Implementation of Super-Twisting Control: Super-Twisting and Higher Order Sliding-Mode Observer-Based Approaches

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***Abstract*—In this paper, an output feedback stabilization of perturbed double-integrator systems using super-twisting control (STC) is studied. It is shown that when STC is implemented based on super-twisting observer (STO), then it is not possible to achieve second-order sliding mode (SOSM) using continuous control on the chosen sliding surface. Two methodologies are proposed to circumvent the above-mentioned problem. In the first method, control input is discontinuous, which may not be desirable for practical systems. In the second method, continuous STC is proposed based on higher order sliding mode observer (HOSMO) that achieves SOSM on the chosen sliding surface. For simplicity, we are considering here only the perturbed double integrator, which can be generalized for an arbitrary-order systems. Numerical simulations and experimental validation are also presented to show the effectiveness of the proposed method.**

***Index Terms*—Higher order sliding-mode observer (HOSMO), super-twisting control (STC), super-twisting observer (STO).**

I. Introduction

SLIDING-MODE control (SMC) [1]–[15] is one of the most promising robust control techniques. It is able to reject bounded matched perturbation theoretically completely. The main disadvantage of SMC is chattering [1]–[3].

To avoid the chattering effect, several methodologies are proposed in sliding mode literature, super-twisting algorithm (STA) [16] is one among them. The STC plays a special role among the sliding mode controllers. Unlike other second-order sliding mode (SOSM) controllers, STC is applicable to a system (in general, any order) where control appears in the first derivative of the sliding variable. It has the following advantages:

1) compensates uncertainties/perturbations that are Lipschitz[1];
2) requires only information of the output (sliding variable) σ;
3) provides finite-time convergence to the origin for σ and $\dot{\sigma}$ simultaneously;
4) generates continuous control signal and, consequently, adjusts the chattering.

For example, if we want to apply STC on second-order mechanical system to adjust the chattering problem, then we have to design a sliding variable such that it has a relative degree one. If we choose a linear surface, then STC ensures the uncertainties/perturbations compensation, finite-time convergence to sliding variable and its derivative, but states converge *asymptotically* to the origin.

The sliding mode approach has been exceptionally successful in the design of state-feedback controllers. However, in most of the physical systems, an output is available for measurement. In that case, other states of the system can be obtained using an observer. The sliding mode observers are widely used due to the finite-time convergence, insensitive with respect to uncertainties and also the estimation of the uncertainty [21].

A new generation of observers based on the cascaded interconnection of the STA has been recently developed [18], [22]. The STA is a well-known SOSM algorithm introduced in [17] and it has been widely used for control, observation [22], and robust exact differentiation. Finite-time convergence and robustness for the STA has been proved by geometrical methods [16] and by means of Lyapunov-based approach [19], [20].

An output feedback finite-time stabilization for a double-integrator system using observer is already studied in [27]–[29]. The controller used in [27]–[29] are able to achieve finite-time stabilization of the states only when system is free from the disturbance. These controllers are not able to reject disturbance theoretically completely in the case of disturbance, so the states

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[1] First-order sliding mode can also compensate discontinuous and uniformly bounded uncertainties/perturbations.

will be remain bounded around origin in the case of disturbance and bound depends on magnitude of the disturbance.

Using output feedback, twisting control [26], and prescribed control law, it is possible to achieve finite-time stabilization of both the states of perturbed double-integrator system. But, the control input is discontinuous in nature and it generates chattering, which is undesirable. Using STC, we can stabilize both the states of the perturbed double-integrator asymptotically using continuous control. Due to continuous control, STC [30], [31] can adjust the chattering problem, which is good from the practical point of view.

A. Main Contribution

If only output is available for perturbed double-integrator system and STC is to be designed, then we need both states' information. Using STO, it is possible to estimate other state in finite time in the presence of disturbance. It has been shown in this paper that when STC is implemented based on STO, then it is not possible to achieve SOSM using continuous control on the chosen sliding surface. In this paper, two methodologies were proposed to circumvent the above problem. In the first method, the control input is discontinuous, which may not be desirable for practical systems. In the second method, continuous STC is proposed based on higher order sliding mode observer (HOSMO) that achieves SOSM on the chosen sliding surface. For simplicity, we are considering here only the perturbed double integrator, which can be generalized for an arbitrary-order systems. The initial idea of this work was reported in [23].

B. Structure of This Paper

This paper is organized as follows. Section II discusses the STC based on STO for perturbed double-integrator system. Section III details the super-twisting output feedback (STOF) control. HOSMO-based STC for perturbed double integrator is discussed in Section IV. Section V contains the application of the proposed method to an industrial plant emulator followed by the conclusion in Section VI.

II. STC Based on STO for Perturbed Double-Integrator System

Consider the dynamical system of the following form:

$$\begin{aligned}\dot{x}_1 &= x_2\\ \dot{x}_2 &= u + \rho_1\\ y &= x_1\end{aligned} \tag{1}$$

where y is an output of the system and ρ_1 is a disturbance. Only the output information is available here, most of the controller needs the all-state information, so first we reconstruct the other state of the system and then we design the STC based on the estimated information. STO is already reported in [22], using that, next we will show that the STC design based on STO does not have a mathematical justification.

