

15-110 Principles of Computing – F19

LECTURE 18:

FUNCTIONS 2, MODULES

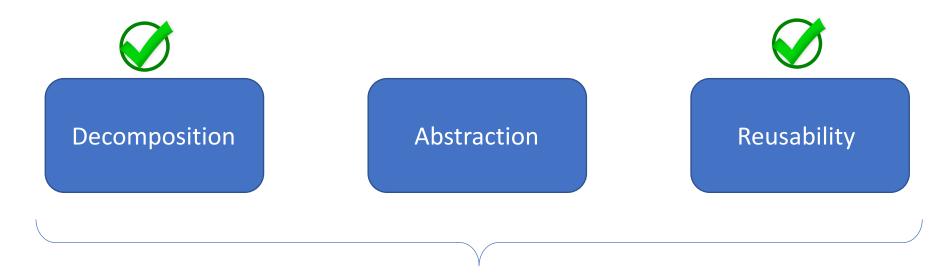
TEACHER:

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Functions: organizing the code, putting aside functionalities

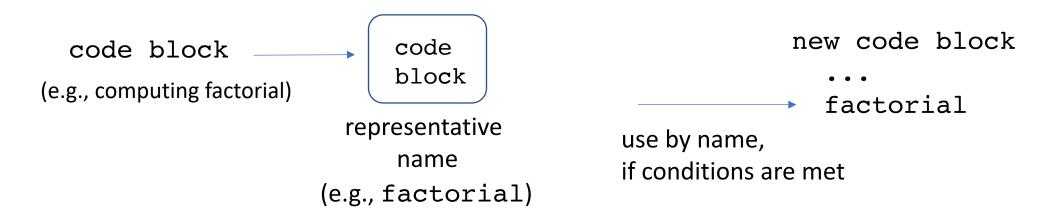
- Functions are a fundamental way to organize the code into procedural elements that can be reused
- Functions provide structure and organization, that facilitate:



Fundamental ingredients in the design of computational solutions

Abstraction

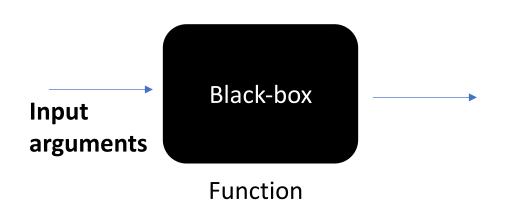
- Abstraction in computing aims to hide details that are not necessary in a given context, preserving only the information that is relevant in the context:
 - Operationally, abstraction is the process that:
 - 1. allows to take/write a *piece of code* (including anything relevant)
 - 2. name it
 - 3. use it as it were a *black-box* as long as the *conditions* for using the code are met



Abstraction types include functions, classes (abstract data types), ...

Abstraction using functions

When we define a function we are performing an abstraction: we take a piece of code, including objects and expressions, we name it, and in principle we can we use it, without caring about how the outputs are precisely obtained in the function body



Outputs

- o returned values
- modifications that propagates outside the function (changes to input parameters that are mutable data types)
- side effects on the program environment

Abstraction using functions

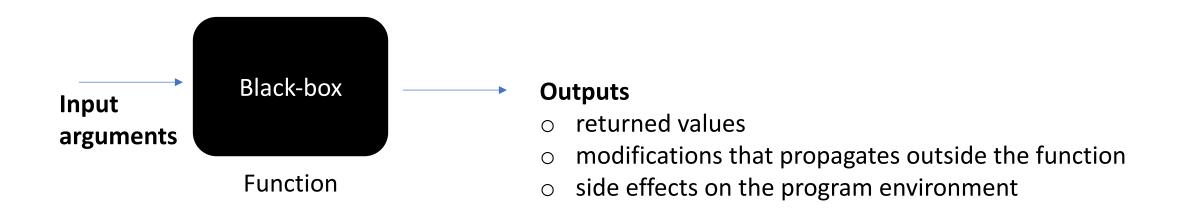
```
def make_product(values):
    prod = 1
    for v in values:
        prod *= v
    return prod
```

```
my_data = [2,4,5]
p = make_product(my_data)
s = sum(my_data)
print('Product:', p, 'Sum:', s)
```

```
def sort_and_get_median(1):
    l.sort()
    return l[len(1)//2]
```

```
my_data = [2, 4, 5, -1, 3]
median = sort_and_get_median(my_data)
min_val = my_data[0]
max_val = my_data[len(my_data)-1]
print('Min:', min_val, 'Max:', max_val,
'Median:', median)
```

