Kochen-Specker Uncolorable Sets:

An Algorithmic Approach to Proving Contradictions

Camilo Morales

Department of Mathematics
University of California, Irvine

Vector Spaces

Vector Spaces

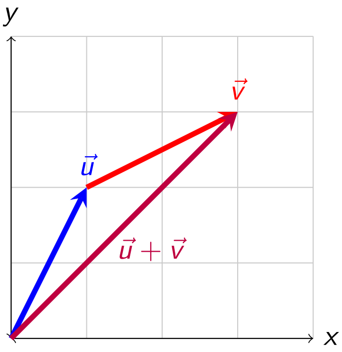


Figure 1: Vector addition

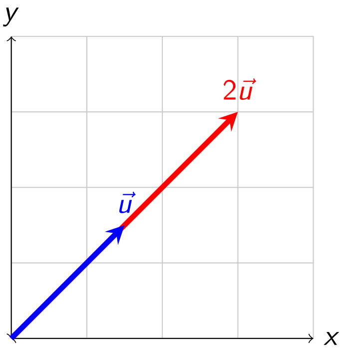


Figure 2: Scalar multiplication

Orthogonality

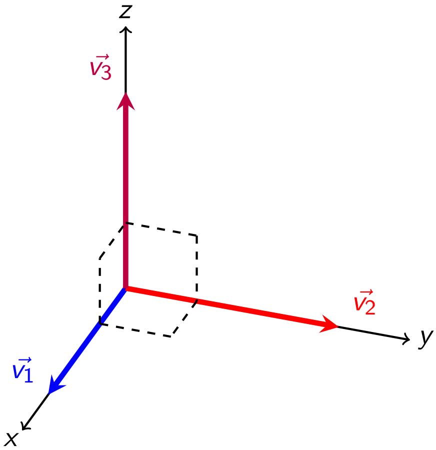
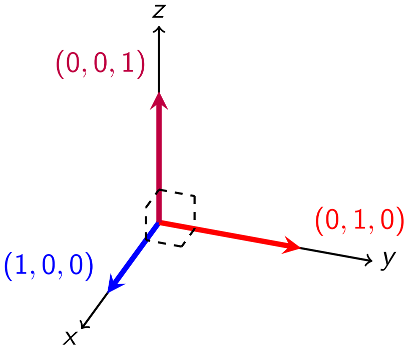
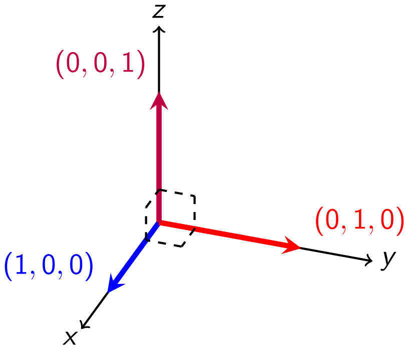


Figure 3: $\vec{v_1}$, $\vec{v_2}$, and $\vec{v_3}$ are pairwise orthogonal.

Orthogonality



Orthogonality



Computing the dot product of each pair of vectors above will always yield 0:

$$(1, 0, 0) \cdot (0, 1, 0) = (1 \cdot 0) + (0 \cdot 1) + (0 \cdot 0) = 0 + 0 + 0 = 0.$$

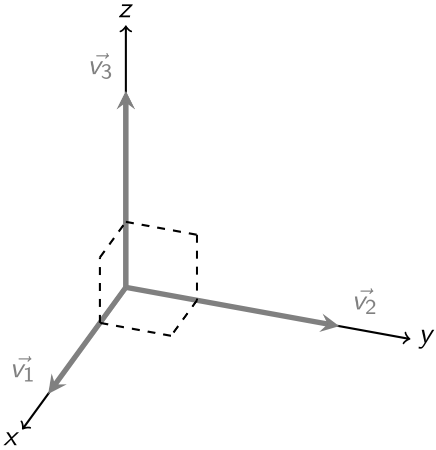
Orthogonal Sets

Definition

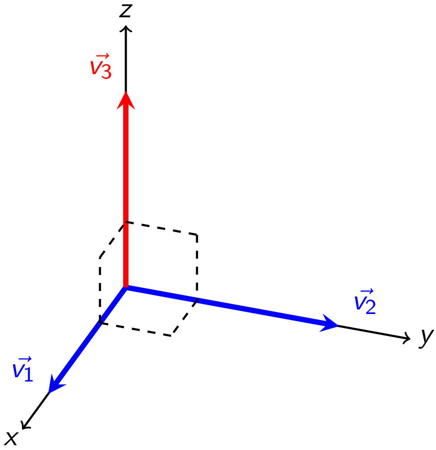
A set of vectors $\{\vec{v_1}, \vec{v_2}, \ldots, \vec{v_k}\}$ in a vector space V is called an **orthogonal set** if all pairs of distinct vectors in the set are orthogonal – that is, if

