_a)}{P_n}\right) + \sum_{n=1}^{2} D_n \cdot \cos\left(\frac{2\pi(T - T_a)}{P_n}\right) \\ y_p &= E + F(T - T_a) + \sum_{n=1}^{2} G_n \cdot \sin\left(\frac{2\pi(T - T_a)}{Q_n}\right) + \sum_{n=1}^{2} H_n \cdot \cos\left(\frac{2\pi(T - T_a)}{Q_n}\right) \end{aligned} \tag{A-21}$$

where the terms in (A-21) are discussed in Section 2.3 above as well as described on the NGA web site:

http://earth-info.nga.mil/GandG/sathtml/eoppdoc.html

NGA, in collaboration with USNO, provides a daily file of Earth Orientation Parameter Predictions to the GPS Operational Control Segment. The specific details describing the content and format of these predictions are found in the Interface Control Document ICD-GPS-211G-001 [63]. These predictive x_p and y_p values are provided for the real-time applications of portion 3.4 in this appendix, while the predictive (UT1-UTC) are for the real-time applications of portion 3.3. An updated version of this interface, designated as ICD-GPS-811, is under development and is being designed to be compatible with the new GPS Control Segment known as OCX.

To use the polar coordinates x_p and y_p , the user needs to apply the diurnal and semi-diurnal ocean tides as described in [1, Chapter 8].

Because the polar motion is variable, the x_p and y_p polar coordinates must be observed. To use the polar coordinates, users should apply the diurnal and semi-diurnal ocean tides as described in [1, Chapter 8]. The application of the diurnal and semi-diurnal ocean tides should be consistent with NGA's usage based on [1, Section 8.2].

3.5 Reformulation of Transformation with Respect to the Rotation about the Z-Axis

As stated in Section 3.1, the transformation from the TRS to the GCRS can be defined as

$$[TRS] = W^T(t)R^T(t)Q^T(t)[GCRS]. \tag{A-22}$$

This expression can be expanded as follows:

$$[TRS] = R_1(-y_p)\ \ R_2(-x_p)\ \ R_3(s')\ \ R_3(ERA)\ \ R_3(-s)\ \ R_3(-E)\ \ R_2(d)\ \ R_3(E)\ \ [GCRS]. \tag{A-23}$$

We can then define parameter Θ(t) to represent the total rotation about the Z-axis, excluding E parameter, as

$$\Theta(t) = s'(t) + ERA(t) - s(t)$$

and

$$R_3(\Theta) = R_3(s')\ \ R_3(ERA)\ \ R_3(-s').$$

We can define

$$\widetilde{W}^T(t) = R_1(-y_p)\ \ R_2(-x_p), \tag{A-24}$$

$$\widetilde{R}^T(t) = R_3(\Theta), \tag{A-25}$$

and

$$\tilde{Q}^T(t) = R_3(-E)R_2(d)R_3(E)\,. \qquad \text{(A-26)}$$

This results in the rapidly changing terms involving rotations about the Z-axis to all be in the $R(t)$ term, the polar motion transformation reduces to the variations of the polar coordinates (x_p and y_p), and the $Q(t)$ is a simpler function of E and d. This means that the position and velocity transformations now can be, respectively, written as

$$[TRS] = W^T(t)R^T(t)Q^T(t)[GCRS] \qquad \text{(Position)} \qquad \text{(A-27)}$$

and

$$[T\dot{R}S] = \tilde{W}^T(t)\dot{\tilde{R}}^T(t)\tilde{Q}^T(t)[GCRS] + \tilde{W}^T(t)\tilde{R}^T(t)\tilde{Q}^T(t)[G\dot{C}RS] \quad \text{(Velocity)} \qquad \text{(A-28)}$$

The reformulation in this section is provided for mathematical correctness, although $\dot{\Theta}$ may insignificantly differ from $E\dot{R}A$ in most applications.

4. Summary of Transformations

The matrices previously described and the equations to compute the angles and other quantities required in these matrices are summarized below. However, the expressions for the IAU 2006/2000A precession-nutation model have too many terms for complete representation here. Users can use the software provided by the IERS to validate these models. The IERS software required for these transformations can be found at:

http://www.iausofa.org/ .

Determine the time argument for the transformation

TAI = UTC - n where n is an integer number of seconds

TT= TAI+32.184 seconds

t = (TT - 2000 January 1d 12h TT) in days / 36525

(Eq. A-2; t is the time elapsed as measured in centuries since 2000 January 1.5.)

Compute the precession-nutation angles

Compute s from Eq. A-13,

Compute X and Y from Eqs. A-09, A-10, A-11, and A-12

Form Precession-Nutation Matrix

Compute Q from Eqs. A-6 and A-6a,

Compute Earth Orientation Parameters

Compute x_p, y_p, and UT1-UTC using Eqs. A-17 and A-21 (Section 2.3)

Compute Earth Rotation Angle

Compute ERA using Eq. A-16

Form Earth Rotation Matrix

$R(t) = R_3(-ERA(t))$ from Eqs. A-1 and A-14

Form Polar Motion Transformation

Compute s' using Eq. A-20,

$W(t) = R_3(-s') \;\; R_2(x_p) \;\; R_1(y_p)$

Form Matrices for Velocity Transformation

Compute $\Theta(t) = s'(t) + ERA(t) - s(t)$

Compute $\dot{\Theta}(t) = \dot{s}'(t) + ERA(t) - \dot{s}(t)$

Compute $\dot{\tilde{R}}(t)$, $\tilde{W}(t)$, and $\tilde{Q}(t)$

Perform Transformation

$$[TRS] = W(t)^T \, R(t)^T \, Q(t)^T \, [GCRS]$$

$$[T\dot{R}S] = \tilde{W}^T(t)\dot{\tilde{R}}^T(t)\tilde{Q}^T(t)[GCRS] + \tilde{W}^T(t)\tilde{R}^T(t)\tilde{Q}^T(t)[G\dot{C}RS]$$

APPENDIX B

CALCULATION AND VALIDATION OF THE WORLD GEODETIC SYSTEM 1984 FUNDAMENTAL PARAMETERS, DERIVED CONSTANTS, AND MISCELLANEOUS PRINCIPAL VALUES

Appendix B
Calculation and Validation of the World Geodetic System 1984 Fundamental Parameters, Derived Constants, and Miscellaneous Principal Values

1. Scope

This appendix is to support Chapter 3 with the purpose of demonstrating the calculations and validation of the various parameters and values associated with the World Geodetic System 1984 (WGS 84) for use by the Department of Defense, Intelligence Community, and the greater scientific and civilian communities at large.

It is strongly recommend that the WGS 84 parameters as presented in Chapter 3 be used for direct implementation. Although the following formulas are well known and published in numerous textbooks and journals, as well as in a previous version of the WGS 84 Technical Report [17], the WGS 84 Development Team felt that these formulas should be presented again.

2. Fundamental Constants

2.1 Overview

The explanation and rationale for the following fundamental WGS 84 Defining Parameters and Special Parameters, and Other Fundamental Constants are presented in Chapter 3; and the values are reproduced here.

Table B.1 WGS 84 Defining Parameters

Parameter	Symbol	Value	Units
Semi-major Axis (Equatorial Radius of the Earth)	a	6378137.0	m
Flattening Factor of the Earth	1/f	298.257223563	
Geocentric Gravitational Constant	GM	$3.986004418 \times 10^{+14}$	m^3 / s^2
Nominal Mean Angular Velocity of the Earth	ω	7.292115×10^{-05}	rads / s

Table B.2 Special WGS 84 Parameters

Parameter	Symbol	Value	Units
WGS 84 Earth Gravitational Model 2008 Dynamic Second Degree Zonal and Sectorial Harmonics	$\bar{C}_{2,0}$dyn	$-4.84165143790815 \times 10^{-04}$	
	$\bar{C}_{2,2}$dyn	$2.43938357328313 \times 10^{-06}$	

Table B.3 Relevant Miscellaneous Constants

Parameter	Symbol	Value	Units
Velocity of Light (in a vacuum)	c	$2.99792458 \times 10^{+08}$	m / s
Universal Constant of Gravitation	G	6.67428×10^{-11}	m^3 / kg s^2
Total mean mass of the atmosphere (with water vapor)	M_A	$5.1480 \times 10^{+18}$	kg
Dynamic Ellipticity	H	$3.2737949 \times 10^{-03}$	

3. Calculation of Geometric Constants

3.1 Flattening Factor of the Earth (f)

The flattening factor of the Earth can be properly expressed as,

$$f = 3.3528106647475 \times 10^{-03} \qquad \text{(B-1)}$$

3.2 Semi-minor Axis (b)

By definition, and from [17](3-28), the semi-major axis, b, can be calculated as,

$$b = a(1 - f) = 6.3567523142 \times 10^{6}\ meters \qquad \text{(B-2)}$$

3.3 First Eccentricity Squared (e^2)

By definition, and from [17](3-23), the first eccentricity squared can be calculated as

$$e^2 = \frac{a^2 - b^2}{a^2} \qquad \text{(B-3)}$$

$$= (2f - f^2) = 6.69437999014 \times 10^{-03} \qquad \text{(B-4)}$$

Equation (B-4) is derived by substitution of (B-2) into (B-3). It should be noted that there is a significant figure sensitivity associated with the b^2 term. Therefore, it is recommended that the calculation of the first eccentricity squared use Equation (B-4).

3.4 First Eccentricity (e)

By definition the first eccentricity can be calculated as

$$e = \sqrt{e^2} = 8.1819190842621 \times 10^{-02} \tag{B-5}$$

3.5 Second Eccentricity Squared (e'^2)

By definition, and from [18](2-94), the second eccentricity squared is given by:

$$e'^2 = \frac{a^2 - b^2}{b^2} \tag{B-6}$$

$$= \frac{e^2}{1 - e^2} = 6.73949674228 \times 10^{-03} \tag{B-7}$$

Equation (B-7) is derived by substitution of (B-2) and (B-3) into (B-6) which agrees with [17](3-26). It is recommended that the calculation of the second eccentricity squared use Equation (B-7).

3.6 Second Eccentricity (e′)

By definition the second eccentricity can be calculated as

$$e' = \sqrt{e'^2} = 8.2094437949696 \times 10^{-02} \tag{B-8}$$

3.7 Linear Eccentricity (E)

The linear eccentricity, E, can be calculated from [17](3-31) or [17](3-32), as

$$E = \sqrt{a^2 - b^2} \tag{B-9}$$

$$= a \times e = 5.2185400842339 \times 10^{5}\ meters \tag{B-10}$$

Equation (B-10) is derived by substitution of (B-3) into (B-9). It is recommended that the linear eccentricity use Equation (B-10).

3.8 Aspect Ratio (AR) The aspect ratio is given simply by:

$$AR = \frac{b}{a} = 9.966471893352525 \times 10^{-01} \tag{B-11}$$

3.9 Polar Radius of Curvature (R_p)

There are several formulas for the polar radius of curvature given by [17](3-33) and [18](2-82). However, these formulas can be reduced using (B-2) and (B-4) to just the ellipsoid terms of *a* and *f* as

$$R_p = \frac{a^2}{b} = \frac{a}{\sqrt{1-e^2}} \tag{B-12}$$

$$= \frac{a}{(1-f)} = 6399593.6258\ meters \tag{B-13}$$

3.10 Mean Radius of the Three Semi-Axes (R_1)

The mean radius of the three semi-axes is given by [17](3-38) and [9] as

$$R_1 = a \times \left(1 - \frac{f}{3}\right) = 6371008.7714\ meters \tag{B-14}$$

3.11 Radius of a Sphere of Equal Area (R_2)

The radius of a sphere of equal area is given by [17](3-41) and by [9] as

$$R_2 = R_p \times \left(1 - \tfrac{2}{3}e'^2 + \tfrac{26}{45}e'^4 - \tfrac{100}{189}e'^6 + \tfrac{7034}{14175}e'^8 - \tfrac{220652}{467775}e'^{10}\right) \tag{B-15}$$

$$= 6371007.1810\ meters$$

3.12 Radius of a Sphere of Equal Volume (R_3)

The radius of a sphere of equal volume is given by [17](3-43) and by [9], and can be reduced using (B-2) to just the ellipsoid terms of *a* and *f* as:

$$R_3 = \sqrt[3]{a^2 b} \tag{B-16}$$

$$= a \times \sqrt[3]{1-f} = 6371000.7900\ meters \tag{B-17}$$

3.13 q_0

A useful intermediate term, q_0, can be calculate by [17](3-66), by combining [18](2-58) and [18](2-71), and by [9] as

$$q_0 = \frac{1}{2}\left[\left(1 + \frac{3}{e'^2}\right)\arctan(e') - \frac{3}{e'}\right]$$
$$= 7.334625787083 \times 10^{-05} \tag{B-18}$$

3.14 q_0’

A second useful intermediate term, q_0', can be calculated by [17](3-65), by combining [18](2-67), where u=b for a level ellipsoid, and [18](2-71), and by [9] as

$$q_0' = 3\left[\left(1+\frac{1}{e'^2}\right)\left(1-\frac{1}{e'}\arctan(e')\right)\right]-1 \\ = 2.688041300461 \times 10^{-03} \tag{B-19}$$

3.15 Comment on Calculations of Terms q_0 and q_0'

Direct calculation of many of the values presented for WGS 84 depend upon the determination of the intermediate terms q_0 and q_0'. Although the formulas for these Eqs. (B-19) and (B-20) are exact, their calculations present numerical challenges. Both of these terms are dependent upon the calculation algorithm used for the *arctan()* function.

3.16 Normal Gravity Formula Constant (m)

A third useful intermediate term, m, can be calculated by [17](3-53), by [18](2-70), and from [9] as

$$m = \frac{\omega^2 a^2 b}{GM} = 3.44978650684084 \times 10^{-03} \tag{B-20}$$

3.17 WGS 84 Dynamical Form Factor (J_2)

The WGS 84 Dynamical Form Factor J_2 is given by combining [18](2-90) and [18](2-92') and (B) above:

$$J_2 = \frac{e^2}{3}\left(1-\frac{2\,m\,e'}{15q_0}\right) = 1.082629821313 \times 10^{-03} \tag{B-21}$$

3.18 WGS 84 Second Degree Zonal Harmonic ($\bar{C}_{2,0}$)

The WGS 84 Second Degree Zonal Harmonic is given by [4](3-12) and by combining the first value from [19](Table3a) with [18](2-41) yields:

$$\bar{C}_{2,0} = -1 \times \frac{J_2}{\sqrt{5}} = -4.84166774985 \times 10^{-04} \tag{B-22}$$

4. Calculation of Physical Constants

4.1 Normal Gravity Potential of the WGS 84 Ellipsoid (U_0)

The normal gravity potential is defined by [17](3-51), [18](2-61), [18](2-71), and [9] as

$$U_0 = \frac{GM}{E} \arctan e' + \frac{\omega^2 a^2}{3} \\ = 6.26368517146 \times 10^{+07} meters^2/seconds^2 \tag{B-23}$$

4.2 Normal Gravity at the Equator (γ_e)

The normal gravity at the equator is given by [17](3-63) ,[18](2-73), and [9] as

$$\gamma_e = \frac{GM}{ab}\left(1 - m - \frac{m\, e'\, q_0'}{6\, q_0}\right) \\ = 9.7803253359\ meters/second^2 \tag{B-24}$$

4.3 Normal Gravity at the Pole (γ_p)

The normal gravity at the pole is given by [17](3-64),[18](2-74), and [9] as

$$\gamma_p = \frac{GM}{a^2}\left(1 + \frac{m\, e'\, q_0'}{3\, q_0}\right) \\ = 9.8321849379\ meters/second^2 \tag{B-25}$$

4.4 k

Another useful intermediate term, k, can be calculated by [17](3-70) and described in a simplified form by [9] as

$$k = \frac{b\gamma_p - a\gamma_e}{a\gamma_e} = \left(\frac{b\gamma_p}{a\gamma_e} - 1\right) = 1.931852652458 \times 10^{-03} \tag{B-26}$$

4.5 Mean Value of Normal Gravity ($\overline{\gamma}$)

The general expression for the average or mean value of normal gravity is usually transformed by series expansion into the following given by [17](3-69) and [9] as

$$\bar{\gamma} = \gamma_e \left(1 + \frac{1}{6}e^2 + \frac{1}{3}k + \frac{59}{360}e^4 + \frac{5}{18}e^2 k + \frac{2371}{15120}e^6 + \frac{259}{1080}e^4 k + \frac{270229}{1814400}e^8 + \frac{9623}{45360}e^6 k\right) \quad \text{(B-27)}$$

$$= 9.7976432223\ meters/second^2$$

4.6 The Mass of the Earth (M)

The mass of the Earth is derived from Table B.1 and Table B.3 as

$$M = \frac{GM}{G} = 5.9721864 \times 10^{+24} kilograms \quad \text{(B-28)}$$

4.7 Gravitational Constant of the Atmosphere (GM_A)

The gravitational constant of the atmosphere is derived using Table B.3 as

$$GM_A = G \times M_A = 3.43592 \times 10^{+08} meters^3/second^2 \quad \text{(B-29)}$$

4.8 Geocentric Gravitational Constant with Earth's Atmosphere Excluded (GM')

The geocentric gravitational constant with the Earth's atmosphere excluded is obtained from Table B.1 and (B-29) as

$$GM' = GM - GM_A = 3.9860000982 \times 10^{+14} meters^3/second^2 \quad \text{(B-30)}$$

5. Calculation of the Moments of Inertia

5.1 Overview

It is possible to determine the moments of inertia either geometrically, using only the defining parameters of an ellipsoid (a, GM, ω, f (using f to derive $\bar{C}_{2,0}$)), or dynamically using Earth gravitational model coefficients and the dynamical ellipticity. Both methods are used here to determine the moments of inertia, though it is generally accepted that the dynamic solution is more accurate.

