Robust Parameter Estimation of Nonlinear Systems Using Sliding-Mode Differentiator Observer

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***Abstract*—This paper presents the design, simulation, and experimental results of a new scheme for the robust parameter estimation of uncertain nonlinear dynamic systems. The technique is established on the estimation of robust time derivatives using a variable-structure differentiator observer. A second-order sliding motion is established along designed sliding manifolds to estimate the time derivatives of flat outputs and inputs, leading to better tracking performance of estimates during transients. The parameter convergence and accuracy analysis is rigorously explored systematically for the proposed class of estimators. The proposed method is validated using two case studies; first, the parameters of an uncertain nonlinear system with known, but uncertain nominal parametric values are estimated to demonstrate the convergence, accuracy, and robustness of the scheme; in the second application, the experimental parameter estimation of an onboard-diagnosis-II-compliant automotive vehicle engine is presented. The estimated parameters of the automotive engine are used to tune the theoretical mean value engine model having inaccuracies due to modeling errors and approximation assumptions. The resulting dynamics of the tuned engine model matches exactly with experimental engine data, verifying the accuracy of the estimates.**

***Index Terms*—Automotive engine, higher order sliding modes (HOSMs), onboard diagnosis-II (OBD-II), parameter estimation, sliding-mode differentiator, throttle discharge coefficient.**

I. Introduction

In SYSTEMS and control theory, models generally contain a number of parameters which are unknown or roughly known. A complete knowledge of these parameters is critical to describe and analyze the dynamics of real-world systems. Also, advanced control and diagnosis algorithms for modern industrial, automotive, and aerospace systems require the accurate knowledge of system parameters. Any control or diagnosis algorithm with poor parameter estimates will have poor performance and could also become unstable. Online parameter-estimation schemes allow these algorithms to have accurate parameter estimates even when subjected to perturbations. Several methods have been used previously to solve the problem of parameter estimation [1]–[10], [18], [19], [22]–[27]. Adaptive estimations using Kalman filters [3], [25], recursive least squares [18], [24], [25], and sliding-mode estimators [1], [5]–[10], [22], [27], [31], [33] are among the frequently used techniques. Parameter estimation using an algebraic differentiator has been studied by Join *et al.* [4]. High-gain differentiators by Ball and Khalil [16] and Chitour [17] provide an exact derivative provided that their gains tend to infinity, which also leads to higher sensitivity to small high-frequency noise. Another drawback of the high-gain differentiators is their peaking effect [11].

More recently, sliding-mode observers and controllers have been used for the control and diagnosis of uncertain dynamic systems due to their intrinsic robustness to parametric and modeling uncertainties. A new generation of controllers and observers based on higher order sliding modes (HOSMs) has been recently developed and applied to industrial and aerospace systems by Zaky *et al.* [1], Spurgeon [5], Alwi and Edwards [6], Iqbal *et al.* [8], Rao *et al.* [9], Butt and Bhatti [10], Levant [11], [12], Fridman *et al.* [14], Defoort *et al.* [26], Proca and Keyhani [27], and Huang and Sung [32]. A very good number of references utilizing the sliding modes for linear and nonlinear systems with tried-and-tested theories are available, and research is ongoing to fully utilize the vast possibilities offered by HOSM approaches to develop algorithms for observer design, parameter estimation, and robust control [5]. Levant [11]–[13] and Fridman *et al.* [14] have demonstrated arbitrary-order sliding-mode controllers (SMCs) and differentiators with finite time convergence. These differentiators provide much better alternatives to algebraic and high-gain differentiators, thus eliminating their associated problems. The robustness, better accuracy, and fast convergence of these differentiators can be utilized for the parameter estimation of uncertain nonlinear dynamic systems.

In this paper, the synthesis of an uncertainty observer for the parameter estimation of uncertain nonlinear dynamic systems using Levant's sliding-mode differentiator [11] is presented. The rest of this paper is structured as follows. In Section II, the problem formulation for the estimation of unknown uncertain parameters is described. The convergence and accuracy analysis of the proposed estimator is presented in Section III. Section IV covers the validation of the proposed scheme using simulation results for a nonlinear multi-input–multioutput (MIMO) three-tank system and using experimental results to estimate the throttle discharge coefficient and load torque of a production model of an automotive engine. Section V contains the concluding remarks followed by references.