$$\vec{v_i} \cdot \vec{v_j} = 0 \quad \text{whenever} \quad i \neq j \text{ for } i, j = 1, 2, \ldots, k.$$

Coloring and Contextuality



Coloring and Contextuality



The Kochen-Specker (KS) Theorem

Kochen-Specker, 1967

There is a finite set $\mathcal{S} \subset \mathbb{R}^3$ such that there is no function $f : \mathcal{S} \to \{0, 1\}$ satisfying

$$f(\vec{u}) + f(\vec{v}) + f(\vec{w}) = 1$$

for all triples $(\vec{u}, \vec{v}, \vec{w})$ of mutually orthogonal vectors in $\mathcal{S}$.

$\mathbb{R}^3 := \{(x, y, z) : x, y, z \in \mathbb{R}\}$

The Kochen-Specker (KS) Theorem

Let $\mathcal{S}$ be a set of vectors and consider the value function $f : \mathcal{S} \to \{0, 1\}$.

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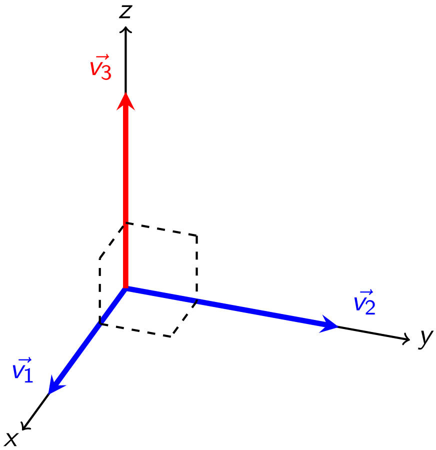

$$f(\vec{v_1}) = 0 \quad f(\vec{v_2}) = 0 \quad f(\vec{v_3}) = 1$$

The Kochen-Specker (KS) Theorem

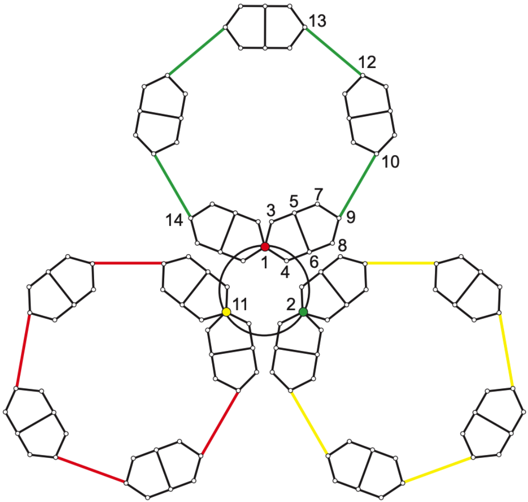

Figure 4: A graphical representation of the set of 117 vectors in the original proof of the KS Theorem (Budroni et al, 2022).

Uncolorable Sets of Integer Vectors

Uncolorable Sets of Integer Vectors

Let N be a positive squarefree integer. We define

$$\begin{aligned}\mathcal{S}_n(N) &= \{\vec{v} \in \mathbb{Z}^n : ||\vec{v}||^2 \text{ is a unit in } \mathbb{Z}[1/N]\} \\ &= \{\vec{v} \in \mathbb{Z}^n : ||\vec{v}||^2 \text{ divides a power of } N\}.\end{aligned}$$

Uncolorable Sets of Integer Vectors

Let N be a positive squarefree integer. We define

$$\mathcal{S}_n(N) = \{\vec{v} \in \mathbb{Z}^n : ||\vec{v}||^2 \text{ is a unit in } \mathbb{Z}[1/N]\}$$
$$= \{\vec{v} \in \mathbb{Z}^n : ||\vec{v}||^2 \text{ divides a power of } N\}.$$

Question

For which positive squarefree integers N is the set of vectors $\mathcal{S}(N) := \mathcal{S}_3(N)$ Kochen-Specker uncolorable?

