A MPC method based on the oval invariant set for PMSM Speed Control System

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***Abstract*— A novel anti-windup model predictive control (MPC) method is proposed to deal with the controller output saturation for the nonlinear permanent magnet synchronous motor (PMSM) speed control system. The nonlinear PMSM speed control system is transformed into the time-varying linear system and the thresholds of system variables are converted to system constraints, furthermore, the linear MPC controller is designed to achieve anti-windup control with the system. At the same time, the oval invariant set is introduced to ensure the stability of the finite horizon MPC control system. With contrast to Anti-Windup PI, The simulation results verify that the MPC has excellent static and dynamic performance and strong robustness.**

***Keywords*— MPC; PMSM; oval invariant set; Anti-Windup**

I. INTRODUCTION

In PMSM speed control system, some constraints are paid close attention because of their affects on system performance, for example, the restrictions on the amplitude of voltage , current or torque [1]. Two commonly used methods to handle the constraints are Anti-Windup and MPC. Wherein, Anti-Windup method has some advantages such as a simple structure, offline design and very small amount of calculation [2]. However, MPC is more potential for the systems containing multivariable constraints and can obtain the optimal control law with solving optimization problems online.

The approximate linearization model of PMSM speed control system is used to design a linear MPC speed controller to meet the constraints on the speed and the current [3]. [4] shows the feedback linearization method is combined with MPC to achieve the speed control and meet current constraints. [5] proposed the piecewise feedback Anti-Windup method to design PID speed controllers and the overshoot of speed is effectively suppressed over a large dynamic range.

This paper presented a simple and accurate linearization method for PMSM speed control system, on this basis, a linear MPC controller to ensure the closed-loop stability of systems is designed by defining the terminal constraint set conditions. Finally, the performance comparison of MPC and Anti-Windup is fulfilled through simulation.

II. LINEAR TIME-VARIANT PMSM MODEL

The commonly used d-q model of a PMSM is giver in terms of its rotor reference frame as

$$\begin{cases} \dfrac{di_d}{dt} = -\dfrac{R_s}{L} i_d + \omega i_q + \dfrac{u_d}{L} \\ \dfrac{di_q}{dt} = -\dfrac{R_s}{L} i_q - \omega i_d - \dfrac{\Psi_r}{L}\omega + \dfrac{u_q}{L} \\ \dfrac{d\omega}{dt} = \dfrac{3p^2\Psi_r}{2J} i_q - \dfrac{B}{J}\omega - \dfrac{pT_l}{J} \end{cases} \tag{1}$$

where i_d , i_q , u_d and u_q are respectively the stator currents and the stator voltages in the d-q frame, R_s denoting the stator resistance, L denoting the stator inductance, ψ_r denoting the permanent magnet flux, B denoting the viscous friction coefficient, J denoting the moment of inertia, ω denoting the electrical speed and being related to the rotor speed by $\omega = p\omega_m$, with p denoting the number of pole pairs, ω_m denoting the mechanical speed, and T_l denoting the load torque. The system state variables and system input variables are defined as

$$\mathbf{x} = \left[i_d, i_q, \omega\right]^T ; \mathbf{u} = \left[u_d, u_q, \frac{pT_l}{J}\right]^T \tag{2}$$

(1) shows the nonlinear of PMSM speed control system mainly derives from the coupling between currents and electrical speeds on the d axis and the q axis respectively. Considering the much greater mechanical constant than the electrical time constant and very small sampling period, the speed variable can be considered as a constant, like $\omega = \omega_k$ and k denoting the sampling instant, over each sampling period. The real-time speed value will substitute for the speed constant at the beginning of each sampling period, so the time-vary linear model is established.

In order to ensure a sufficiently small dynamic error of the discrete current, the discrete-time constant must be enough small relative to the current dynamic. Meanwhile, in order to avoid the considerable computational complexity and numerical problems, the discrete-time constant is not too small and is defined as 100 μs . With the input and output constraints, the linear discrete model is gotten by forward Euler discrete law.

