Astron. Astrophys. 105, 359–361 (1982)

ASTRONOMY AND ASTROPHYSICS

The New Definition of Universal Time

S. Aoki[1], B. Guinot[2], G. H. Kaplan[3], H. Kinoshita[1], D. D. McCarthy[3], and P. K. Seidelmann[3]

[1] Tokyo Astronomical Observatory, University of Tokyo, Mitaka, Tokyo 181, Japan
[2] Bureau International de l'Heure, 61, avenue de l'Observatoire, F-75014 Paris, France
[3] U.S. Naval Observatory, Washington, D.C. 20390, USA

Received November 19, 1980; accepted September 15, 1981

Summary. The planned improvement to the astronomical reference system, scheduled for 1 January 1984, will have profound effects on astrometry and timekeeping. In this paper, a new expression for the relationship between universal and sidereal time is developed for use with the new reference system.

Key words: universal time – sidereal time – Earth rotation – FK5

1. Introduction

Universal time, UT1, which forms the basis for the world-wide system of civil time, is indirectly derived from the transit times of stars. UT1 is formally defined by an expression which relates it to mean sidereal time, the latter quantity being directly obtained from the apparent right ascensions of transiting stars. The current formula which relates universal and sidereal time is based on Newcomb's (1895b) expression for the right ascension of the fictitious mean sun.

The planned introduction on 1 January 1984 of improvements to the definition of the astronomical reference system, embodied in the introduction of the IAU (1976) System of Astronomical Constants, the 1980 IAU Theory of Nutation, and the equinox of the FK5 necessitate a reexamination of the definition of UT1. It is important that the new definition be consistent with the improved constants and that it provides a clear relationship between UT1 determined before and after the introduction of the changes. For this reason, IAU Commissions 4, 19, and 31 have recommended (*IAU Information Bulletin* No. 43, p. 10) that the expression for Greenwich mean sidereal time at 0^h UT1 be modified to provide continuity in the value and rate of UT1 as determined from observations, neglecting the small periodic differences due to the change in the theory of nutation. The resolution also included a definition of UT1 based on preliminary values for the equinox correction. In this paper, the problem of the re-definition of universal time is addressed, and a new expression relating universal time and sidereal time is developed for use with the new astronomical reference system. The new expression is based on the final values adopted for the position of the FK5 equinox, and is given to the same precision as that of the new precession expressions.

2. The Relationship Between Universal and Sidereal Time

The currently adopted expression for the relationship between universal time, UT1, and the observed Greenwich mean sidereal time, GMST1, given on p. 75 of the *Explanatory Supplement to the Ephemeris*, is as follows:

$$\text{GMST1 of } 0^h \text{ UT1} = 23925\overset{s}{.}836 + 8640184\overset{s}{.}542\, T_U + 0\overset{s}{.}0929\, T_U^2 , \quad (1)$$

where $T_U = d_U/36525$, and d_U is the number of days of universal time elapsed since JD 2415020.0 UT1, taking on values of ± 0.5, ± 1.5, ± 2.5, ... This expression effectively defines UT1 since UT1 can be considered as the hour angle, reckoned from the prime meridian, of a point on the celestial equator the right ascension of which is

$$R_\odot = \text{GMTS1 of } 0^h \text{ UT1} + 12^h . \quad (2)$$

This point is referred to as the fictitious mean Sun. The coefficients of (1) are taken directly from Newcomb's expression for the right ascension of the fictitious mean Sun (Newcomb 1895a, b).

Conceptually, the right ascension of the fictitious mean Sun was intended to represent the right ascension of a point possessing a strictly uniform sidereal motion. Specifically, the right ascension at any instant was to equal the Sun's mean longitude, so that it would provide the basis for mean solar time. However, Newcomb realized that these two requirements are mutually inconsistent. Mainly because of the difference in the T^2-terms of precession in longitude and right ascension, a point which has uniform motion in right ascension with respect to an inertial frame tends to drift away from a point whose right ascension is equal to the true Sun's mean longitude by $0\overset{s}{.}020$ per century squared (see Newcomb, 1895a, p. 188).

Newcomb defined his fictitious mean Sun to have strict uniform motion in right ascension in the reference frame defined by the precessional constants in use in early 1890's. However, his expression was not adjusted when the precessional quantities were revised in 1897, so that Newcomb's fictitious mean Sun does not possess strictly uniform motion in the reference frame defined by the precessional constants in current use; the error is $0\overset{s}{.}0002$ per century squared. Therefore, Newcomb's expression for the fictitious mean Sun represents neither the mean motion of the true Sun nor the motion of a point with uniformly increasing right ascension.