(Un)colorable Sets of Integer Vectors

Let M be an integer such that M divides a squarefree integer N. It follows that $\mathcal{S}(M) \subseteq \mathcal{S}(N)$. Furthermore,

$$\mathcal{S}(N) \text{ colorable} \implies \mathcal{S}(M) \text{ colorable},$$
$$\mathcal{S}(M) \text{ uncolorable} \implies \mathcal{S}(N) \text{ uncolorable}.$$

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Ben-Zvi et al., 2017

If $\mathcal{S}(N)$ is KS uncolorable, then 6 divides N.

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$$\mathcal{S}(M) \text{ uncolorable } \implies \mathcal{S}(N) \text{ uncolorable}.$$

Ben-Zvi et al., 2017

If $\mathcal{S}(N)$ is KS uncolorable, then 6 divides N.

Bub, 1996

If 30 divides N, then the set $\mathcal{S}(N)$ is KS uncolorable.

Diophantine Equations

Diophantine Equations

Let N be a positive squarefree integer.

$$x^2 + y^2 + z^2 = N \rightarrow (x, y, z)^T \in \mathcal{S}(N)$$

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Let N be a positive squarefree integer.

$$x^2 + y^2 + z^2 = N \to (x, y, z)^T \in \mathcal{S}(N)$$

```
from sympy import diophantine
from sympy.abc import i, j, k
vec_sol = diophantine(i**2 + j**2 + k**2 - n, permute
= True)
```

Primitive and Well-Signed Vectors

Definition

An integer vector is *primitive* if its entries have a greatest common divisor equal to 1.

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Definition

A vector $\vec{v} = (v_1, v_2, v_3) \in \mathbb{Z}^3 \setminus \{0\}$ is *well-signed* if either:

- $\vec{v}$ has only one nonzero entry which is positive,
- $\vec{v}$ has two nonzero entries and its first nonzero entry is positive, or
- $\vec{v}$ has three nonzero entries, at least two of which are positive.

Coloring the Vectors

Let N be a positive squarefree integer.

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```
color_dict = {(1, 0, 0): 1, (0, 0, 1): 0,
              (0, 1, 0): 0}
```

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```
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              (0, 1, 0): 0}
```

If $N \equiv 0 \pmod{2}$, then

```
color_dict[(1, 0, -1)] = 1
color_dict[(1, 0, 1)] = 0
color_dict[(1, 1, 0)] = 0
color_dict[(1, -1, 0)] = 1
color_dict[(0, 1, 1)] = 0
color_dict[(0, 1, -1)] = 0.
```

Coloring the Vectors

Let `n` $= N$ be a positive squarefree integer and recall that
`vec_sol = diophantine(i**2 + j**2 + k**2 - n, permute = True)`.

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Let $\vec{v}_1$ be a primitive, well-signed vector in `vec_sol` and $\vec{v}_2$ be a vector in `color_dict`.

Coloring the Vectors

Let $\texttt{n} = N$ be a positive squarefree integer and recall that `vec_sol = diophantine(i**2 + j**2 + k**2 - n, permute = True)`.

Let $\vec{v}_1$ be a primitive, well-signed vector in `vec_sol` and $\vec{v}_2$ be a vector in `color_dict`.

If $\vec{v}_1 \cdot \vec{v}_2 = 0$ and $\vec{v}_2$ was assigned the value 1, then $\vec{v}_1$ must be assigned the value 0.

Coloring the Vectors

Let $\vec{v}_1$ and $\vec{v}_2$ be vectors in `color_dict`, which was updated in the previous slide.

Coloring the Vectors

Let $\vec{v}_1$ and $\vec{v}_2$ be vectors in `color_dict`, which was updated in the previous slide.
If $\vec{v}_1 \cdot \vec{v}_2 = 0$ and both $\vec{v}_1$ and $\vec{v}_2$ were assigned the value 0, then we take the cross product of $\vec{v}_1$ and $\vec{v}_2$.

Coloring the Vectors

Let $\vec{v}_1$ and $\vec{v}_2$ be vectors in `color_dict`, which was updated in the previous slide.

