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Terminal sliding mode control based on super-twisting algorithm

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Abstract: An improved non-singular terminal sliding mode control based on the super-twisting algorithm is proposed for a class of second-order uncertain nonlinear systems. This method can effectively avoid the singularity problem and obviously reduce the chattering phenomenon. The stability of the proposed procedure is proven to be finite-time convergence using the Lyapunov theory against uncertain unmodeled dynamic and external disturbances. An example is given to show the proposed improved non-singular terminal sliding mode control (SMC) law effectively.

Keywords: sliding mode control (SMC), super-twisting, finite-time convergence, second-order nonlinear system.

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1. Introduction

Sliding mode control (SMC) is well known for its fine robustness to system parameter variations and external disturbances [1 – 4]. SMC has been widely used in many fields, such as robotics, power systems, synchronization of chaotic systems, machine control, motion control, networked systems control, time-delay system and process control [5 – 8]. In spite of the conventional SMC claimed robustness properties, one of the drawbacks of SMC is the chattering phenomenon. Besides, the convergence of the system states to the equilibrium point for the conventional SMC is usually asymptotical. In order to overcome the two drawbacks, the higher-order SMC (HOSMC) method has been proposed which removes discontinuous control acts on its higher order time derivative [9 – 12]. The advantage of HOSMC imposing an rth order sliding mode requires the information of $s, \dot{s}, \ddot{s}, \ldots, s^{(r-1)}$. However, as a special case of second order sliding modes, we do not require the derivative information.

Compared to the conventional linear sliding modes control surface, the controller is used to constrain the system states motion into an asymptotically stable sliding mode surface manifold. Terminal sliding mode (TSM) control can offer various superior properties, for example, fast and finite-time convergence, high steady-state tracking precision. Recently, many papers are devoted to the theoretical research of TSM control. Reference [13] has achieved a continuous non-smooth TSM controller. The continuous non-smooth second-order fast TSM control results have been applied to the control of robot manipulators. However, the robustness cannot be fully addressed [14]. Moreover, because of the negative fractional existing in the TSM control, the singularity problem is caused around the equilibrium.

A discontinuous nonsingular TSM control method is developed to avoid this problem [15]. Some other methods such as backstepping based nonsingular TSM control [16] and derivative and integral TSM control [17]. References [18,19] proposed a global nonsingular terminal SMC strategy for nonlinear systems which can eliminate the singularity and guarantee the finite-time reachability of the systems to the TSM surface in the finite-time convergence.

However, to the best of authors' knowledge, there are many TSM control results to consider the singularity and finite-time convergence problem in design of control system [15 – 19]. Few terminal sliding mode control papers consider alleviating the chattering effect. In this paper, a novel TSM control law based on the super-twisting algorithm is designed in the presence of the unmodeled dynamics and external disturbances, which can effectively avoid the singularity problem and obviously alleviate the chattering phenomenon. The proof is based on recently proposed Lyapunov function. The current paper is extending the result of [15,19,20] to a class of second-order nonlinear uncertain systems.

The rest of this paper is organized as follows. In Sec-

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tion 2, the design of conventional TSM control is introduced. In Section 3, the novel TSM controller is developed under the existence of system unmodeled dynamics and external disturbances. Numerical simulations are performed to verify the effectiveness of the presented schemes in Section 4 and conclusion is made in the final section.

2. Design of conventional TSM control

A nonlinear second-order system with unmodeled dynamics and external disturbancces is

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = f(x) + g(x)u + d(t) \end{cases} \tag{1}$$

where the system states are denoted as vector $\boldsymbol{x} = [x_1, x_2]^{\mathrm{T}}$ and $d(t)$ denotes bounded disturbance.

Assumption 1 The uncertain nonlinear functions $f(x)$ and $g(x)$ are partitioned into two parts. One is the nominal part; the other is the uncertain bounded function. They can be expressed as follows:

$$f(x) = f_0(x) + \Delta f(x)$$

$$g(x) = g_0(x) + \Delta g(x). \tag{2}$$

Suppose the uncertain functions $\Delta f(x)$ and $\Delta g(x)$ are priori known bounded functions which are designed in the latter.

In the presence of the unmodeled dynamics and external disturbances, we design a non-singular TSM controller to stabilize the states of system (1) and reduce the chattering phenomenon.

A first-order terminal sliding variable of the conventional TSM is described [14] as follows:

$$s = x_2 + \beta x_1^{\frac{q}{p}} \tag{3}$$

where $\beta > 0$ and $p > q$ are positive odd integers.