The STO dynamics to estimate the states of the system (1) is given in the following form:

$$\begin{aligned}\dot{\hat{x}}_1 &= z_1 + \hat{x}_2\\ \dot{\hat{x}}_2 &= z_2 + u\end{aligned} \tag{2}$$

where z_1 and z_2 are the correction terms. Let, define the error variable as $e_1 = x_1 - \hat{x}_1$ and $e_2 = x_2 - \hat{x}_2$. The correction terms are selected as $z_1 = k_1|e_1|^{\frac{1}{2}}\text{sign}(e_1)$ and $z_2 = k_2\text{sign}(e_1)$. Then, we can represent error dynamics in the following form:

$$\begin{aligned}\dot{e}_1 &= -\,k_1|e_1|^{\frac{1}{2}}\text{sign}(e_1) + e_2\\ \dot{e}_2 &= -\,k_2\text{sign}(e_1) + \rho_1.\end{aligned} \tag{3}$$

It is assumed that $|\rho_1| < \Delta_0$. If we choose $k_1 = 1.5\sqrt{\Delta_0}$ and $k_2 = 1.1\Delta_0$, then both the errors e_1 and e_2 will go to zero simultaneously. The above equation's finite-time stability is already proved in [20] and [22] so we can conclude, both e_1 and e_2 will converge to zero in finite time $t > T_0$. Once the errors e_1 and e_2 are zero, we can say that $x_1 = \hat{x}_1$ and $x_2 = \hat{x}_2$ after finite time $t > T_0$.

Now, for the controller design here, an output of the system (1) has a relative degree two, therefore one cannot apply direct STC, because STC is applicable for only relative degree one system. Therefore, we have to define a sliding manifold of the following form to obtain a relative degree one:

$$\hat{s} = c_1 x_1 + \hat{x}_2, \quad \text{where } c_1 > 0. \tag{4}$$

To synthesize the control law (for designing STC), taking the time derivative of (4), then we can write as

$$\begin{aligned}\dot{\hat{s}} &= c_1\dot{x}_1 + \dot{\hat{x}}_2\\ \dot{\hat{s}} &= c_1 x_2 + u + k_2\text{sign}(e_1).\end{aligned} \tag{5}$$

Substituting the $x_2 = e_2 + \hat{x}_2$ in (5), we can further write as

$$\dot{\hat{s}} = c_1\hat{x}_2 + c_1 e_2 + u + k_2\text{sign}(e_1). \tag{6}$$

Now, transforming the system (1) in the co-ordinates of x_1 and $\hat{s}$ by using (4) and (6), which can be written as

$$\begin{aligned}\dot{x}_1 &= \hat{s} - c_1 x_1 + e_2\\ \dot{\hat{s}} &= c_1\hat{x}_2 + c_1 e_2 + u + k_2\text{sign}(e_1).\end{aligned} \tag{7}$$

Now, if we select the control u to obtain SOSM on $\hat{s}$ as

$$u = -c_1\hat{x}_2 - \lambda_1|\hat{s}|^{\frac{1}{2}}\text{sign}(\hat{s}) - \int_0^t \lambda_2\text{sign}(\hat{s})d\tau \tag{8}$$

where λ_1 and λ_2 are the designed parameters for the controller. This parameter can be selected as a procedure given in [17] and [20]. After substituting the control input (8) into (7), we can obtain

$$\begin{aligned}\dot{x}_1 &= \hat{s} - c_1 x_1 + e_2\\ \dot{\hat{s}} &= c_1 e_2 - \lambda_1|\hat{s}|^{\frac{1}{2}}\text{sign}(\hat{s}) - \int_0^t \lambda_2\text{sign}(\hat{s})d\tau + k_2\text{sign}(e_1).\end{aligned} \tag{9}$$

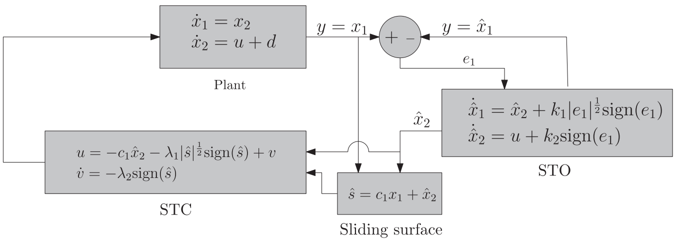

Fig. 1. Block diagram of the STC based on STO.

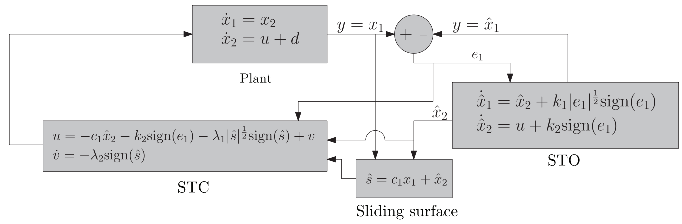

Fig. 2. Block diagram of Proposition 1 control based on STO.