If $\vec{v}_1 \cdot \vec{v}_2 = 0$ and both $\vec{v}_1$ and $\vec{v}_2$ were assigned the value 0, then we take the cross product of $\vec{v}_1$ and $\vec{v}_2$.

Let $\vec{v}_{\text{cross}}$ be the normalized cross product of $\vec{v}_1$ and $\vec{v}_2$, which must be assigned the value 1.

Preliminary Results

Kochen_Specker_Colorability > Vector_Coloring.py

```python
def color_assignment(N):
    """Return True if all vectors (constructed) from primative_well_signed_solutions(N) has been designate
      either a 1 or 0 and False if we identify a contradiction.
    """
    # white = 1 and black = 0
    color_dict = {(1, 0, 0): 1, (0, 0, 1): 0, (0, 1, 0): 0} # Q_{1} is a subset of S_{n} (N) for all n and
    if N % 2 == 0:
        color_dict[(1, 0, -1)] = 1  # Now, we add vectors from Q_{2} that form an orthogonal set with (0,
        color_dict[(1, 0, 1)] = 0
        color_dict[(1, 1, 0)] = 0   # Add vectors from Q_{2} that form an orthogonal set with (0, 0, 1) in
        color_dict[(1, -1, 0)] = 1
        color_dict[(0, 1, 1)] = 0   # Finally, add vectors from Q_{2} that form an orthogonal set  with (1
        color_dict[(0, 1, -1)] = 0

    # vec_set = primative_well_signed_solutions(N)
    vec_set = vectors_to_color(N)
```

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```
In [3]: run Vector_Coloring.py

In [4]: color_assignment(30)
A contradiction has been identified from coloring (2, 1, 0), the cross product of (0, 0, 1) and (1, -2, 0).
We colored the other vectors as follows: {(1, 0, 0): 1, (0, 0, 1): 0, (0, 1, 0): 0, (1, 0, -1): 1, (1, 0, 1): 0, (1, 1, 0): 0, (1, -1, 0): 1, (0, 1, 1): 0, (0, 1, -1): 0, (1, -1,
 1): 0, (1, -2, 1): 0, (0, 1, -3): 0, (0, 1, 3): 0, (0, 2, -1): 0, (0, 2, 1): 0, (1, 1, -1): 0, (0, 1, 2): 0, (1, 2, 1): 0, (1, 1, 2): 0, (0, 1, -2): 0, (0, 3, -1): 0, (0, 3, 1):
 0, (1, 1, -2): 0, (1, 1, 1): 0, (-1, 0, 0): 1, (-1, 1, 0): 1, (-1, 0, 1): 1, (-1, 2, 1): 1, (1, -1, -2): 1, (2, 1, -1): 1, (-2, 1, -1): 1, (2, -1, -1): 1, (-1, 1, 2): 1, (-2, -1
, 1): 1, (-5, -2, 1): 1, (5, -1, -2): 1, (-5, 1, -2): 1, (1, -2, -1): 1, (2, -1, 1): 1, (5, 2, -1): 1, (-5, 2, 1): 1, (-5, 1, 2): 1, (5, -2, -1): 1, (5, -1, 2): 1, (-2, 1, 1): 1,
 (1, 5, -2): 0, (2, 1, 0): 0, (2, 5, -1): 0, (2, 5, 1): 0, (5, 2, 1): 0, (-2, 5, 1): 0, (1, 0, -2): 0, (5, 1, 2): 0, (2, 0, 1): 0, (2, 1, 5): 0, (1, -2, 0): 0, (-2, 1, 5): 0, (1,
 2, 0): 0, (1, -2, 5): 0, (2, -1, 5): 0, (1, 0, 2): 0, (-1, 2, 0): 1}
Out[4]: False