A new expression relating universal and sidereal time must be developed for use with the new astronomical reference system. It is important that the new expression: 1. be consistent with the origin of right ascension (equinox) of the FK5; 2. maintain the continuity of UT1, as determined from observations, both in value and rate at the epoch of change; and 3. represent a fiducial point with uniform sidereal motion in the new system. Such an expression is developed in the next section. All terms in the new expression differ from those in Eq. (1); in particular, a change of

Send offprint requests to: D. D. McCarthy

$0^s.0002\, T_U^2$ is required in order for the new expression to be consistent with the new precessional formulas. The new expression is independent of the concept of the fictitious mean Sun, reflecting the reality of the independence of the rotation of the Earth from its orbital motion around the Sun. Nevertheless, the difference between the right ascensions of the mean Sun and that of the fiducial point defined by the new expression remains negligibly small for a period of many centuries.

3. The Determination of Universal Time

Universal time is determined observationally from the transit times of stars of known right ascension. A world-wide ensemble of transit telescopes, zenith tubes, and astrolabes is currently used for this purpose; the data from these instruments is collected by the Bureau International de l'Heure (BIH) in Paris, which distributes the resulting information on UT1 and polar motion on a regular basis.

The star positions used are based on the FK4 system. On any given date, a star's computed apparent place, upon which its contribution to the UT1 determination depends, is a function not only of its catalog position and proper motion, but also of the adopted constants of precession, nutation, and aberration. Therefore, a change of reference systems and astronomical constants can have complex and subtle effects resulting in steps of UT1 and of its rate at the date of the change. It was agreed that the "relationship between mean sidereal time and UT1 be modified so that there is no change in either value or rate of UT1". However, in realizing these conditions only the effects of the changes of the star catalog equinox and of the precession quantities are considered.

Let us explain first the reasons for this simplification. The improvement of the internal consistency of the positions and proper motions of the stars, i.e., the more accurate representation of their relative positions and motions, will cause steps in the UT0 obtained using various instruments, depending on the observing programs. These steps cannot be predicted; however, they will be compensated for by corrections to the longitudes of the instruments. As the astronomical longitudes are determined by the condition that each instrument should give the same value of UT1, the new longitude system should be an improvement. The longitude corrections could also compensate for the small step due to the change of the nutation coefficients.

In the following, we will reckon the time, t, in Julian centuries from the date of the introduction of the FK5 and of the new precession constant (1984 January 1, 0^h UT1):

$$t = (\mathrm{JD} - 2445700.5)/36525 .$$

(t should be a uniform time scale, e.g., ET or TDB, but it is sufficient for this application to measure it in Julian centuries of UT1 days.)

The equinox correction, which represents the right ascension of the FK4 equinox in the FK5 system (see Fricke, 1980), is given for the argument t by

$$E = E_0 + \dot{E} t , \tag{3}$$

with

$$E_0 = 0.06390\,\mathrm{s} \quad (E_0 = 0.035\,\mathrm{s} \text{ at } 1950.0)$$
$$\dot{E} = 0.08500\,\mathrm{s}\ \mathrm{century}^{-1} . \tag{4}$$

The equinox correction will be implicitly included in the positions and proper motions of all stars in the FK5.

The new expression relating universal and sidereal time must represent a fiducial point with uniform sidereal motion on the equator of the new system. Let α_0 be the right ascension of such a point. It is easily demonstrated that

$$\alpha_0 = \text{linear function of } t + z_A + \zeta_A - 1/2 \int \theta_A^2 d\zeta_A , \tag{5}$$

z_A, ζ_A, θ_A being the usual precession quantities (Lieske et al., 1977). Then

$$\alpha_0 = A_0 + A_1 t + 0^s.093107\, t^2 - 6^s.2\, 10^{-6}\, t^3 , \tag{6}$$

where A_0 and A_1 are constants. The right ascension of the new fiducial point with uniform sidereal motion must therefore take this form:

$$\alpha_0(\mathrm{new}) = B_0 + B_1 t + 0^s.093107\, t^2 - 6^s.2\, 10^{-6}\, t^3 , \tag{7}$$

where B_0 and B_1 are constants to be determined.