Abstraction using functions



- The only information <u>relevant to use a function</u>:
 - input parameters: types and admitted values
 - returned objects: types, range of values, adopted conventions
 - description of what the function does, including possible side effects

The precise details about how processing is performed are hidden by the abstraction

Example of two functions doing the same task but in different way

Find the (one) value of x that satisfies the equation f(x) = 0, for $x \in [a, b]$

```
def find root(f, a, b, N):
   if f(a) * f(b) >= 0:
       print("Secant method fails.")
       return None
   a n = a
   b n = b
   for n in range(1,N+1):
       m n = a n - f(a n)*(b n - a n)/(f(b n) - f(a n))
       f m n = f(m n)
       if f(a n)*f m n < 0:
           a n = a n
           b n = m n
       elif f(b n)*f m n < 0:
           a n = m n
           b n = b n
        elif f m n == 0:
            print("Found exact solution.")
           return m n
        else:
            print("Secant method fails.")
           return None
   root = a n - f(a n)*(b n - a n)/(f(b n) - f(a n))
   return root
```

```
def find root(f, a, b, N):
    if f(a) * f(b) >= 0:
       print("Bisection method fails.")
      return None
    a n = a
   b n = b
    for n in range(1,N+1):
       m n = (a n + b n)/2
       f m n = f(m n)
        if f(a n)*f m n < 0:
           a n = a n
           b n = m n
        elif f(b n)*f m n < 0:
           a n = m n
           b n = b n
        elif f m n == 0:
            print("Found exact solution.")
            return m n
        else:
            print("Bisection method fails.")
            return None
    root = (a n + b n)/2
    return root
```

Houston we have a problem!

- The (basic) abstraction through a <u>function code</u> only provides (exposes) in an *explicit way*:
 - input parameters: names
 - returned objects: names / expressions
 - description of what the function does: name of the function

def quadratic_roots(xx, x, c):

$$x1 = -x / (2 * xx)$$

 $x2 = math.sqrt(x**2 - (4 * xx * c)) / (2 * xx)$
 $x2 = math.sqrt(x**2 - (4 * xx * c)) / (2 * xx)$
 $x2 = math.sqrt(x**2 - (4 * xx * c)) / (2 * xx)$
 $x2 = math.sqrt(x**2 - (4 * xx * c)) / (2 * xx)$

- ➤ I want to use the function (possibly developed by somebody else)
- ➤ I want to be a **client** for the numerical services offered by the function

- O What is the precise meaning of those input parameters?
- Can I set them to any numeric value?
- O What about the outputs?
- Will I always get an admissible numeric result?

Specifications

Parameters and names of the abstraction aren't enough!

We also need to **describe the abstraction**, the function and its elements in our case, in order to properly <u>use it and reuse it!</u>

- Specification: defines a contract between the provider/implementer of an abstraction (a function) and those who will be using the abstraction (the function), the users
- User

 Client for the services provided by the abstraction (the function)



Specifications: example

Assumptions: what the client has to do to use the function

Guarantees: what the provider guarantees about outputs and effects of the function

Numeric examples: show function usage and provide basic test cases

def find root(f, a, b, N): 'Approximate solution of f(x)=0 on interval [a,b] by the bisection method. Parameters f : function The function for which we are trying to approximate a solution f(x)=0. a,b: numbers The interval in which to search for a solution. The function returns None if f(a)*f(b) >= 0 since a solution is not guaranteed. N: (positive) integer The number of iterations to implement. Returns x N : number The midpoint of the Nth interval computed by the bisection method. The initial interval $[a\ 0,b\ 0]$ is given by [a,b]. If $f(m\ n) == 0$ for some midpoint m n = (a n + b n)/2, then the function returns this solution. If all signs of values f(a n), f(b n) and f(m n) are the same at any iteration, the bisection method fails and return None. Examples

>>> f = lambda x: x**2 - x - 1 >>> bisection(f,1,2,25)

1.618033990263939 >>> f = lambda x: (2*x - 1)*(x - 3)

>>> bisection(f,0,1,10)