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$$
\mathbf{x}(k+1)=\begin{bmatrix}1-T\frac{R_s}{L} & T\omega_k & 0\\ -T\omega_k & 1-T\frac{R_s}{L} & -T\frac{\Psi_r}{L}\\ 0 & T\frac{3p^2\Psi_r}{2J} & 1-T\frac{B}{J}\end{bmatrix}\mathbf{x}(k)
$$

$$
+\,diag(\frac{T}{L},\frac{T}{L},-T)\mathbf{u}=A\mathbf{x}+B\mathbf{u} \tag{3}
$$

$$
\mathbf{y}(k)=E\mathbf{x}(k)=C\mathbf{x},\ \ |\mathbf{u}|\le U,\ \ |\mathbf{y}|\le Y
$$

where diag(.) denotes the diagonal matrix and E denotes identity matrix.

III. MPC DESIGN METHODOLOGY

For the speed control of PMSM drives , the main objective is that the electrical speed ω is able to accurately and quickly track the reference electrical speed ω_r , moreover, the stator currents and voltages don't exceed rated ones and maintain the optimal ones. By translating current system operating point into the coordinate origin, the objective can be represented as a quadratic performance index

$$
\mathrm{V}(\mathbf{x},k,\mathbf{u})=\sum_{i=k}^{k+N-1}\left\{\left\|\mathbf{Cx}(i\mid k)\right\|_{\mathbf{Q}}^2+\left\|\mathbf{u}(i\mid k)\right\|_{\mathbf{R}}^2\right\} \tag{4}
$$

where $\mathbf{x}(i\mid k)$、$\mathbf{u}(i\mid k), i=k,\cdots,k+N-1$ denote the N predicted values of the system state vector and the system manipulated vector (2) at sampling instant k, $\mathbf{Q}\in R^{n\times n},\mathbf{R}\in R^{m\times m},\mathbf{P}\in R^{n\times n}$ denoting the positive definite symmetric weight matrix, N denoting the length of the prediction horizon and the control horizon having same length as the prediction horizon. Based on the aforementioned objectives with the linear model(3), the MPC problem is formulated as

$$
\begin{aligned}&\min_{\mathbf{u}} V(\mathbf{x},k,\mathbf{u})\\ &s.t.\ \ (3),\ \ \mathbf{u}(k)\in U,\ \ \forall k=0,1,\cdots,n,\cdots\end{aligned} \tag{5}
$$

The linear model (3) is used to predict the effect of the future actions of the manipulated variables on the PMSM speed control system and U denoting the constraints on system variables. at sampling instant k, $\mathbf{x}(i\mid k)$ are expressed as the function of the initial state x(0) and $\mathbf{u}(i\mid k)$ and the x(0) are either measured or estimated using an observer.

By solving the open-loop optimization problem(5), the optimization manipulated variables are collected in the vector

$$
U^o(k)=\left\{\mathbf{u}^o(k\mid k),\cdots,\mathbf{u}^o(k+N-1\mid k)\right\} \tag{6}
$$

$\mathbf{u}(k\mid k)$ is applied to the system as the current manipulated variables. At sampling instant k+1, the initial state is date, and then repeat the presented process.

However the foregoing receding horizon optimization method can ensure system stability with the linear time-invariant system rather than the linear time-variant system.

IV. MPC LYAPUNOV STABILITY DESIGN

At present, the Lyapunov method is commonly used to research the system stability for the finite horizon MPC system[6]. Specifically, the optimization manipulated variables(6) are firstly gotten by solving the open-loop optimization problem(5) , drive the initial state x(0) into the invariant set O_∞ through N receding horizons and the terminal state x(k+N|k) is gotten. Then, the local linear state feedback manipulated variables u (k) = Kx(k) are designed to further drive the terminal state x(k+N|k) to the coordinate origin. Consequently, any initial system states are driven to the coordinate origin finally and ensure the system stability. Meanwhile, the quadratic performance index(4) and the MPC problem(5) transform the following forms respectively

$$
\begin{aligned}\mathrm{V}(\mathbf{x},k,\mathbf{u})=&\sum_{i=k}^{k+N-1}\left\{\left\|\mathbf{Cx}(i\mid k)\right\|_{\mathbf{Q}}^2+\left\|\mathbf{u}(i\mid k)\right\|_{\mathbf{R}}^2\right\}\\ &+\left\|\mathbf{x}(k+N\mid k)\right\|_{\mathbf{P}}^2\end{aligned} \tag{7}
$$

$$
\begin{aligned}&\min_{\mathbf{u}} V(\mathbf{x},k,\mathbf{u})\\ &s.t.\ \ (3),\ \ \ \mathbf{x}(k+N\mid k)\in O_\infty\end{aligned} \tag{8}
$$

where $\left\|\mathbf{x}(k+N\mid k)\right\|_{\mathbf{P}}^2$ denotes the terminal function which is approximately considered as the upper limit of the infinite horizon performance index function and ensure (7) to be bounded.

