Problem Chosen	2021 MCM/ICM Summary Sheet	Team Control Number
A		2120943

Analysis and Prediction of Fungi Decomposition Process

As an indespensable part of our terrestrial ecosystem, fungi free the carbon and other elements out from remains and debris and drive them into the cycle of ecosystem. Fungi tend to live in warm and humid environment,and are sensitive to the smallest changes.In this study, we focus on the interaction between different population of fungi and how they interact with microenvironment around them on woddy fibres.

In our first model, we use cmpetitive Lotka-Voterra model to demostrate the competition among different types of fungi. After the estimation of some important parameters, we simulate the gorwth of several population of fungi fed on infinite nutrition with no competition. The result shows that.

In our second model, we mainly sudy the comprtition behavior of the fungi when woody fibres are not infinite. We add the influence of model to demonstrate the competition the competition among different types of fungi.

Keywords:

Contents

1 Introduction

1.1 Problem Background

As an indespensable part of our terrestrial ecosystem, fungi free the carbon and other elements out from remains and debris and drive them into the ecosystem. Fungi tend to live in warm and humid environment,and are sensitive with the smallest changes.

In this study, we focus on the interaction between different population of fungi and how they interact with microenvironment around them on woddy fibres. We use cmpetitive Lotka-Voterra model to demostrate the competition among different types of fungi.

1.2 Restatement of the Problem

Restatement

1.3 Our Work

Our work mainly includes the following:

1. We use Lotka-Volterra Equations to build a model to demonstrate the competition among different types of fungi.

2. We build a model to explain how the process of decomposition is affected by temperature, moisture, and distribution of different tuypes of fungi.

3. We analyse the sensitivity and robustness of the models above.

2 Model Preperation

2.1 Assumptions

1. Fungi strains grow in a stable density;

2. Fungi grow in a 2-D plane;

3. The rate of decomposition of woody fibres is propotional to the number of fungi on it.

2.2 Notations

Important notations used in this paper are listed in Table 1

2.3 Data Collection

The data we use mainly include several kinds of fungi's growth rate with temperature and moisture. The data sources are summarized in Table 2.

Table 1: Notations

Symbol	Definition	Unit
T	Temperature in Celcius	$^\circ C$
M	Moisture	MPa
n_i	Fungi's Competitive Rank	n is scaled to $[0,1]$
ρ_{hyphae}	Hyphae Density	$\mu g/cm^2$
S_i	Popluation Size	Population of Fungi
$v_{\text{decomposition}}$	Decomposition Rate	% Dry weight loss in 122 days
K_i	Bioligical Capacity	μg
M_{woody}	Weight of the Woody	g

Table 2: Data Sources

Data	Source
Different fungi trait	github.com/dsmaynard/
Different fungi growth rate	github.com/dsmaynard/

3 Model 1:Fungi Population Prediction Model

3.1 Competitive Lotca-Voterra Equations

We assume that there is only one kind of fungi, Phlebia acerina DR60 A8A,and using data as follows:

$$T = 22^\circ C, M = -0.5\text{MPa}, n = 0.97, V_{ex} = 8.51\text{mm/day},$$
$$\rho_{hy} = 0.27\mu g/cm^2, v_{de} = 73.39 \pm 10.22\%/122\text{days}$$

where r is pamrameter of describing the speed of growth, and k is bioligical capacity.

We assume that there are N kinds of different fungi, and the biomass of each kind of fungi is S_i. We only concern about the relative biomass of each kind of fungi, so we substitute S_i with $x_i = S_i/K_i$.

We use the Competitive Lotca-Voterre equations, a model for the population dynamics of species competing for some resource.

$$\frac{dS_i}{dt} = r_i S_i \left(1 - \frac{\sum_{j=1}^{N} \alpha_{ij} S_j}{K_i}\right) \tag{1}$$

Using the substitution $x_i = S_i/K_i$, the equations can be written as below:

$$\frac{dx_i}{dt} = r_i x_i \left(1 - \frac{\sum_{j=1}^{N} \alpha_{ij} x_j K_j}{K_i}\right) \tag{2}$$

where

$$r_i = \frac{v_{extension}}{R} \tag{3}$$
$$K_i = C\rho_i, C = const \tag{4}$$
$$\alpha_{ij} = \exp 1 - \frac{n_i}{n_j} \tag{5}$$
$$\varepsilon = 0.33 \tag{6}$$

ε is efficiency, according to the reference[3].

3.2 Parameter Estimation of LV Equations

We assume that $\alpha_{ij} = \exp(1 - n_i/n_j)$, making the fungi with larger n having advantages over fungi with smaller n.

