0.1 Numerical evaluation of $I_{\lambda\mu}(\vartheta, \xi)$

Since the values of $I_{\lambda\mu}(\vartheta, \xi)$ are only tabulated for certain values of λ, μ, ϑ and ξ in references [1, 2] and [3], the first objective of this work was to create a program which allows the user to evaluate the $I_{\lambda\mu}(\vartheta, \xi)$ numerically for arbitrary values of λ, μ, ϑ and ξ. Various approximations for $I_{\lambda\mu}(\vartheta, \xi)$ for different limiting cases are well estabilshed and documented in references [2] and [3]. This was done by first creating functions to calculate the reaction kinematics within the semiclassical approximation and symmetrized values of the parameters, a_{if}, ϑ, ξ, η and ϵ according to the expressions given in Appendix A of reference [1], these are then used to compute the $f_{\lambda,\mu}(\epsilon, \omega)$ given by [4, 2].

$$f_{\lambda,\mu}(\epsilon, \omega) = \frac{(\cosh\omega + \epsilon + i\sqrt{\epsilon^2 - 1}\sinh\omega)^{\mu}}{(\epsilon\cosh\omega + 1)^{\lambda+\mu}} \tag{1}$$

Equation 1 is then multiplied by $e^{i\xi g(\epsilon,\omega)}$, where,

$$g(\epsilon, \omega) = \epsilon sinh(\omega) + \omega, \tag{2}$$

so that the classical orbital integral $I_{\lambda,\mu}(\vartheta, \xi)$ [2, 1, 3], is expressed as,

$$I_{\lambda,\mu}(\vartheta, \xi) = \int_{-\infty}^{\infty} f_{\lambda,\mu}(\epsilon, \omega) e^{i\xi g(\epsilon,\omega)} d\omega. \tag{3}$$

The integration range of the parameter, ω is then initially calulated according to Section 6.1 of reference [4]. The integrand in Equation 3 is then evaluated numerically using Gauss-Legendre quadrature from 0 to N, where N is the number of integration steps by first computing the nodes, $n(i)$ according to,

$$n(i) = \frac{(\omega_{max} + \omega_{min})}{2} + \frac{(\omega_{max} - \omega_{min})}{2x(i)} \tag{4}$$

where ω_{max} and ω_{min} the maximum and minimum values of the parameter ω, i denotes a iterative index and,

$$x(i) = \cos\big(\frac{\pi}{N}(\frac{2i+1}{2})\big). \tag{5}$$

The weights, $w(i)$, involved in the numerical integration are computed according to,

$$w(i) = \frac{\pi}{N}\sin\big(\frac{\pi}{N}(\frac{2i+1}{2})\big). \tag{6}$$

The numerical integration is then carried out computing the sum of the product of the $w(i)$ with the integrand given in Equation 4 the initial values of ω, ω_{max} and ω_{min} are then replaced by those for which adjusting the $n(i)$ and $w(i)$ reproduce the values of $I_{\lambda,\mu}(\vartheta, \xi)$ given in reference [2] and these are then used to compute $I_{\lambda,\mu}(\vartheta, \xi)$ for user defined values of λ, μ, ϑ and ξ.

Bibliography

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