Now the closed-loop stability conditions with the oval invariant set is presented and proved.

Lemma1: Assume that (3) is stabilized, that is the linear state feedback manipulated variables $\mathbf{u}=K\mathbf{x}$ ensure $A_k := A+BK$ to be asymptotically stable, $\varepsilon=\left\{\mathbf{x}\in R^n \mid \mathbf{x}^T P\mathbf{x}\le 1\right\}$ denoting the oval set. Then, the closed-loop asymptotic stability of the controlled system can be guaranteed if the following conditions are met:

1) there exists a constant $\alpha\in(0,\infty)$, and the neighbourhood of the origin is determined as $O_\infty := \left\{\mathbf{x}\in R^n \mid \mathbf{x}^T P\mathbf{x}\le\alpha\right\}$ For all $\mathbf{x}\in O_\infty$, the linear state feedback manipulated variables meet the input control constraints, namely $K\mathbf{x}\in U$;

2) the positive definite symmetric matrix is uniquely determined by Lyapunov function $A_k{}^T P A_k - P = -Q^*$, in which $Q^* = Q + K^T R K$ is positive definite symmetric matrix;

3) O_∞ is the control invariant set of systems(3) which is associated with the linear state feedback manipulated variables $\mathbf{u}=K\mathbf{x}$;

4) $\Delta F(\mathbf{x},\mathbf{u})\le -l_1$, known as $F(\mathbf{x},\mathbf{u})=\left\|\mathbf{x}(k+N\mid k)\right\|_P^2$, is a local Lyapunov function.

Proof.

The proof is divided into four parts in order to show separately that the corresponding conclusions hold.

1) Because of $0 \in$ U, a sufficiently large constant $\alpha \in (0,\infty)$ can always be obtained for the determined P> 0, which ensures $\mathbf{x} \in O_\infty := \{\mathbf{x} \in R^n \mid \mathbf{x}^T P\mathbf{x} \le \alpha\}$, $K\mathbf{x} \in U$ hold.

2) According to the condition 1),when $\mathbf{x}(k+N \mid k) \in O_\infty$ holds, the manipulated variables $K\mathbf{x} \in U$ ensures A_k stability. the terminal function in(7) is reformulated as

$$\begin{aligned} \left\|\mathbf{x}(k+N \mid k)\right\|_P^2 &= \sum_{i=k+N}^{\infty}\left\{\left\|C\mathbf{x}(i \mid k)\right\|_Q^2 + \left\|\mathbf{u}(i \mid k)\right\|_R^2\right\} \\ &= \sum_{i=k+N}^{\infty} \mathbf{x}(i \mid k)^T \left(C^TQC + K^TRK\right)\mathbf{x}(i \mid k) \\ &= \mathbf{x}(k+N \mid k)^T \left[\sum_{i=0}^{\infty}\left(A_k^T\right)^i Q^* \left(A_k\right)^i\right]\mathbf{x}(k+N \mid k) \end{aligned} \tag{9}$$

Let $P = \sum_{i=0}^{\infty}\left(A_k^T\right)^i Q^* \left(A_k\right)^i$. Because A_k is asymptotically stable, the above series converges. Then, both sides of the equation multiply by A_k and the Lyapunov equation , known as $A_k^T P A_k = \sum_{i=1}^{\infty}\left(A_k^T\right)^i Q^* \left(A_k\right)^i = P - Q^*$, is obtained. Furthermore, because of $Q^* \succ 0$, based on the general conditions on the solvability of the Lyapunov equation that is the Lyapunov equation has the unique symmetric positive definite solution if all the eigenvalues of A_k have negative real parts, condition 2) holds.