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II. Problem Formulation

A. Preliminaries

Consider a nonlinear system of the form

$$\dot{x} = f(x, p, t) + g(x, t)\Phi(u) \tag{1}$$

$$y = h(x, p, t) + \pi(t) \tag{2}$$

where $x \in \mathbb{R}^n$ is a state vector, $u \in \mathbb{R}^m$ is a control input vector, $g(x, t)$ is a known nonlinear function with $g(x, t) \neq 0$, $p \in \mathbb{R}^q$ is the unknown/uncertain parameter vector in parameter space $P\{p_i \in [p_{i\min}, p_{i\max}], i = 1, 2, \ldots, q\}$, $y \in \mathbb{R}^p$ is the output vector, $\Phi(u)$ can be a nonlinear continuous function, $\pi(t)$ is a finite set of perturbation variables which are assumed here as high-frequency noises, and $f(x, p, t)$ is a smooth nonlinear function. Also assume that the functions $f(x, p, t)$ satisfy the following assumptions [8].

Assumption 1: The function $f(x, p, t)$ can be decomposed in the following form:

$$f(x, p, t) = \alpha^{\mathrm{T}}(p)\xi(x, t), \tag{3}$$

$$\alpha^{\mathrm{T}} = [\alpha_1, \alpha_2, \ldots, \alpha_q]; \xi^{\mathrm{T}} = [\xi_1, \xi_2, \ldots, \xi_q] \tag{4}$$

where $\xi_i = \xi_i(x, t)$ and $\xi_i(x, t) \neq 0$ are known nonlinear functions and are linearly independent. $\alpha_i = \alpha_i(p)$ denotes the combinations of p; q denotes the dimension of α. As the bounds of p are given, the bounds of $\alpha(p)$ can also be obtained, and the following assumption can be introduced.

Assumption 2: The uncertain parameters α_i satisfy the bounds

$$\alpha_i \in [\alpha_{i\min}, \alpha_{i\max}] \qquad \forall\, p \in P. \tag{5}$$

Suppose that $\hat{\alpha}$ is an uncertain parameter vector and that $\hat{\alpha}_0$ is a time-varying parameter vector which is calculated in terms of the uncertain parameter and its bounds are

$$\hat{\alpha}_0(t) = \begin{cases} \alpha_{i\min}, & \text{if } \hat{\alpha}_i < \alpha_{i\min} \\ \hat{\alpha}_i(t), & \text{if } \alpha_{i\min} \leq \hat{\alpha}_i \leq \alpha_{i\max} \\ \alpha_{i\max}, & \text{if } \hat{\alpha}_i > \alpha_{i\max}. \end{cases} \tag{6}$$

It is important to consider *a priori* identifiability; whether the parameters can be determined uniquely in the ideal noise-free case from a given model and available outputs or measurements, consider the following assumption.

Assumption 3: Assume that system (1) is observable and identifiable, i.e., it satisfies the following rank test conditions for observability and identifiability, respectively ($i = 1, 2, \ldots, p,\ j_1 = 1, 2, \ldots, n,\ j_2 = 1, 2, \ldots, q,$ and $k = 1, 2, \ldots, n-1$) [8]

$$\operatorname{rank}(J_O) = \operatorname{rank}\left(\left[\frac{\partial}{\partial x_{j_1}} L_f^k h_i\right]\right) = n \tag{7}$$

$$\operatorname{rank}(J_I) = \operatorname{rank}\left(\left[\frac{\partial}{\partial p_{j_2}} L_f^k h_i\right]\right) = n \tag{8}$$

where J_o and J_I are observability and identifiability Jacobians, respectively, and $L_f h$ is the Lie derivative. Furthermore, system (1) is also assumed to be a differentially flat system as per the following definition.

Definition 1 (Flat Systems): Any dynamic system is said to be flat if any of its parameters, states, or inputs can be represented as a function of flat outputs and their derivatives up to some finite order and if any component of the flat output can be written as a function of system variables and their derivatives up to some finite order such that [28]

$$y = h\left(x, u, \dot{u}, \ldots, u^{(r)}\right) \tag{9}$$

$$x = \varphi_1\left(L_f^0 h, L_f^1 h, \ldots, L_f^p h\right) \tag{10}$$

$$u = \varphi_2\left(L_f^0 h, L_f^1 h, \ldots, L_f^q h\right) \tag{11}$$

where p and q are positive integers. Flatness property is exhibited by a wide variety of mechanical and chemical systems. The flatness for a class of linear and nonlinear systems using differential algebra is discussed in detail by Sira-Ramirez and Agarwal [28].