The overall closed-loop system controller observer together can be represented as

$$\Pi : \begin{cases} \dot{x}_1 & = \hat{s} - c_1 x_1 + e_2 \\ \dot{\hat{s}} & = c_1 e_2 - \lambda_1 |\hat{s}|^{\frac{1}{2}} \text{sign}(\hat{s}) - \int_0^t \lambda_2 \text{sign}(\hat{s}) d\tau + k_2 \text{sign}(e_1) \end{cases}$$
$$\Xi : \begin{cases} \dot{e}_1 & = -k_1 |e_1|^{\frac{1}{2}} \text{sign}(e_1) + e_2 \\ \dot{e}_2 & = -k_2 \text{sign}(e_1) + \rho_1. \end{cases}$$

It is discussed earlier that the estimation error of system Ξ converges to zero in finite time, i.e., there exists a $T_0 > 0$ such that for all $t > T_0$, it follows that $e_1 = e_2 = 0$. Note that, the trajectories of system Π (above) cannot escape to infinity in finite time [26, Theorem 5.1]. Usually, observer gains are chosen in such a way that observation error converges faster. After finite time $t > T_0$, the closed-loop system can further be written as follows:

$$\begin{aligned} \dot{x}_1 &= \hat{s} - c_1 x_1 \\ \dot{\hat{s}} &= -\lambda_1 |\hat{s}|^{\frac{1}{2}} \text{sign}(\hat{s}) - \int_0^t \lambda_2 \text{sign}(\hat{s}) d\tau + k_2 \text{sign}(e_1). \end{aligned} \tag{10}$$

In another way, by adding some new fictitious state variable L, we can represent the dynamics as

$$\begin{aligned} \dot{x}_1 &= \hat{s} - c_1 x_1 \\ \dot{\hat{s}} &= -\lambda_1 |\hat{s}|^{\frac{1}{2}} \text{sign}(\hat{s}) + L + k_2 \text{sign}(e_1) \\ \dot{L} &= -\lambda_2 \text{sign}(\hat{s}). \end{aligned} \tag{11}$$

A. Discussion of the Above Mathematical Transformation

Now, we can conclude from the above mathematical transformation, SOSM never achieved in (11), because $\dot{\hat{s}}$ contains the nondifferentiable term $k_2\text{sign}(e_1)$, which prevents the possibility of lower two equations of (11) to act as the STA. Thus, second-order sliding motion (so that, $\hat{s} = \dot{\hat{s}} = 0$ in finite time) never begins. The block diagram of STC based on STO for system (1) is given in Fig. 1.

Next, we are going to propose the possible methodology of the control design such that nondifferentiable term $k_2\text{sign}(e_1)$ cancels out and then the lower two subsystems of (11) act as the STA and finally SOSM is established. For this purpose, control is selected according to the following proposition,

Proposition 1: The following control input leads to the establishment SOSM in finite time for (7), which further implies asymptotic stability of x_1 and x_2

$$u = -c_1 \hat{x}_2 - k_2 \text{sign}(e_1) - \lambda_1 |\hat{s}|^{\frac{1}{2}} \text{sign}(\hat{s}) - \int_0^t \lambda_2 \text{sign}(\hat{s}) d\tau \tag{12}$$

where $\lambda_1 > 0$ and $\lambda_2 > 0$.

Proof: The system dynamics after substituting (12) into (7)

$$\begin{aligned} \dot{x}_1 &= \hat{s} - c_1 x_1 + e_2 \\ \dot{\hat{s}} &= c_1 e_2 - \lambda_1 |\hat{s}|^{\frac{1}{2}} \text{sign}(\hat{s}) - \int_0^t \lambda_2 \text{sign}(\hat{s}) d\tau. \end{aligned} \tag{13}$$

Now, the overall closed-loop system can be represented as

$$\begin{aligned} &\Pi_1 : \begin{cases} \dot{x}_1 & = \hat{s} - c_1 x_1 + e_2 \\ \dot{\hat{s}} & = c_1 e_2 - \lambda_1 |\hat{s}|^{\frac{1}{2}} \text{sign}(\hat{s}) - \int_0^t \lambda_2 \text{sign}(\hat{s}) d\tau \end{cases} \\ &\Xi : \begin{cases} \dot{e}_1 & = -k_1 |e_1|^{\frac{1}{2}} \text{sign}(e_1) + e_2 \\ \dot{e}_2 & = -k_2 \text{sign}(e_1) + \rho_1. \end{cases} \end{aligned} \tag{14}$$

The observer error of system Ξ converges to zero in finite time, which is discussed earlier. Note that, the trajectories of system Π_1 (above) cannot escape to infinity in finite time [26, Theorem 5.1]. So, we can substitute $e_2 = 0$, we can further write the closed-loop system as

$$\begin{aligned} \dot{x}_1 &= \hat{s} - c_1 x_1 \\ \dot{\hat{s}} &= -\lambda_1 |\hat{s}|^{\frac{1}{2}} \text{sign}(\hat{s}) + \nu \\ \dot{\nu} &= -\lambda_2 \text{sign}(\hat{s}) \end{aligned} \tag{15}$$

where $\nu = -\int_0^t \lambda_2 \text{sign}(\hat{s}) d\tau$. The last two equations of (15) have the structure same as STA. Therefore, one can easily conclude that after finite time $t > T_1$, $\hat{s} = \dot{\hat{s}} = 0$. Later, the remaining system dynamics can be written as

$$\begin{aligned} \dot{x}_1 &= -c_1 x_1 \\ x_2 &= -c_1 x_1. \end{aligned} \tag{16}$$