The control law is designed as follows:

$$u = g^{-1}[-f - \beta x_1^{\frac{q}{p}-1} x_2 - (l_d + \eta) sign(s)] \tag{4}$$

where $\|d(x)\| \leqslant l_d$ and $sign(\cdot)$ denotes sign function.

The finite-time t_s which the system states can reach the sliding mode surface $s = 0$ is given by

$$t_s = \frac{p}{\beta(p-q)} |x_1(t_r)|^{1-\frac{q}{p}}. \tag{5}$$

The TSM control term containing $x_1^{\frac{q}{p}-1} x_2$ in (4) may cause a singularity problem in the reaching phase, because $x_2 \neq 0$ while $x_1 = 0$ cannot be sufficiently ensured.

A simple non-singular TSM (NTSM) is proposed in order to overcome the singularity problem [14] as follows:

$$s = x_1 + \beta x_2^{\frac{q}{p}}. \tag{6}$$

The control law is designed as

$$u = g^{-1}[-f - \beta \frac{q}{p} x_2^{2-\frac{p}{q}} - (l_d + \eta) sign(s)]. \tag{7}$$

These non-singular TSM methods are further studied in the second-order control systems, which have many advantages, such as non-singular, fast and finite-time convergence, robustness and accuracy [15–19]. However, chattering phenomenon may cause high-frequency control switching, in other words, dangerous high-frequency vibrations. In the next sections, we design a non-singular TSM control law based on the super-twisting algorithm to reduce the chattering effect keeping the main advantages of the standard TSM control in the second-order uncertain dynamic system (1).

3. Design of TSM controller using super-twisting method

We can describe NTSM surface as

$$s = x_2 + x_{aux} \tag{8}$$

$$\dot{x}_{aux} = \alpha_1 |x_1|^{\lambda_1 - 1} x_1 + \alpha_2 |x_2|^{\lambda_2 - 1} x_2 \tag{9}$$

where constant parameters should satisfy $0 < \lambda_i < 1$, $\alpha_i > 0, i = 1, 2$.

The time derivative of NTSM surface (8), (9) yields

$$\dot{s} = \dot{x}_2 + \dot{x}_{aux} \tag{10}$$

$$\dot{s} = f(x) + g(x)u + d(t) + \alpha_1 |x_1|^{\lambda_1 - 1} x_1 + \alpha_2 |x_2|^{\lambda_2 - 1} x_2. \tag{11}$$

First of all, the following assumption is made for the system (1).

Taking (2) into (11), we can get

$$\dot{s} = f_0(\boldsymbol{x}) + g_0(\boldsymbol{x})u + \alpha_1 |x_1|^{\lambda - 1} x_1 +$$

$$\alpha_2 |x_2|^{\lambda_2 - 1} + \Delta f(\boldsymbol{x}) + \Delta g(\boldsymbol{x})u + d(t). \tag{12}$$

Denote $\psi(\boldsymbol{x}, t) := \Delta f(\boldsymbol{x}) + \Delta g(\boldsymbol{x})u + d(t)$. Despite the control law designed in [15], [19] can ensure stability at the equilibrium point in finite-time; the chattering phenomenon is still serious. The following theory designs a TSM controller based super-twisting to weaken the chattering phenomenon.

A preliminary feedback is designed as follows:

$$u = g_0^{-1}[-f_0 - (\alpha_1 |x_1|^{\lambda_1 - 1} x_1 +$$

$$\alpha_2 |x_2|^{\lambda_2 - 1} x_2) + v]. \tag{13}$$

Using (13), we can rewrite (12) as

$$\dot{s} = \psi(x, t) + v. \tag{14}$$

Assumption 2 $\psi(x,t)$ in (14) is assumed to satisfy

$$\|\psi(x,t)\| \leqslant \delta\|s\|^{\frac{1}{2}}. \tag{15}$$

We design v based on the super-twisting method which was considered in [11] as

$$v = -k_1\|s\|^{-\frac{1}{2}} \cdot s + w \tag{16}$$

$$\dot{w} = -k_2\|s\|^{-\frac{1}{2}} \cdot s. \tag{17}$$

Using (14) and (16) one obtains

$$\dot{s} = -k_1\|s\|^{-\frac{1}{2}} \cdot s + w + \psi(x,t). \tag{18}$$

Remark 1 In [11], the sign function $sign(s)$ is replaced by $\dfrac{s}{\|s\|}$ which is equivalent to sign function.