B. Estimation of the Parameters

The proposed parameter-estimation approach judiciously exploits the properties of differentially flat systems. Let the unknown parameter vector $\alpha^{\mathrm{T}} = [\alpha_1, \alpha_2, \ldots, \alpha_q]$ of differentially flat system (1) under Assumptions 1 and 2 satisfy identifiability Assumption 3; then, the system parameters can take the following form [4], [8]:

$$\alpha_j = \gamma_j\left(t, L_f^0 h, L_f^1 h, \ldots, L_f^k h, u, \dot{u}, \ldots, u^{(m)}\right), \qquad j = 1, \ldots, q \tag{12}$$

where γ_j is a nonlinear function of flat inputs and outputs and of their derivatives up to some finite order. Considering that the inputs and outputs are measurable, their derivatives in (12) are estimated with Levant's differentiator observer [11] using the HOSMs discussed in the following section.

C. Sliding-Mode Differentiator Observer

For system (1) having relative degree "r," the r-sliding mode is determined by $L_g^0\sigma = L_g\sigma = L_g L_f\sigma = \cdots = L_g L_f^{r-2}\sigma = 0$ and $L_g L_f^{r-1}\sigma \neq 0$, which forms an r-dimensional condition on the sliding manifold of the dynamic system. Consider the HOSM-based Levant's robust differentiator observer [11] based on the modified supertwisting algorithm

$$\begin{aligned}
\dot{z}_0 &= v_0 \\
v_0 &= -\lambda_0|\sigma|^{n/(n+1)}\operatorname{sign}\sigma + z_1 \\
\dot{z}_1 &= v_1 \\
v_1 &= -\lambda_1|z_1 - v_0|^{n-1/n}\operatorname{sign}(z_1 - v_0) + z_2 \\
&\vdots \\
\dot{z}_{n-1} &= v_{n-1} \\
v_{n-1} &= -\lambda_{n-1}|z_{n-1} - v_{n-2}|^{1/2}\operatorname{sign}(z_{n-1} - v_{n-2}) + z_n \\
\dot{z}_n &= -\lambda_n\operatorname{sign}(z_n - v_{n-1}).
\end{aligned} \tag{13}$$

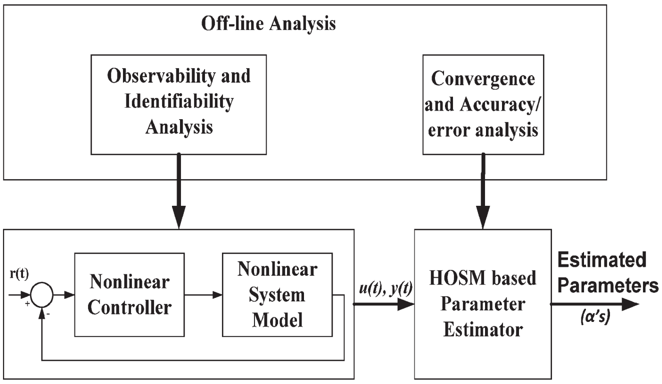

Fig. 1. Schematic diagram of the sliding-mode differentiator-observer-based parameter estimator.

Subject to the condition that $|f^{(n)}(t)| < C_n$, where C_n is a Lipschitz constant, $\sigma(t, z_0) = z_0 - f(t)$ and the associated differential inclusions discussed in [13] exists as a solution to (1). Then, z_{n-1} converges to $f^{(n-1)}(t)$ in finite time if the sliding variables σ and u are measured without noise. If σ and u are measured with noise bounded by $\epsilon \geq 0$ and $\epsilon^{(n-i)/n}$, respectively, then according to Levant [11]

$$\left| z_i - f_0^{(i)}(t) \right| \leq \ell_i C^{i/(n)} \epsilon^{(n-i)/(n)}, \qquad \ell_i > 0. \tag{14}$$

Remark 1: The Lipschitz constant C can be computed by taking into account the maximum frequency component and maximum amplitude of the function $f(t)$, i.e., $C = (2\pi f_{\max})^2 A_{\max}$ [15], where $f_{\max}$ is the maximum frequency component and $A_{\max}$ is the maximum amplitude of $f(t)$.