Therefore, both the states x_1 and x_2 are asymptotically stable by choosing $c_1 > 0$. The block diagram of the controller (12) based on STO is shown in Fig. 2. ∎

It is clear from the mathematical derivation of the control input (12) that, if one can use STO to estimate the state of the second-order uncertain system (1) and then design STC by selecting sliding manifold as (4), the control becomes discontinuous, because it contains the discontinuous term $k_2\text{sign}(e_1)$. Therefore, continuous control design based on STO–STC is not possible. It is also to be noted that in this method only

$|\rho_1| < \Delta_0$ is needed. Before going to propose the solution of the above-mentioned problem, it is necessary to discuss the existing methodology of super-twisting output feedback (STOF) control.

III. Super-Twisting Output Feedback Control

In the existing references, it is reported that consider the following sliding surface first:

$$s = c_1 x_1 + x_2 \tag{17}$$

assuming that the entire state vector is available. After that for realizing the control expression based on STA, take the first time derivative of sliding surface s (17)

$$\dot{s} = c_1 \dot{x}_1 + \dot{x}_2. \tag{18}$$

Substituting $\dot{x}_1$ and $\dot{x}_2$ from (1) into (18), it follows that:

$$\dot{s} = c_1 x_2 + u + \rho_1. \tag{19}$$

Now, select the control input u as

$$u = -c_1 x_2 - \lambda_1 |s|^{\frac{1}{2}} \text{sign}(s) - \int_0^t \lambda_2 \text{sign}(s) d\tau \tag{20}$$

assuming both the states are available for measurement. After substituting the control input (20) into (19), one can write

$$\dot{s} = -\lambda_1 |s|^{\frac{1}{2}} \text{sign}(s) - \int_0^t \lambda_2 \text{sign}(s) d\tau + \rho_1 \tag{21}$$

or

$$\begin{aligned} \dot{s} &= -\lambda_1 |s|^{\frac{1}{2}} \text{sign}(s) + z \\ \dot{z} &= -\lambda_2 \text{sign}(s) + \dot{\rho}_1 \end{aligned} \tag{22}$$

where $z = \nu_1 + \rho_1$ and $\nu_1 = -\int_0^t \lambda_2 \text{sign}(s) d\tau$. It is assumed that $|\dot{\rho}_1| < \Delta_1$. By selecting $\lambda_1 = 1.5\sqrt{\Delta_1}$ and $\lambda_2 = 1.1\Delta_1$ according to [17] and [20], which leads to SOSM in finite time on s. Once $s = \dot{s} = 0$, then x_1 and x_2 both converge to zero asymptotically, which is discussed in the earlier section.

The control input (20) is based on full-state information, so we cannot implement it directly on system (1) because we do not have the information of x_2. If we use STO to estimate the $\hat{x}_2$ and using it in control (20) by replacing x_2 with its estimated value $\hat{x}_2$, then the control input (20) becomes

$$u = -c_1 \hat{x}_2 - \lambda_1 |\hat{s}|^{\frac{1}{2}} \text{sign}(\hat{s}) - \int_0^t \lambda_2 \text{sign}(\hat{s}) d\tau \tag{23}$$

where $\hat{s} = c_1 x_1 + \hat{x}_2$. If the controller (23) is applied to (1), then it is not possible to get SOSM on the chosen surface. It is already shown in Section II that control input (8), which is same as (23) is not able to achieve SOSM on the sliding surface $\hat{s}$. If the control is applied to the system, then system becomes (11) where the discontinuous term is present in the first derivative of the sliding surface, which prevents the SOSM on the chosen surface. So, the method is not mathematically correct to obtain SOSM on chosen sliding surface.

Remark 1: If we use this method for practical implementation, it may work (which may not be true for all system), because most of the time controller implemented digitally through computer. It means that controller is implemented at some fix sampling time, so the value of discontinuous term $k_2 \text{sign}(e_1)$ will be constant during sampling interval.

In STOF control approach, one has to choose STO gains k_1 and k_2 based on the upper bound of the disturbance and STC gains λ_1 and λ_2 based on the upper bound of the derivative of the disturbance.

Now, in the following section, we are going to propose the new strategy that gives the correct way to implement STC, when only output information of the perturbed double integrator (1) is available.