0.5

... program code continue in the next slide

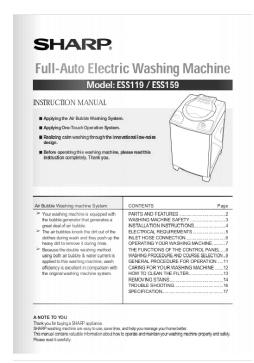
Specifications: example

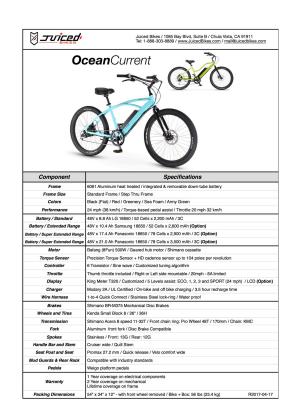
```
if f(a)*f(b) >= 0:
       print("Secant method fails.")
       return None
    a n = a
    b n = b
    for n in range(1,N+1):
       m n = a n - f(a n)*(b n - a n)/(f(b n) - f(a n))
       f m n = f(m n)
       if f(a_n)*f_m < 0:
            a n = a n
           b n = m n
        elif f(b_n)*f_m n < 0:
            a n = m n
           b n = b n
        elif f m n == 0:
            print("Found exact solution.")
            return m n
        else:
            print("Secant method fails.")
           return None
    root = a n - f(a n)*(b n - a n)/(f(b n) - f(a n))
    return root
```

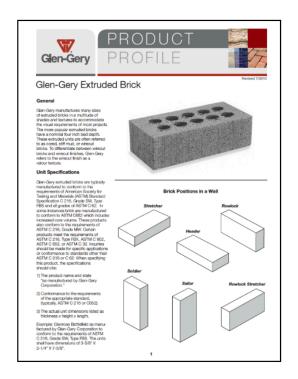
Specifications: for every product we trade or share











The specification contract

- Specification: defines a contract between the <u>provider/implementer</u> of an abstraction (e.g., function) and those who will be using the abstraction (e.g., the function), the <u>users / clients</u>
- The specification contract has <u>two mandatory parts</u>:
 - ✓ **Assumptions:** describe the <u>conditions</u> that must be met by the <u>users</u> of the function
 - What are the admissible types and values / ranges of the input parameters
 - What other functions must have been executed first
 - What is supposed to be already in place before calling the function
 - 0 ...
 - ✓ **Guarantees:** describe the <u>conditions</u> that must be met by the <u>function</u> (given that the invoking conditions have been satisfied), what the providers ensures about the function
 - What are returning types and/or range of values
 - What are the side effects (e.g., on the input parameters) caused by the function
 - Under which conditions the function fails or may fail

O ...

The moral is ...

Functions (methods, classes) need to be <u>documented</u>, accompanied by a <u>specification for the client</u>



Specifications fully support abstraction and facilitate / make it possible code reusability, maintenance, and sharing

docstrings for providing specifications

- docstring: documentation strings (or docstrings) provide a structured way for providing specifications within functions, modules, classes, and methods
- A docstring is defined by including a string constant as the first statement in the object's definition

```
def add(x, y):
    "Sum two objects x and y and returns the sum."
    return x + y
def add(x, y):
    'Sum two objects and returns the sum.'
    return x + y
                                                              ''' triple (single) quote
def add(x, y):
                                                              delimiters are the first choice for
    '''Sum two objects and returns the sum.'''
                                                              docstrings, since they allow to
    return x + y
                                                              write multiline descriptions
def add(x, y):
    '''Sum two objects and returns the sum based on their data types.
       If the + operator cannot be used for the two objects, an error is thrown.
    return x + y
```

docstrings for providing specifications: help(f)function

help(f) function: returns the docstring information about the function

```
def add(x, y):
    '''Sum two objects and returns the sum based on their data types.
    If the + operator cannot be used for the two objects, an error is thrown.'''
    return x + y

help(add)
Help on function add in module __main__:
add(x, y)
    Sum two objects and returns the sum based on their data types.
    If the + operator cannot be used for the two objects, an error is thrown.
```

docstrings for providing specifications: help(f)function

Only the string constant following the definition is returned by help()!

```
def add2(x, y):
   1 1 1
   Sum two numeric data types.
   Input: x and y must be numeric data types and/or boolean. Type can be mixed.
   Output: The sum of x and y is returned as an object whose data type is
           defined by the data type of the two inputs based on python's rules.
           If x and y aren't both numeric or boolean, None is returned.
   1 1 1
   if (type(x) != int and type(x) != float and type(x) != bool) or \
       (type(y) != int and type(y) != float and type(y) != bool):
       return None
   '''The provided inputs are fine, the + operator can be applied'''
   return x + y
```

docstrings for providing specifications: help(f)function

```
help(add2)
Help on function add2 in module __main__:
add2(x, y)
Sum two numeric data types.
   Input: x and y must be numeric data types and/or boolean. Type can be mixed.
   Output: The sum of x and y is returned as an object whose data type is
        defined by the data type of the two inputs based on python's rules.
        If x and y aren't both numeric or boolean, None is returned.
```