3) along the state trajectory of $\mathbf{x}(k+1) = A\mathbf{x}(k) + BK\mathbf{x}(k)$, the difference of $\mathbf{x}^T P\mathbf{x}$ is solved and is represented as $\begin{aligned}&\mathbf{x}(k+1)^T P\mathbf{x}(k+1) - \mathbf{x}(k)^T P\mathbf{x}(k) \\ &= \mathbf{x}(k)^T\left(A_k^T P A_k - P\right)\mathbf{x}(k)\end{aligned}$. Based on the condition 2), $A_k^T P A_k - P = -Q^* \prec 0$ holds because $P \succ 0,\ Q^* \succ 0$ holds. These imply that the system state variables is decreasing in O_∞ and O_∞ is control invariant set [7] for the system (1) and the local linear state feedback manipulated variables $\mathbf{u} = K\mathbf{x}$. Condition 3) is proved.

4) Let $l(\mathbf{x}, K\mathbf{x}) = \mathbf{x}(k+N \mid k)^T Q^* \mathbf{x}(k+N \mid k)$, and $\begin{aligned}\Delta F(\mathbf{x}, K\mathbf{x}) &= \left\|\mathbf{x}(k+N+1 \mid k)\right\|_P^2 - \left\|\mathbf{x}(k+N \mid k)\right\|_P^2 \\ &= \mathbf{x}(k+N \mid k)^T\left(A_k^T P A_k - P\right)\mathbf{x}(k+N \mid k)\end{aligned}$.

According to condition 2), $\Delta F(\mathbf{x}, K\mathbf{x}) + l(\mathbf{x}, K\mathbf{x}) = 0,\ \forall \mathbf{x} \in O_\infty$ holds. Condition 4) is proved. □

In summary, the design of the stable MPC controller finally boils down to the calculation for the terminal function matrix P and the associated invariant set O_∞ of the controlled system.

V. SYSTEM DESIGN AND SIMULATION ANALYSIS

Based on the above conclusions, the simulation models of the MPC control system and the Anti-Windup PI control system are established in Matlab environment. The configuration chart of the Anti-Windup with single input and single output is shown as figure1.

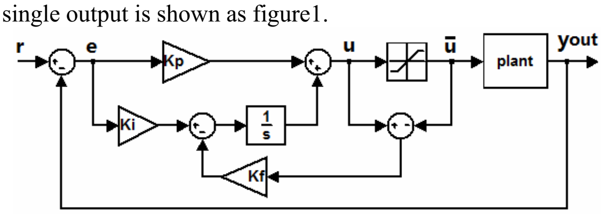
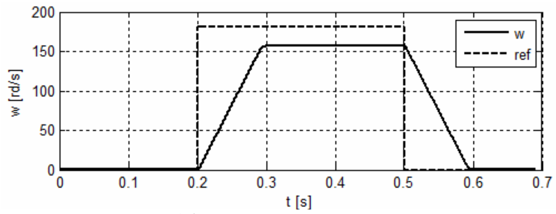


Fig.1 Algorithm structure of Anti-Windup

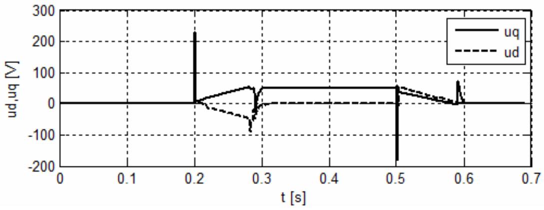
The parameters of the used PMSM are as follows: the rated torque is 28.4N.m, the rated current is 16.5A, the rated speed is 2355rpm, the phase resistance is 0.25 Ω, the phase inductance is 7mH, the rotor flux is 0.32V/(rad/s), the pole-pairs are 4, the total moment of inertia is 0.053kg.m^2, the friction coefficient is 0.005kg.m^2.s^{-1}, the DC bus voltage is 400V. According to the above values, the constraints of input variables and state variables are defined as $|\mathbf{u}| \le 230\ V, |\mathbf{i}| \le 23\ A, \omega \le 157\ rad/s$. The weight matrix of MPC are $Q = diag(100,1,30)$, $R = diag(1,1,1)$, the receding horizon is 10, the control horizon is 2 and the prediction step is T=100 μs.

Because the parameter ω_k of the system model (3) is updated at each sampling instant, the terminal function matrix P and the associated invariant set O_∞ is recalculated each time.