5.2 Dynamic Moment of Inertia (C), with respect to the Z-Axis of Rotation

From [17] Table 3.11 the dynamic moment of inertia C can be calculated as

$$C_{dyn} = -\sqrt{5}Ma^2\,\frac{\overline{C}_{2,0}dyn}{H} \qquad \text{(B-31)}$$
$$= 8.0340094 \times 10^{37} kilograms\ meters^2$$

5.3 Dynamic Moment of Inertia (A), with respect to the X-Axis in the Equatorial Plane

From [17] Table 3.11 the dynamic moment of inertia A can be calculated as

$$A_{dyn} = \sqrt{5}Ma^2\left[\left(1-\frac{1}{H}\right)\bar{C}_{2,0}dyn - \frac{\bar{C}_{2,2}dyn}{\sqrt{3}}\right] \qquad \text{(B-32)}$$
$$= 8.00792178 \times 10^{37} kilgorams\ meters^2$$

5.4 Dynamic Moment of Inertia (B), with respect to the Y-Axis in the Equatorial Plane

From [17] Table 3.11 the dynamic moment of inertia B can be calculated as

$$B_{dyn} = \sqrt{5}Ma^2\left[\left(1-\frac{1}{H}\right)\bar{C}_{2,0}dyn + \frac{\bar{C}_{2,2}dyn}{\sqrt{3}}\right] \qquad \text{(B-33)}$$
$$= 8.00807480 \times 10^{37} kilograms\ meters^2$$

5.5 Geometric Moment of Inertia (C), with respect to the Z-Axis of Rotation

From [17] Table 3.10 or [17](3-80) the geometric moment of inertia C can be calculated as

$$C_{geo} = \frac{2}{3}Ma^2\left(1-\frac{2}{5}\sqrt{\frac{5m}{2f}-1}\right) \qquad \text{(B-34)}$$
$$= 8.07302937 \times 10^{37} kilograms\ meters^2$$

5.6 Geometric Moment of Inertia (A), with respect to Any Axis in the Equatorial Plane

Due to symmetry of the rotational ellipsoid, A = B, from [17](3-77). From [17] Table 3.10, or by combining [17](3-12) and [17] (3-83), the geometric moment of inertia A can be calculated as

$$
\begin{aligned}
A_{geo} &= C_{geo} + \sqrt{5} M a^2 \cdot \bar{C}_{2,0} geo \\
&= 8.04672663 \times 10^{37} kilograms\ meters^2
\end{aligned}
\tag{B-35}
$$

5.7 Geometric Solution for Dynamical Ellipticity (H)

As previously stated, the dynamical ellipticity is more accurately determined through other techniques, it can be solved for geometrically from A and C using [17] Table 3.10 or [17](3-87) as

$$
H_{geo} = \frac{C_{geo} - A_{geo}}{C_{geo}} = 3.2581004 \times 10^{-03} \tag{B-36}
$$

APPENDIX C

LIST OF REFERENCE ELLIPSOID NAMES AND PARAMETERS (USED FOR GENERATING DATUM TRANSFORMATIONS)

REFERENCE ELLIPSOIDS
FOR
LOCAL GEODETIC DATUMS

1. General

This appendix lists the reference ellipsoids and their constants (a, f^{-1}) associated with the local geodetic datums which are tied to WGS 84 through datum transformation constants and/or MREs (Appendices D, E and F).

2. Constant Characteristics

In Appendix C.1, some of the reference ellipsoids have more than one semi-major axis (a) associated with them. These different values of axis (a) vary from one region or country to another or from one year to another within the same region or country.

A typical example of such an ellipsoid is Everest whose semi-major axis (a) was originally defined in yards. Changes in the yard to meter conversion ratio over the years have resulted in five different values for the constant (a), as identified in Appendix C.1.

To facilitate correct referencing, a standardized two letter code is also included to identify the different ellipsoids and/or their “versions” pertaining to the different values of the semi-major axis (a).

Appendix C.1

Reference Ellipsoid Names and Constants
Used for Datum Transformations*

Reference Ellipsoid Name	ID Code	a (Meters)	f^{-1}
Airy 1830	AA	6377563.396	299.3249646
Australian National 1966	AN	6378160	298.25
Bessel 1841			
Ethiopia, Indonesia, Japan and Korea	BR	6377397.155	299.1528128
Namibia	BN	6377483.865	299.1528128
Clarke 1866	CC	6378206.4	294.9786982
Clarke 1880**	CD	6378249.145	293.465
Clarke 1880 (IGN)	CG	6378249.2	293.4660208
Everest			
Brunei and E. Malaysia (Sabah and Sarawak)	EB	6377298.556	300.8017
India 1830	EA	6377276.345	300.8017
India 1956***	EC	6377301.243	300.8017
Pakistan***	EF	6377309.613	300.8017
W. Malaysia and Singapore 1948	EE	6377304.063	300.8017
W. Malaysia 1969***	ED	6377295.664	300.8017
Geodetic Reference System 1980	RF	6378137	298.257222101
Helmert 1906	HE	6378200	298.3

* Refer to Appendices D, E, and F.

** As accepted by NGA.

*** Through adoption of a new yard to meter conversion factor in the referenced country.

Appendix C.1

Reference Ellipsoid Names and Constants
Used for Datum Transformations*

Reference Ellipsoid Name	ID Code	a (Meters)	f^{-1}
Hough 1960	HO	6378270	297
Indonesian 1974	ID	6378160	298.247
International 1924	IN	6378388	297
Krassovsky 1940	KA	6378245	298.3
Modified Airy	AM	6377340.189	299.3249646
Modified Fischer 1960	FA	6378155	298.3
South American 1969	SA	6378160	298.25
War Office 1924	WO	6378300.58	296
WGS 1972	WD	6378135	298.26
WGS 1984	WE	6378137	298.257223563

* Refer to Appendices D, E, and F.

APPENDIX D

DATUM TRANSFORMATIONS DERIVED USING SATELLITE TIES TO GEODETIC DATUMS/SYSTEMS

DATUM TRANSFORMATION CONSTANTS
GEODETIC DATUMS/SYSTEMS TO WGS 84
(THROUGH SATELLITE TIES)

1. General

This appendix provides the details about the reference ellipsoids (Appendix C) which are used as defining parameters for the geodetic datums and systems.

There are 125 local geodetic datums which are currently related to WGS 84 through satellite ties.

2. Local Datum Ellipsoids

Appendix D.1 lists, alphabetically, the local geodetic datums with their associated ellipsoids. Two letter ellipsoidal codes (Appendix C) have also been included against each datum to indicate which specific "version" of the ellipsoid was used in determining the transformation constants.

3. Transformation Constants

Appendices D.2 through D.7 list the constants for local datums for continental areas. The continents and the local geodetic datums are arranged alphabetically.

Appendices D.8 through D.10 list the constants for local datums which fall within the ocean areas. The ocean areas and the geodetic datums are also arranged alphabetically.

The year of initial publication and cycle numbers have been provided. This makes it possible for a user to determine when a particular set of transformation parameters first became available and if the current set has replaced an outdated set.

A cycle number of zero indicates that the set of parameters is as it was published in DMA TR 8350.2, Second Edition, 1 September 1991, including Insert 1, 30 August 1993, or that the parameters is new to this publication. A cycle number of one (or greater) indicates that the current parameters have replaced outdated parameters that were in the previous edition. A change page is included for convenience.

If transformation parameter sets are updated in future editions of this publication, the cycle numbers for each parameter set that is updated will increment by one.

4. Error Estimates

The 1σ error estimates for the datum transformation constants ($\Delta X, \Delta Y, \Delta Z$), obtained from the computed solutions, are also tabulated. These estimates do not include the errors of the common control station coordinates which were used to compute the shift constants.

For datums having <u>four or less</u> common control stations, the 1σ errors for shift constants are non-computed estimates.

The current set of error estimates has been reevaluated and revised after careful consideration of the datum transformation solutions and the related geodetic information: the intent has been to assign the most realistic estimates possible.

Appendix D.1
Geodetic Datums/Reference Systems
Related to World Geodetic System 1984
(Through Satellite Ties)

Local Geodetic Datum	Associated Reference Ellipsoid*	Code
Accra (Ghana)	War Office 1924	WO
Adindan	Clarke 1880	CD
Afgooye	Krassovsky 1940	KA
Ain el Abd 1970	International 1924	IN
American Samoa 1962	Clarke 1866	CC
Anna 1 Astro 1965	Australian National 1966	AN
Antigua Island Astro 1943	Clarke 1880	CD
Arc 1950	Clarke 1880	CD
Arc 1960	Clarke 1880	CD
Ascension Island 1958	International 1924	IN
Astro Beacon "E" 1945	International 1924	IN
Astro DOS 71/4	International 1924	IN
Astro Tern Island (FRIG) 1961	International 1924	IN
Astronomical Station 1952	International 1924	IN
Australian Geodetic 1966	Australian National 1966	AN
Australian Geodetic 1984	Australian National 1966	AN
Ayabelle Lighthouse	Clarke 1880	CD
Bellevue (IGN)	International 1924	IN
Bermuda 1957	Clarke 1866	CC
Bioko	International 1924	IN
Bissau	International 1924	IN
Bogota Observatory	International 1924	IN
Campo Inchauspe	International 1924	IN
Canton Astro 1966	International 1924	IN
Cape	Clarke 1880	CD
Cape Canaveral	Clarke 1866	CC
Carthage	Clarke 1880	CD
Chatham Island Astro 1971	International 1924	IN
Chua Astro	International 1924	IN
Circuit (Zimbabwe)	Clarke 1880	CD
Co-Ordinate System 1937 of Estonia	Bessel 1841	BR
Corrego Alegre	International 1924	IN
Dabola	Clarke 1880	CD
DCS-3 (Astro 1955)	Clarke 1880	CD
Deception Island	Clarke 1880	CD
Djakarta (Batavia)	Bessel 1841	BR

* See Appendix C.1 for associated constants a, f^{-1}.

Appendix D.1

Geodetic Datums/Reference Systems
Related to World Geodetic System 1984
(Through Satellite Ties)

Local Geodetic Datum	Associated Reference Ellipsoid*	Code
DOS 1968	International 1924	IN
Easter Island 1967	International 1924	IN
European 1950	International 1924	IN
European 1979	International 1924	IN
Fiji 1956	International 1924	IN
Fort Thomas 1955	Clarke 1880	CD
Gambia	Clarke 1880	CD
Gan 1970	International 1924	IN
Geodetic Datum 1949	International 1924	IN
Graciosa Base SW 1948	International 1924	IN
Guam 1963	Clarke 1866	CC
GUX 1 Astro	International 1924	IN
Hjorsey 1955	International 1924	IN
Hong Kong 1963	International 1924	IN
Hu-Tzu-Shan	International 1924	IN
Indian	Everest	EA/EC**
Indian 1954	Everest	EA
Indian 1960	Everest	EA
Indian 1975	Everest	EA
Indonesian 1974	Indonesian 1974	ID
Ireland 1965	Modified Airy	AM
ISTS 061 Astro 1968	International 1924	IN
ISTS 073 Astro 1969	International 1924	IN
Johnston Island 1961	International 1924	IN
Kandawala	Everest	EA
Kerguelen Island 1949	International 1924	IN
Kertau 1948	Everest	EE
Korean Geodetic System 1995	WGS 84	WE
Kusaie Astro 1951	International 1924	IN
L. C. 5 Astro 1961	Clarke 1866	CC
Leigon	Clarke 1880	CD
Liberia 1964	Clarke 1880	CD
Lisbon	International 1924	IN
Luzon	Clarke 1866	CC
Mahe 1971	Clarke 1880	CD

* See Appendix C.1 for associated constants a, f^{-1}.

** Due to different semi-major axes. See Appendix C.1.

Appendix D.1

Geodetic Datums/Reference Systems
Related to World Geodetic System 1984
(Through Satellite Ties)

Local Geodetic Datum	Associated Reference Ellipsoid*	Code
Massawa	Bessel 1841	BR
Merchich	Clarke 1880	CD
Midway Astro 1961	International 1924	IN
Minna	Clarke 1880	CD
Montserrat Island Astro 1958	Clarke 1880	CD
M'Poraloko	Clarke 1880	CD
Nahrwan	Clarke 1880	CD
Naparima, BWI	International 1924	IN
North American 1927	Clarke 1866	CC
North American 1983	GRS 80**	RF
North Sahara 1959	Clarke 1880	CD
Observatorio Campos Rodrigues 1907	Clarke 1866	CC
Observatorio Meteorologico 1939	International 1924	IN
Old Egyptian 1907	Helmert 1906	HE
Old Hawaiian	Clarke 1866	CC
Old Hawaiian	International 1924	IN
Oman	Clarke 1880	CD
Ordnance Survey of Great Britain 1936	Airy 1830	AA
Pico de las Nieves	International 1924	IN
Pitcairn Astro 1967	International 1924	IN
Point 58	Clarke 1880	CD
Pointe Noire 1948	Clarke 1880	CD
Porto Santo 1936	International 1924	IN
Provisional South American 1956	International 1924	IN
Provisional South Chilean 1963***	International 1924	IN
Puerto Rico	Clarke 1866	CC
Qatar National	International 1924	IN
Qornoq	International 1924	IN
Reunion	International 1924	IN

* See Appendix C.1 for associated constants a, f^{-1}.
** Geodetic Reference System 1980
*** Also known as Hito XVIII 1963

Appendix D.1
Geodetic Datums/Reference Systems
Related to World Geodetic System 1984
(Through Satellite Ties)

Local Geodetic Datum	**Associated Reference Ellipsoid***	**Code**
Rome 1940	International 1924	IN
S-42 (Pulkovo 1942)	Krassovsky 1940	KA
Santo (DOS) 1965	International 1924	IN
Sao Braz	International 1924	IN
Sapper Hill 1943	International 1924	IN
Schwarzeck	Bessel 1841	BN
Selvagem Grande 1938	International 1924	IN
Sierra Leone 1960	Clark 1880	CD
S-JTSK	Bessel 1841	BR
South American 1969	South American 1969	SA
South American Geocentric Reference System (SIRGAS)	GRS 80**	RF
South Asia	Modified Fischer 1960	FA
South East Island	Clarke 1880	CD
Tananarive Observatory 1925	International 1924	IN
Tete 1960	Clarke 1866	CC
Timbalai 1948	Everest	EB
Timbalai 1968	Everest	EB
Tokyo	Bessel 1841	BR
Tristan Astro 1968	International 1924	IN
Viti Levu 1916	Clarke 1880	CD
Voirol 1960	Clarke 1880	CD
Wake-Eniwetok 1960	Hough 1960	HO
Wake Island Astro 1952	International 1924	IN
Yof Astro 1967 (Datum 200)	Clarke 1880	CD
Zanderij	International 1924	IN

* See Appendix C.1 for associated constants a,f^{-1}.