IV. HOSMO-Based STC for Perturbed Double-Integrator System

The dynamics of HOSMO to estimate the states of perturbed double-integrator system (1) is given as

$$\begin{aligned} \dot{\hat{x}}_1 &= \hat{x}_2 + z_1 \\ \dot{\hat{x}}_2 &= \hat{x}_3 + u + z_2 \\ \dot{\hat{x}}_3 &= z_3 \end{aligned} \tag{24}$$

where z_1, z_2, and z_3 are the correction terms. For simplicity, initial value of $\hat{x}_3$ is assumed to be a zero. Let us define the error variable $e_1 = x_1 - \hat{x}_1$ and $e_2 = x_2 - \hat{x}_2$. The correction terms are defined as $z_1 = k_1 |e_1|^{\frac{2}{3}} \text{sign}(e_1)$, $z_2 = k_2 |e_1|^{\frac{1}{3}} \text{sign}(e_1)$, and $z_3 = k_3 \text{sign}(e_1)$, where k_1, k_2, and k_3 are the positive constant. Then, error dynamics can be written as

$$\begin{aligned} \dot{e}_1 &= -k_1 |e_1|^{\frac{2}{3}} \text{sign}(e_1) + e_2 \\ \dot{e}_2 &= -k_2 |e_1|^{\frac{1}{3}} \text{sign}(e_1) - \hat{x}_3 + \rho_1 \\ \dot{\hat{x}}_3 &= k_3 \text{sign}(e_1). \end{aligned} \tag{25}$$

Now, defining the new variable as $e_3 = -\hat{x}_3 + \rho_1$. We also assumed that disturbance ρ_1 is a Lipschitz and $|\dot{\rho}_1| < \Delta_1$. Then, we can further rewrite (25) as

$$\begin{aligned} \dot{e}_1 &= -k_1 |e_1|^{\frac{2}{3}} \text{sign}(e_1) + e_2 \\ \dot{e}_2 &= -k_2 |e_1|^{\frac{1}{3}} \text{sign}(e_1) + e_3 \\ \dot{e}_3 &= -k_3 \text{sign}(e_1) + \dot{\rho}_1. \end{aligned} \tag{26}$$

The above equation is finite-time stable that is already proved in [24] and [25], so we can conclude that e_1, e_2, and e_3 will converge to zero in finite time $t > T_2$, by selecting the appropriate gains k_1, k_2, and k_3 [18]. After the convergence of error, one can find that $x_1 = \hat{x}_1$, $x_2 = \hat{x}_2$, and $\hat{x}_3 = \rho_1$ after finite time $t > T_2$.

Now, we will design an STC based on the estimated state information for system (1). For that, consider the same sliding surface (4) and taking its time derivative, it follows that:

$$\begin{aligned} \dot{\hat{s}} &= c_1 \dot{x}_1 + \dot{\hat{x}}_2 \\ \dot{\hat{s}} &= c_1 \hat{x}_2 + c_1 e_2 + u + k_2 |e_1|^{\frac{1}{3}} \text{sign}(e_1) + \int_0^t k_3 \text{sign}(e_1) d\tau. \end{aligned} \tag{27}$$

Representing system (1) in the co-ordinate of x_1 and $\hat{s}$ by using (4) and (27), then we can write the dynamics as

$$\dot{x}_1 = \hat{s} - c_1 x_1 + e_2$$
$$\dot{\hat{s}} = c_1 \hat{x}_2 + c_1 e_2 + u + k_2 |e_1|^{\frac{1}{3}} \operatorname{sign}(e_1) + \int_0^t k_3 \operatorname{sign}(e_1) d\tau. \tag{28}$$

The main aim here is to design a continuous control u, such that the SOSM occurs in finite time on the sliding surface. For this purpose, control is selected according to the following proposition.

Proposition 2: The following control input leads to the establishment of SOSM on $\hat{s}$ in finite time, once $\hat{s} = 0$ it further implies asymptotic stability of x_1 and x_2

$$u = -c_1 \hat{x}_2 - k_2 |e_1|^{\frac{1}{3}} \operatorname{sign}(e_1) - \int_0^t k_3 \operatorname{sign}(e_1) d\tau - \lambda_1 |\hat{s}|^{\frac{1}{2}} \operatorname{sign}(\hat{s}) - \int_0^t \lambda_2 \operatorname{sign}(\hat{s}) d\tau \tag{29}$$

or

$$u = -c_1 \hat{x}_2 - \int_0^t k_3 \operatorname{sign}(e_1) d\tau - \lambda_1 |\hat{s}|^{\frac{1}{2}} \operatorname{sign}(\hat{s}) - \int_0^t \lambda_2 \operatorname{sign}(\hat{s}) d\tau \tag{30}$$

where $\lambda_1 > 0$ and $\lambda_2 > 0$.

Proof: Substituting the control input (29) in (28), we can obtain

$$\dot{x}_1 = \hat{s} - c_1 x_1 + e_2$$
$$\dot{\hat{s}} = c_1 e_2 - \lambda_1 |\hat{s}|^{\frac{1}{2}} \operatorname{sign}(\hat{s}) + v$$
$$\dot{v} = -\lambda_2 \operatorname{sign}(\hat{s}). \tag{31}$$

Now, the overall closed-loop system can be represented as

$$\Pi_2 : \begin{cases} \dot{x}_1 &= \hat{s} - c_1 x_1 + e_2 \\ \dot{\hat{s}} &= c_1 e_2 - \lambda_1 |\hat{s}|^{\frac{1}{2}} \operatorname{sign}(\hat{s}) + v \\ \dot{v} &= -\lambda_2 \operatorname{sign}(\hat{s}) \end{cases}$$
$$\Xi_1 : \begin{cases} \dot{e}_1 &= -k_1 |e_1|^{\frac{2}{3}} \operatorname{sign}(e_1) + e_2 \\ \dot{e}_2 &= -k_2 |e_1|^{\frac{1}{3}} \operatorname{sign}(e_1) + e_3 \\ \dot{e}_3 &= -k_3 \operatorname{sign}(e_1) + \dot{\rho}_1. \end{cases} \tag{32}$$