Assumption 3 Solutions of the differential equation about NTSM surface (8), (9) with discontinuous right-hand side are supposed in the sense of Filippov.

Before the main analysis, we give a lemma to be used for design of controller and stability analysis in finite time.

Lemma 1 [20] The following system is considered:

$$\dot{x} = f(x), x(0) = x_0, x \in \mathbf{R}^n, f(0) = 0 \tag{19}$$

where $f(x) : \boldsymbol{D} \to \mathbf{R}^n$ is continuous on an open neighborhood $\boldsymbol{D} \subset \mathbf{R}^n$. Suppose there is a continuous positive definite function $V(x) : \boldsymbol{D} \to \mathbf{R}$, and exists positive constants $\eta > 0$ and $0 < \gamma < 1$, such that

$$\dot{V}(x) + \eta V^{\gamma}(x) \leqslant 0. \tag{20}$$

Then, the system (19) is locally finite-time stable. Depending on the initial state $x(0) = x_0$, the settling time T satisfies the following inequality as

$$T \leqslant \frac{V^{1-\gamma}(x_0)}{\eta(1-\gamma)}. \tag{21}$$

Especially, when $\boldsymbol{D} = \mathbf{R}^n$ and $V(x)$ is also radially unbounded, the state of system (19) is globally finite-time stable.

Theorem 1 Consider the system (1) with unmodeled dynamics and external disturbances. Suppose that Assumptions 1 – 3 are satisfied, the NTSM surface (8), (9) can ensure the establishment of a second order sliding mode with respect to s in finite-time. If we design the control law as (13) and v is defined as (16), (17), and then the following conditions are satisfied:

$$\begin{cases} k_1 > \max\left[\dfrac{(\mu+4\varepsilon^2+3\varepsilon\delta)^2+4\mu\delta+16\delta\varepsilon^2}{4\varepsilon(\mu+3\varepsilon)}, \dfrac{4}{3}\delta\right] \\ k_2 = \dfrac{\varepsilon}{2}k_1 \end{cases}. \tag{22}$$

The novel control based on the NTSM surface (8), (9) can ensure that the states of (1) converge to an equilibrium point in finite-time, where α, δ, μ, ε are arbitrary positive constants.

Proof A Lyapunov function is chosen as follows:

$$V = (\mu + 4\varepsilon^2) \cdot \|s\| + \|w\|^2 - 4\varepsilon\|s\|^{-\frac{1}{2}} \cdot w^{\mathrm{T}}s. \tag{23}$$

Define $\boldsymbol{X} := \left(\|s\|^{-\frac{1}{2}} \cdot s, w\right)^{\mathrm{T}}$. So, $V = \boldsymbol{X}^{\mathrm{T}}\boldsymbol{P}\boldsymbol{X}$, where $\boldsymbol{P} = \begin{pmatrix} \mu+4\varepsilon^2 & -2\varepsilon \\ -2\varepsilon & 1 \end{pmatrix}$. It is obviously that $\boldsymbol{P}$ is the positive definite matrix when $\mu > 0, \varepsilon > 0$.

The time derivative of the candidate Lyapunov function (23) is expressed as

$$\dot{V} = (\mu+4\varepsilon^2)\|s\|^{-1}s^{\mathrm{T}}\dot{s} - 2\varepsilon[-\frac{1}{2}\cdot\|s\|^{-\frac{5}{2}}(s^{\mathrm{T}}\dot{s})(w^{\mathrm{T}}s)+$$
$$\|s\|^{-\frac{1}{2}}(\dot{w}^{\mathrm{T}}s + w^{\mathrm{T}}\dot{s})] + 2w^{\mathrm{T}}\dot{w}^{\mathrm{T}}. \tag{24}$$

Substituting (17), (18) into (24), and through straightforward algebra we can obtain the following equation:

$$\dot{V} = -[(\mu+4\varepsilon^2)k_1 - 2\varepsilon k_2]\|s\|^{-\frac{1}{2}}-$$
$$[2k_2 + \varepsilon k_1 - (\mu+4\varepsilon^2)]\cdot\|s\|\cdot w^{\mathrm{T}}s + (\mu+4\varepsilon^2)\cdot$$
$$\|s\|^{-1}\cdot s^{\mathrm{T}}\psi + \varepsilon\cdot\|s\|^{-\frac{5}{2}}[\|w^{\mathrm{T}}s\| + (s^{\mathrm{T}}\psi)\cdot(w^{\mathrm{T}}s)]-$$
$$2\varepsilon\|s\|^{-\frac{1}{2}}(w^{\mathrm{T}}w + w^{\mathrm{T}}\psi).$$