** Geodetic Reference System 1980

Appendix D.2
Transformation Parameters
Local Geodetic Datums to WGS 84

Continent: AFRICA															
Local Geodetic Datums		**Reference Ellipsoids and Parameter Differences**			**No. of Satellite Stations Used**	**Transformation Parameters**									
Name	**Code**	**Name**	**Δa(m)**	**Δf x 10⁴**		**Cycle Number**	**Pub. Date**	**ΔX(m)**		**ΔY(m)**		**ΔZ(m)**			
ACCRA Ghana	ACC	War Office 1924	-163.58	-0.25567714	4	0	2012	-170	±3	33	±4	326	±3		
ADINDAN	ADI	Clarke 1880	-112.145	-0.54750714											
Mean Solution (Ethiopia and Sudan)	ADI-M				22	0	1991	-166	±5	-15	±5	204	±3		
Burkina Faso	ADI-E				1	0	1991	-118	±25	-14	±25	218	±25		
Cameroon	ADI-F				1	0	1991	-134	±25	-2	±25	210	±25		
Ethiopia	ADI-A				8	0	1991	-165	±3	-11	±3	206	±3		
Mali	ADI-C				1	0	1991	-123	±25	-20	±25	220	±25		
Senegal	ADI-D				2	0	1991	-128	±25	-18	±25	224	±25		
Sudan	ADI-B				14	0	1991	-161	±3	-14	±5	205	±3		
AFGOOYE	AFG	Krassovsky 1940	-108	0.00480795	1	0	1987	-43	±25	-163	±25	45	±25		

Appendix D.2

Transformation Parameters

Local Geodetic Datums to WGS 84

Continent: AFRICA										
Local Geodetic Datums		**Reference Ellipsoids and Parameter Differences**			**No. of Satellite Stations Used**	**Transformation Parameters**				
Name	**Code**	**Name**	**Δa(m)**	**Δf x 10^4**		**Cycle Number**	**Pub. Date**	**ΔX(m)**	**ΔY(m)**	**ΔZ(m)**
ARC 1950	ARF	Clarke 1880	-112.145	-0.54750714						
Mean Solution (Botswana, Lesotho, Malawi, Swaziland, Zaire, Zambia and Zimbabwe)	ARF-M				41	0	1987	-143 ±20	-90 ±33	-294 ±20
Botswana	ARF-A				9	0	1991	-138 ±3	-105 ±5	-289 ±3
Burundi	ARF-H				3	0	1991	-153 ±20	-5 ±20	-292 ±20
Lesotho	ARF-B				5	0	1991	-125 ±3	-108 ±3	-295 ±8
Malawi	ARF-C				6	0	1991	-161 ±9	-73 ±24	-317 ±8
Swaziland	ARF-D				4	0	1991	-134 ±15	-105 ±15	-295 ±15
Dem Rep of the Congo (Zaire)	ARF-E				2	0	1991	-169 ±25	-19 ±25	-278 ±25
Zambia	ARF-F				5	0	1991	-147 ±21	-74 ±21	-283 ±27
Zimbabwe	ARF-G				38	1	2012	-145 ±10	-97 ±10	-292 ±10

Appendix D.2

Transformation Parameters

Local Geodetic Datums to WGS 84

Continent: AFRICA

Local Geodetic Datums		Reference Ellipsoids and Parameter Differences			No. of Satellite Stations Used	Transformation Parameters				
Name	**Code**	**Name**	**Δa(m)**	**Δf x 10^4**		**Cycle Number**	**Pub. Date**	**ΔX(m)**	**ΔY(m)**	**ΔZ(m)**
ARC 1960	ARS	Clarke 1880	-112.145	-0.54750714						
Mean Solution (Kenya and Tanzania)	ARS-M				25	0	1991	-160 ±20	-6 ±20	-302 ±20
Kenya	ARS-A				24	0	1997	-157 ±4	-2 ±3	-299 ±3
Tanzania	ARS-B				12	0	1997	-175 ±6	-23 ±9	-303 ±10
Malawi	ARS-C				7	0	2012	-179 ±13	-81 ±25	-314 ±7
AYABELLE LIGHTHOUSE	PHA	Clarke 1880	-112.145	-0.54750714						
Djibouti					2	1	2012	-77 ±10	-128 ±10	142 ±10
BISSAU	BID	International 1924	-251	-0.14192702						
Guinea-Bissau					2	0	1991	-173 ±25	253 ±25	27 ±25
CAPE	CAP	Clarke 1880	-112.145	-0.54750714						
South Africa					5	0	1987	-136 ±3	-108 ±6	-292 ±6

Appendix D.2
Transformation Parameters
Local Geodetic Datums to WGS 84

Continent: AFRICA															
Local Geodetic Datums		**Reference Ellipsoids and Parameter Differences**			**No. of Satellite Stations Used**	**Transformation Parameters**									
Name	**Code**	**Name**	**Δa(m)**	**Δf x 10^4**		**Cycle Number**	**Pub. Date**	**ΔX(m)**		**ΔY(m)**		**ΔZ(m)**			
CARTHAGE	CGE	Clarke 1880	-112.145	-0.54750714											
Tunisia					5	0	1987	-263	±6	6	±9	431	±8		
CIRCUIT	CIR	Clarke 1880	-112.145	-0.54750714											
Zimbabwe					27	0	2012	-144	±10	-97	±10	-291	±10		
DABOLA	DAL	Clarke 1880	-112.145	-0.54750714											
Guinea					4	0	1991	-83	±15	37	±15	124	±15		
EUROPEAN 1950	EUR	International 1924	-251	-0.14192702											
Egypt	EUR-F				14	0	1991	-130	±6	-117	±8	-151	±8		
Tunisia	EUR-T				4	0	1993	-112	±25	-77	±25	-145	±25		
GAMBIA	GAI	Clarke 1880	-112.145	-0.54750714											
Gambia					1	0	2012	-63	±25	176	±25	185	±25		
LEIGON	LEH	Clarke 1880	-112.145	-0.54750714											
Ghana					8	0	1991	-130	±2	29	±3	364	±2		

Appendix D.2

Transformation Parameters
Local Geodetic Datums to WGS 84

Continent: AFRICA										
Local Geodetic Datums		**Reference Ellipsoids and Parameter Differences**			**No. of Satellite Stations Used**	**Transformation Parameters**				
Name	**Code**	**Name**	**Δa(m)**	**Δf x 10⁴**		**Cycle Number**	**Pub. Date**	**ΔX(m)**	**ΔY(m)**	**ΔZ(m)**
LIBERIA 1964	LIB	Clarke 1880	-112.145	-0.54750714						
Liberia					4	0	1987	-90 ±15	40 ±15	88 ±15
MASSAWA	MAS	Bessel 1841	739.845	0.10037483						
Eritrea (Ethiopia)					1	0	1987	639 ±25	405 ±25	60 ±25
MERCHICH	MER	Clarke 1880	-112.145	-0.54750714						
Morocco					9	0	1987	31 ±5	146 ±3	47 ±3
MINNA	MIN	Clarke 1880	-112.145	-0.54750714						
Cameroon	MIN-A				2	0	1991	-81 ±25	-84 ±25	115 ±25
Nigeria	MIN-B				6	0	1987	-92 ±3	-93 ±6	122 ±5
M'PORALOKO	MPO	Clarke 1880	-112.145	-0.54750714						
Gabon					1	0	1991	-74 ±25	-130 ±25	42 ±25
NORTH SAHARA 1959	NSD	Clarke 1880	-112.145	-0.54750714						

Appendix D.2
Transformation Parameters
Local Geodetic Datums to WGS 84

Continent: AFRICA															
Local Geodetic Datums		**Reference Ellipsoids and Parameter Differences**			**No. of Satellite Stations Used**	**Transformation Parameters**									
Name	**Code**	**Name**	**Δa(m)**	**$\Delta f \times 10^4$**		**Cycle Number**	**Pub. Date**	**ΔX(m)**		**ΔY(m)**		**ΔZ(m)**			
Algeria					3	0	1993	-186	±25	-93	±25	310	±25		
OBSERVATORIO CAMPOS RODRIGUES 1907	CPR	Clarke 1866	-69.4	-0.37264639											
Mozambique					3	0	2012	-132	±10	-110	±10	-335	±10		
OLD EGYPTIAN 1907	OEG	Helmert 1906	-63	0.00480795											
Egypt					14	0	1987	-130	±3	110	±6	-13	±8		
POINT 58	PTB	Clarke 1880	-112.145	-0.54750714											
Mean Solution (Burkina Faso and Niger)					2	0	1991	-106	±25	-129	±25	165	±25		
POINTE NOIRE 1948	PTN	Clarke 1880	-112.145	-0.54750714											
Congo					1	0	1991	-148	±25	51	±25	-291	±25		

Appendix D.2
Transformation Parameters
Local Geodetic Datums to WGS 84

Continent: AFRICA										
Local Geodetic Datums		**Reference Ellipsoids and Parameter Differences**			**No. of Satellite Stations Used**	**Transformation Parameters**				
Name	**Code**	**Name**	**Δa(m)**	**Δf x 10^4**		**Cycle Number**	**Pub. Date**	**ΔX(m)**	**ΔY(m)**	**ΔZ(m)**
SCHWARZECK	SCK	Bessel 1841	653.135*	0.10037483						
Namibia					3	0	1991	616 ±20	97 ±20	-251 ±20
SIERRA LEONE 1960	SRL	Clark 1880	-112.145	-0.54750714						
Sierra Leone					8	0	1997	-88 ±15	4 ±15	101 ±15
TETE 1960	TEC	Clarke 1866	-69.4	-0.37264639						
Mozambique					4	0	2012	-80 ±10	-100 ±10	-228 ±10
VOIROL 1960	VOR	Clarke 1880	-112.145	-0.54750714						

Appendix D.2
Transformation Parameters
Local Geodetic Datums to WGS 84

Continent: AFRICA											
Local Geodetic Datums		**Reference Ellipsoids and Parameter Differences**			**No. of Satellite Stations Used**	**Transformation Parameters**					
Name	**Code**	**Name**	**Δa(m)**	**Δf x 10^4**		**Cycle Number**	**Pub. Date**	**ΔX(m)**	**ΔY(m)**	**ΔZ(m)**	
Algeria					2	0	1993	-123 ±25	-206 ±25	219 ±25	
YOF ASTRO 1967 (DATUM 200)	YOF	Clarke 1880	-112.145	-0.54750714							
Senegal					7	0	2012	-30 ±3	190 ±3	89 ±3	

* This Δa value reflects an a-value of 6377483.865 meters for the Bessel 1841 Ellipsoid in Namibia.

Appendix D.3
Transformation Parameters
Local Geodetic Datums to WGS 84

Continent: ASIA										
Local Geodetic Datums		**Reference Ellipsoids and Parameter Differences**			**No. of Satellite Stations Used**	**Transformation Parameters**				
Name	**Code**	**Name**	**Δa(m)**	**Δf x 10⁴**		**Cycle Number**	**Pub. Date**	**ΔX(m)**	**ΔY(m)**	**ΔZ(m)**
AIN EL ABD 1970	AIN	International 1924	-251	-0.14192702						
Bahrain Island	AIN-A				2	0	1991	-150 ±25	-250 ±25	-1 ±25
Saudi Arabia	AIN-B				9	0	1991	-143 ±10	-236 ±10	7 ±10
DJAKARTA (BATAVIA)	BAT	Bessel 1841	739.845	0.10037483						
Sumatra (Indonesia)					5	0	1987	-377 ±3	681 ±3	-50 ±3
EUROPEAN 1950	EUR	International 1924	-251	-0.14192702						
Iran	EUR-H				27	0	1991	-117 ±9	-132 ±12	-164 ±11
HONG KONG 1963	HKD	International 1924	-251	-0.14192702						
Hong Kong					2	0	1987	-156 ±25	-271 ±25	-189 ±25
HU-TZU-SHAN	HTN	International 1924	-251	-0.14192702						
Taiwan					4	0	1991	-637 ±15	-549 ±15	-203 ±15

Appendix D.3

Transformation Parameters
Local Geodetic Datums to WGS 84

Continent: ASIA

Local Geodetic Datums		Reference Ellipsoids and Parameter Differences			No. of Satellite Stations Used	Transformation Parameters				
Name	**Code**	**Name**	**Δa(m)**	**Δf x 10^4**		**Cycle Number**	**Pub. Date**	**ΔX(m)**	**ΔY(m)**	**ΔZ(m)**
INDIAN	IND	Everest								
Bangladesh	IND-B	Everest (1830)	860.655*	0.28361368	6	0	1991	282 ±10	726 ±8	254 ±12
India and Nepal	IND-I	Everest (1956)	835.757*	0.28361368	7	0	1991	295 ±12	736 ±10	257 ±15
INDIAN 1954	INF	Everest (1830)	860.655*	0.28361368						
Thailand	INF-A				11	0	1993	217 ±15	823 ±6	299 ±12
INDIAN 1960	ING	Everest (1830)	860.655*	0.28361368						
Vietnam (near 16°N)	ING-A				2	0	1993	198 ±25	881 ±25	317 ±25
Con Son Island (Vietnam)	ING-B				1	0	1993	182 ±25	915 ±25	344 ±25
INDIAN 1975	INH	Everest (1830)	860.655*	0.28361368						
Thailand	INH-A				6	0	1991	209 ±12	818 ±10	290 ±12
Thailand	INH-A1				62	1	1997	210 ±3	814 ±2	289 ±3

* See Appendix C

Appendix D.3
Transformation Parameters
Local Geodetic Datums to WGS 84

Continent: ASIA										
Local Geodetic Datums		**Reference Ellipsoids and Parameter Differences**			**No. of Satellite Stations Used**	**Transformation Parameters**				
Name	**Code**	**Name**	**Δa(m)**	**Δf x 10^4**		**Cycle Number**	**Pub. Date**	**ΔX(m)**	**ΔY(m)**	**ΔZ(m)**
INDONESIAN 1974	IDN	Indonesian 1974	-23	-0.00114930						
Indonesia					1	0	1993	-24 ±25	-15 ±25	5 ±25
KANDAWALA	KAN	Everest (1830)	860.655*	0.28361368						
Sri Lanka					3	0	1987	-97 ±20	787 ±20	86 ±20
KERTAU 1948	KEA	Everest (1948)	832.937*	0.28361368						
West Malaysia and Singapore					6	0	1987	-11 ±10	851 ±8	5 ±6
KOREAN GEODETIC SYSTEM 1995	KGS	WGS 84	0	0						
South Korea					29	0	2000	0 ±1	0 ±1	0 ±1

* See Appendix C

Appendix D.3
Transformation Parameters
Local Geodetic Datums to WGS 84

Continent: ASIA

Local Geodetic Datums		Reference Ellipsoids and Parameter Differences			No. of Satellite Stations Used	Transformation Parameters				
Name	**Code**	**Name**	**Δa(m)**	**Δf x 10^4**		**Cycle Number**	**Pub. Date**	**ΔX(m)**	**ΔY(m)**	**ΔZ(m)**
NAHRWAN	NAH	Clarke 1880	-112.145	-0.54750714						
Masirah Island (Oman)	NAH-A				2	0	1987	-247 ±25	-148 ±25	369 ±25
United Arab Emirates	NAH-B				2	0	1987	-249 ±25	-156 ±25	381 ±25
Saudi Arabia	NAH-C				3	0	1991	-243 ±20	-192 ±20	477 ±20
OMAN	FAH	Clarke 1880	-112.145	-0.54750714						
Oman					11	1	2012	-345 ±3	3 ±3	223 ±6
QATAR NATIONAL	QAT	International 1924	-251	-0.14192702						
Qatar					3	0	1987	-128 ±20	-283 ±20	22 ±20
SOUTH ASIA	SOA	Modified Fischer 1960	-18	0.00480795						
Singapore					1	0	1987	7 ±25	-10 ±25	-26 ±25

