Optimal Parking (EN)

🔍 Project Overview

This project aims to compute and visualize an optimal, collision-free parking trajectory and control inputs for a vehicle in a complex parking environment, considering its physical constraints.

🔧 Development Environment and Technologies

- **Programming Language**: C++
- **Computational Libraries**: Eigen, OSQP
- **Development Tools**: VSCode, Docker
- **Development Environment**: Ubuntu 22.04

❓ Objective

Develop an autonomous parking system using optimization techniques:

- Generate a safe parking trajectory in tight parking spaces with various obstacles, accounting for the vehicle's kinematic properties.
- Solve the optimization problem caused by the vehicle's nonlinear dynamics and various constraints using the Sequential Quadratic Programming (SQP) method.

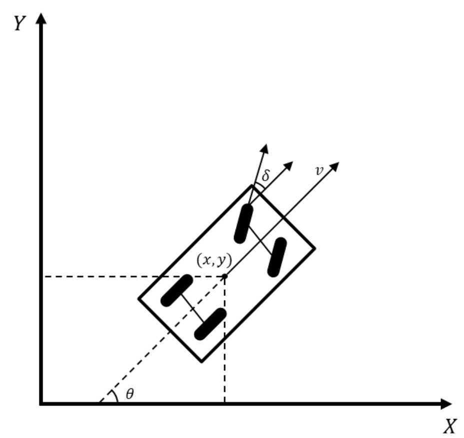
🧮 Algorithm

To generate an optimal parking trajectory, the following algorithms are implemented:

- **RRT**(Rapidly-exploring Random Tree Star): Generates an initial path considering obstacle avoidance constraints.
- **SQP**: Performs optimization based on the initial path generated by RRT*.

🧠 System Design

The system model to represent and control the vehicle's state is designed as follows



$$\mathbf{x}(t) = \begin{bmatrix} p_x(t) \\ p_y(t) \\ \theta(t) \\ v(t) \\ \delta(t) \end{bmatrix}$$

$$\mathbf{u}(t) = \begin{bmatrix} a(t) \\ \dot{\delta}(t) \end{bmatrix}$$

- A 5-dimensional state space is defined to represent the vehicle's position, heading angle, velocity, and steering angle.
- The vehicle's acceleration and steering angle rate are used as control inputs.

🔢 Optimization Problem Formulation and QP Transformation

1. 최적화 문제 설계

- **Objective Function**

  Minimize the weighted combination of control input magnitudes $(u_1^2(t) + u_2^2(t))$and slack variables for goal tracking and obstacle avoidance $(s_{\text{goal}}, s_{\text{obs}}(t))$ with weights $(\lambda_{\text{goal}}, \lambda_{\text{obs}})$

  $J(\mathbf{u}(t), s_{\text{goal}}, s_{\text{obs}}(t)) = \int_0^T \left(u_1^2(t) + u_2^2(t) + \lambda_{\text{goal}} s_{\text{goal}} + \lambda_{\text{obs}} s_{\text{obs}}(t)\right) \mathrm{d}t$

- **Constraints**

- **Dynamics**: The state ($\mathbf{x}(t)$) follows the dynamics function $f(\mathbf{x}(t), \mathbf{u}(t))$

  $\dot{\mathbf{x}}(t) = f(\mathbf{x}(t), \mathbf{u}(t))$

- **초기 및 최종 조건**:
  - Initial state: $\mathbf{x}(0) = \mathbf{x}_{\text{init}}$
  - Final state: $\mathbf{x}(T) = \mathbf{x}_{\text{goal}} + s_{\text{goal}}$
  - Initial and final control inputs: $\mathbf{u}(0) = \mathbf{u}_{\text{init}}, \mathbf{u}(T) = \mathbf{u}_{\text{goal}}$

- **Obstacle Avoidance**: The distance between the state and obstacles $d(\mathbf{x}(t), \mathcal{O}j)$*) must be at least* $d_{\min}$*, relaxed by the slack variable* $s_{\text{obs}}(t) \geq 0$.