In [5]: color_assignment(462)
A contradiction has been identified from coloring (-3, 2, 8), the cross product of (2, -5, 2) and (4, 2, 1).
We colored the other vectors as follows: {(1, 0, 0): 1, (0, 0, 1): 0, (0, 1, 0): 0, (1, 0, -1): 1, (1, 0, 1): 0, (1, 1, 0): 0, (1, -1, 0): 1, (0, 1, 1): 0, (0, 1, -1): 0, (3, 3,
2): 0, (5, 5, -4): 0, (4, 1, 4): 0, (4, 4, -1): 0, (1, 2, 1): 0, (3, 2, 3): 0, (1, 3, 1): 0, (5, 4, 5): 0, (3, -2, 3): 0, (3, 3, -2): 0, (1, 1, 8): 0, (1, 1, 1): 0, (4, 4, 1): 0,
 (1, 1, 3): 0, (5, -4, 5): 0, (1, -3, 1): 0, (1, 1, -1): 0, (1, 1, -2): 0, (1, -1, 1): 0, (2, 2, 5): 0, (1, -2, 1): 0, (2, -5, 2): 0, (1, 1, 2): 0, (1, 1, -3): 0, (2, 2, -5): 0,
(4, -1, 4): 0, (1, 1, -8): 0, (1, -8, 1): 0, (5, 5, 4): 0, (1, 8, 1): 0, (2, 5, 2): 0, (-1, 0, 0): 1, (-1, 1, 0): 1, (-1, 0, 1): 1, (-1, 2, 1): 1, (1, -1, -2): 1, (-2, 1, -1): 1,
 (2, 1, -1): 1, (2, -1, -1): 1, (-7, 4, -1): 1, (-2, 1, 1): 1, (-7, -1, 4): 1, (1, -2, -1): 1, (2, -1, 1): 1, (-1, 1, 2): 1, (-2, -1, 1): 1, (7, 1, -4): 1, (7, -4, 1): 1, (2, 1,
-3): 0, (-1, 3, 1): 0, (4, 1, -7): 0, (6, -4, 5): 0, (3, 8, -2): 0, (4, 1, 2): 0, (2, 4, -1): 0, (-1, 2, 4): 0, (5, 4, 6): 0, (4, 7, 1): 0, (4, -7, 1): 0, (5, 4, -6): 0, (-3, 2,
8): 0, (4, 7, -1): 0, (-2, 17, 13): 0, (2, 3, 1): 0, (2, 8, -3): 0, (7, 4, -1): 0, (10, 1, 19): 0, (3, 2, -1): 0, (5, -4, 6): 0, (8, 3, 2): 0, (2, 3, -1): 0, (8, 2, 3): 0, (4, 2,
 1): 0, (1, -2, 4): 0, (19, 10, -1): 0, (-2, 13, 17): 0, (17, 13, 2): 0, (1, 10, -19): 0, (5, 6, 4): 0, (1, 2, 4): 0, (3, -1, 2): 0, (2, 17, 13): 0, (1, 2, -3): 0, (19, -1, 10):
0, (3, 2, 8): 0, (2, -13, 17): 0, (10, 19, 1): 0, (10, -19, 1): 0, (1, -1, 3): 0, (-1, 4, 2): 0, (2, -1, 3): 0, (13, -2, 17): 0, (10, 1, -19): 0, (3, -2, 8): 0, (5, -6, 4): 0, (4
, -1, 7): 0, (10, 19, -1): 0, (5, 6, -4): 0, (6, 5, -4): 0, (2, 13, 17): 0, (2, 17, -13): 0, (4, -6, 5): 0, (2, -3, 8): 0, (-1, 1, 3): 0, (1, -3, 2): 0, (1, 4, -2): 0, (10, -1, 1
9): 0, (2, -3, 1): 0, (3, 8, 2): 0, (1, -19, 10): 0, (2, 1, 3): 0, (4, 5, -6): 0, (1, 4, -7): 0, (2, -1, 4): 0, (1, 4, 2): 0, (13, 17, -2): 0, (3, 1, 1): 0, (1, -7, 4): 0, (4, 1,
 7): 0, (7, -1, 4): 0, (1, 3, -1): 0, (-3, 8, 2): 0, (17, 2, 13): 0, (-3, -2, 3): 1, (-3, 2, 3): 1, (3, -3, -2): 1, (3, -3, 2): 1, (2, -3, -3): 1, (-2, 4, 1): 1, (-6, 4, 5): 1, (
6, -5, 4): 1, (5, -6, -4): 1, (-4, 5, -1): 1, (-5, 4, 1): 1, (5, -1, -4): 1, (4, 1, -5): 1, (6, -5, -4): 1, (2, -1, -4): 1, (1, -5, -4): 1, (4, -5, -1): 1, (7, 1, 4): 1, (-4, 1,
5): 1, (-1, 4, 5): 1, (-8, 3, 2): 1, (-7, -4, -1): 1, (-4, -2, -1): 1}
Out[5]: False