We can obtain the bounding arguments with Cauchy-Schwarz inequality and Assumptions 3 as follows:

$$\dot{V} \leqslant -[(\mu+4\varepsilon^2)(k_1-\delta) - 2\varepsilon k_2]\cdot\|s\|^{\frac{1}{2}} - \varepsilon|s\|^{-\frac{1}{2}}\|w\|^2+$$
$$(\mu + 4\varepsilon^2 - 2k_2 + k_1\varepsilon + 3\varepsilon\delta)\|w\|. \tag{25}$$

k_2 is designed by $k_2 = \dfrac{\varepsilon}{2}k_1$. In order to obtain a positive definite matrix, we define $\boldsymbol{x} := \left(\|s\|^{\frac{1}{2}}, \|w\|\right)^{\mathrm{T}}$, where $\|\boldsymbol{X}\| = \|\boldsymbol{x}\|$.

The above inequality can change as

$$\dot{V} \leqslant -|s\|^{-\frac{1}{2}}\boldsymbol{x}^{\mathrm{T}}\boldsymbol{\Omega}\boldsymbol{x} \tag{26}$$

where

$$\boldsymbol{\Omega} := \begin{pmatrix} (\mu+3\varepsilon^2)k_1 - (\mu+4\varepsilon^2)\delta & -\dfrac{1}{2}(\mu+4\varepsilon^2+3\varepsilon\delta) \\ * & \varepsilon \end{pmatrix}.$$

In order to satisfy $\boldsymbol{\Omega} > 0$, we can choose k_1 and k_2 to fulfill conditions (22). The inequality (26) can be rewritten using Rayleigh's inequality and the relation between $\boldsymbol{x}$ and $\boldsymbol{X}$. One obtains

$$\dot{V} \leqslant -\lambda_{\min}(\boldsymbol{\Omega})\|s\|^{-\frac{1}{2}}\|\boldsymbol{x}\|^2 - \lambda_{\min}(\boldsymbol{\Omega})\|s\|^{-\frac{1}{2}}\|\boldsymbol{X}\|^2. \tag{27}$$

Using Rayleigh's inequality to V. One obtains

$$\lambda_{\min}(\boldsymbol{P})\|\boldsymbol{X}\|^2 \leqslant V = \boldsymbol{X}^{\mathrm{T}}\boldsymbol{P}\boldsymbol{X} \leqslant \lambda_{\max}(\boldsymbol{P})\|\boldsymbol{X}\|^2. \tag{28}$$

Using the definition of $\boldsymbol{X}$, $\|\boldsymbol{X}\|^2 = \|s\| + \|\omega\|^2 \geqslant \|s\|$, we get

$$\|s\|^{\frac{1}{2}} \leqslant \lambda_{\min}^{-\frac{1}{2}}(\boldsymbol{P})V^{\frac{1}{2}}. \tag{29}$$

Combining inequalities (27) – (29), we obtain

$$\dot{V} \leqslant -\eta V^{\frac{1}{2}} \tag{30}$$

where $\eta = \dfrac{\lambda_{\min}(\boldsymbol{\Omega}) \cdot (\lambda_{\min}(\boldsymbol{P}))^{\frac{1}{2}}}{\lambda_{\max}(\boldsymbol{P})}$. As $\boldsymbol{P}$ and $\boldsymbol{\Omega}$ are all positive definite matrices, we can conclude $\eta > 0$, which satisfies Lemma 1. From the above analysis, we can conclude that the sliding mode surface $s = 0$ which is defined in (8), (9) can be reached from anywhere in the finite-time. If $\|s\|^{\frac{1}{2}}$, $\|\omega\| \to 0$ in finite time, we can obtain that $s \to 0$ and $\dot{s} \to 0$ using the definition of x. Using Lemma 1, we can get that the settling time T satisfies the following inequality as

$$T \leqslant 2\eta^{-1}V^{\frac{1}{2}}(s(0), \omega(0)) \tag{31}$$

where (31) can assure the states of system converge to equilibrium point in finite time. Then, the requirements depicted in this theorem are satisfied to design as the control law (13), (16), (17). □

Remark 2 From the NTSM surface (8), (9) and the control law as (13), (16), (17), we can find that the $sign()$ function introduces an integral term, which can effectively reduce the chattering phenomenon.