  $d(\mathbf{x}(t), \mathcal{O}j) \geq d\min - s_{\text{obs}}(t), \quad s_{\text{obs}}(t) \geq 0, \quad \forall t \in [0, T],\ \forall j \in \{1, \ldots, N_{\text{obs}}\}$

- **Physical Constraints**:
  - Velocity: $v_{\min} \leq v(t) \leq v_{\max}$
  - Steering angle:$\delta_{\min} \leq \delta(t) \leq \delta_{\max}$
  - Acceleration: $a_{\min} \leq a(t) \leq a_{\max}$
  - Steering angle rate: $\dot{\delta}_{\min} \leq \dot{\delta}(t) \leq \dot{\delta}_{\max}$

$$
\begin{aligned}
\min_{\mathbf{u}, s_{\text{goal}}, s_{\text{obs}}} \quad & J(\mathbf{u}(t), s_{\text{goal}}, s_{\text{obs}}(t)) = \int_0^T \left(u_1^2(t) + u_2^2(t) + \lambda_{\text{goal}} s_{\text{goal}} + \lambda_{\text{obs}} s_{\text{obs}}(t)\right) \mathrm{d}t && (1) \\
\text{s.t.} \quad & \dot{\mathbf{x}}(t) = f(\mathbf{x}(t), \mathbf{u}(t)), && (2) \\
& \mathbf{x}(0) = \mathbf{x}_{\text{init}}, \quad \mathbf{x}(T) = \mathbf{x}_{\text{goal}} + s_{\text{goal}}, && (3) \\
& \mathbf{u}(0) = \mathbf{u}_{\text{init}}, \quad \mathbf{u}(T) = \mathbf{u}_{\text{goal}}, && (4) \\
& d(\mathbf{x}(t), \mathcal{O}_j) \geq d_{\min} - s_{\text{obs}}(t), \quad s_{\text{obs}}(t) \geq 0, \quad \forall t \in [0, T],\ \forall j \in \{1, \ldots, N_{\text{obs}}\}, && (5) \\
& v_{\min} \leq v(t) \leq v_{\max}, && (6) \\
& \delta_{\min} \leq \delta(t) \leq \delta_{\max}, && (7) \\
& a_{\min} \leq a(t) \leq a_{\max}, && (8) \\
& \dot{\delta}_{\min} \leq \dot{\delta}(t) \leq \dot{\delta}_{\max}, && (9)
\end{aligned}
$$

2. Discretization and SQP Transformation

The continuous-time problem is discretized and transformed into an SQP framework, approximating the nonlinear optimization problem as a series of linearized Quadratic Programming (QP) problems.

- **Discretized Objective Function**
  Divide the time interval
  $[0, T]$ into Nsegments, minimizing the weighted sum of control inputs and slack variables in each segment.

  $J = \sum_{k=0}^{N-1} \left(u_1^2[k] + u_2^2[k] + \lambda_{\text{goal}} s_{\text{goal}} + \lambda_{\text{obs}} s_{\text{obs}}[k]\right) \Delta t$

- **Discretized Constraints**
  - **Dynamics**: State changes follow linearized dynamics

    $\Delta\mathbf{x}[k+1] = \mathbf{A}_k \Delta\mathbf{x}_k + \mathbf{B}_k \Delta\mathbf{u}_k + \mathbf{g}_k, \quad k = 0, 1, \ldots, N-1$

  - **Initial and Final Conditions**
    - $\mathbf{x}[0] = \mathbf{x}_{\text{init}}, \quad \mathbf{x}[N] = \mathbf{x}_{\text{goal}} + s_{\text{goal}}$
    - $\mathbf{u}[0] = \mathbf{u}_{\text{init}} \quad \mathbf{u}[N] = \mathbf{u}_{\text{goal}}$

  - **Obstacle Avoidance**: Distance constraints are expressed in linearized form.