It is discussed earlier that the estimation error of system Ξ_1 converges to zero in finite time. Note that, the trajectories of system Π_2 (above) cannot escape to infinity in finite time [26, Theorem 5.1]. So, we can substitute $e_1 = e_2 = 0$. Once the error becomes zero, the closed-loop system is given by the following expression:

$$\dot{x}_1 = \hat{s} - c_1 x_1$$
$$\dot{\hat{s}} = -\lambda_1 |\hat{s}|^{\frac{1}{2}} \operatorname{sign}(\hat{s}) + v$$
$$\dot{v} = -\lambda_2 \operatorname{sign}(\hat{s}). \tag{33}$$

The lower two equation of (33) is an STA, by selecting appropriate gains $\lambda_1 > 0$ and $\lambda_2 > 0$, then $\hat{s} = \dot{\hat{s}} = 0$ in finite time, which further implies that the closed-loop system is given as

$$\dot{x}_1 = -c_1 x_1$$
$$x_2 = -c_1 x_1. \tag{34}$$

Therefore, both the states x_1 and x_2 are asymptotically stable by choosing $c_1 > 0$. The block diagram for super-twisting control (STC) based on HOSM observer is depicted in Fig. 3. ■

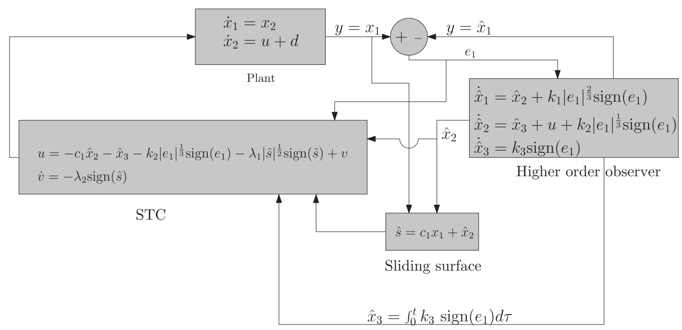

Fig. 3. Block diagram of the STC based on HOSM observer.

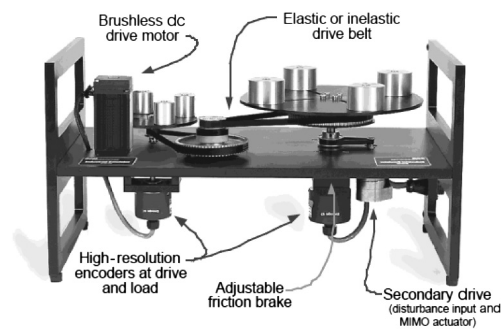

Fig. 4. Industrial emulator setup [32].

A. Discussion of HOSMO-Based STC Design

It is clear from the STC control (29) and (30) expression based on HOSMO (24) is continuous. Also, when we design STC control based on HOSMO, then one has to tune only the observer gains, according to the first derivative of disturbance, because it is necessary for the convergence of the error variables of the HOSMO. However, during controller design, there is no explicit gain condition for the λ_2 with respect to the disturbances.

One can also observe that super-twisting output feedback controller (23) design based on super-twisting observer (STO), requires two gains, one is STO gain means observer gain k_2 based on the explicit maximum bound of the direct disturbance and another is λ_2, STC gain based on the maximum bound of the derivative of disturbance.

Therefore, one can conclude from the above observation that sound mathematical analysis reduces the two gains' conditions with respect to disturbance by simply one gain condition. Also, the precision of the sliding manifold is much improved by using the HOSMO-based STC rather than STO-based STC. Due to