4. Simulation

In order to analyze the effectiveness of the proposed improved NTSM control law, we consider the following simple second-order dynamical nonlinear system in [19]:

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = x_2^3 + 0.1\sin(20t) + u \end{cases}. \tag{32}$$

An NTSM surface chosen in [19] is as follows:

$$s = x_2 + x_1^{\frac{3}{5}}. \tag{33}$$

According to the description of [19], the following control law and control parameters are chosen as

$$u = -x_2^3 + sat(u_f, 3) - (0.1 + 5)sign(s) \tag{34}$$

where $u_f = \dfrac{3}{5}x_1^{\frac{3}{5}-1}x_2$. The initial values are chosen as $\boldsymbol{x}_0 = (-0.1, 2)$.

The NTSM surface is designed as (8), (9) in the proposed method, where parameters are chosen as $\alpha_1 = 2, \alpha_2 = 1.5, \lambda_1 = \lambda_2 = 0.5$. The NTSM control law based on the super-twisting algorithm is chosen as (13), (16), (17), where the control parameters are chosen as $k_1 = 5, k_2 = 1.5$. The simulation results are shown to compare the result between [19] and our proposed control algorithm in Fig. 1 – Fig. 6.

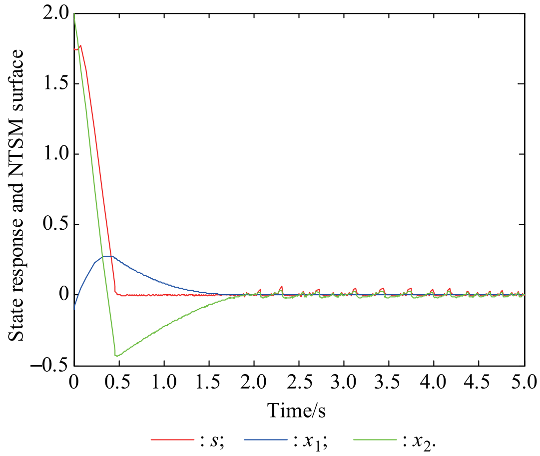

Fig. 1 System states of system (32) and NTSM surface in [19]

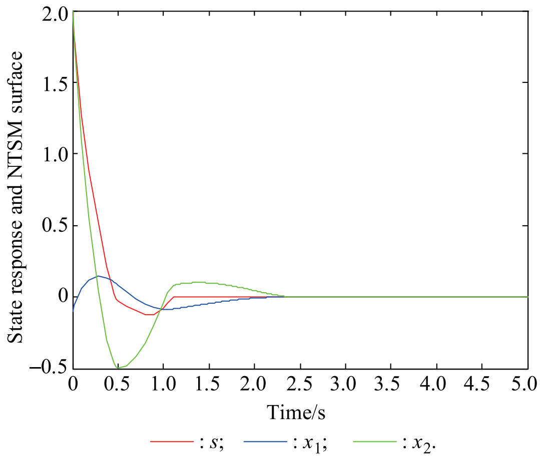

Fig. 2 System states of system (32) and NTSM surface in proposed control algorithm

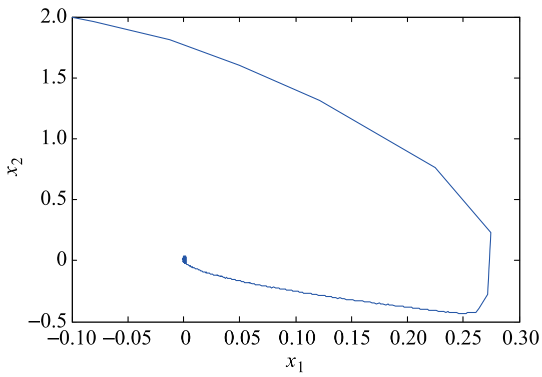

Fig. 3 Phase plot of NTSM states $\boldsymbol{x_1}$ and $\boldsymbol{x_2}$ based on NTSM control law in [19]