Parameter estimates $[\alpha_i]_e$ are obtained by replacing the derivatives of the control input and output in (12) with their estimates using differentiator observer (13)

$$[\alpha_i]_e = \Upsilon_i \left(t, L_f^0 h, \left[L_f^1 h\right]_e, \ldots, \left[L_f^k h\right]_e, u, [\dot{u}]_e, \ldots, [u^m]_e \right). \tag{15}$$

Here, $[\alpha_i]_e$ is the estimate of the actual system parameter denoted as $\widehat{\alpha}_i$ for the rest of the paper. Parameter estimator (15) must follow Assumption 1 subject to $\xi_i(x,t) \neq 0$. The schematic diagram of the HOSM-based parameter estimator is shown in Fig. 1. The convergence and accuracy of the parameter estimates is analyzed in Section III as follows.

Remark 2: The finite-time transient process is defined as the time required during the reachability phase to the sliding manifold from arbitrary initial conditions until sliding motion is established.

III. Convergence and Accuracy Analysis

For the convergence and accuracy of parameter estimates, Theorems 1–4 and Lemma 1 are described as follows.

Theorem 1: Consider a differentially flat system (1) subject to Assumptions 1, 2, and 3 and let the variables u and y be Lebesgue measurable and differentiable, respectively, with known Lipschitz constant C. Consider the first-order differentiator observer using the modified supertwisting algorithm with the following structure [13]:

$$\begin{aligned}
\dot{z}_0 &= v_0 \\
v_0 &= z_1 - 2C^{1/2} \left| v_0 - f(t) \right|^{3/4} \operatorname{sign}\left(v_0 - f(t)\right) \\
\dot{z}_1 &= v_1 \\
v_1 &= z_2 - 1.5C^{1/2} |z_1 - v_0|^{2/3} \operatorname{sign}(z_1 - v_0) \\
\dot{z}_2 &= -1.1C^{1/2} \operatorname{sign}(z_2 - v_1)
\end{aligned} \tag{16}$$

where $z_1 \mapsto \dot{f}(t)$ with properly chosen parameters; then, after a finite-time transient process, the parameter estimates (15) converge to the actual parameters of the system, i.e., $\hat{\alpha}_i \to \alpha_i$.

Proof: Assumptions 1, 2, and 3 guarantee the separability, boundedness, and identifiability of the parameters to be estimated. Assume that the system (1) is operating in feedback closed-loop configuration to track the desired system trajectories. The solutions and associated differential inclusions of (16) are understood in a Filippov sense [21]. Consider the following simple parameter estimator with only first-order derivative estimates:

$$\hat{\alpha}_i = \gamma_i(u, y, \dot{u}_e, \dot{y}_e). \tag{17}$$

In order to prove the convergence of parameter estimates to their nominal values, the following theorems are quoted as follows.

Theorem 2 [11]: With the parameters of differentiator (13) being properly chosen, the following equalities are true in the absence of input noise after a finite-time transient process:

$$z_0 = f_0(t) \quad z_i = v_{i-1} = f_0^{(i)}(t), \qquad i = 1, \ldots, n.$$

Moreover, the corresponding solutions of the dynamic system are Lyapunov stable, i.e., finite-time stable. Theorem 2 means that the equalities $z_i = f_0^{(i)}(t)$ are kept in 2-sliding mode $i = 1, \ldots, n-1$.

Theorem 3 [11]: Let the input noise satisfy the inequality $|f(t) - f_0(t)| \leq \epsilon$; then, the following inequalities are established in finite time for some positive constants ℓ_i and ν_i, depending exclusively on the parameters of the differentiator (13)

$$\begin{aligned}
\left| z_i - f_0^{(i)}(t) \right| &\leq \ell_i \epsilon^{(n-i+1)/(n+1)}, \qquad i = 0, \ldots, n \\
\left| v_i - f_0^{(i+1)}(t) \right| &\leq v_i \epsilon^{(n-i)/(n+1)}, \qquad i = 0, \ldots, n-1.
\end{aligned}$$

The proof of Theorems 2 and 3 is given in [11].