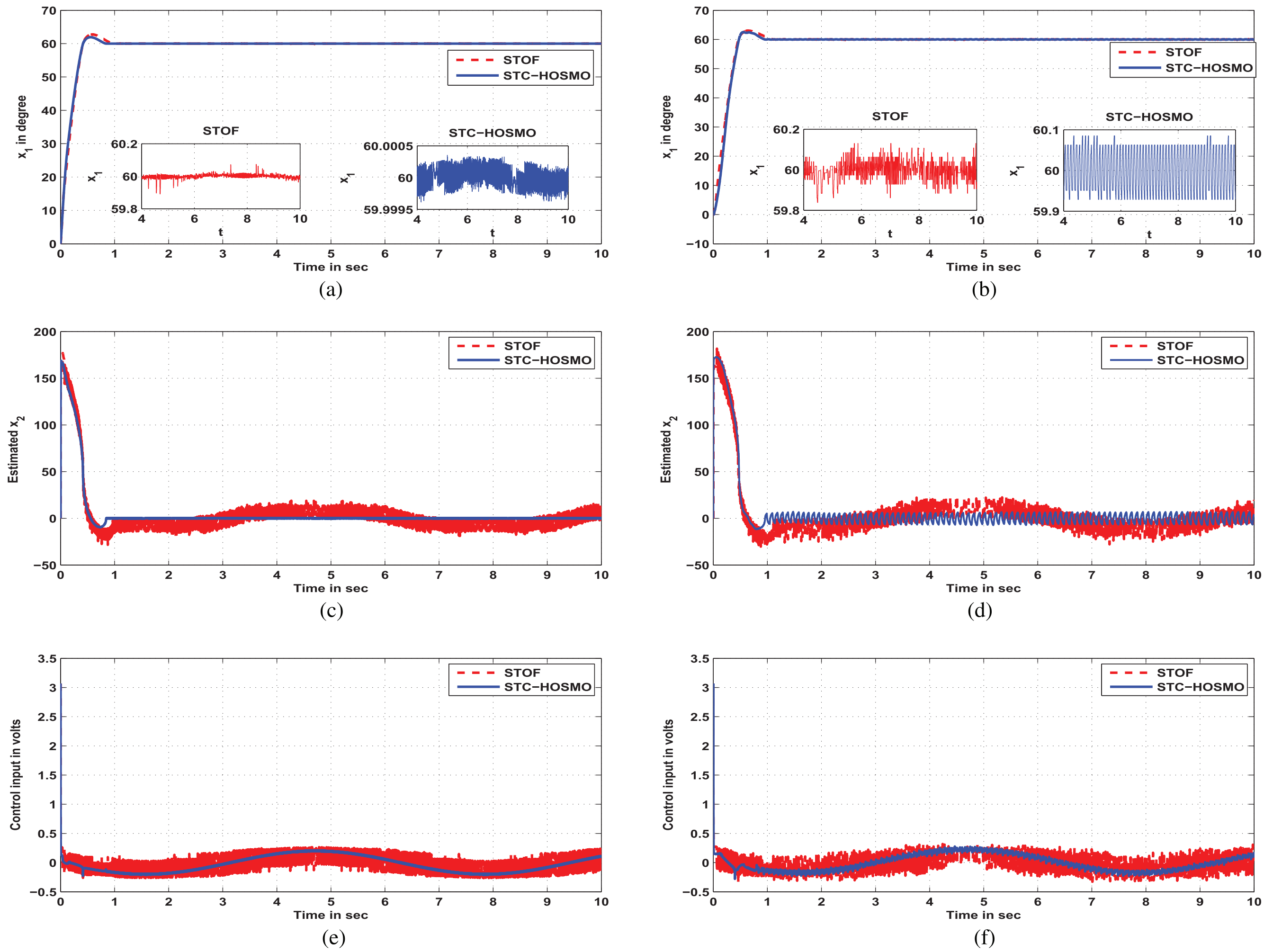

Fig. 5. Simulation and experimental results: industrial plant emulator. (a) Evolution of position (simulation). (b) Evolution of position (experimental). (c) Evolution of estimated state x_2 (simulation). (d) Evolution of estimated state x_2 (experimental). (e) Evolution of control input u (simulation). (f) Evolution of control input u (experimental).

the increase of this precision of sliding variable precision of the states are also much affected. In other words, if we talk about stabilization problem, then states are much closer to the origin in the case of HOSMO-based STC rather than STO-based STC. We only talk about the closeness of state variables with respect to an equilibrium point, because only asymptotic stability is possible in both the design methodology.

V. Application of HOSMO-Based STC for the Position Control of Industrial Emulator

The industrial emulator system shown in Fig. 4 is an electromechanical system that represents the important classes of systems such as conveyors, machine tools, spindle drives, and automated assembly machines. The setup is provided by Educational Control Products (ECPs) [32], California, USA. The system consists of a drive disk that is driven through a drive motor (servo actuator). The drive disk is coupled to the drive motor through a timing belt. The motion of the drive disk is transferred to another disk called load disk that is used to load the system. The motion from the drive disk to load disk is transferred through a speed reduction assembly (idler pulleys) and a timing belt. The load and the drive disk inertias are adjustable. High-resolution encoder is used to measure the position of the drive disk and load disk. The drive motor is driven by a servo amplifier. The system equation in state-space form is given by

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix} = \begin{bmatrix}0 & 1\\ 0 & -8.4344\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix} + \begin{bmatrix}0\\ 458.46\end{bmatrix}(u+d)$$
$$y = \begin{bmatrix}1 & 0\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix} \tag{35}$$

where x_1 and x_2 are the angular position and the angular velocity of the load disk, u is the input voltage to the drive motor, and d is the disturbance voltage signal injected externally to perturb the plant. For the simulation and experimental results, $d = 0.2\sin(t)$ is chosen. The control input in Proposition 1 is discontinuous, which is not good for practical setup, so we have compared the two control techniques STOF and STC based on HOSMO for the implementation. The controller and observer gains are selected as follows:

1) STOF
   a) STC gains $\lambda_1 = 2$ and $\lambda_2 = 2$.
   b) STO gains $k_1 = 1.5\sqrt{m}$ and $k_2 = 1.1m$, where $m = 100$.