$d(\mathbf{x}[k], \mathcal{O}j) + \frac{\partial d(\mathbf{x}[k], \mathcal{O}j)}{\partial \mathbf{x}} \Delta\mathbf{x} \geq d_{\min} - s_{\text{obs}}[k], \quad s_{\text{obs}}[k] \geq 0$

- **Physical Constraints:**
  - Velocity: $v_{\min} \leq \mathbf{c}_v^T(\mathbf{x}[k] + \Delta\mathbf{x}[k]) \leq v_{\max}$
  - Steering angle: $\delta_{\min} \leq \mathbf{c}_\delta^T(\mathbf{x}[k] + \Delta\mathbf{x}[k]) \leq \delta_{\max}$
  - Acceleration: $a_{\min} \leq \mathbf{c}_a^T(\mathbf{u}[k] + \Delta\mathbf{u}[k]) \leq a_{\max}$
  - Steering angle rate: $\dot{\delta}_{\min} \leq \mathbf{c}_{\dot{\delta}}^T(\mathbf{u}[k] + \Delta\mathbf{u}[k]) \leq \dot{\delta}_{\max}$

$$
\begin{aligned}
\min_{\Delta\mathbf{u}, s_{\text{goal}}, \mathbf{s}_{\text{obs}}} \quad & J = \sum_{k=0}^{N-1} \left(u_1^2[k] + u_2^2[k] + \lambda_{\text{goal}} s_{\text{goal}} + \lambda_{\text{obs}} s_{\text{obs}}[k]\right) \Delta t && (10) \\
\text{s.t.} \quad & \Delta\mathbf{x}[k+1] = \mathbf{A}_k \Delta\mathbf{x}_k + \mathbf{B}_k \Delta\mathbf{u}_k + \mathbf{g}_k, \quad k = 0, 1, \ldots, N-1, && (11) \\
& \mathbf{x}[0] = \mathbf{x}_{\text{init}}, \quad \mathbf{x}[N] = \mathbf{x}_{\text{goal}} + s_{\text{goal}}, && (12) \\
& \mathbf{u}[0] = \mathbf{u}_{\text{init}}, \quad \mathbf{u}[N] = \mathbf{u}_{\text{goal}}, && (13) \\
& d(\mathbf{x}[k], \mathcal{O}_j) + \frac{\partial d(\mathbf{x}[k], \mathcal{O}_j)}{\partial \mathbf{x}} \Delta\mathbf{x}_k \geq d_{\min} - s_{\text{obs}}[k], \quad s_{\text{obs}}[k] \geq 0, && (14) \\
& v_{\min} \leq \mathbf{c}_v^T(\mathbf{x}[k] + \Delta\mathbf{x}[k]) \leq v_{\max}, && (15) \\
& \delta_{\min} \leq \mathbf{c}_\delta^T(\mathbf{x}[k] + \Delta\mathbf{x}[k]) \leq \delta_{\max}, && (16) \\
& a_{\min} \leq \mathbf{c}_a^T(\mathbf{u}[k] + \Delta\mathbf{u}[k]) \leq a_{\max} && (17) \\
& \dot{\delta}_{\min} \leq \mathbf{c}_{\dot{\delta}}^T(\mathbf{u}[k] + \Delta\mathbf{u}[k]) \leq \dot{\delta}_{\max} && (18)
\end{aligned}
$$

3. QP Problem Transformation

The SQP problem is converted into a standard QP form for solving.

- **Variable Vector**

$$
\mathbf{z} = \begin{bmatrix} \Delta\mathbf{x}_1 \\ \Delta\mathbf{x}_2 \\ \vdots \\ \Delta\mathbf{x}_{N+1} \\ \Delta\mathbf{u}_1 \\ \Delta\mathbf{u}_2 \\ \vdots \\ \Delta\mathbf{u}_N \\ s_{\text{goal}} \\ s_{\text{obs},1} \\ s_{\text{obs},2} \\ \vdots \\ s_{\text{obs},N} \end{bmatrix}
$$

- **QP Objective Function**

  Minimize the quadratic cost of the variable vector $\mathbf{z}$ (state changes $\Delta\mathbf{x}$, control input changes $\Delta\mathbf{u}$, and slack variables $s_{\text{goal}}, s_{\text{obs}}$).

$$
\min_{\mathbf{z}} \quad \frac{1}{2}\mathbf{z}^T \begin{bmatrix} \mathbf{Q} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{R} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{\Lambda_{goal}} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{\Lambda_{obs}} \end{bmatrix} \mathbf{z}
$$

  where $\mathbf{Q}, \mathbf{R}, \mathbf{\Lambda_{goal}}, \mathbf{\Lambda_{obs}}$ are weighting matrices for states, control inputs, goal slack, and obstacle slack, respectively.