The comment strings other than the first one are not displayed, being not part of the specification

Of course comments to well document the code <u>inline</u> are most welcome / necessary!

Inline comments: explaining the how part of the code

- Inline comments describe how things are done, at a certain step of the program code
- Many different ways to document the code inline
 - Use of the hashtag # symbol for a <u>line comment</u>
 - Use of the triple quotes for single or multiline string comments
 - In general, use of string constants for line comments

Inline comments: explaining the how part of the code

```
# let's start by defining an empty list
x = []
# fill the list with numbers from the user
n = int(input("How many numbers? "))
''' We should check if n is a number or not!
    As it is the program could crash if a wrong input is given'''
# get the user numbers, checking that they are actual numbers
while len(x) < n:
    nn = input("Next number: ")
    'check if it is a number or not'
    if nn.isnumeric():
        x.append(int(nn))
# compute the arithmetic average
s = sum(x) # the sum first
avg = s / len(x) # then the division by the number of elements
# print out the result with some message
print("The average of the given", len(x), "numbers is:", avg)
```

Storing code in *modules* for flexible reusing and sharing

```
def get average(1):
    if type(l) != list:
        return None
    if len(1) == 0:
        return 0
    avq = 0
    count = 0
    for i in range(len(l)):
        if (type(l[i]) != int and\
            type(l[i]) != float):
            continue
        else:
            avg += l[i]
            count += 1
    if count > 0:
        return avg / count
    else:
        return None
```

```
def get median(1):
    if type(1) != list:
        return None
    if len(1) == 0:
        return 0
    for i in range(len(l)):
        if (type(l[i]) != int and type(l[i]) != float):
            return None
    l sorted = sorted(1)
    median = l sorted[ len(l sorted)//2 ]
    return median
```

- Let's put them together and store in a module
- → file named my_stats.py

import (the entire module) and dot notation

■ How do I use the functions in the module in another program? → import statement

```
import module name
```

- ✓ import my_stats
- > How do I invoke the functions inside my stats?
- > **Dot notation!** ... the functions are *methods of the modules*

```
import my_stats

x = [2,7,5]
avg = my_stats.get_average(x)
mdn = my_stats.get_median(x)
print("Average:", avg, "Median:", mdn)
```

import module_name imports
(loads) in the current program all the
functions of the module

import only specific functions (no need dot notation)

 From large modules made by third parties, it might be heavy, time and memory taking, to load all functions, such as if only one or more functions are needed, they can be directly selected for import

```
from module_name import function_name

from module_name import function_1, function_2, function_3

from my_stats import get_average

x = [2,7,5]
avg = get_average(x)
print("Average:", avg)
```

- We don't need in this case the dot notation for invoking the function!
- > The functions get actually *copied* into the current program module

import only specific functions (no need dot notation)

```
from my_stats import get_average

x = [2,7,5]
avg = get_average(x)
mdn = my_stats.get_median(x)
print("Average:", avg, "Median:", mdn)
```

Error!! my stats isn't defined

Use aliases to refer to the module

- When in the code we need to refer many times to functions from a module, and maybe the name of the module is even long, it can be bothering / heavy to read to have such long names everywhere
- Juse an <u>alias for the module name</u>

```
import module name as alias name
import my stats as st
                                                Vs.
x = [2, 7, 5]
avg = st.get average(x)
mdn = st.get median(x)
                                         import my stats
print("Average:", avg, "Median:", mdn)
                                         x = [2, 7, 5]
                                         avg = my stats.get average(x)
                                         mdn = my stats.get median(x)
                                         print("Average:", avg, "Median:", mdn)
```

Importing all functions and getting rid of dots

- We can extend the from notation to import (i.e., copy) all functions from a module
- Since we make a copy into the current module, we don't need the dot notation anymore

```
from module name import *
```

```
from my_stats import *

x = [2,7,5]
avg = get_average(x)
mdn = get_median(x)
print("Average:", avg, "Median:", mdn)
```