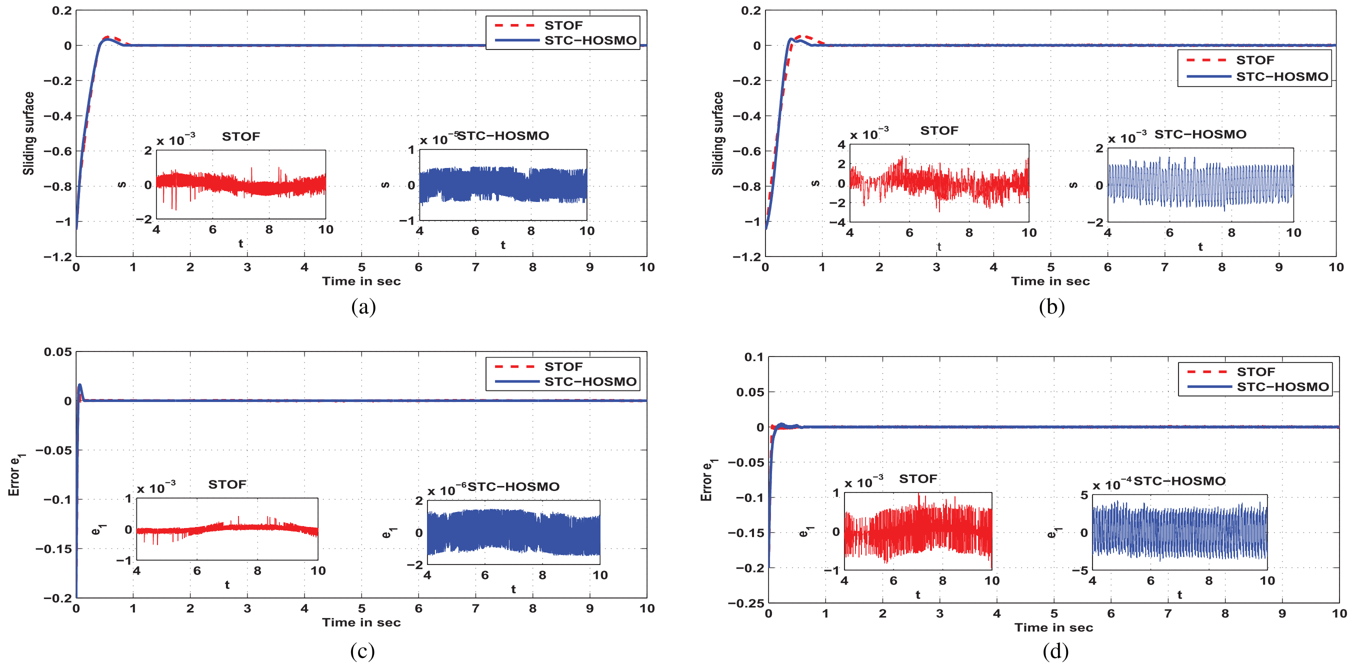

Fig. 6. Simulation and experimental results: industrial plant emulator. (a) Evolution of sliding surface $\hat{s}$ (simulation). (b) Evolution of sliding surface $\hat{s}$ (experimental). (c) Evolution of observer error e_1 (simulation). (d) Evolution of observer error e_1 (experimental).

2) STC-HOSMO
   a) STC gains $\lambda_1 = 2$ and $\lambda_2 = 2$.
   b) HOSMO gains $k_1 = 6n^{\frac{1}{3}}$, $k_2 = 11n^{\frac{1}{2}}$, and $k_3 = 6n$, where $n = 50$.

The sliding surface is chosen as $\hat{s} = x_1 + \frac{1}{458.46}\hat{x}_2$. The controller and observer are implemented in MATLAB with a sampling time of 1 ms. Control objective is to bring the load disk from zero position to the desired position, we have selected $60°$ as the desired position. The simulation and experimental results comparison of STOF and STC-HOSMO methods are illustrated in Figs. 5 and 6. One can see that both the method achieves desired load disk position, but from zoom version, we can say that precision of position in STC-HOSMO is more compared to STOF. The experimental results for the same are shown in Fig. 5(b). Fig. 5(c) and (d) shows the simulation and experimental results of the estimated states using STO and HOSMO. One can see that the estimated state using HOSMO is more correct than the STO. The control input plot in the simulation and experimental results is depicted in Fig. 5(e) and (f), respectively.

From the observation, one can say that the control input is more smoother in the case of STC-HOSMO compared to STOF. The sliding surface simulation plot is shown in Fig. 6(a), one can see that precision of the sliding variable (in zoom plot) is more (up to the order of τ^2) in the case of STC-HOSMO, while in the case of STOF, precision is only the order of τ. So, we can say that in STOF method, we are not able to obtain exact SOSM, but in STC-HOSMO, we are getting the exact SOSM on the sliding surface. The plot of sliding surface in experiment is depicted in Fig. 6(b). The evolution of observer error in simulation and experiment is shown in Fig. 6(c) and (d). The precision of error in HOSMO is more compared to STO that we can see from the zoom version.

VI. Conclusion

It has been shown in this paper that when STC is implemented based on STO, then it is not possible to achieve SOSM using continuous control on the chosen sliding surface. Two methodologies were proposed to circumvent the above-mentioned problem. For simplicity, only the perturbed double integrator was considered here, which can be generalized for an arbitrary-order systems. Numerical simulations and experimental validation were also presented to show the effectiveness of the proposed method.

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