Math 53: Project

There are two main goals of this project: (1) To gain some understanding and experience in a linear algebra application using eigenvalues and eigenvectors, and to (2) practice writing clear and understandable mathematical prose. For this project you should:

1. Choose a 2×2 matrix of numbers that represent the changes in the size of a predator-prey relationship. You may make up any numbers, but they should be reasonable given whatever sorts of predator-prey relationship on which you decide to base your project.
2. Conduct and eigenvalue/eigenvector analysis of your matrix, following the model of Section 5.6 of your text. Use this to determine the long-term scaling factor for your populations and the long-term distribution of the predators to prey.
3. Compute "trajectories" for several different starting values of predators and prey. It will be helpful to use your eigenvectors to simplify the computation of these trajectories.
4. Write up a careful analysis of the set-up of your project, the mathematical analysis of it, your predictions, and a graphical analysis of your trajectories. You should assume you are writing this for other members of your linear algebra class, but you should still be complete and clear in the mathematical steps you show, and how you incorporate equations and graphs in your writing.

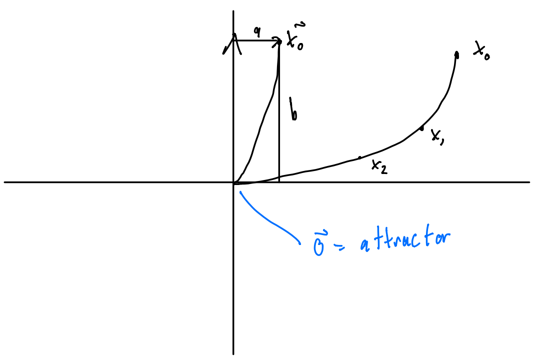
You may show your instructor drafts of the project at any time, and are encouraged to do so. Most points are lost by incomplete projects, or sloppy organization, so be careful.

The project may be turned in at any time, but is due no later than ~~at the start of the final exam Monday~~. 5:00 PM Friday

Graphing

1) let $A = \begin{array}{cc} \lambda_1 \downarrow & \\ \begin{bmatrix} 0.8 & 0 \\ 0 & 0.6 \end{bmatrix} \end{array}$ $\qquad \vec{v}_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

$\vec{v}_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$



$$\begin{bmatrix} 0.8 & 0 \\ 0 & 0.6 \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 0.8a \\ 0.6b \end{bmatrix}$$

2) $A = \begin{bmatrix} 1.4 & 0 \\ 0 & 1.2 \end{bmatrix}$

instruction
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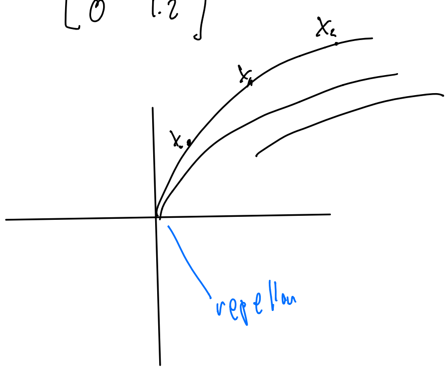
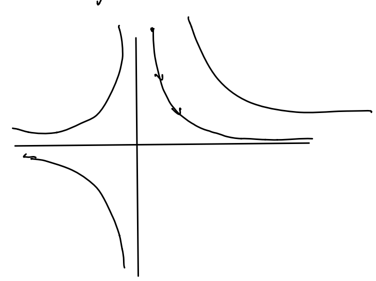
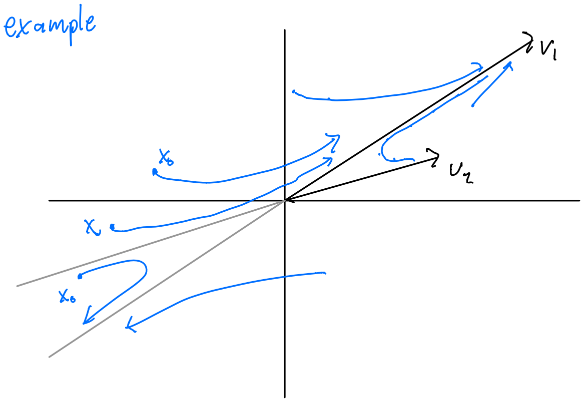