Appendix D.3

Transformation Parameters
Local Geodetic Datums to WGS 84

Continent: ASIA										
Local Geodetic Datums		**Reference Ellipsoids and Parameter Differences**			**No. of Satellite Stations Used**	**Transformation Parameters**				
Name	**Code**	**Name**	**Δa(m)**	**Δf x 10^4**		**Cycle Number**	**Pub. Date**	**ΔX(m)**	**ΔY(m)**	**ΔZ(m)**
TIMBALAI 1948	TIL	Everest	838.444*	0.28361368						
Brunei and East Malaysia (Sarawak and Sabah)					8	0	1987	-679 ±10	669 ±10	-48 ±12
TIMBALAI 1968	TIN	Everest	838.444*	0.28361368						
Brunei and East Malaysia					9	0	2012	-679 ±1	667 ±6	-49 ±2
TOKYO	TOY	Bessel 1841	739.845	0.10037483						
Mean Solution (Japan, Okinawa and South Korea)	TOY-M				31	0	1991	-148 ±20	507 ±5	685 ±20
Japan	TOY-A				16	0	1991	-148 ±8	507 ±5	685 ±8
Okinawa	TOY-C				3	0	1991	-158 ±20	507 ±5	676 ±20
South Korea	TOY-B				12	0	1991	-146 ±8	507 ±5	687 ±8

* See Appendix C

Appendix D.4

Transformation Parameters

Local Geodetic Datums to WGS 84

Continent: AUSTRALIA										
Local Geodetic Datums		**Reference Ellipsoids and Parameter Differences**			**No. of Satellite Stations Used**	**Transformation Parameters**				
Name	**Code**	**Name**	**$\Delta a(m)$**	**$\Delta f \times 10^4$**		**Cycle Number**	**Pub. Date**	**$\Delta X(m)$**	**$\Delta Y(m)$**	**$\Delta Z(m)$**
AUSTRALIAN GEODETIC 1966	AUA	Australian National 1966	-23	-0.00081204						
Australia and Tasmania					161	1	2012	-128 ± 5	-52 ± 5	153 ± 5
AUSTRALIAN GEODETIC 1984	AUG	Australian National 1966	-23	-0.00081204						
Australia and Tasmania					90	0	1987	-134 ± 2	-48 ± 2	149 ± 2

Appendix D.5
Transformation Parameters
Local Geodetic Datums to WGS 84

Continent: EUROPE											
Local Geodetic Datums		**Reference Ellipsoids and Parameter Differences**			**No. of Satellite Stations Used**	**Transformation Parameters**					
Name	**Code**	**Name**	**Δa(m)**	**Δf x 10^4**		**Cycle Number**	**Pub. Date**	**ΔX(m)**	**ΔY(m)**	**ΔZ(m)**
CO-ORDINATE SYSTEM 1937 OF ESTONIA	EST	Bessel 1841	739.85	0.10037483						
Estonia					19	0	1997	374 ±2	150 ±3	588 ±3
EUROPEAN 1950	EUR	International 1924	-251	-0.14192702						
Mean Solution {Austria, Belgium, Denmark, Finland, France, FRG (Federal Republic of Germany)*, Gibraltar, Greece, Italy, Luxembourg, Netherlands, Norway, Portugal, Spain, Sweden and Switzerland}	EUR-M				85	0	1987	-87 ±3	-98 ±8	-121 ±5

* Prior to 1 January 1993

Appendix D.5
Transformation Parameters
Local Geodetic Datums to WGS 84

Continent: EUROPE										
Local Geodetic Datums		**Reference Ellipsoids and Parameter Differences**			**No. of Satellite Stations Used**	**Transformation Parameters**				
Name	**Code**	**Name**	**Δa(m)**	**$\Delta f \times 10^4$**		**Cycle Number**	**Pub. Date**	**ΔX(m)**	**ΔY(m)**	**ΔZ(m)**
EUROPEAN 1950 (cont'd)	EUR	International 1924	-251	-0.14192702						
Western Europe {Limited to Austria, Denmark, France, FRG (Federal Republic of Germany)*, Netherlands and Switzerland}	EUR-A				52	0	1991	-87 ±3	-96 ±3	-120 ±3
Cyprus	EUR-E				4	0	1991	-104 ±15	-101 ±15	-140 ±15
Egypt	EUR-F				14	0	1991	-130 ±6	-117 ±8	-151 ±8
England, Channel Islands, Scotland and Shetland Islands**	EUR-G				40	0	1991	-86 ±3	-96 ±3	-120 ±3
England, Ireland, Scotland and Shetland Islands**	EUR-K				47	0	1991	-86 ±3	-96 ±3	-120 ±3

* Prior to 1 January 1993
** European Datum 1950 coordinates developed from Ordnance Survey of Great Britain (OSGB) Scientific Network 1980 (SN 80) coordinates.

Appendix D.5
Transformation Parameters
Local Geodetic Datums to WGS 84

Continent: EUROPE														
Local Geodetic Datums		**Reference Ellipsoids and Parameter Differences**			**No. of Satellite Stations Used**	**Transformation Parameters**								
Name	**Code**	**Name**	**Δa(m)**	**Δf x 10^4**		**Cycle Number**	**Pub. Date**	**ΔX(m)**		**ΔY(m)**		**ΔZ(m)**		
EUROPEAN 1950 (cont'd)	EUR	International 1924	-251	-0.14192702										
Greece	EUR-B				2	0	1991	-84	±25	-95	±25	-130	±25	
Iran	EUR-H				27	0	1991	-117	±9	-132	±12	-164	±11	
Italy														
Sardinia	EUR-I				2	0	1991	-97	±25	-103	±25	-120	±25	
Sicily	EUR-J				3	0	1991	-97	±20	-88	±20	-135	±20	
Malta	EUR-L				1	0	1991	-107	±25	-88	±25	-149	±25	
Norway and Finland	EUR-C				20	0	1991	-87	±3	-95	±5	-120	±3	
Portugal and Spain	EUR-D				18	0	1991	-84	±5	-107	±6	-120	±3	
Tunisia	EUR-T				4	0	1993	-112	±25	-77	±25	-145	±25	

Appendix D.5

Transformation Parameters
Local Geodetic Datums to WGS 84

Continent: EUROPE										
Local Geodetic Datums		Reference Ellipsoids and Parameter Differences			No. of Satellite Stations Used	Transformation Parameters				
Name	Code	Name	Δa(m)	Δf x 10^4		Cycle Number	Pub. Date	ΔX(m)	ΔY(m)	ΔZ(m)
EUROPEAN 1979	EUS	International 1924	-251	-0.14192702						
Mean Solution (Austria, Finland, Netherlands, Norway, Spain, Sweden and Switzerland)					22	0	1987	-86 ±3	-98 ±3	-119 ±3
HJORSEY 1955	HJO	International 1924	-251	-0.14192702						
Iceland					16	1	2012	-73 ±3	47 ±3	-83 ±6
IRELAND 1965	IRL	Modified Airy	796.811	0.11960023						
Ireland					7	0	1987	506 ±3	-122 ±3	611 ±3
LISBON	LIS	International 1924	-251	-0.14192702						
Portugal					1	0	2012	-306 ±25	-62 ±25	105 ±25

Appendix D.5

Transformation Parameters
Local Geodetic Datums to WGS 84

Continent: EUROPE														
Local Geodetic Datums		**Reference Ellipsoids and Parameter Differences**			**No. of Satellite Stations Used**	**Transformation Parameters**								
Name	**Code**	**Name**	**Δa(m)**	**Δf x 10^4**		**Cycle Number**	**Pub. Date**	**ΔX(m)**		**ΔY(m)**		**ΔZ(m)**		
ORDNANCE SURVEY OF GREAT BRITAIN 1936	OGB	Airy	573.604	0.11960023										
Mean Solution (England, Isle of Man, Scotland, Shetland Islands and Wales)	OGB-M				38	0	1987	375	±10	-111	±10	431	±15	
England	OGB-A				21	0	1991	371	±5	-112	±5	434	±6	
England, Isle of Man and Wales	OGB-B				25	0	1991	371	±10	-111	±10	434	±15	
Scotland and Shetland Islands	OGB-C				13	0	1991	384	±10	-111	±10	425	±10	
Wales	OGB-D				3	0	1991	370	±20	-108	±20	434	±20	
.														

Appendix D.5
Transformation Parameters
Local Geodetic Datums to WGS 84

Continent: EUROPE										
Local Geodetic Datums		**Reference Ellipsoids and Parameter Differences**			**No. of Satellite Stations Used**	**Transformation Parameters**				
Name	**Code**	**Name**	**Δa(m)**	**Δf x 10⁴**		**Cycle Number**	**Pub. Date**	**ΔX(m)**	**ΔY(m)**	**ΔZ(m)**
ROME 1940	MOD	International 1924	-251	-0.14192702						
Sardinia					1	0	1987	-225 ±25	-65 ±25	9 ±25
S-42 (PULKOVO 1942)	SPK	Krassovsky 1940	-108	0.00480795						
Hungary	SPK-A				5	0	1993	28 ±2	-121 ±2	-77 ±2
Poland	SPK-B				11	0	1997	23 ±4	-124 ±2	-82 ±4
Czechoslovakia*	SPK-C				6	0	1997	26 ±3	-121 ±3	-78 ±2
Latvia	SPK-D				5	0	1997	24 ±2	-124 ±2	-82 ±2
Kazakhstan	SPK-E				2	0	1997	15 ±25	-130 ±25	-84 ±25
Albania	SPK-F				7	0	1997	24 ±3	-130 ±3	-92 ±3
Romania	SPK-G				4	0	1997	28 ±3	-121 ±5	-77 ±3
Estonia	SPK-H				19	0	2012	22 ±2	-126 ±3	-85 ±3

Appendix D.5
Transformation Parameters
Local Geodetic Datums to WGS 84

Continent: EUROPE										
Local Geodetic Datums		**Reference Ellipsoids and Parameter Differences**			**No. of Satellite Stations Used**	**Transformation Parameters**				
Name	**Code**	**Name**	**Δa(m)**	**$\Delta f \times 10^4$**		**Cycle Number**	**Pub. Date**	**ΔX(m)**	**ΔY(m)**	**ΔZ(m)**
S-JTSK Czechoslovakia *	CCD	Bessel 1841	739.845	0.10037483	6	0	1993	589 $\pm$4	76 $\pm$2	480 $\pm$3

* Prior to 1 January 1993

Appendix D.6

Transformation Parameters

Local Geodetic Datums to WGS 84

Continent: NORTH AMERICA										
Local Geodetic Datums		**Reference Ellipsoids and Parameter Differences**			**No. of Satellite Stations Used**	**Transformation Parameters**				
Name	**Code**	**Name**	**Δa(m)**	**Δf x 10^4**		**Cycle Number**	**Pub. Date**	**ΔX(m)**	**ΔY(m)**	**ΔZ(m)**
CAPE CANAVERAL	CAC	Clarke 1866	-69.4	-0.37264639						
Mean Solution (Florida and Bahamas)					19	0	1991	-2 $\pm$3	151 $\pm$3	181 $\pm$3
NORTH AMERICAN 1927	NAS	Clarke 1866	-69.4	-0.37264639						
Mean Solution (CONUS)	NAS-C				405	0	1987	-8 $\pm$5	160 $\pm$5	176 $\pm$6
Western United States (Arizona, Arkansas, California, Colorado, Idaho, Iowa, Kansas, Montana, Nebraska, Nevada,New Mexico, North Dakota, Oklahoma, Oregon, South Dakota, Texas, Utah, Washington and Wyoming)	NAS-B				276	0	1991	-8 $\pm$5	159 $\pm$3	175 $\pm$3

Appendix D.6
Transformation Parameters
Local Geodetic Datums to WGS 84

Continent: NORTH AMERICA

Local Geodetic Datums		**Reference Ellipsoids and Parameter Differences**			**No. of Satellite Stations Used**	**Transformation Parameters**				
Name	**Code**	**Name**	**Δa(m)**	**Δf x 10^4**		**Cycle Number**	**Pub. Date**	**ΔX(m)**	**ΔY(m)**	**ΔZ(m)**
NORTH AMERICAN 1927 (cont'd)	NAS	Clarke 1866	-69.4	-0.37264639						
Eastern United States (Alabama, Connecticut, Delaware, District of Columbia, Florida, Georgia, Illinois, Indiana, Kentucky, Louisiana, Maine, Maryland, Massachusetts, Michigan, Minnesota, Mississippi, Missouri, New Hampshire, New Jersey, New York, North Carolina, Ohio, Pennsylvania, Rhode Island, South Carolina, Tennessee, Vermont, Virginia, West Virginia and Wisconsin)	NAS-A				129	0	1991	-9 ±5	161 ±5	179 ±8

Appendix D.6
Transformation Parameters
Local Geodetic Datums to WGS 84

Continent: NORTH AMERICA														
Local Geodetic Datums		**Reference Ellipsoids and Parameter Differences**			**No. of Satellite Stations Used**	**Transformation Parameters**								
Name	**Code**	**Name**	**Δa(m)**	**Δf x 10^4**		**Cycle Number**	**Pub. Date**	**ΔX(m)**		**ΔY(m)**		**ΔZ(m)**		
NORTH AMERICAN 1927 (cont'd)	NAS	Clarke 1866	-69.4	-0.37264639										
Alaska (Excluding Aleutian Islands)	NAS-D				47	0	1987	-5	±5	135	±9	172	±5	
Aleutian Islands														
East of 180°W	NAS-V				6	0	1993	-2	±6	152	±8	149	±10	
West of 180°W	NAS-W				5	0	1993	2	±10	204	±10	105	±10	
Bahamas (Excluding San Salvador Island)	NAS-Q				11	0	1987	-4	±5	154	±3	178	±5	
San Salvador Island	NAS-R				1	0	1987	1	±25	140	±25	165	±25	
Canada Mean Solution (Including Newfoundland)	NAS-E				112	0	1987	-10	±15	158	±11	187	±6	
Alberta and British Columbia	NAS-F				25	0	1991	-7	±8	162	±8	188	±6	

Appendix D.6
Transformation Parameters
Local Geodetic Datums to WGS 84

Continent: NORTH AMERICA														
Local Geodetic Datums		**Reference Ellipsoids and Parameter Differences**			**No. of Satellite Stations Used**	**Transformation Parameters**								
Name	**Code**	**Name**	**Δa(m)**	**$\Delta f \times 10^4$**		**Cycle Number**	**Pub. Date**	**ΔX(m)**		**ΔY(m)**		**ΔZ(m)**		
NORTH AMERICAN 1927 (cont'd)	NAS	Clarke 1866	-69.4	-0.37264639										
Eastern Canada (Newfoundland, New Brunswick, Nova Scotia and Quebec)	NAS-G				37	0	1991	-22	±6	160	±6	190	±3	
Manitoba and Ontario	NAS-H				25	0	1991	-9	±9	157	±5	184	±5	
Northwest Territories and Saskatchewan	NAS-I				17	0	1991	4	±5	159	±5	188	±3	
Yukon	NAS-J				8	0	1991	-7	±5	139	±8	181	±3	
Canal Zone	NAS-O				3	0	1987	0	±20	125	±20	201	±20	
Caribbean (Antigua Island, Barbados, Barbuda, Caicos Islands, Cuba, Dominican Republic, Grand Cayman, Jamaica and Turks Islands)	NAS-P				15	0	1991	-3	±3	142	±9	183	±12	