The parameter estimates $\hat{\alpha}_i$ in (17) depend exclusively on u and y and on their time derivatives in a nonlinear fashion. Since u and y are measurable quantities, their first derivatives can be observed using Levant's first-order differentiator observer (16). The convergence of the differentiator observer is ensured by Theorems 2 and 3. Since the differentiator (16) is homogeneous [11], then the parameter estimator is invariant under the following transformation:

$$\begin{aligned}
G_\eta : &\left(t, f, f^{(i)}, u, u^{(i)}\right) \\
&\mapsto \left(\eta t, \eta^{n+1} f, \eta^{n-i+1} f^{(i)}, \eta^{n-i} u, \eta^{n-i} u^{(i)}\right).
\end{aligned}$$

The convergence of parameter estimates depends only on the convergence of the differentiator observer which is guaranteed in finite time; thus, under the conditions of Theorems 2 and 3, the convergence of estimates (17) $\hat{\alpha}_i \to \alpha_i$ is established in finite time. ■

Lemma 1: Consider the nonlinear function γ_i in (17) to be Lipschitz with the Lipschitz constant κ_i independent of t, u, and χ such that

$$|\gamma_i(t,u,\chi_1) - \gamma_i(t,u,\chi_2)| \le \kappa_i|\chi_1 - \chi_2|$$

where $\chi \in \mathbb{R}^2$ and $|\cdot|$ is a norm in $\mathbb{R}^2$. If the parameters of the differentiator observer (16) are properly chosen in the absence of measurement noise, then the parameter estimates $\hat{\alpha}_i$ converge to actual parameters α_i in finite time, i.e., $\hat{\alpha}_i \to \alpha_i$. If u and y are measured with noise bounded by $\epsilon \ge 0$, then the estimation accuracy provided by the parameter estimator is

$$|\hat{\alpha}_i - \alpha_i| \le \kappa_i \xi(C,\epsilon), \qquad \kappa_i > 0.$$

Theorem 4: Let the measurement noise satisfy the inequality $|f(t) - f_0(t)| \le \epsilon$ and the parameters λ_i of the differentiator observer (16) be chosen properly; then, the estimator (17) provides the accuracy $|\hat{\alpha}_i - \alpha_i| \le \kappa_i \ell_i C^{i/(n+1)} \epsilon^{(n-i+1)/(n+1)}$ for some $\kappa_i \ge 1$.

Proof: Consider the parameter estimator (17), the differentiator observer (16), and the following propositions.

Proposition 1 [12]: Let $W(C,n)$ be the set of all input signals having $(n-1)$th derivatives and Lipschitz constant $C > 0$, any sufficiently small measurement noise bounded by $\epsilon > 0$. Then, no differentiator of order $i \le n$ on $W(c,n)$, where $n > 0$, exists, which may provide accuracy better than $C^{i/n}\epsilon^{(n-1)/n}$.

Proposition 2 [11]: Let the parameters λ_i, $i = 0, 1, \ldots, n$, of differentiator (13) provide for the exact nth-order differentiation with $C = 1$. Then, the parameters $\lambda_i = \lambda_{0i} C^{1/(n-i+1)}$ are valid for $C > 0$ and provide for the accuracy

$$\left| z_i - f_0^{(i)}(t) \right| \le \ell_i C^{i/(n+1)} \epsilon^{(n-i+1)/(n+1)}, \qquad \ell_i \ge 1. \tag{18}$$

The proof of Theorem 4 is a direct consequence of Theorems 1 and 3, Lemma 1, and Propositions 1 and 2 with the estimation accuracy given as follows:

$$|\hat{\alpha}_i - \alpha_i| \le \kappa_i \ell_i C^{i/(n+1)} \epsilon^{(n-i+1)/(n+1)} \tag{19}$$

where gain $\kappa_i \ge 1$ depends on system (1) in the sense of nonlinear function γ_i. Equation (19) depicts that the estimation error is algebraically associated with the accuracy of the derivative's estimates of the input and output and the measurement noise ϵ. ■

IV. Validation of Parameter-Estimation Scheme

To demonstrate the performance of the proposed parameter-estimation scheme, two case studies are presented, validating the parameter estimation of uncertain nonlinear systems. In Case I, a MIMO three-tank system is considered for its nominal values of uncertain parameters (flow or viscosity coefficients) are known. With the actual parameters known, the convergence, robustness, and accuracy of their estimates are demonstrated with and without measurement noise. In Case II, the parameters of an automotive engine are estimated experimentally using a 1.3-l onboard-diagnosis-II (OBD-II)-compliant real vehicle engine. The parameter estimates are validated by tuning the theoretical mean value engine model (MVEM). The tuned theoretical model's dynamics match exactly with the experimental results with zero steady-state error.