In [6]: 
```

Preliminary Results

Kochen_Specker_Colorability > Vector_Coloring.py

```python
def color_assignment(N):
    """Return True if all vectors (constructed) from primative_well_signed_solutions(N) has been designate
      either a 1 or 0 and False if we identify a contradiction.
    """
    # white = 1 and black = 0
    color_dict = {(1, 0, 0): 1, (0, 0, 1): 0, (0, 1, 0): 0} # Q_{1} is a subset of S_{n} (N) for all n and
    if N % 2 == 0:
        color_dict[(1, 0, -1)] = 1  # Now, we add vectors from Q_{2} that form an orthogonal set with (0,
        color_dict[(1, 0, 1)] = 0
        color_dict[(1, 1, 0)] = 0   # Add vectors from Q_{2} that form an orthogonal set with (0, 0, 1) in
        color_dict[(1, -1, 0)] = 1
        color_dict[(0, 1, 1)] = 0   # Finally, add vectors from Q_{2} that form an orthogonal set  with (1
        color_dict[(0, 1, -1)] = 0

    # vec_set = primative_well_signed_solutions(N)
    vec_set = vectors_to_color(N)
```

PROBLEMS OUTPUT DEBUG CONSOLE TERMINAL

bash

```
Out[5]: False

In [6]: color_assignment(6)
The vectors we have left to color are:
{(2, 1, 1), (1, -1, 2), (1, 2, -1), (-1, 1, 1)}
Let's assume (2, 1, 1) is colored 0.
The vectors we have left to color are:
set()
Every element in our original finite set of vectors has been colored.
Out[6]: False

In [7]: color_assignment(6**2)
The vectors we have left to color are:
{(-2, 1, 2), (1, 4, -1), (2, 1, 1), (-1, 4, 1), (1, -1, 2), (1, -2, 2), (-1, 1, 1), (4, 1, -1), (-2, 2, 1), (4, -1, 1), (-1, 1, 4), (1, -1, 4), (1, 2, 2), (-4, 1, 1), (1, 2, -1),
 (2, -2, 1), (1, 2, -2), (2, 1, -2)}
Let's assume (-2, 1, 2) is colored 0.
The vectors we have left to color are:
{(-1, 4, 1), (2, 1, 1), (1, -1, 2), (-1, 1, 1), (-2, 2, 1), (4, -1, 1), (-1, 1, 4), (1, -1, 4), (1, 2, 2), (-4, 1, 1), (1, 2, -1), (2, -2, 1), (1, 2, -2), (2, 1, -2)}
Let's assume (-1, 4, 1) is colored 0.
The vectors we have left to color are:
{(2, 1, 1), (1, -1, 2), (-1, 1, 1), (-2, 2, 1), (4, -1, 1), (1, -1, 4), (1, 2, -1), (1, 2, -2)}
Let's assume (2, 1, 1) is colored 0.
The vectors we have left to color are:
{(-2, 2, 1), (4, -1, 1), (1, 2, -2), (1, -1, 4)}
Let's assume (-2, 2, 1) is colored 0.
The vectors we have left to color are:
set()
Every element in our original finite set of vectors has been colored.
Out[7]: False

In [8]:
```

Preliminary Results

Kochen_Specker_Colorability > Vector_Coloring.py

```python
def color_assignment(N):
    """Return True if all vectors (constructed) from primative_well_signed_solutions(N) has been designate
      either a 1 or 0 and False if we identify a contradiction.
    """
    # white = 1 and black = 0
    color_dict = {(1, 0, 0): 1, (0, 0, 1): 0, (0, 1, 0): 0} # Q_{1} is a subset of S_{n} (N) for all n and
    if N % 2 == 0:
        color_dict[(1, 0, -1)] = 1  # Now, we add vectors from Q_{2} that form an orthogonal set with (0,
        color_dict[(1, 0, 1)] = 0
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        color_dict[(0, 1, -1)] = 0