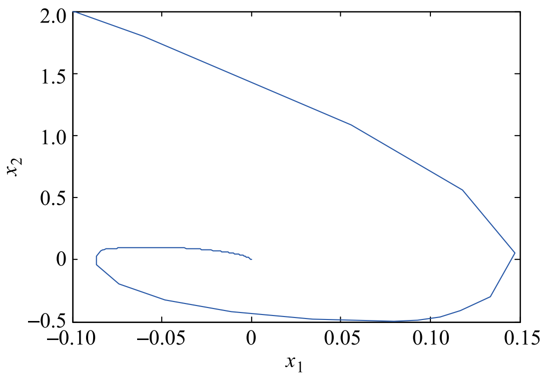

Fig. 4 Phase plot of NTSM states x_1 and x_2 in proposed control algorithm

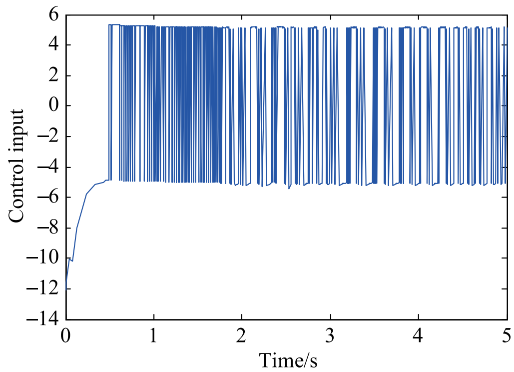

Fig. 5 Control input of system (32) based on NTSM control law in [19]

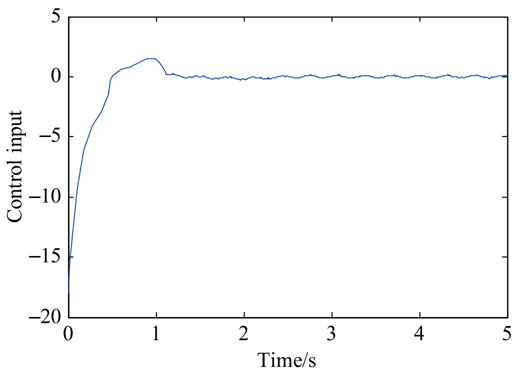

Fig. 6 Control input of system (32) in proposed control algorithm

Figs. 1 and 2 depict the two state variables of the second-order dynamical nonlinear system based on the two different NTSM control methods. We can see that the states of system (32) reach the NTSM surface in finite-time. By comparing the method in [19] with our proposed control method, the control effects are good. Figs. 3 and 4 display the phase plane response of second-order dynamical nonlinear system based on two different NTSM control methods, and the two different control NTSM inputs are shown in Figs. 5 and 6. We can obtain that the chattering phenomenon is reduced compared with [19], because the chattering phenomenon is caused by the $sign()$ function. We take the $sign()$ function into the integral term in order to reduce the chattering effect.

We can get that the system states reach the surface $s = 0$ in finite-time, and move to the origin along $s = 0$ in finite-time from Figs. 1 and 2 using the proposed control algorithm and [19]. The convergence times are more similar to two different methods. No singularity occurs from phase plot of Figs. 3 and 4, and the second-order dynamical nonlinear system states finally converge to the equilibrium point in finite-time. Thus, the proposed control law with the super-twisting algorithm is not singularity terminal sliding mode from Figs. 1 – 4.

The phase plane response of the improved NTSM control system is obtained in control law (14) and [15], as shown in Fig. 1. We can get that the system states reach the improved NTSM surface $s = 0$ in finite-time, and move to the origin along $s = 0$ in finite-time from Fig. 1. As can be seen, although there are external disturbances in the control system (32), the system states accuracy and fast convergence are still held, namely the external disturbances are well suppressed. From Figs. 5 and 6, we can see that the chattering problem in [19] is obviously reduced compared with the proposed method. Our proposed method keeps the main advantages of the standard TSM control and the chattering problem is obviously reduced.

5. Conclusions

In this paper, a second-order improved TSM control based super-twisting scheme for suppressing and stabilizing the unmodeled dynamic and external disturbances is proposed. The improved TSMC based on the super-twisting algorithm is proved using Lyapunov technique and holds robustness and stability in the presence unmodeled dynamic and external disturbance. The singularity problem can be effectively avoided in design of control law. This proposed control scheme can converge to zero in a finite-time and the chattering effect is obviously reduced. Numerical simulations and results have verified the effeciency of the proposed improved NTSM control law. The further study will be extended to industrial process control problem [21,22] and control input saturation problem [23,24], which will bring big challenges for TSM control design.

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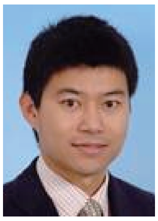
Biographies



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