Appendix D.6

Transformation Parameters
Local Geodetic Datums to WGS 84

Continent: NORTH AMERICA										
Local Geodetic Datums		**Reference Ellipsoids and Parameter Differences**			**No. of Satellite Stations Used**	**Transformation Parameters**				
Name	**Code**	**Name**	**Δa(m)**	**Δf x 10^4**		**Cycle Number**	**Pub. Date**	**ΔX(m)**	**ΔY(m)**	**ΔZ(m)**
NORTH AMERICAN 1927 (cont'd)	NAS	Clarke 1866	-69.4	-0.37264639						
Central America (Belize, Costa Rica, El Salvador, Guatemala, Honduras and Nicaragua)	NAS-N				19	0	1987	0 ±8	125 ±3	194 ±5
Cuba	NAS-T				1	0	1987	-9 ±25	152 ±25	178 ±25
Greenland (Hayes Peninsula)	NAS-U				2	0	1987	11 ±25	114 ±25	195 ±25
Mexico	NAS-L				22	0	1987	-12 ±8	130 ±6	190 ±6
NORTH AMERICAN 1983	NAR	GRS 80	0	-0.00000016						
Alaska (Excluding Aleutian Islands)	NAR-A				42	0	1987	0 ±2	0 ±2	0 ±2
Aleutian Islands	NAR-E				4	0	1993	-2 ±5	0 ±2	4 ±5
Canada	NAR-B				96	0	1987	0 ±2	0 ±2	0 ±2

Appendix D.6
Transformation Parameters
Local Geodetic Datums to WGS 84

Continent: NORTH AMERICA										
Local Geodetic Datums		**Reference Ellipsoids and Parameter Differences**			**No. of Satellite Stations Used**	**Transformation Parameters**				
Name	**Code**	**Name**	**Δa(m)**	**Δf x 10^4**		**Cycle Number**	**Pub. Date**	**ΔX(m)**	**ΔY(m)**	**ΔZ(m)**
NORTH AMERICAN 1983 (cont'd)	NAR	GRS 80	0	-0.00000016						
CONUS	NAR-C				216	0	1987	0 ±2	0 ±2	0 ±2
Hawaii	NAR-H				6	0	1993	1 ±2	1 ±2	-1 ±2
Mexico and Central America	NAR-D				25	0	1987	0 ±2	0 ±2	0 ±2

Appendix D.7
Transformation Parameters
Local Geodetic Datums to WGS 84

Continent: SOUTH AMERICA

Local Geodetic Datums		**Reference Ellipsoids and Parameter Differences**			**No. of Satellite Stations Used**	**Transformation Parameters**				
Name	**Code**	**Name**	**Δa(m)**	**$\Delta f \times 10^4$**		**Cycle Number**	**Pub. Date**	**ΔX(m)**	**ΔY(m)**	**ΔZ(m)**
BOGOTA OBSERVATORY	BOO	International 1924	-251	-0.14192702						
Colombia					7	0	1987	307 ±6	304 ±5	-318 ±6
CAMPO INCHAUSPE 1969	CAI	International 1924	-251	-0.14192702						
Argentina					20	0	1987	-148 ±5	136 ±5	90 ±5
CHUA ASTRO	CHU	International 1924	-251	-0.14192702						
Paraguay					6	0	1987	-134 ±6	229 ±9	-29 ±5
CORREGO ALEGRE	COA	International 1924	-251	-0.14192702						
Brazil					17	0	1987	-206 ±5	172 ±3	-6 ±5

Appendix D.7
Transformation Parameters
Local Geodetic Datums to WGS 84

Continent: SOUTH AMERICA											
Local Geodetic Datums		**Reference Ellipsoids and Parameter Differences**			**No. of Satellite Stations Used**	**Transformation Parameters**					
Name	**Code**	**Name**	**Δa(m)**	**$\Delta f \times 10^4$**		**Cycle Number**	**Pub. Date**	**ΔX(m)**	**ΔY(m)**	**ΔZ(m)**
PROVISIONAL SOUTH AMERICAN 1956	PRP	International 1924	-251	-0.14192702						
Mean Solution (Bolivia, Chile, Colombia, Ecuador, Guyana, Peru and Venezuela)	PRP-M				63	0	1987	-288 ±17	175 ±27	-376 ±27
Bolivia	PRP-A				5	0	1991	-270 ±5	188 ±11	-388 ±14
Chile										
17° 30’S to 26°S	PRP-B1				5	0	2014	-302 ±10	272 ±10	-360 ±10
26°S to 36°S	PRP-B2				7	0	2014	-328 ±10	340 ±10	-329 ±10
36°S to 44°S	PRP-C1				6	0	2014	-352 ±10	403 ±10	-287 ±10
Colombia	PRP-D				4	0	1991	-282 ±15	169 ±15	-371 ±15
Ecuador	PRP-E				11	0	1991	-278 ±3	171 ±5	-367 ±3

Appendix D.7
Transformation Parameters
Local Geodetic Datums to WGS 84

Continent: SOUTH AMERICA

Local Geodetic Datums		Reference Ellipsoids and Parameter Differences			No. of Satellite Stations Used	Transformation Parameters								
Name	**Code**	**Name**	**Δa(m)**	**Δf x 10^4**		**Cycle Number**	**Pub. Date**	**ΔX(m)**		**ΔY(m)**		**ΔZ(m)**		
PROVISIONAL SOUTH AMERICAN 1956 (cont'd)	PRP	International 1924	-251	-0.14192702										
Guyana	PRP-F				9	0	1991	-298	±6	159	±14	-369	±5	
Peru	PRP-G				6	0	1991	-279	±6	175	±8	-379	±12	
Venezuela	PRP-H				24	0	1991	-295	±9	173	±14	-371	±15	
PROVISIONAL SOUTH CHILEAN 1963*	HIT	International 1924	-251	-0.14192702										
Southern Chile (near 53°S)					2	0	1987	16	±25	196	±25	93	±25	

Continent: SOUTH AMERICA

Local Geodetic Datums		Reference Ellipsoids and Parameter Differences			No. of Satellite Stations Used	Transformation Parameters				
Name	**Code**	**Name**	**Δa(m)**	**Δf x 10^4**		**Cycle Number**	**Pub. Date**	**ΔX(m)**	**ΔY(m)**	**ΔZ(m)**
SOUTH AMERICAN 1969	SAN	South American 1969	-23	-0.00081204						
Mean Solution (Argentina, Bolivia, Brazil, Chile, Colombia, Ecuador, Guyana, Paraguay, Peru, Trinidad and Tobago and Venezuela)	SAN-M				84	0	1987	-57 ±15	1 ±6	-41 ±9
Argentina	SAN-A				10	0	1991	-62 ±5	-1 ±5	-37 ±5
Bolivia	SAN-B				4	0	1991	-61 ±15	2 ±15	-48 ±15
Brazil	SAN-C				22	0	1991	-60 ±3	-2 ±5	-41 ±5
Chile	SAN-D				9	0	1991	-75 ±15	-1 ±8	-44 ±11
					8	0	2014	-59 ±2	-11 ±2	-52 ±2
					6	0	2014	-64 ±2	0 ±2	-32 ±2
					4	0	2014	-72 ±4	10 ±4	-32 ±4
					6	0	2014	-79 ±3	13 ±3	-14 ±4

Appendix D.7
Transformation Parameters
Local Geodetic Datums to WGS 84

Continent: SOUTH AMERICA										
Local Geodetic Datums		**Reference Ellipsoids and Parameter Differences**			**No. of Satellite Stations Used**	**Transformation Parameters**				
Name	**Code**	**Name**	**$\Delta a(m)$**	**$\Delta f \times 10^4$**		**Cycle Number**	**Pub. Date**	**$\Delta X(m)$**	**$\Delta Y(m)$**	**$\Delta Z(m)$**
SOUTH AMERICAN 1969 (cont'd)	SAN	South American 1969	-23	-0.00081204						
Colombia	SAN-E				7	0	1991	-44 ±6	6 ±6	-36 ±5
Ecuador (Excluding Galapagos Islands)	SAN-F				11	0	1991	-48 ±3	3 ±3	-44 ±3
Baltra and Galapagos Islands	SAN-J				1	0	1991	-47 ±25	26 ±25	-42 ±25
Guyana	SAN-G				5	0	1991	-53 ±9	3 ±5	-47 ±5
Paraguay	SAN-H				4	0	1991	-61 ±15	2 ±15	-33 ±15
Peru	SAN-I				6	0	1991	-58 ±5	0 ±5	-44 ±5
Trinidad and Tobago	SAN-K				1	0	1991	-45 ±25	12 ±25	-33 ±25
Venezuela	SAN-L				5	0	1991	-45 ±3	8 ±6	-33 ±3

Appendix D.7
Transformation Parameters
Local Geodetic Datums to WGS 84

Continent: SOUTH AMERICA														
Local Geodetic Datums		**Reference Ellipsoids and Parameter Differences**			**No. of Satellite Stations Used**	**Transformation Parameters**								
Name	**Code**	**Name**	**Δa(m)**	**$\Delta f \times 10^4$**		**Cycle Number**	**Pub. Date**	**ΔX(m)**		**ΔY(m)**		**ΔZ(m)**		
SOUTH AMERICAN GEOCENTRIC REFERENCE SYSTEM (SIRGAS)	SIR	GRS 80	0	-0.00000016										
South America					66	0	2000	0	±1	0	±1	0	±1	
ZANDERIJ	ZAN	International 1924	-251	-0.14192702										
Suriname					5	0	1987	-265	±5	120	±5	-358	±8	

Appendix D.8

Transformation Parameters

Local Geodetic Datums to WGS 84

Continent: ATLANTIC OCEAN

Local Geodetic Datums		Reference Ellipsoids and Parameter Differences			No. of Satellite Stations Used	Transformation Parameters								
Name	**Code**	**Name**	**Δa(m)**	**Δf x 10^4**		**Cycle Number**	**Pub. Date**	**ΔX(m)**		**ΔY(m)**		**ΔZ(m)**		
ANTIGUA ISLAND ASTRO 1943	AIA	Clarke 1880	-112.145	-0.54750714										
Antigua and Leeward Islands					1	0	1991	-270	±25	13	±25	62	±25	
ASCENSION ISLAND 1958	ASC	International 1924	-251	-0.14192702										
Ascension Island					2	0	1991	-205	±25	107	±25	53	±25	
ASTRO DOS 71/4	SHB	International 1924	-251	-0.14192702										
St. Helena Island					1	0	1987	-320	±25	550	±25	-494	±25	
BERMUDA 1957	BER	Clarke 1866	-69.4	-0.37264639										
Bermuda Islands					3	0	1987	-73	±20	213	±20	296	±20	
BIOKO	BIO	International 1924	-251	-0.14192702										
Bioko Island					6	0	2012	-235	±5	-110	±17	393	±38	

Appendix D.8
Transformation Parameters
Local Geodetic Datums to WGS 84

Continent: ATLANTIC OCEAN										
Local Geodetic Datums		**Reference Ellipsoids and Parameter Differences**			**No. of Satellite Stations Used**	**Transformation Parameters**				
Name	**Code**	**Name**	**Δa(m)**	**Δf x 10^4**		**Cycle Number**	**Pub. Date**	**ΔX(m)**	**ΔY(m)**	**ΔZ(m)**
CAPE CANAVERAL	CAC	Clarke 1866	-69.4	-0.37264639						
Mean Solution (Bahamas and Florida)					19	0	1991	-2 ±3	151 ±3	181 ±3
DECEPTION ISLAND	DID	Clarke 1880	-112.145	-0.54750714						
Deception Island and Antarctica					3	0	1993	260 ±20	12 ±20	-147 ±20
DCS-3 (ASTRO 1955)	DCS	Clarke 1880	-112.145	-0.54750714						
St Lucia					3	0	2012	-153 ±1	153 ±1	307 ±1
FORT THOMAS 1955	FOT	Clarke 1880	-112.145	-0.54750714						
Nevis, St. Kitts and Leeward Islands					2	0	1991	-7 ±25	215 ±25	225 ±25
GRACIOSA BASE SW 1948	GRA	International 1924	-251	-0.14192702						
Faial, Graciosa, Pico, Sao Jorge and Terceira Islands (Azores)					5	0	1991	-104 ±3	167 ±3	-38 ±3

Appendix D.8

Transformation Parameters

Local Geodetic Datums to WGS 84

Continent: ATLANTIC OCEAN															
Local Geodetic Datums		**Reference Ellipsoids and Parameter Differences**			**No. of Satellite Stations Used**	**Transformation Parameters**									
Name	**Code**	**Name**	**Δa(m)**	**Δf x 10⁴**		**Cycle Number**	**Pub. Date**	**ΔX(m)**		**ΔY(m)**		**ΔZ(m)**			
HJORSEY 1955	HJO	International 1924	-251	-0.14192702											
Iceland					16	1	2012	-73	±3	47	±3	-83	±6		
ISTS 061 ASTRO 1968	ISG	International 1924	-251	-0.14192702											
South Georgia Island					1	0	1991	-794	±25	119	±25	-298	±25		
L. C. 5 ASTRO 1961	LCF	Clarke 1866	-69.4	-0.37264639											
Cayman Brac Island					1	0	1987	42	±25	124	±25	147	±25		
MONTSERRAT ISLAND ASTRO 1958	ASM	Clarke 1880	-112.145	-0.54750714											
Montserrat and Leeward Islands					1	0	1991	174	±25	359	±25	365	±25		
NAPARIMA, BWI	NAP	International 1924	-251	-0.14192702											
Trinidad and Tobago					4	0	1991	-10	±15	375	±15	165	±15		

Appendix D.8
Transformation Parameters
Local Geodetic Datums to WGS 84

Continent: ATLANTIC OCEAN										
Local Geodetic Datums		**Reference Ellipsoids and Parameter Differences**			**No. of Satellite Stations Used**	**Transformation Parameters**				
Name	**Code**	**Name**	**Δa(m)**	**Δf x 10^4**		**Cycle Number**	**Pub. Date**	**ΔX(m)**	**ΔY(m)**	**ΔZ(m)**
OBSERVATORIO METEOROLOGICO 1939	FLO	International 1924	-251	-0.14192702						
Corvo and Flores Islands (Azores)					3	0	1991	-425 ±20	-169 ±20	81 ±20
PICO DE LAS NIEVES	PLN	International 1924	-251	-0.14192702						
Canary Islands					1	0	1987	-307 ±25	-92 ±25	127 ±25
PORTO SANTO 1936	POS	International 1924	-251	-0.14192702						
Porto Santo and Madeira Islands					2	0	1991	-499 ±25	-249 ±25	314 ±25
PUERTO RICO	PUR	Clarke 1866	-69.4	-0.37264639						
Puerto Rico and Virgin Islands					11	0	1987	11 ±3	72 ±3	-101 ±3
QORNOQ	QUO	International 1924	-251	-0.14192702						
South Greenland					2	0	1987	164 ±25	138 ±25	-189 ±32

Appendix D.8
Transformation Parameters
Local Geodetic Datums to WGS 84

Continent: ATLANTIC OCEAN										
Local Geodetic Datums		**Reference Ellipsoids and Parameter Differences**			**No. of Satellite Stations Used**	**Transformation Parameters**				
Name	**Code**	**Name**	**Δa(m)**	**Δf x 10^4**		**Cycle Number**	**Pub. Date**	**ΔX(m)**	**ΔY(m)**	**ΔZ(m)**
SAO BRAZ	SAO	International 1924	-251	-0.14192702						
Sao Miguel and Santa Maria Islands (Azores)					2	0	1987	-203 ±25	141 ±25	53 ±25
SAPPER HILL 1943	SAP	International 1924	-251	-0.14192702						
East Falkland Island					5	0	1991	-355 ±1	21 ±1	72 ±1
SELVAGEM GRANDE 1938	SGM	International 1924	-251	-0.14192702						
Salvage Islands					1	0	1991	-289 ±25	-124 ±25	60 ±25
TRISTAN ASTRO 1968	TDC	International 1924	-251	-0.14192702						
Tristan da Cunha					1	0	1987	-632 ±25	438 ±25	-609 ±25