3) $A = \begin{bmatrix} 1.4 & 0 \\ 0 & 0.5 \end{bmatrix}$

$x \uparrow \quad y \downarrow$



Our example



$$X_{k+1} = \begin{bmatrix} L_{k+1} \\ H_{k+1} \end{bmatrix} = \begin{bmatrix} 0.45 & 0.54 \\ -0.2 & 1.3 \end{bmatrix} \begin{bmatrix} L_k \\ H_k \end{bmatrix}$$

$$\det(A - \lambda I)$$
$$= \det \begin{bmatrix} 0.45-\lambda & 0.54 \\ -0.2 & 1.3-\lambda \end{bmatrix}$$
$$= (0.45-\lambda)(1.3-\lambda) - (-0.2)(0.54)$$
$$= \lambda^2 - 1.75\lambda + 0.585 + 0.108$$
$$= \lambda^2 - 1.75\lambda + 0.693$$

$$\lambda = \frac{1.75 \pm \sqrt{(1.75)^2 - 4(1)(0.693)}}{2(1)}$$

$$\lambda_1 = 1.1445 \qquad \lambda_2 = 0.6055$$

$(A-\lambda_1 I)\vec{V}_1 = 0$

$(A - 1.1445 I)\vec{V}_1 = 0$

$$\begin{bmatrix} -0.6945 & 0.54 \\ -0.2 & 0.1555 \end{bmatrix} \vec{V}_1 = 0$$

$$\begin{bmatrix} -0.6945 & 0.54 \\ -0.6945 & 0.54 \end{bmatrix} \vec{V}_1 = 0$$

let $V_1 = \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}$

$$\begin{bmatrix} -0.6945 & 0.54 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \end{bmatrix} = 0$$

$-0.6945 a_1 + 0.54 a_2 = 0$

let $a_1 = 1$

$0.54 a_2 = 0.6945$

$a_2 = 1.2861$

$$V_1 = \begin{bmatrix} 1 \\ 1.2861 \end{bmatrix}$$

$(A-\lambda I)\vec{V} = 0$

$(A - 0.6055 I)\vec{V} = 0$

$$\begin{bmatrix} -0.1555 & 0.54 \\ -0.2 & 0.6945 \end{bmatrix} \vec{V}_2 = 0$$

$$\begin{bmatrix} -0.1555 & 0.54 \\ -0.1555 & 0.54 \end{bmatrix} \vec{V}_2 = 0$$

let $V_2 = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$

$$\begin{bmatrix} -0.1555 & 0.54 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = 0$$

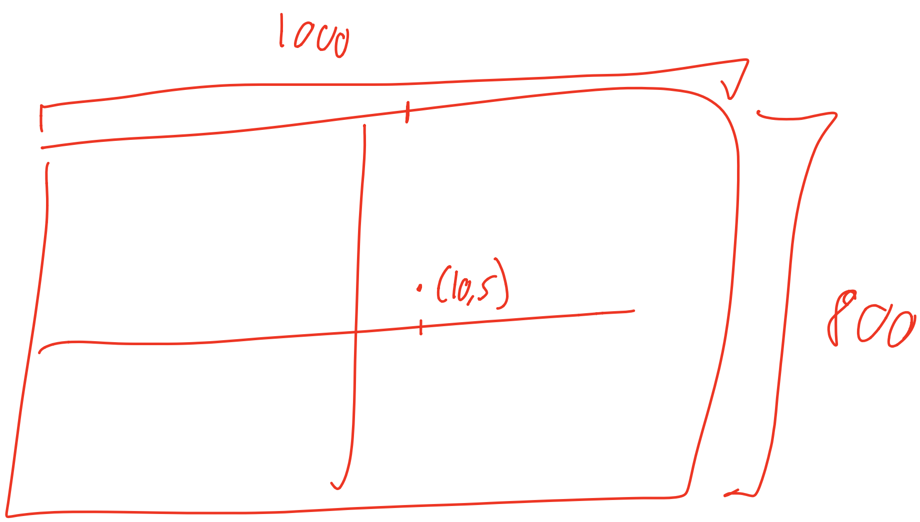
$-0.1555 b_1 + 0.54 b_2 = 0$

let $b_1 = 1$

$0.54 b_2 = 0.1555$

$b_2 = 0.2880$

$$V_2 = \begin{bmatrix} 1 \\ 0.2880 \end{bmatrix}$$



(510, 395)

(width/2 + x, heigh/2 - y)