    # vec_set = primative_well_signed_solutions(N)
    vec_set = vectors_to_color(N)
```

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```
The vectors we have left to color are:
{(-1, 4, 1), (2, 1, 1), (1, -1, 2), (-1, 1, 1), (-2, 2, 1), (4, -1, 1), (-1, 1, 4), (1, -1, 4), (1, 2, 2), (-4, 1, 1), (1, 2, -1), (2, -2, 1), (1, 2, -2), (2, 1, -2)}
Let's assume (-1, 4, 1) is colored 0.
The vectors we have left to color are:
{(2, 1, 1), (1, -1, 2), (-1, 1, 1), (-2, 2, 1), (4, -1, 1), (1, -1, 4), (1, 2, -1), (1, 2, -2)}
Let's assume (2, 1, 1) is colored 0.
The vectors we have left to color are:
{(-2, 2, 1), (4, -1, 1), (1, 2, -2), (1, -1, 4)}
Let's assume (-2, 2, 1) is colored 0.
The vectors we have left to color are:
set()
Every element in our original finite set of vectors has been colored.
Out[7]: False

In [8]: color_assignment(6**3)
The vectors we have left to color are:
{(-2, 5, 5), (-1, 7, 2), (1, 5, -1), (-5, 5, 2), (2, 5, -5), (-1, 1, 1), (5, -5, 2), (1, 7, -2), (-1, 1, 4), (5, 2, -5), (1, 2, -7), (1, -1, 4), (2, 7, -1), (1, 2, 2), (1, -2, 7)
, (-7, 2, 1), (2, 7, 1), (1, -7, 2), (-5, 1, 1), (2, -2, 1), (-7, 1, 2), (2, -1, 7), (7, -1, 2), (-1, 5, 1), (2, -5, 5), (2, 1, -7), (2, -7, 1), (1, 2, -2), (1, 2, -1), (1, 2, 7)
, (-5, 2, 5), (1, 4, -1), (4, 1, -1), (-2, 2, 1), (5, 1, -1), (5, 1, 1), (-1, 2, 7), (2, 1, -2), (1, 7, 2), (-2, 1, 2), (2, 1, 1), (2, 1, 7), (-1, 4, 1), (1, -1, 2), (1, -2, 2),
(2, 5, 5), (-1, 1, 5), (1, -1, 5), (4, -1, 1), (-4, 1, 1), (5, -1, 1), (7, 2, -1), (7, 1, 2), (7, 2, 1)}
Let's assume (-2, 5, 5) is colored 0.
A contradiction has been identified from taking the dot product of (2, -2, 1) and (-2, -1, 2).
We colored the other vectors as follows: {(1, 0, 0): 1, (0, 0, 1): 0, (0, 1, 0): 0, (1, 0, -1): 1, (1, 0, 1): 0, (1, 1, 0): 0, (1, -1, 0): 1, (0, 1, 1): 0, (0, 1, -1): 0, (1, 5,
1): 0, (1, -1, 1): 0, (1, -2, 1): 0, (2, 2, -1): 0, (1, -5, 1): 0, (5, 5, 2): 0, (5, -2, 5): 0, (1, 1, 2): 0, (5, 5, -2): 0, (1, 1, -5): 0, (1, 1, 4): 0, (1, 1, 1): 0, (1, -4, 1)
: 0, (1, 1, -2): 0, (2, 1, 2): 0, (2, 2, 1): 0, (1, 4, 1): 0, (1, 2, 1): 0, (1, 1, -4): 0, (1, 1, 5): 0, (5, 2, 5): 0, (2, -1, 2): 0, (1, 1, -1): 0, (-1, 0, 0): 1, (-1, 1, 0): 1,
 (-1, 0, 1): 1, (-1, 2, 1): 1, (1, -1, -2): 1, (2, 1, -1): 1, (-2, 1, -1): 1, (2, -1, -1): 1, (-1, 1, 2): 1, (-2, -1, 1): 1, (1, -2, -2): 1, (-7, -1, 2): 1, (7, -2, 1): 1, (7, 1,
 -2): 1, (-2, 1, 1): 1, (-7, 2, -1): 1, (-1, 2, 2): 1, (1, -2, -1): 1, (2, -1, 1): 1, (4, 1, 1): 0, (-2, 1, 7): 1, (2, -7, -1): 1, (2, -1, -7): 1, (-2, 7, 1): 1, (-2, 5, 5): 0, (
5, 1, 1): 1, (-5, -1, -1): 1, (-1, 1, 4): 0, (1, -7, 2): 0, (1, 2, -7): 0, (-1, 4, 1): 0, (-2, -1, 2): 1, (2, -2, 1): 1, (-2, 2, -1): 1, (2, 1, -2): 1}
Out[8]: False

In [9]:
```

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