Appendix D.9
Transformation Parameters
Local Geodetic Datums to WGS 84

Continent: INDIAN OCEAN

Local Geodetic Datums		Reference Ellipsoids and Parameter Differences			No. of Satellite Stations Used	Transformation Parameters				
Name	**Code**	**Name**	**Δa(m)**	**Δf x 10^4**		**Cycle Number**	**Pub. Date**	**ΔX(m)**	**ΔY(m)**	**ΔZ(m)**
ANNA 1 ASTRO 1965	ANO	Australian National 1966	-23	-0.00081204						
Cocos Islands					1	0	1987	-491 ±25	-22 ±25	435 ±25
GAN 1970	GAA	International 1924	-251	-0.14192702						
Republic of Maldives					1	0	1987	-133 ±25	-321 ±25	50 ±25
ISTS 073 ASTRO 1969	IST	International 1924	-251	-0.14192702						
Diego Garcia					2	0	1987	208 ±25	-435 ±25	-229 ±25
KERGUELEN ISLAND 1949	KEG	International 1924	-251	-0.14192702						
Kerguelen Island					1	0	1987	145 ±25	-187 ±25	103 ±25
MAHE 1971	MIK	Clarke 1880	-112.145	-0.54750714						
Mahe Island					1	0	1987	41 ±25	-220 ±25	-134 ±25
REUNION	REU	International 1924	-251	-0.14192702						
Mascarene Islands					1	0	1987	94 ±25	-948 ±25	-1262 ±25

Appendix D.9
Transformation Parameters
Local Geodetic Datums to WGS 84

Continent: INDIAN OCEAN										
Local Geodetic Datums		**Reference Ellipsoids and Parameter Differences**			**No. of Satellite Stations Used**	**Transformation Parameters**				
Name	**Code**	**Name**	**Δa(m)**	**$\Delta f \times 10^4$**		**Cycle Number**	**Pub. Date**	**ΔX(m)**	**ΔY(m)**	**ΔZ(m)**
SOUTH EAST ISLAND	SEI	Clarke 1880	-112.145	-0.54750714						
Seychelles					10	0	2012	-44 $\pm$1	-180 $\pm$1	-268 $\pm$1
TANANARIVE OBSERVATORY 1925	TAN	International 1924	-251	-0.14192702						
					9	0	2012	-191 $\pm$6	-232 $\pm$5	-111 $\pm$2
Madagascar										

Appendix D.10
Transformation Parameters
Local Geodetic Datums to WGS 84

Continent: PACIFIC OCEAN

Local Geodetic Datums		Reference Ellipsoids and Parameter Differences			No. of Satellite Stations Used	Transformation Parameters				
Name	**Code**	**Name**	**Δa(m)**	**Δf x 10^4**		**Cycle Number**	**Pub. Date**	**ΔX(m)**	**ΔY(m)**	**ΔZ(m)**
AMERICAN SAMOA 1962	AMA	Clarke 1866	-69.4	-0.37264639						
American Samoa Islands					2	0	1993	-115 ±25	118 ±25	426 ±25
ASTRO BEACON "E" 1945	ATF	International 1924	-251	-0.14192702						
Iwo Jima					1	0	1987	145 ±25	75 ±25	-272 ±25
ASTRO TERN ISLAND (FRIG) 1961	TRN	International 1924	-251	-0.14192702						
Tern Island					1	0	1991	114 ±25	-116 ±25	-333 ±25
ASTRONOMICAL STATION 1952	ASQ	International 1924	-251	-0.14192702						
Marcus Island					1	0	1987	124 ±25	-234 ±25	-25 ±25
BELLEVUE (IGN)	IBE	International 1924	-251	-0.14192702						
Efate and Erromango Islands					3	0	1987	-127 ±20	-769 ±20	472 ±20

Appendix D.10
Transformation Parameters
Local Geodetic Datums to WGS 84

Continent: PACIFIC OCEAN																
Local Geodetic Datums		**Reference Ellipsoids and Parameter Differences**			**No. of Satellite Stations Used**	**Transformation Parameters**										
Name	**Code**	**Name**	**Δa(m)**	**Δf x 10^4**		**Cycle Number**	**Pub. Date**	**ΔX(m)**		**ΔY(m)**		**ΔZ(m)**				
CANTON ASTRO 1966	CAO	International 1924	-251	-0.14192702												
Phoenix Islands					4	0	1987	298	±15	-304	±15	-375	±15			
CHATHAM ISLAND ASTRO 1971	CHI	International 1924	-251	-0.14192702												
Chatham Island (New Zealand)					4	0	1987	175	±15	-38	±15	113	±15			
DOS 1968	GIZ	International 1924	-251	-0.14192702												
Gizo Island (New Georgia Islands)					1	0	1987	230	±25	-199	±25	-752	±25			
EASTER ISLAND 1967	EAS	International 1924	-251	-0.14192702												
Easter Island					1	0	1987	211	±25	147	±25	111	±25			
FIJI 1956	FJI	International 1924	-251	-0.14192702												
Fiji					20	0	2012	265	±5	385	±3	-194	±2			

Appendix D.10

Transformation Parameters
Local Geodetic Datums to WGS 84

Continent: PACIFIC OCEAN										
Local Geodetic Datums		**Reference Ellipsoids and Parameter Differences**			**No. of Satellite Stations Used**	**Transformation Parameters**				
Name	**Code**	**Name**	**Δa(m)**	**Δf x 10^4**		**Cycle Number**	**Pub. Date**	**ΔX(m)**	**ΔY(m)**	**ΔZ(m)**
GEODETIC DATUM 1949	GEO	International 1924	-251	-0.14192702						
New Zealand					14	0	1987	84 ±5	-22 ±3	209 ±5
GUAM 1963	GUA	Clarke 1866	-69.4	-0.37264639						
Guam					5	0	1987	-100 ±3	-248 ±3	259 ±3
GUX l ASTRO	DOB	International 1924	-251	-0.14192702						
Guadalcanal Island					1	0	1987	252 ±25	-209 ±25	-751 ±25
INDONESIAN 1974	IDN	Indonesian 1974	-23	-0.00114930						
Indonesia					1	0	1993	-24 ±25	-15 ±25	5 ±25
JOHNSTON ISLAND 1961	JOH	International 1924	-251	-0.14192702						
Johnston Island					2	0	1991	189 ±25	-79 ±25	-202 ±25

Appendix D.10
Transformation Parameters
Local Geodetic Datums to WGS 84

Continent: PACIFIC OCEAN										
Local Geodetic Datums		**Reference Ellipsoids and Parameter Differences**			**No. of Satellite Stations Used**	**Transformation Parameters**				
Name	**Code**	**Name**	**Δa(m)**	**Δf x 10^4**		**Cycle Number**	**Pub. Date**	**ΔX(m)**	**ΔY(m)**	**ΔZ(m)**
KUSAIE ASTRO 1951	KUS	International 1924	-251	-0.14192702						
Caroline Islands, Fed. States of Micronesia					1	0	1991	647 ±25	1777 ±25	-1124 ±25
LUZON	LUZ	Clarke 1866	-69.4	-0.37264639						
Philippines (Excluding Mindanao Island)	LUZ-A				6	0	1987	-133 ±8	-77 ±11	-51 ±9
Mindanao Island	LUZ-B				1	0	1987	-133 ±25	-79 ±25	-72 ±25
MIDWAY ASTRO 1961	MID	International 1924	-251	-0.14192702						
Midway Islands					1	1	2003	403 ±25	-81 ±25	277 ±25
OLD HAWAIIAN	OHA	Clarke 1866	-69.4	-0.37264639						
Mean Solution	OHA-M				15	0	1987	61 ±25	-285 ±20	-181 ±20
Hawaii	OHA-A				2	0	1991	89 ±25	-279 ±25	-183 ±25
Kauai	OHA-B				3	0	1991	45 ±20	-290 ±20	-172 ±20
Maui	OHA-C				2	0	1991	65 ±25	-290 ±25	-190 ±25

Appendix D.10
Transformation Parameters
Local Geodetic Datums to WGS 84

Continent: PACIFIC OCEAN														
Local Geodetic Datums		**Reference Ellipsoids and Parameter Differences**			**No. of Satellite Stations Used**	**Transformation Parameters**								
Name	**Code**	**Name**	**Δa(m)**	**Δf x 10^4**		**Cycle Number**	**Pub. Date**	**ΔX(m)**		**ΔY(m)**		**ΔZ(m)**		
OLD HAWAIIAN (cont'd)	OHA	Clarke 1866	-69.4	-0.37264639										
Oahu	OHA-D				8	0	1991	58	±10	-283	±6	-182	±6	
OLD HAWAIIAN	OHI	International 1924	-251	-0.14192702										
Mean Solution	OHI-M				15	0	2000	201	±25	-228	±20	-346	±20	
Hawaii	OHI-A				2	0	2000	229	±25	-222	±25	-348	±25	
Kauai	OHI-B				3	0	2000	185	±20	-233	±20	-337	±20	
Maui	OHI-C				2	0	2000	205	±25	-233	±25	-355	±25	
Oahu	OHI-D				8	0	2000	198	±10	-226	±6	-347	±6	
PITCAIRN ASTRO 1967	PIT	International 1924	-251	-0.14192702										
Pitcairn Island					1	0	1987	185	±25	165	±25	42	±25	

Appendix D.10
Transformation Parameters
Local Geodetic Datums to WGS 84

Continent: PACIFIC OCEAN										
Local Geodetic Datums		**Reference Ellipsoids and Parameter Differences**			**No. of Satellite Stations Used**	**Transformation Parameters**				
Name	**Code**	**Name**	**Δa(m)**	**Δf x 10^4**		**Cycle Number**	**Pub. Date**	**ΔX(m)**	**ΔY(m)**	**ΔZ(m)**
SANTO (DOS) 1965	SAE	International 1924	-251	-0.14192702						
Espirito Santo Island					1	0	1987	170 ±25	42 ±25	84 ±25
VITI LEVU 1916	MVS	Clarke 1880	-112.145	-0.54750714						
Viti Levu Island (Fiji Islands)					9	1	2012	98 ±3	390 ±3	-22 ±3
WAKE-ENIWETOK 1960	ENW	Hough	-133	-0.14192702						
Marshall Islands					10	0	1991	102 ±3	52 ±3	-38 ±3
WAKE ISLAND ASTRO 1952	WAK	International 1924	-251	-0.14192702						
Wake Atoll					2	0	1991	276 ±25	-57 ±25	149 ±25

APPENDIX E

DATUM TRANSFORMATIONS DERIVED USING NON-SATELLITE INFORMATION

DATUM TRANSFORMATION CONSTANTS
LOCAL GEODETIC DATUMS TO WGS 84
(THROUGH NON-SATELLITE TIES)

1. General

This appendix provides the details about the reference ellipsoids (Appendix C) used as defining parameters for the local geodetic datums which are related to WGS 84 through non-satellite ties to the local control.

There are sixteen such local/regional geodetic datums, one region under the European Datum 1950 (EUR-S), and two regional realizations of the Indian Datum (IND).

2. Local Datum Ellipsoids

Appendix E.1 lists, alphabetically, the local geodetic datums and their associated ellipsoids. Two letter ellipsoidal codes (Appendix B) have also been included to clearly indicate which "version" of the ellipsoid has been used to determine the transformation constants.

3. Transformation Constants

Appendix E.2 lists, alphabetically, the local geodetic datums and the three special areas with the associated shift constants.

The year of initial publication and cycle numbers have been provided as a new feature in this edition. This makes it possible for a user to determine when a particular set of transformation parameters first became available and if the current set has replaced an outdated set.

A cycle number of zero indicates that the set of parameters are as they were published in DMA TR 8350.2, Second Edition, 1 September 1991, including Insert 1, 30 August 1993, or that the parameters are new to this publication. A cycle number of one indicates that the current parameters have replaced outdated parameters that were in the previous edition.

If transformation parameter sets are updated in future editions of this publication, the cycle numbers for each parameter set that is updated will increment by one.

4. Error Estimates

The error estimates are not available for the datum transformation constants listed in the Appendix E.2.

Appendix E.1
Local Geodetic Datums and their Associated Ellipsoids
Related to World Geodetic System 1984
(Through non-Satellite Ties)

Local Geodetic Datum	Associated* Reference Ellipsoid	Code
Aden	Clarke 1880	CD
Beijing (Peking) 1954	Krassovsky 1940	KA
Bekaa Valley 1920	Clarke 1880	CD
Bukit Rimpah	Bessel 1841	BR
Camp Area Astro	International 1924	IN
Conakry 1905	Clarke 1880 (IGN)	CG
European 1950	International 1924	IN
Gunung Segara	Bessel 1841	BR
Herat North	International 1924	IN
Hermannskogel	Bessel 1841	BR
Indian	Everest	EA
Indian	Everest	EF
Mayotte Combani	International 1924	IN
Ocotopeque	Clarke 1866	CC
Pulkovo 1942	Krassovsky 1940	KA
New Triangulation of France	Clarke 1880 (IGN)	CG
St Pierre et Miquelon 1950	Clarke 1866	CC
Voirol 1874	Clarke 1880	CD
Yacare	International 1924	IN

* See Appendix C.1 for associated constants a,f^{-1}.

Appendix E.2

Non-Satellite Derived Transformation Parameters
Local Geodetic Datums to WGS 84

Local Geodetic Datums		Reference Ellipsoids and Parameter Differences			Transformation Parameters				
Name	Code	Name	Δa(m)	$\Delta f \times 10^4$	Cycle Number	Pub. Date	ΔX(m)	ΔY(m)	ΔZ(m)
ADEN	ADN	Clarke 1880	-112.145	-0.54750714					
Yemen					0	2012	-24	-203	268
BEIJING (PEKING) 1954	PED	Krassovsky 1940	-108	0.00480795					
China					0	2012	-11	-113	-41
BEKAA VALLEY 1920	BVD	Clarke 1880	-112.145	-0.54750714					
Lebanon					0	2012	-183	-15	273
BUKIT RIMPAH	BUR	Bessel 1841	739.845	0.10037483					
Bangka and Belitung Islands (Indonesia)					0	1987	-384	664	-48
CAMP AREA ASTRO	CAZ	International 1924	-251	-0.14192702					
Camp McMurdo Area, Antarctica					0	1987	-104	-129	239
CONAKRY 1905	COU	Clarke 1880 (IGN)	-112.2	-0.54738861					
Guinea					0	2012	-23	259	-9

Appendix E.2

Non-Satellite Derived Transformation Parameters
Local Geodetic Datums to WGS 84

Local Geodetic Datums		Reference Ellipsoids and Parameter Differences			Transformation Parameters				
Name	**Code**	**Name**	**Δa(m)**	**Δf x 10^4**	**Cycle Number**	**Pub. Date**	**ΔX(m)**	**ΔY(m)**	**ΔZ(m)**
EUROPEAN 1950	EUR-S	International 1924	-251	-0.14192702					
Iraq, Israel, Jordan Kuwait, Lebanon, Saudi Arabia and Syria					0	1991	-103	-106	-141
GUNUNG SEGARA	GSE	Bessel 1841	739.845	0.10037483					
Kalimantan (Indonesia)					0	1987	-403	684	41
HERAT NORTH	HEN	International 1924	-251	-0.14192702					
Afghanistan					0	1987	-333	-222	114
HERMANNSKOGEL	HER	Bessel 1841	739.845	0.10037483					
Yugoslavia (Prior to 1990) Slovenia, Croatia, Bosnia and Herzegovina and Serbia					0	1997	682	-203	480
INDIAN									
Pakistan	IND-P	Everest	827.387*	0.28361368	0	1993	283	682	231
Sri Lanka	IND-S	Everest	860.655*	0.28361368	0	2012	272	706	242

*See Appendix E.1

Appendix E.2

Non-Satellite Derived Transformation Parameters

Local Geodetic Datums to WGS 84

Local Geodetic Datums		Reference Ellipsoids and Parameter Differences			Transformation Parameters				
Name	**Code**	**Name**	**Δa(m)**	**Δf x 10^4**	**Cycle Number**	**Pub. Date**	**ΔX(m)**	**ΔY(m)**	**ΔZ(m)**
MAYOTTE COMBANI	MCX	International 1924	-251	-0.14192702					
Mayotte (Comoros)					0	2012	-382	-59	-262
NEW TRIANGULATION OF FRANCE	NTF	Clarke 1880 (IGN)	-112.2	-0.54738861					
France					0	2012	-168	-60	320
OCOTEPEQUE	OCE	Clarke 1866	-69.4	-0.37264639					
Costa Rica					0	2012	205	96	-98
PULKOVO 1942	PUK	Krassovsky 1940	-108	0.00480795					
Russia					0	1993	28	-130	-95
ST PIERRE et MIQUELON 1950	SPX	Clarke 1866	-69.4	-0.37264639					
St Pierre and Miquelon					0	2012	30	430	368
VOIROL 1874	VOI	Clarke 1880	-112.145	-0.54750714					
Tunisia and Algeria					0	1997	-73	-247	227
YACARE	YAC	International 1924	-251	-0.14192702					
Uruguay					0	1987	-155	171	37

APPENDIX F

MULTIPLE REGRESSION EQUATIONS FOR SPECIAL CONTINENT-SIZED LOCAL GEODETIC DATUMS

MULTIPLE REGRESSION EQUATIONS

1. General

This appendix provides the Multiple Regression Equations' (MREs) parameters for continent-sized datums and large, contiguous large land areas (Table F.1).