- **Constraints**
  - **Linear Equality Constraints**: Dynamics and initial/final conditions

    $\mathbf{Az} = \mathbf{b}$

- **Linear Inequality Constraints**: Obstacle avoidance and physical constraints.

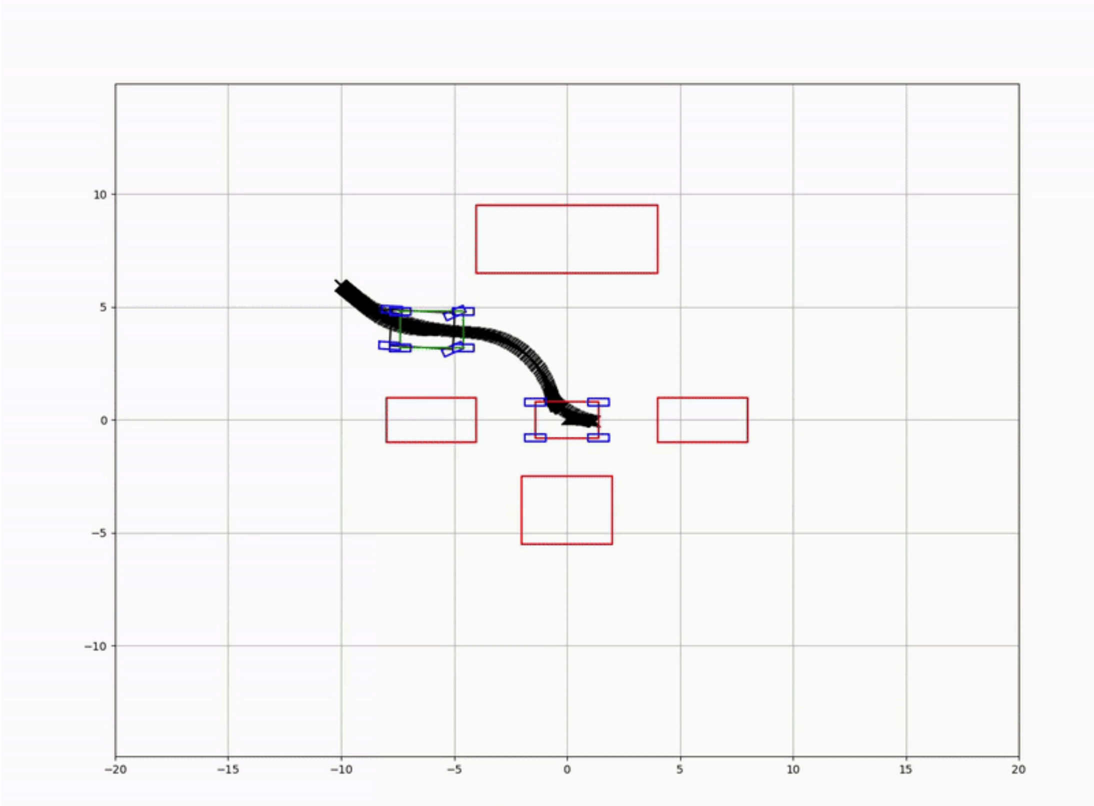
  $$\mathbf{G}\mathbf{z} \le \mathbf{h}$$

4. Trajectory Update

Using the solution $\mathbf{z}$ from the QP problem, update the states $\mathbf{x}$ and control inputs $\mathbf{u}$. Repeat this process to generate the optimal trajectory, ultimately solving the optimization problem.

This process discretizes the nonlinear optimization problem and transforms it into a QP problem for efficient solving.

📊 Results



- Simulations are performed using the generated control inputs.
- The system can be simulated for various parking scenarios by adjusting parameters (via YAML files).