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What can we do with ellipticity gradients and isophote twists?

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ABSTRACT

Some informal notes exploring what we can do with ellipticity gradients and isophote twists in galaxies, the latter of which seem to result in different levels of intrinsic alignments depending on what radii are used for shape measurement. Can we exploit this to learn something?

1. Introduction

Sukhdeep's latest paper on intrinsic alignments showed a different IA amplitude for different shape measurements, with shapes measured using outer isophotes showing a higher level of alignment with the large-scale density field. The outer isophotes also seem to be rounder on average. These two phenomena indicate that early-type galaxies that dominate the IA signal have both ellipticity gradients and isophotal twisting.

The questions I would like to ask are whether we can use this to do some kind of IA self-calibration, or at least to make a better intrinsic alignments model on small scales.

2. Formalism

Let's assume we have per-galaxy shear estimates, $\hat{g}$, such that their ensemble average is

$$\langle \hat{g} \rangle = g + \gamma_{\mathrm{IA}}, \tag{1}$$

where g is the lensing (reduced) shear and γ_{IA} is the intrinsic alignment shear. And we assume we also have another set of per-galaxy shear estimates, $\hat{g}'$, measured using a different radial weight function, with ensemble average

$$\langle \hat{g}' \rangle = g + a\gamma_{\mathrm{IA}} \tag{2}$$

where a is some value that has to do with the relative contributions of large and small-scale information in the light profile when measuring $\hat{g}$ and $\hat{g}'$.

Let's assume that in tomographic bins indexed with i and j, we can define sets of shear cross-correlations[1], where the notation $\langle \hat{g}\hat{g}' \rangle$ means that we always correlate a lower-redshift $\hat{g}$ with a higher-redshift $\hat{g}'$. We don't want to assume that a is the same for each pair of bins, since it depends not only on the radial weight but also the galaxy population, so we'll also give it a subscript i or j. In that case, the cross-correlations are

$$\langle \hat{g}\hat{g} \rangle_{ij}(\theta) = \langle gg \rangle_{ij}(\theta) + \langle \gamma_{\mathrm{IA}} g \rangle_{ij}(\theta) + \langle \gamma_{\mathrm{IA}}\gamma_{\mathrm{IA}} \rangle_{ij}(\theta) \quad (3)$$

$$\langle \hat{g}\hat{g}' \rangle_{ij}(\theta) = \langle gg \rangle_{ij}(\theta) + \langle \gamma_{\mathrm{IA}} g \rangle_{ij}(\theta) + a_j \langle \gamma_{\mathrm{IA}}\gamma_{\mathrm{IA}} \rangle_{ij}(\theta) \quad (4)$$

$$\langle \hat{g}'\hat{g} \rangle_{ij}(\theta) = \langle gg \rangle_{ij}(\theta) + a_i \langle \gamma_{\mathrm{IA}} g \rangle_{ij}(\theta) + a_i \langle \gamma_{\mathrm{IA}}\gamma_{\mathrm{IA}} \rangle_{ij}(\theta) \quad (5)$$

$$\langle \hat{g}'\hat{g}' \rangle_{ij}(\theta) = \langle gg \rangle_{ij}(\theta) + a_i \langle \gamma_{\mathrm{IA}} g \rangle_{ij}(\theta) + a_i a_j \langle \gamma_{\mathrm{IA}}\gamma_{\mathrm{IA}} \rangle_{ij}(\theta). \quad (6)$$

We measure four functions of radius for each pair of bins ij, and want to constrain three functions of radius ($\langle gg \rangle_{ij}$, $\langle \gamma_{\mathrm{IA}} g \rangle_{ij}$, $\langle \gamma_{\mathrm{IA}}\gamma_{\mathrm{IA}} \rangle_{ij}$) plus two numbers a_i and a_j which don't depend on θ. There's some low-hanging fruit here, like the difference between 1st and 2nd lines is

$$\langle \hat{g}\hat{g}' \rangle_{ij}(\theta) - \langle \hat{g}\hat{g} \rangle_{ij}(\theta) = (a_j - 1)\langle \gamma_{\mathrm{IA}}\gamma_{\mathrm{IA}} \rangle_{ij}(\theta), \quad (7)$$

the difference between the 3rd and 4th is

$$\langle \hat{g}'\hat{g}' \rangle_{ij}(\theta) - \langle \hat{g}'\hat{g} \rangle_{ij}(\theta) = a_i(a_j - 1)\langle \gamma_{\mathrm{IA}}\gamma_{\mathrm{IA}} \rangle_{ij}(\theta). \quad (8)$$

So we could first get $(a_j - 1)\langle \gamma_{\mathrm{IA}}\gamma_{\mathrm{IA}} \rangle_{ij}(\theta)$, then compare with $a_i(a_j - 1)\langle \gamma_{\mathrm{IA}}\gamma_{\mathrm{IA}} \rangle_{ij}(\theta)$ to estimate a_i. Once we have that, we know the a's for each bin, and therefore we know the a_j's in all the correlations that we've calculated. We can start plugging in for other things and try to get non-parametric estimates of scale-dependent IA models, which is a nice test of our other means of making small-scale IA models. I think it would take some thinking to decide how far we want to go with non-parametric estimation vs. estimating the parameters of known models. Once we assume parametric models, many aspects of the relationship between these terms becomes fixed by the model, and presumably we have lots of constraining power. Even if we can't sort out all the amplitudes, having a non-parametric IA reconstruction, to test the validity of whatever halo model / Blazek et al (2015)-style approach / other parametric model we are using, sounds really useful.