Table F.1
Datums with Multiple Regression Equations

DATUM NAME	AREA COVERED
Australian Geodetic 1966	Australian Mainland
Australian Geodetic 1984	Australian Mainland
Campo Inchauspe	Argentina
Corrego Alegre	Brazil
European 1950	Western Europe (Austria, Denmark, France, W.Germany*, The Netherlands and Switzerland.)
North American 1927	CONUS and Canadian Mainland
South American 1969	South American Mainland (Argentina, Bolivia, Brazil, Chile, Colombia, Ecuador, Guyana, Peru, Paraguay, Uruguay and Venezuela.

* Prior to October 1990.

2. Applications

The coverage areas for MREs' application are defined in detail for each datum. MREs' coverage area should never be extrapolated and are not to be used over islands and/or isolated land areas.

The main advantage of MREs lies in their modeling of distortions for datums, which cover continent-sized land areas, to obtain better transformation fits in geodetic applications.

Multiple Regression Equations (MREs)
for Transforming
Australian Geodetic Datum 1966 (AUA) to WGS 84

Area of Applicability : **Australian Mainland (excluding Tasmania)**

MRE coefficients for ϕ and λ are :

$$\Delta\phi'' = 5.19238 + 0.12666\,U + 0.52309\,V - 0.42069\,U^2 - 0.39326\,UV + 0.93484\,U^2V + 0.44249\,UV^2 - 0.30074\,UV^3 + 1.00092\,U^5 - 0.07565\,V^6 - 1.42988\,U^9 - 16.06639\,U^4V^5 + 0.07428\,V^9 + 0.24256\,UV^9 + 38.27946\,U^6V^7 - 62.06403\,U^7V^8 + 89.19184\,U^9V^8$$

$$\Delta\lambda'' = 4.69250 - 0.87138\,U - 0.50104\,V + 0.12678\,UV - 0.23076\,V^2 - 0.61098\,U^2V - 0.38064\,V^3 + 2.89189\,U^6 + 5.26013\,U^2V^5 - 2.97897\,U^8 + 5.43221\,U^3V^5 - 3.40748\,U^2V^6 + 0.07772\,V^8 + 1.08514\,U^8V + 0.71516\,UV^8 + 0.20185\,V^9 + 5.18012\,U^2V^8 - 1.72907\,U^3V^8 - 1.24329\,U^2V^9$$

Where : $U = K(\phi + 27°)$; $V = K(\lambda - 134°)$; $K = 0.05235988$

<u>NOTE</u> : Input ϕ as (-) from 90°S to 0°N in degrees.

Input λ as (-) from 180°W to 0°E in degrees.

Quality of fit = ± 2.0 m

<u>Test Case</u> :

<u>AUA</u>	<u>Shift</u>	<u>WGS 84</u>
ϕ = (-)17° 00′ 32.78″S	$\Delta\phi$ = 5.48″	ϕ = (-)17° 00′ 27.30″S
λ = 144° 11′ 37.25″E	$\Delta\lambda$ = 3.92″	λ = 144° 11′ 41.17″E

Multiple Regression Equations (MREs)
for Transforming
Australian Geodetic Datum 1984 (AUG) to WGS 84

Area of Applicability : **Australian Mainland (excluding Tasmania)**

MRE coefficients for ϕ and λ are :

$$\Delta\phi'' = 5.20604 + 0.25225\,U + 0.58528\,V - 0.41584\,U^2 - 0.38620\,UV - 0.06820\,V^2 + 0.38699\,U^2V + 0.07934\,UV^2 + 0.37714\,U^4 - 0.52913\,U^4V + 0.38095\,V^7 + 0.68776\,U^2V^6 - 0.03785\,V^8 - 0.17891\,U^9 - 4.84581\,U^2V^7 - 0.35777\,V^9 + 4.23859\,U^2V^9$$

$$\Delta\lambda'' = 4.67877 - 0.73036\,U - 0.57942\,V + 0.28840\,U^2 + 0.10194\,U^3 - 0.27814\,UV^2 - 0.13598\,V^3 + 0.34670\,UV^3 - 0.46107\,V^4 + 1.29432\,U^2V^3 + 0.17996\,UV^4 - 1.13008\,U^2V^5 - 0.46832\,U^8 + 0.30676\,V^8 + 0.31948\,U^9 + 0.16735\,V^9 - 1.19443\,U^3V^9$$

Where : $U = K\,(\phi + 27°)$; $V = K\,(\lambda - 134°)$; $K = 0.05235988$

NOTE : Input ϕ as (-) from 90°S to 0°N in degrees.

Input λ as (-) from 180°W to 0°E in degrees.

Quality of fit = ± 2.0 m

Test Case :

AUG	Shift	WGS 84
ϕ = (-)20° 38′ 00.67″S	$\Delta\phi$ = 5.50″	ϕ = (-)20° 37′ 55.17″S
λ = 144° 24′ 29.29″E	$\Delta\lambda$ = 4.11″	λ = 144° 24′ 33.40″E

Multiple Regression Equations (MREs)
for Transforming
Campo Inchauspe Datum (CAI) to WGS 84

Area of Applicability : **Argentina (Continental land areas only)**

MRE coefficients for ϕ and λ are :

$$\begin{aligned}\Delta \phi'' = {} & 1.67470 + 0.52924\,U - 0.17100\,V + 0.18962\,U^2 + 0.04216\,UV + 0.19709\,UV^2 \\ & - 0.22037\,U^4 - 0.15483\,U^2V^2 - 0.24506\,UV^4 - 0.05675\,V^5 + 0.06674\,U^6 \\ & + 0.01701\,UV^5 - 0.00202\,U^7 + 0.08625\,V^7 - 0.00628\,U^8 + 0.00172\,U^8V^4 \\ & + 0.00036\,U^9V^6\end{aligned}$$

$$\begin{aligned}\Delta\lambda'' = {} & - 2.93117 + 0.18225\,U + 0.69396\,V - 0.04403\,U^2 + 0.07955\,V^2 + 1.48605\,V^3 \\ & - 0.00499\,U^4 - 0.02180\,U^4V - 0.29575\,U^2V^3 + 0.20377\,UV^4 - 2.47151\,V^5 \\ & + 0.09073\,U^3V^4 + 1.33556\,V^7 + 0.01575\,U^3V^5 - 0.26842\,V^9\end{aligned}$$

Where : $U = K\,(\phi + 35°)$; $V = K\,(\lambda + 64°)$; $K = 0.15707963$

NOTE : Input ϕ as (-) from 90°S to 0°N in degrees.

Input λ as (-) from 180°W to 0°E in degrees.

Quality of fit = ± 2.0 m

Test Case :

CAI	Shift	WGS 84
ϕ = (-)29° 47′ 45.68″S	$\Delta\phi$ = 1.95″	ϕ = (-)29° 47′ 43.73″S
λ = (-)58° 07′ 38.20″W	$\Delta\lambda$ = -1.96″	λ = (-)58° 07′ 40.16″W

Multiple Regression Equations (MREs)
for Transforming
Corrego Alegre Datum (COA) to WGS 84

Area of Applicability : **Brazil (Continental land areas only)**

MRE coefficients for ϕ and λ are :

$$\begin{aligned}\Delta\phi'' = {} & - 0.84315 + 0.74089\ U - 0.21968\ V - 0.98875\ U^2 + 0.89883\ UV + 0.42853\ U^3 \\ & + 2.73442\ U^4 - 0.34750\ U^3V + 4.69235\ U^2V^3 - 1.87277\ U^6 + 11.06672\ U^5V \\ & - 46.24841\ U^3V^3 - 0.92268\ U^7 - 14.26289\ U^7V + 334.33740\ U^5V^5 \\ & - 15.68277\ U^9V^2 - 2428.8586\ U^8V^8\end{aligned}$$

$$\begin{aligned}\Delta\lambda'' = {} & - 1.46053 + 0.63715\ U + 2.24996\ V - 5.66052\ UV + 2.22589\ V^2 - 0.34504\ U^3 \\ & - 8.54151\ U^2V + 0.87138\ U^4 + 43.40004\ U^3V + 4.35977\ UV^3 + 8.17101\ U^4V \\ & + 16.24298\ U^2V^3 + 19.96900\ UV^4 - 8.75655\ V^5 - 125.35753\ U^5V \\ & - 127.41019\ U^3V^4 - 0.61047\ U^8 + 138.76072\ U^7V + 122.04261\ U^5V^4 \\ & - 51.86666\ U^9V + 45.67574\ U^9V^3\end{aligned}$$

Where : $U = K\ (\phi + 15°)$; $V = K\ (\lambda + 50°)$; $K = 0.05235988$

NOTE : Input ϕ as (-) from 90°S to 0°N in degrees.

Input λ as (-) from 180°W to 0°E in degrees.

Quality of fit = ± 2.0 m

Test Case :

COA	Shift	WGS 84
ϕ = (-)20° 29′ 01.02″S	$\Delta\phi$ = -1.03″	ϕ = (-)20° 29′ 02.05″S
λ = (-)54° 47′ 13.17″W	$\Delta\lambda$ = -2.10″	λ = (-)54° 47′ 15.27″W

Multiple Regression Equations (MREs)
for Transforming
European Datum 1950 (EUR) to WGS 84

Area of Applicability : **Western Europe* (Continental contiguous land areas only)**

MRE coefficients for ϕ and λ are :

$$\begin{aligned}\Delta\phi'' = &- 2.65261 + 2.06392\, U + 0.77921\, V + 0.26743\, U^2 + 0.10706\, UV + 0.76407\, U^3 \\ &- 0.95430\, U^2V + 0.17197\, U^4 + 1.04974\, U^4V - 0.22899\, U^5V^2 - 0.05401\, V^8 \\ &- 0.78909\, U^9 - 0.10572\, U^2V^7 + 0.05283\, UV^9 + 0.02445\, U^3V^9\end{aligned}$$

$$\begin{aligned}\Delta\lambda'' = &- 4.13447 - 1.50572\, U + 1.94075\, V - 1.37600\, U^2 + 1.98425\, UV + 0.30068\, V^2 \\ &- 2.31939\, U^3 - 1.70401\, U^4 - 5.48711\, UV^3 + 7.41956\, U^5 - 1.61351\, U^2V^3 \\ &+ 5.92923\, UV^4 - 1.97974\, V^5 + 1.57701\, U^6 - 6.52522\, U^3V^3 + 16.85976\, U^2V^4 \\ &- 1.79701\, UV^5 - 3.08344\, U^7 - 14.32516\, U^6V + 4.49096\, U^4V^4 + 9.98750\, U^8V \\ &+ 7.80215\, U^7V^2 - 2.26917\, U^2V^7 + 0.16438\, V^9 - 17.45428\, U^4V^6 - 8.25844\, U^9V^2 \\ &+ 5.28734\, U^8V^3 + 8.87141\, U^5V^7 - 3.48015\, U^9V^4 + 0.71041\, U^4V^9\end{aligned}$$

Where : $U = K (\phi - 52°)$; $V = K (\lambda - 10°)$; $K = 0.05235988$

NOTE Input ϕ as (-) from 90°S to 0°N in degrees.

Input λ as (-) from 180°W to 0°E in degrees.

Quality of fit = ± 2.0 m

Test Case :

EUR	Shift	WGS 84
ϕ = 46° 41′ 42.89″N	$\Delta\phi$ = -3.08″	ϕ = 46° 41′ 39.81″N
λ = 13° 54′ 54.09″E	$\Delta\lambda$ = -3.49″	λ = 13° 54′ 50.60″E

* See Table F.1 (Page F-2) for the list of countries covered by the above set of MREs.

Multiple Regression Equations (MREs)
for Transforming
North American Datum 1927 (NAS) to WGS 84

Area of Applicability : **Canada (Continental contiguous land areas only)**

MRE coefficients for ϕ and λ are :

$$\begin{aligned}\Delta\phi'' = {} & 0.79395 + 2.29199\,U + 0.27589\,V - 1.76644\,U^2 + 0.47743\,UV + 0.08421\,V^2 \\ & - 6.03894\,U^3 - 3.55747\,U^2V - 1.81118\,UV^2 - 0.20307\,V^3 + 7.75815\,U^4 \\ & - 3.1017\,U^3V + 3.58363\,U^2V^2 - 1.31086\,UV^3 - 0.45916\,V^4 + 14.27239\,U^5 \\ & + 3.28815\,U^4V + 1.35742\,U^2V^3 + 1.75323\,UV^4 + 0.44999\,V^5 - 19.02041\,U^4V^2 \\ & - 1.01631\,U^2V^4 + 1.47331\,UV^5 + 0.15181\,V^6 + 0.41614\,U^2V^5 - 0.80920\,UV^6 \\ & - 0.18177\,V^7 + 5.19854\,U^4V^4 - 0.48837\,UV^7 - 0.01473\,V^8 - 2.26448\,U^9 \\ & - 0.46457\,U^2V^7 + 0.11259\,UV^8 + 0.02067\,V^9 + 47.64961\,U^8V^2 + 0.04828\,UV^9 \\ & + 36.38963\,U^9V^2 + 0.06991\,U^4V^7 + 0.08456\,U^3V^8 + 0.09113\,U^2V^9 \\ & + 5.93797\,U^7V^5 - 2.36261\,U^7V^6 + 0.09575\,U^5V^8\end{aligned}$$

$$\begin{aligned}\Delta\lambda'' = {} & - 1.36099 + 3.61796\,V - 3.97703\,U^2 + 3.09705\,UV - 1.15866\,V^2 - 13.28954\,U^3 \\ & - 3.15795\,U^2V + 0.68405\,UV^2 - 0.50303\,V^3 - 8.81200\,U^3V - 2.17587\,U^2V^2 \\ & - 1.49513\,UV^3 + 0.84700\,V^4 + 31.42448\,U^5 - 14.67474\,U^3V^2 + 0.65640\,UV^4 \\ & + 17.55842\,U^6 + 6.87058\,U^4V^2 - 0.21565\,V^6 + 62.18139\,U^5V^2 + 1.78687\,U^3V^4 \\ & + 2.74517\,U^2V^5 - 0.30085\,UV^6 + 0.04600\,V^7 + 63.52702\,U^6V^2 + 7.83682\,U^5V^3 \\ & + 9.59444\,U^3V^5 + 0.01480\,V^8 + 10.51228\,U^4V^5 - 1.42398\,U^2V^7 - 0.00834\,V^9 \\ & + 5.23485\,U^7V^3 - 3.18129\,U^3V^7 + 8.45704\,U^9V^2 - 2.29333\,U^4V^7 \\ & + 0.14465\,U^2V^9 + 0.29701\,U^3V^9 + 0.17655\,U^4V^9\end{aligned}$$

Where : $U = K(\phi - 60°)$; $V = K(\lambda + 100°)$; $K = 0.05235988$

<u>NOTE</u> : Input ϕ as (-) from 90°S to 0°N in degrees.