To fulfill the condition of continuity of UT1 in value and rate, the fiducial point should maintain the same position and motion with respect to the stars. Hence, the same correction should be applied to it as to the stars' right ascensions and proper motions in right ascension. If we write:

$$\alpha_0(\mathrm{old}) = B_0' + B_1' t + B_2' t^2 \tag{8}$$

the new coefficients should be:

$$B_0 = B_0' + E_0 ,$$
$$B_1 = B_1' + \dot{E} . \tag{9}$$

Expressing (1) with t as the argument,

$$\mathrm{GMST1\ (old)\ of\ } 0^h\ \mathrm{UT1} = R_\odot(\mathrm{old}) - 12^h = \alpha_0(\mathrm{old})$$
$$= 23962^s.63915 + 8640184^s.698069\, t + 0^s.0929\, t^2 . \tag{10}$$

Then, using the values (4) of E_0 and $\dot{E}$ and Eq. (7),

$$\mathrm{GMST1\ (new)\ of\ } 0^h\ \mathrm{UT1} = \alpha_0\ (\mathrm{new})$$
$$= 23962^s.70305 + 8640184^s.783069\, t + 0^s.093107\, t^2$$
$$- 6^s.2\, 10^{-6}\, t^3 . \tag{11}$$

Now, adopting J2000.0 as the reference epoch, let d_U' represent the number of days of universal time elapsed since JD 2451545.0 UT1 (2000 January 1, 12^h UT1), taking on values of ± 0.5, ± 1.5, ± 2.5, ..., and

$$T_U' = d_U'/36525 \tag{12}$$

then the new expression for Greenwich mean sidereal time is

$$\mathrm{GMST1\ of\ } 0^h\ \mathrm{UT1} = 24110^s.54841 + 8640184^s.812866\, T_U'$$
$$+ 0^s.093104\, T_U'^2 - 6^s.2\, 10^{-6}\, T_U'^3 . \tag{13}$$

This expression represents the new definition of universal time, to be used with the FK5-based astronomical reference system.

4. Derived Quantities

Equation (13) relates universal time, UT1, to the observed Greenwich mean sidereal time, GMST1. In this section we derive some other useful expressions and quantities consistent with (13), which is assumed to be the defining relation.

Equation (13) may be re-expressed in terms of a "uniform" time scale such as Ephemeris Time (ET) or Barycentric Dynamical Time (TDB), the latter having been recently defined by the IAU. This yields:

$$S(T') = 67310^s.54841 + (876600^h + 8640184^s.812866) T'$$
$$+ 0^s.093104\, T'^2 - 6^s.2\, 10^{-6}\, T'^3 , \tag{14}$$

where T' is the number of Julian centuries (consisting of 36,525 d of 86,400 s of dynamical time each) elapsed since JD 2451545.0 TDB. In this form, the expression provides the relationship between Barycentric Dynamical Time, T', and what may be called Dynamical Sidereal Time, S. It predicts the rotational phase angle of the Earth in a precessing frame of reference (defined by the new precessional quantities) in a uniform time scale, assuming that the rotation rate of the Earth is constant. Since the Earth's rotation is not constant, the same expression, in the form of Eq. (13), is used to define the relationship between UT1 and the observed sidereal time, GMST1.

The rate at which Dynamical Sidereal Time advances in one day of Barycentric Dynamical Time is given by

$$s=\frac{d}{dT'}S(T')/36525\,. \tag{15}$$

This is numerically equal to the rate at which observed sidereal time advances in one day of UT1:

$$s=\frac{d}{dT'_U}(\text{GMST1 of } 0^{\mathrm{h}}\,\text{UT1})/36525+86400^{\mathrm{s}}\,. \tag{16}$$

From (15) and (14) we obtain

$$s=86636^{\mathrm{s}}.55536790872+5^{\mathrm{s}}.098097\,10^{-6}\,T' -5^{\mathrm{s}}.09\,10^{-10}\,T'^2\,. \tag{17}$$

The quantity s represents the number of sidereal seconds in one solar day; therefore we immediately obtain the ratio of solar to sidereal time. Specifically, given two events, e_1 and e_2,

$$r'=\frac{\text{measure of solar time between } e_1 \text{ and } e_2}{\text{measure of sidereal time between } e_1 \text{ and } e_2} =0.997269566329084-5.8684\,10^{-11}\,T'+5.9\,10^{-15}\,T'^2 \tag{18}$$

and,

$$\frac{1}{r'}=\frac{\text{measure of sidereal time between } e_1 \text{ and } e_2}{\text{measure of solar time between } e_1 \text{ and } e_2} =1.002737909350795+5.9006\,10^{-11}\,T'-5.9\,10^{-15}\,T'^2\,, \tag{19}$$

where "solar time" refers to either Barycentric Dynamical Time or UT1, and "sidereal time" refers to either Dynamical Sidereal Time or observed sidereal time, respectively.