Not the best approach since this *copy* might import variables, functions, etc. that might clash with variables and function names that we already have in the calling program

Importing modules from third parties: math

- Python offers a large number of specialized modules: you name it, it exists!
- Popular / useful modules (libraries):

Returns e**x - 1

math: mathematical functions https://docs.python.org/3/library/math.html

Function	Description	log(x[, base])	Returns the logarithm of x to the base (defaults to e)		
ceil(x)	Returns the smallest integer greater than or equal to x.	log1p(x)	Returns the natural logarithm of 1+x		
copysign(x, y)	Returns x with the sign of y	log2(x)	Returns the base-2 logarithm of x		
fabs(x)	Returns the absolute value of x	log10(x)	Returns the base-10 logarithm of x	cosh(x)	Returns the hyperbolic cosine of x
factorial(x)	Returns the factorial of x	pow(x, y)	Returns x raised to the power y	sinh(x)	Returns the hyperbolic cosine of x
		sqrt(x)	Returns the square root of x	tanh(x)	Returns the hyperbolic tangent of x
floor(x)	Returns the largest integer less than or equal to x	acos(x)	Returns the arc cosine of x	erf(x)	Returns the error function at x
fmod(x, y)	Returns the remainder when x is divided by y	asin(x)	Returns the arc sine of x	erfc(x)	Returns the complementary error function at x
frexp(x)	Returns the mantissa and exponent of x as the pair (m, e)	atan(x)	Returns the arc tangent of x	gamma(x)	Returns the Gamma function at x
fsum(iterable)	Returns an accurate floating point sum of values in the iterable	atan2(y, x)	Returns atan(y / x)	lgamma(x)	Returns the natural logarithm of the absolute value of the Gamma function at \boldsymbol{x}
isfinite(x)	Returns True if x is neither an infinity nor a NaN (Not a Number)	cos(x)	Returns the cosine of x	pi	Mathematical constant, the ratio of circumference of a circle to it's
isinf(x)	Returns True if x is a positive or negative infinity	hypot(x, y)	Returns the Euclidean norm, sqrt(x*x + y*y)		iameter (3.14159)
isnan(x)	Returns True if x is a NaN	sin(x)	Returns the sine of x		mathematical constant e (2.71828)
ldexp(x, i)	Returns x * (2**i)	tan(x)	Returns the tangent of x Converts angle x from radians to degrees		
	` <i>'</i>	degrees(x)			
modf(x)	Returns the fractional and integer parts of x	radians(x)	Converts angle x from degrees to radians		
trunc(x)	Returns the truncated integer value of x	acosh(x)	Returns the inverse hyperbolic cosine of x		
exp(x)	Returns e**x	asinh(x)	Returns the inverse hyperbolic sine of x		
1 ()	Determine white 1				27

Returns the inverse hyperbolic tangent of x

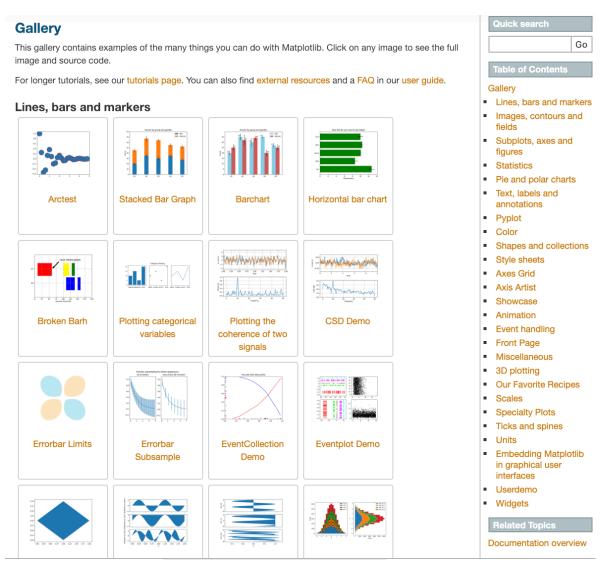
Importing modules from third parties: numpy

numpy: numerical/scientific computing http://www.numpy.org/

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Importing modules from third parties: matplotlib

matplotlib: plotting library https://matplotlib.org/



Importing modules from third parties: random, CSV

- random: generating random numbers
- CSV: handle files in CSV format