Input λ as (-) from 180°W to 0°E in degrees.

Quality of fit = ± 2.0 m

<u>Test Case</u> :

<u>NAS</u>	<u>Shift</u>	<u>WGS 84</u>
ϕ = 54° 26′ 08.67″N	$\Delta\phi$ = 0.29″	ϕ = 54° 26′ 08.96″N
λ = (-)110° 17′ 02.41″W	$\Delta\lambda$ = -3.16″	λ = (-)110° 17′ 05.57″W

Multiple Regression Equations (MREs)
for Transforming
North American Datum 1927 (NAS) to WGS 84

Area of Applicability : **USA (Continental contiguous land areas only; excluding Alaska and Islands)**

MRE coefficients for ϕ and λ are :

$$\begin{aligned}\Delta\phi'' = {} & 0.16984 - 0.76173\,U + 0.09585\,V + 1.09919\,U^2 - 4.57801\,U^3 - 1.13239\,U^2V \\ & + 0.49831\,V^3 - 0.98399\,U^3V + 0.12415\,UV^3 + 0.11450\,V^4 + 27.05396\,U^5 \\ & + 2.03449\,U^4V + 0.73357\,U^2V^3 - 0.37548\,V^5 - 0.14197\,V^6 - 59.96555\,U^7 \\ & + 0.07439\,V^7 - 4.76082\,U^8 + 0.03385\,V^8 + 49.04320\,U^9 - 1.30575\,U^6V^3 \\ & - 0.07653\,U^3V^9 + 0.08646\,U^4V^9\end{aligned}$$

$$\begin{aligned}\Delta\lambda'' = {} & - 0.88437 + 2.05061\,V + 0.26361\,U^2 - 0.76804\,UV + 0.13374\,V^2 - 1.31974\,U^3 \\ & - 0.52162\,U^2V - 1.05853\,UV^2 - 0.49211\,U^2V^2 + 2.17204\,UV^3 - 0.06004\,V^4 \\ & + 0.30139\,U^4V + 1.88585\,UV^4 - 0.81162\,UV^5 - 0.05183\,V^6 - 0.96723\,UV^6 \\ & - 0.12948\,U^3V^5 + 3.41827\,U^9 - 0.44507\,U^8V + 0.18882\,UV^8 - 0.01444\,V^9 \\ & + 0.04794\,UV^9 - 0.59013\,U^9V^3\end{aligned}$$

Where : $U = K(\phi - 37°)$; $V = K(\lambda + 95°)$; $K = 0.05235988$

NOTE : Input ϕ as (-) from 90°S to 0°N in degrees.

Input λ as (-) from 180°W to 0°E in degrees.

Quality of fit = ± 2.0 m

Test Case :

NAS	Shift	WGS 84
ϕ = 34° 47′ 08.83″N	$\Delta\phi$ = 0.36″	ϕ = 34° 47′ 09.19″N
λ = (-)86° 34′ 52.18″W	$\Delta\lambda$ = 0.08″	λ = (-)86° 34′ 52.10″W

Multiple Regression Equations (MREs)
for Transforming
South American Datum 1969 (SAN) to WGS 84

Area of Applicability : **South America (Continental contiguous land areas only)**

MRE coefficients for ϕ and λ are :

$$\begin{aligned}\Delta\phi'' = {} & - 1.67504 - 0.05209\, U + 0.25158\, V + 1.10149\, U^2 + 0.24913\, UV - 1.00937\, U^2V \\ & - 0.74977\, V^3 - 1.54090\, U^4 + 0.14474\, V^4 + 0.47866\, U^5 + 0.36278\, U^3V^2 \\ & - 1.29942\, UV^4 + 0.30410\, V^5 + 0.87669\, U^6 - 0.27950\, U^5V - 0.46367\, U^7 \\ & + 4.31466\, U^4V^3 + 2.09523\, U^2V^5 + 0.85556\, UV^6 - 0.17897\, U^8 - 0.57205\, UV^7 \\ & + 0.12327\, U^9 - 0.85033\, U^6V^3 - 4.86117\, U^4V^5 + 0.06085\, U^9V - 0.21518\, U^3V^8 \\ & + 0.31053\, U^5V^7 - 0.09228\, U^8V^5 - 0.22996\, U^9V^5 + 0.58774\, U^6V^9 \\ & + 0.87562\, U^9V^7 + 0.39001\, U^8V^9 - 0.81697\, U^9V^9\end{aligned}$$

$$\begin{aligned}\Delta\lambda'' = {} & - 1.77967 + 0.40405\, U + 0.50268\, V - 0.05387\, U^2 - 0.12837\, UV - 0.54687\, U^2V \\ & - 0.17056\, V^3 - 0.14400\, U^3V + 0.11351\, U^5V - 0.62692\, U^3V^3 - 0.01750\, U^8 \\ & + 1.18616\, U^3V^5 + 0.01305\, U^9 + 1.01360\, U^7V^3 - 0.29059\, U^8V^3 + 5.12370\, U^6V^5 \\ & - 5.09561\, U^7V^5 - 5.27168\, U^6V^7 + 4.04265\, U^7V^7 - 1.62710\, U^8V^7 \\ & + 1.68899\, U^9V^7 + 2.07213\, U^8V^9 - 1.76074\, U^9V^9\end{aligned}$$

Where : $U = K\,(\phi + 20°)$; $V = K\,(\lambda + 60°)$; $K = 0.05235988$

NOTE : Input ϕ as (-) from 90°S to 0°N in degrees.

Input λ as (-) from 180°W to 0°E in degrees.

Quality of fit = ± 2.0 m

Test Case

SAN	Shift	WGS 84
ϕ = (-)31° 56′ 33.95″S	$\Delta\phi$ = -1.36″	ϕ = (-)31° 56′ 35.31″S
λ = (-)65° 06′ 18.66″W	$\Delta\lambda$ = -2.16″	λ = (-)65° 06′ 20.82″W

APPENDIX G

WGS 72 TO WGS 84 TRANSFORMATION

WGS 72 to WGS 84 TRANSFORMATION

Situations arise where only WGS 72 coordinates are available for a site. In such instances, the WGS 72 to WGS 84 transformations listed in Table G.1 can be used with the following equations to obtain WGS 84 coordinates for the sites:

$$\phi_{WGS\ 84} = \phi_{WGS\ 72} + \Delta\phi$$

$$\lambda_{WGS\ 84} = \lambda_{WGS\ 72} + \Delta\lambda$$

$$h_{WGS\ 84} = h_{WGS\ 72} + \Delta h$$

. As indicated in Table G.1, when proceeding directly from WGS 72 coordinates to obtain WGS 84 values, the WGS 84 coordinates will differ from the WGS 72 coordinates due to a shift in the coordinate system origin, a change in the longitude reference, a scale change (treated through Δr) and changes in the size and shape of the ellipsoid. In addition, it is important to be aware that $\Delta\phi$, $\Delta\lambda$, and Δh values calculated using Table G.1 do not reflect the effect of differences between the WGS 72 and WGS 84 EGMs and geoids. The following cases are important to note:

Table G.1 equations are to be used for direct transformation of Doppler-derived WGS 72 coordinates. These transformed coordinates should agree to within approximately ± 2 meters with the directly surveyed WGS 84 coordinates using TRANSIT or GPS point positioning.

Table G.1 should not be used for local satellite geodetic stations whose WGS 72 coordinates were determined using datum shifts from [37]. The preferred approach is to transform such WGS 72 coordinates, using datum shifts from [37], back to their respective local datums, and then transform the local datum coordinates to WGS 84 using Appendices D or E.

. Table G.1 should be used only when no other approach is applicable.

Table G.1
Formulas and Parameters
to Transform WGS 72 Coordinates
to WGS 84 Coordinates

FORMULAS	$\Delta\phi'' = (4.5 \cos \phi) / (a \sin 1'') + (\Delta f \sin 2\phi) / (\sin 1'')$ $\Delta\lambda'' = 0.554$ $\Delta h = 4.5 \sin \phi + a \Delta f \sin^2 \phi - \Delta a + \Delta r$ (Units = Meters)
PARAMETERS	$\Delta f = 0.3121057 \times 10^{-7}$ $a = 6378135$ m $\Delta a = 2.0$ m $\Delta r = 1.4$ m
INSTRUCTIONS	To obtain WGS 84 coordinates, add the $\Delta\phi$, $\Delta\lambda$, Δh changes calculated using WGS 72 coordinates to the WGS 72 coordinates (ϕ, λ, h, respectively). Latitude is positive north and longitude is positive east (0° to 180°).

APPENDIX H

DATUMS/SYSTEMS EQUIVALENT TO WGS 84 FOR MAPPING AND CHARTING

Appendix H

Datums/Systems Equivalent to WGS 84 for Mapping and Charting

Scales of 1:50000 and Smaller

Name	Ellipsoid	Area Covered
Australian Antarctic 1998, 2000	GRS 80	Antarctica
CHTRF 95	GRS 80	Switzerland
Datum Geodesi Nasional 1995 (DGN95)	WGS 84	Indonesia
Estonia 1992, 1997	GRS 80	Estonia
European Terrestrial Reference System 1989	GRS 80	Europe
Geocentric Datum of Australia 1994, 2000	GRS 80	Australia
Geocentric Datum of Malaysia 2000	GRS 80	Malaysia
Hartebeesthoek 1994	WGS 84	South Africa
Systems Based on ITRF 1988-2000	GRS 80	Global
Irenet 95	GRS 80	Ireland
ISN 93, ISN 2004	GRS 80	Iceland
Istituto Geografico Militare Italiano (IGM95)	GRS 80	Italy
Japan Geodetic Datum 2000	GRS 80	Japan
Korean Geodetic System 1995	WGS 84	South Korea
Korean Geodetic Datum 2002	GRS 80	South Korea
Latvia 1992 (LKS 92)	GRS 80	Latvia
Lithuanian Coordinate System (LKS 94)	GRS 80	Lithuania
MONREF 97	GRS 80	Mongolia
MOZNET	WGS 84	Mozambique
New Zealand Geodetic Datum 2000	GRS 80	New Zealand
North American Datum of 1983	GRS 80	North America
• National Spatial Reference System 2007	GRS 80	United States
• Canadian Spatial Reference System	GRS 80	Canada
Reseau Geodesique Francais 1993	GRS 80	France

Appendix H

Datums/Systems Equivalent to WGS 84 for Mapping and Charting

Scales of 1:50000 and Smaller

Name	Ellipsoid	Area Covered
SIRGAS 95, SIRGAS 2000 & Regional Densifications	GRS 80	South & Central America
• POSGAR/RAMSAC	GRS 80	Argentina
• MARGEN/SIRGAS-CON	GRS 80	Bolivia
• SIRGAS2000/RBMC	GRS 80	Brazil
• SIRGAS-Chile/SIRGAS-CON	GRS 80	Chile
• MAGNA-SIRGAS/MAGNA-ECO	GRS 80	Columbia
• CR05	GRS 80	Costa Rica
• Red basica GPS/REGME	GRS 80	Ecuador
• SIRGAS_ES2007	GRS 80	El Salvador
• RGFG	GRS 80	French Guyana
• RGNA	GRS 80	Mexico
• MACARIO SOLIS	GRS 80	Panama
• PERU96	GRS 80	Peru
• SIRGAS-ROU98/Red de estaciones permanente de referncia	GRS 80	Uruguay
• SIRGAS-REGVEN/REMOS	GRS 80	Venezuela
SWEREF 99	GRS 80	Sweden
TAIWAN DATUM 1997	GRS 80	Taiwan

APPENDIX I

ACRONYMS

BIH	= Bureau International de l'Heure
BTS	= BIH Terrestrial System
CCRS	= Conventional Celestial Reference System
CEP	= Celestial Ephemeris Pole
CIO	= Celestial Intermediate Origin
CIS	= Conventional Inertial System
CONUS	= Contiguous United States
CCRS	= Conventional Celestial Reference System
CRF	= Celestial Reference Frame
CRS	= Celestial Reference System
CTP	= Conventional Terrestrial Pole
CTRF	= Conventional Terrestrial Reference Frame
CTRS	= Conventional Terrestrial Reference System
CTS	= Conventional Terrestrial System
DMA	= Defense Mapping Agency
DoD	= Department of Defense
DISR	= Department of Defense Information Technology Standards Register
DoT	= Department of Transportation
DTM	= Digital Terrain Model
ECEF	= Earth-Centered Earth-Fixed
ECI	= Earth-Centered Inertial
ECM	= Earth's Center of Mass
EGM	= Earth Gravitational Model
EGM96	= Earth Gravitational Model 1996
EGM2008	= Earth Gravitational Model 2008
EOP	= Earth Orientation Parameters
FRG	= Federal Republic of Germany
GCRS	= Geocentric Celestial Reference System
GEOINT	= Geospatial Intelligence
GEOPS	= Geopotential Surfaces
GIS	= Geographic Information System
GSFC	= Goddard Space Flight Center
GLONASS	= Global Navigation Satellite System

GMST	= Greenwich Mean Sidereal Time
GNSS	= Global Navigation Satellite Systems (generic)
GPS	= Global Positioning System
GRACE	= Gravity Recovery and Climate Experiment (satellite mission)
GRS 80	= Geodetic Reference System 1980
GST	= Greenwich Sidereal Time
GV	= Grid Variation (Grivation)
GWG	= Geospatial Intelligence Standards Working Group
HTDP	= Horizontal Time-Dependent Positioning
IAG	= International Association of Geodesy
IAU	= International Astronomical Union
IC	= Intelligence Community
ICRS	= International Celestial Reference System
IERS	= International Earth Rotation Service
IGeS	= International Geoid Service
IGS	= International GPS Service
IRM	= IERS Reference Meridian
IRP	= IERS Reference Pole
ITRF	= IERS Terrestrial Reference Frame
ITS	= Instantaneous Terrestrial System
IUGG	= International Union of Geodesy and Geophysics
JGP95E	= Joint Gravity Project 1995 Elevation
LAGEOS	= LAser GEOdynamics Satellites
MREs	= Multiple Regression Equations
MSL	= Mean Sea Level
MST	= Mean Sidereal Time
NAD 27	= North American Datum 1927
NAD 83	= North American Datum 1983
NASA	= National Aeronautics and Space Administration
NAVSTAR GPS	= Navigation Satellite Timing and Ranging GPS
NGA	= National Geospatial-Intelligence Agency
NGS	= National Geodetic Survey
NIMA	= National Imagery and Mapping Agency
NNSS	= Navy Navigation Satellite System

NSG	= National System for Geospatial Intelligence
NSWC	= Naval Surface Warfare Center (formerly Naval Surface Weapons Center)
NSWCDD	= Naval Surface Warfare Center Dahlgren Division
OCS	= Operational Control Segment
PNT	= Position, Navigation, and Timing
PPS	= Precise Positioning Service
RMS	= Root-Mean-Square
SA	= Selective Availability
SV	= Secular Variation, other usage Space Vehicle
SLR	= Satellite Laser Ranging
SPS	= Standard Positioning Service
SRTM	= Shuttle Radar and Topography Mission
TAI	= International Atomic Time
TDB	= Barycentric Dynamic Time
TDRSS	= Tracking and Data Relay Satellite System
TIO	= Terrestrial Intermediate Origin
TOPEX	= ocean TOPography EXperiment (satellite mission)
TR	= Technical Report
TRF	= Terrestrial Reference Frame
TRS	= Terrestrial Reference System
TT	= Terrestrial Time
UK	= United Kingdom
UPS	= Universal Polar Stereographic
US	= United States
USAF	= Unites States Air Force
USG	= Unites States Government
USNO	= United States Naval Observatory
UT	= Universal Time
UTC	= Coordinated Universal Time
UTM	= Universal Transverse Mercator
VLBI	= Very Long Baseline Interferometry
WGS	= World Geodetic System
WGS 60	= World Geodetic System 1960
WGS 66	= World Geodetic System 1966

WGS 72	= World Geodetic System 1972
WGS 84	= World Geodetic System 1984
WMM	= World Magnetic Model