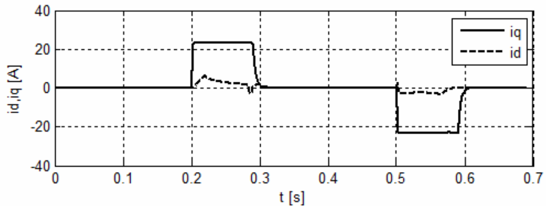
Figure2 and figure3 separately correspond to the system response of the MPC controller and the Anti-Windup controller , which are simulated with the square wave speed reference signal from 0 rad/s to 180rad/s and without the load disturbance and the friction disturbance. The simulation is designed to verify the ability of the two dealing with the system constraints.



（a）speed reference ref and speed w



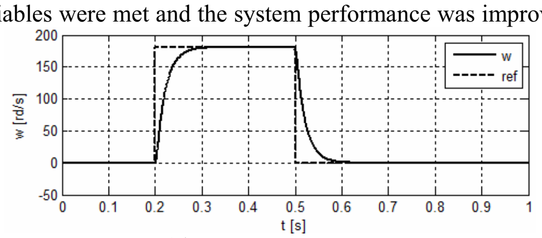
(b) d-axis voltage ud and q-axis voltage uq



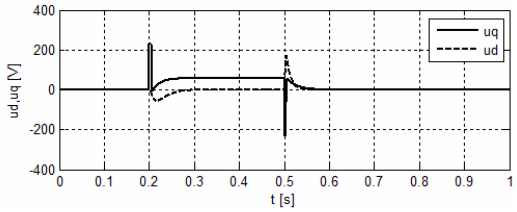
(c) d-axis current id and q-axis current iq

Fig.2 Responses of the MPC controller

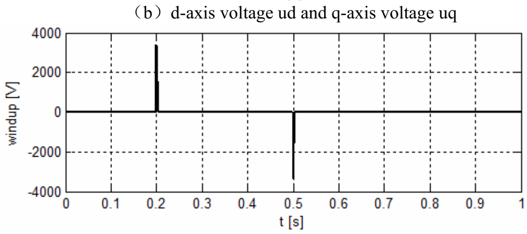
Figure2 shows that the input voltage uq reached the maximum value of the constraint conditions in the start-up phase and the q-axis current iq maintained the maximum value of the constraint conditions during the speed rise stage, so the PMSM started with the maximum starting current. Meanwhile, the speed variable subjected to the defined constraint conditions and reached the reference speed. In conclusion, the constraint conditions of the input and output variables were met and the system performance was improved.



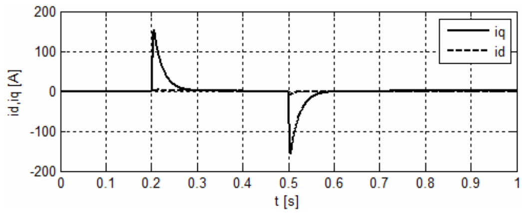
(a) speed reference ref and speed w



(b) d-axis voltage ud and q-axis voltage uq



(c) Anti-Windup feedback signal



(d) d-axis current id and q-axis current iq

Fig.3 Responses of the Anti-Windup controller

Contrary to the above results, figure3 shows that the input voltage uq reached saturation, and then the input voltage withdrew from the saturation state due to the role of the anti saturation compensator. It is different from figure2 that the rate of withdrawing from the saturation state was more slow . The reason is mainly that the saturation was compensated after the occurrence of saturation with the Anti-Windup controller. In addition, after the speed exceed the reference value r=180 rad/s, the PI controller was just out of the saturation state. Because the Anti-Windup controller had not effective ways to constraint the output variables on the basis of the defined constraint conditions, the values of the q-axis current and the speed are beyond the maximum value of the system constraints which is not allowed in practice.

VI. Conclusions

The time-variant linear model of the PMSM speed control system is proposed. Based on the analysis of the MPC control principle and the stability condition, a stable MPC control strategy is designed with the time-variant linear model. The new strategy not only improves the system performance, but also can deal with the constraint conditions of the system input and output variables simultaneously. By comparing the simulation results, the stable MPC controller is proved to have good processing ability for the multivariable and multi constraint conditions system and have the excellent dynamic performance and robustness.

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