The quantity r' can be thought of as the length of one sidereal day in units of solar days. The length of one sidereal day is defined by two successive transits of the mean equinox; while the Earth is rotating eastward, the mean equinox is moving westward due to precession. Therefore, one sidereal day is shorter than the Earth's rotational period by about 0.008 s, the amount of precession in right ascension in one day. To obtain the Earth's rotational period, we must first subtract the accumulated precession in right ascension, $M(T')$, from $S(T')$ given by Eq. (14):

$$S(T')-M(T')=67310^{\mathrm{s}}.54841+(876600^{\mathrm{h}}+8639877^{\mathrm{s}}.317119)T'\,, \tag{20}$$

where

$$M(T')=\int_0^{T'} m(T)dT$$

and

$$m(T)=307^{\mathrm{s}}.495747+0^{\mathrm{s}}.186208\,T-1^{\mathrm{s}}.85\,10^{-5}\,T^2\,. \tag{22}$$

In the above, $m(T)$ is the rate of precession in right ascension (in sidereal seconds per Julian century) from Lieske et al. (1977). The expression (20) is the rotation measure with respect to an inertial frame, from which we obtain the rotational angular velocity of the Earth, ω:

$$\omega=\frac{d}{dT'}(S(T')-M(T')) =(36625-0.00141994075)\ \text{rev century}^{-1} =15''.04106717866910\,\mathrm{s}^{-1}\,. \tag{23}$$

If we define p, the rotation period, by $p=2\pi/\omega$, then we have, using $2\pi=1296000''$,

$$p=86164.09890369732\,\mathrm{s}\,.$$

Thus, the rotational period of the Earth, p, implied by (13) is 86164.09890369732 seconds of UT1. In comparison, the value of p implied by Eq. (1) is 86164.0989041 seconds of UT1, a difference of only a few parts in 10^{12}. Either value of p is consistent with the value of the Earth's angular velocity of rotation adopted by the International Association of Geodesy in 1979: $\omega=7.292115\,10^{-5}$ radian s^{-1} (Moritz, 1980). It must be emphasized that these values of p are derived, not fundamental, constants. The observed value of p, measured in uniform time, is subject to irregular variations at the millisecond level. Furthermore, the decrease in the angular velocity of the Earth due to tidal friction results in a secular increase in the true value of p of 1–5 ms per century. The relationship between the value of p given above, derived from Eq. (13), and the true average rotational period of the Earth at the present epoch is not known.

5. Conclusion

The changes in the system of astronomical constants and the introduction of the FK5 stellar reference system, scheduled for 1 January 1984, will have profound effects on astrometry, dynamical astronomy, timekeeping, and geodesy. The IAU has recommended that the value and rate of the UT1 time scale determined from stellar observations, which forms the basis for civil time, be continuous across this date.

In this paper, a development of the expression defining the relationship between universal and sidereal time has been presented. A new expression for this relationship, Eq. (13), has been derived for use with the new system, which maintains the continuity of the current UT1 time scale. This expression shall be communicated to the IAU for adoption in 1982.

References

Fricke, W.: 1980, Definition and Practical Realization of the Reference Frame in the FK5 – The Role of Planetary Dynamics and Stellar Kinematics in the Definition, in *Reference Coordinate Systems for Earth Dynamics*, eds. E. M. Gaposchkin and B. Kolaczek, D. Reidel, Dordrecht, pp. 331–340

IAU Information Bulletin No. 43, 1980

Lieske, J.H., Lederle, T., Fricke, W., Morando, B.: 1977, *Astron. Astrophys.* **58**, 1

Moritz, H.: 1980, *Bull. Geodesique* **54**, 395

Newcomb, S.: 1895a, The Elements of the Four Inner Planets and the Fundamental Constants of Astronomy, Supplement to the American Ephemeris and Nautical Almanac for 1897

Newcomb, S.: 1895b, *Astron. Papers Prepared for the Use of the American Ephemeris and Nautical Almanac*, VI, Part I

Explanatory Supplement to the Astronomical Ephemeris and the American Ephemeris and Nautical Almanac, 1961, Her Majesty's Stationery Office, London