3.2.1 Carrying Capacity:K

3.2.2 Inherent Per-capita Growth Rate:r

When there's only one kind of fungi, it evolve as he equation below:

$$\frac{dx(t)}{dt} = r(T,M)x(t)(1 - \frac{S(t)}{K(T,M)}) \tag{7}$$

So we assume that $r = \left\langle \frac{dS}{dt} \right\rangle$

3.3 Results

We choose several kinds of fungi, the data are shown as follow:

and the result after enough long time of evolution is shown in Figure 1, where temperature $T = 25°C$, moisture $M = -0.5MPa$

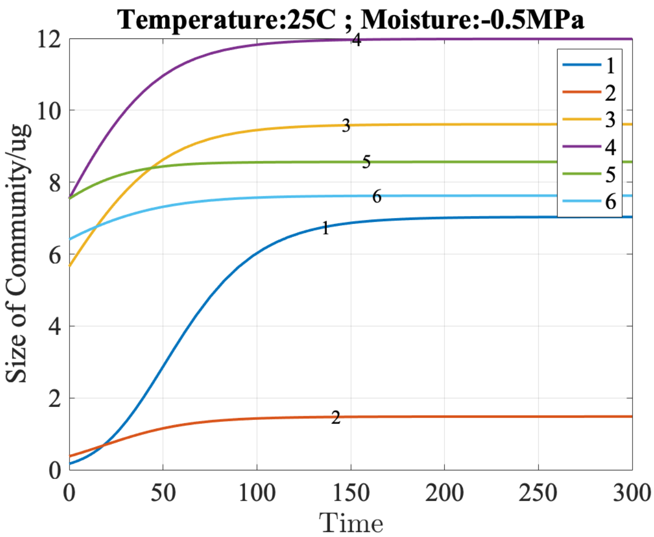

Figure 1: The result of Model 1

3.4 Detail 1 about Model 1

4 Model 2:Woody Fibres Decomposition Model

4.1 Decomposition Equations

4.2 Single Population

4.2.1 Population Equation

According to the reference[3], the decomposition of wood confirms to the following model:

$$\frac{dS}{dt} = rS(1 - S/K) \times M_{woody} \tag{8}$$

$$\frac{dM_{woody}}{dt} = -\frac{r}{\varepsilon} S M_{woody} \tag{9}$$

4.2.2 Results

4.3 Multi Population

4.3.1 Population Equation

4.3.2 Results

We assume that there are N kinds of different fungi, and the biomass of each kind of fungi is S_i. We only concern about the relative biomass o feach kind of fungi, so we substitute S_i with $x_i = S_i/K_i$.

We use hte Competitive Lotca-Voterre equations, a model for the population dynamics of species competing for some resource.

$$\frac{dS_i}{dt} = r_i S_i (1 - \frac{\sum_{j=1}^{N} \alpha_{ij} S_j}{K_i}) \tag{10}$$

Using the substitution $x_i = S_i/K_i$, the equations are shown as below:

$$\frac{dx_i}{dt} = r_i x_i (1 - \frac{\sum_{j=1}^{N} \alpha_{ij} x_j K_j}{K_i}) \tag{11}$$

where

$$r_i = \frac{v_{extension}}{R} \tag{12}$$

$$K_i = C\rho_i, C = const \tag{13}$$

$$\alpha_{ij} = \exp 1 - \frac{n_i}{n_j} \tag{14}$$

$$\varepsilon = 0.33 \text{is efficiency, according to the reference[3]} \tag{15}$$

The results are shown in Figure 2

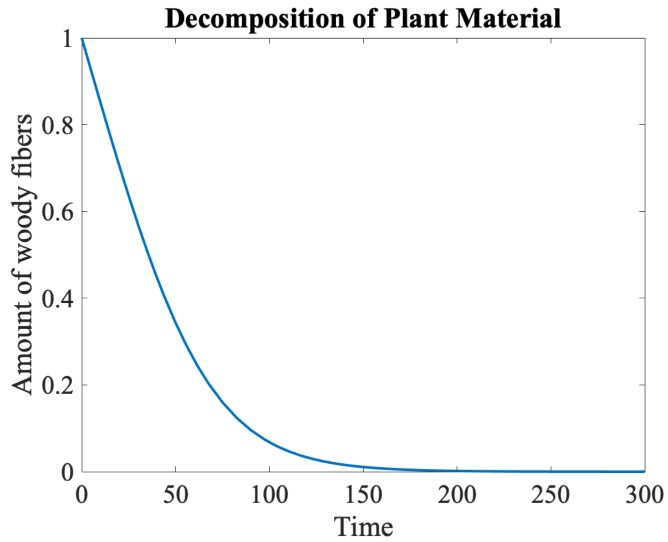

Figure 2: Decomposition Model

5 Discussion

5.1 Rapid Fluctuations

5.2 Prediction Under Different Environment

6 Test the Models

6.1 Sensitivity Analysis

6.2 Robustness

7 Conclusion

7.1 Summary of Results

7.2 Strength

7.3 Possible Improvements

8 Referencese

References

[1] Dang, Christian K. , et al. "Temperature oscillation coupled with fungal community shifts can modulate warming effects on litter decomposition." *Ecology* 90.1(2009).