There are also other correlations we could use. For example, using density tracers δ_g (assuming we have perfect density tracers with no magnification contribution; this is wrong and needs to be properly treated) we can make correlations like

$$\langle \delta_g \hat{g} \rangle_{ij}(\theta) = \langle \delta_g g \rangle_{ij}(\theta) + \langle \delta_g \gamma_{\mathrm{IA}} \rangle_{ij}(\theta) \quad (9)$$

$$\langle \delta_g \hat{g}' \rangle_{ij}(\theta) = \langle \delta_g g \rangle_{ij}(\theta) + a_j \langle \delta_g \gamma_{\mathrm{IA}} \rangle_{ij}(\theta). \quad (10)$$

[1] I'm writing these as correlation functions, but they could be power spectra or whatever you want as your favorite two-point function.

In the case where $i = j$, the IA contribution should be substantial, while for $i \neq j$ it will be a small correction. In the Blazek et al (2012) and Chisari et al (2014) papers, the essence of the algorithm was to measure these for one shear algorithm, for the cases of $i = j$ and for $i \neq j$, using the latter to predict the gg lensing "contamination" of the former (modulo uncertainties in the dN/dz), and thereby estimate the IA as a function of separation. Here, with our extra information, if we take the difference between the two of these measurements for a given i and j, we get

$$\langle \delta_g \hat{g}' \rangle_{ij}(\theta) - \langle \delta_g \hat{g} \rangle_{ij}(\theta) = (a_j - 1) \langle \delta_g \gamma_{\rm IA} \rangle_{ij}(\theta). \tag{11}$$

If we have a parametric IA model, we could constrain its parameters with this. Or, again, we could use this to non-parametrically determine the scale-dependence of IA, and test other means of making these models. Combined with the above approach to learn the a's, we could even get a non-parametric reconstruction of the amplitude of $\langle \delta_g \gamma_{\rm IA} \rangle_{ij}(\theta)$.

3. Alternate formulation

Another way to think about this, which doesn't give different information from the previous section but might be conceptually cleaner, is as follows:

If we define $\hat{\Delta}_g \equiv \hat{g}' - \hat{g}$, we could think of this as a quantity in which we have "nulled" any lensing shear. We've also nulled out some but not all of the shape noise. $\hat{\Delta}_g$ should only include intrinsic alignments (plus measurement noise of course, but not lensing shear). So, for example, in the context of the Blazek et al (2012) and Chisari et al (2014) indirect IA measurement methods, we have to choose a "source" sample and an "associated" sample, and use the "source" sample to remove the gg lensing contribution in the shear of the "associated" sample. But a new way to do this would be to correlate lens position with $\hat{\Delta}_g$ of an "associated" sample, and we've already removed the gg lensing contribution automatically (without even having to know the correct redshift distribution of the "source" and "associated" samples, which is one challenge of the Blazek et al and Chisari et al methods).

In the context of what I wrote in the previous section, we can imagine correlating the $\hat{\Delta}_g$ values in different tomographic slices (or $\hat{\Delta}_g$ in one slice with $\hat{g}$ or $\hat{g}'$ from the other slice), to get statistics that tell us about the 2-point stats of the intrinsic alignment shears, modulo the amplitude parameter. The formalism is the same and the equations are rearranged slightly, but it's the same math, just expressed more cleanly. The downside is that I prefer only to talk about ensemble shear estimates (previous section), not differences between per-object quantities (this section).

4. What do we learn

I think it depends on how much we want to try to reconstruct without a model, vs. constraining model parameters. We clearly learn something though!

5. Validity of assumptions

I started with assumptions about the type of shear estimators that you could define with different radial weights. We know from Sukhdeep's paper that on data with high-S/N and resolution, it's possible to achieve tens of % difference in IA amplitudes. However, on lower S/N and resolution data, it's unclear how much contrast we'll achieve.

Also, I've assumed that there's no scale-dependence in the a's, i.e., that all the scale-dependence goes into the two-point functions only:

$$\langle \hat{g}\hat{g}' \rangle_{ij}(\theta) = \langle gg \rangle_{ij}(\theta) + \langle \gamma_{\rm IA} g \rangle_{ij}(\theta) + a_j \langle \gamma_{\rm IA}\gamma_{\rm IA} \rangle_{ij}(\theta). \tag{12}$$

I wonder if it's possible to break this assumption if you have a shape estimate $\hat{g}'$ that behaves differently for early and late type galaxies, which live in different environments and could therefore have a different small-scale density-shape correlation.

For red galaxies, we could at least test this by looking at whether the LOWZ density-shape correlations for isophotal and regaussianization shapes have the same scale-dependence on all scales (not just the large scales where we did NLA model fits). Visually they look very similar, but we should check quantitatively.