A. Simulation Results

Case I—Three-Tank System: Three-tank-system parameters like viscosity coefficients (μ_1, μ_2, and μ_3) are subject to uncertainty for changes in liquid characteristics, aging effects, corrosion, or change in operating environments and conditions. Viscosity coefficients play a critical role in the control and diagnosis of fuel management systems or chemical processes. Minor changes in these parameters can seriously affect the performance of dynamic systems. Consider the full-state system model [8] given by (20)–(22) shown at the bottom of the page, where $\mathcal{C}_i = (1/s)\mu_i S_p \sqrt{2g}$, μ_i's are uncertain viscosity coefficients to be estimated, x_i's are liquid levels in three tanks, and y_i's are the outputs of the plant. The function $f(x,p,t)$ can be easily represented in terms of the α's and ξ's referred in Assumption 1. The identifiability Jacobian (8) of system (20) is given as follows:

$$J = \left[\frac{\partial}{\partial p_j} L_f h_i \right], \qquad i = j = 1, 2, 3.$$

Here, for system (20) and $(p_1, p_2, p_3 \to \mu_1, \mu_2, \mu_3)$, $(\partial/\partial\mu_2)L_f h_1 = (\partial/\partial\mu_3)L_f h_1 = (\partial/\partial\mu_1)L_f h_2 = (\partial/\partial\mu_2)L_f h_3 = 0$

$$f(x,t) = \begin{bmatrix} -C_1 \text{sign}(x_1 - x_3)\sqrt{|x_1 - x_3|} \\ C_3 \text{sign}(x_3 - x_2)\sqrt{|x_3 - x_2|} - C_2 \text{sign}(x_2)\sqrt{|x_2|} \\ C_1 \text{sign}(x_1 - x_3)\sqrt{|x_1 - x_3|} - C_3 \text{sign}(x_3 - x_2)\sqrt{|x_3 - x_2|} \end{bmatrix} \tag{20}$$

$$g(x,t) = \begin{bmatrix} 1/S \\ 1/S \\ 0 \end{bmatrix} \tag{21}$$

$$h(x,t) = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \tag{22}$$

and the determinant of the identifiability Jacobian comes out to be

$$
\begin{aligned}
\det(J) =& -\frac{\partial}{\partial \mu_2} L_f h_2 \frac{\partial}{\partial \mu_1} L_f h_1 \frac{\partial}{\partial \mu_3} L_f h_3 \\
=& -K\left(-\operatorname{sign}(x_2)\sqrt{|x_2|}\right) \\
& \times \left(-C_1 \operatorname{sign}(x_1 - x_3)\sqrt{|x_1 - x_3|}\right) \\
& \times \left(-C_3 \operatorname{sign}(x_3 - x_2)\sqrt{|x_3 - x_2|}\right) \neq 0, \\
& \qquad \text{for } x_1 > x_2 > x_3 \qquad (23)
\end{aligned}
$$

where $K = \pi(S_p\sqrt{2g}/S)^3$, the identifiability Jacobian has full rank for $x_1 > x_2 > x_3$, ensuring the identifiability of μ_1, μ_2, and μ_3. The uncertain flow coefficients are estimated using the following [3], [8]:

$$[\mu_1]_e = -\left(S[\dot{y}_1]_e - u1\right)/\psi_1 \qquad (24)$$

$$[\mu_2]_e = -\left(S[\dot{y}_1]_e + S[\dot{y}_2]_e + S[\dot{y}_3]_e - u_1 - u_2\right)/\psi_2 \qquad (25)$$

$$[\mu_3] = -\left(S[\dot{y}_1]_e + S[\dot{y}_3]_e - u_2\right)/\psi_2 \qquad (26)$$

where denominator functions ψ_1, ψ_2, ψ_3 are defined as

$$
\begin{aligned}
\psi_1 &= S_p \operatorname{sign}(y_1 - y_3)\sqrt{2g|y_1 - y_3|} \\
\psi_2 &= S_p \operatorname{sign}(y_2)\sqrt{2g|y_2|} \\
\psi_3 &= S_p \operatorname{sign}(y_3 - y_2)\sqrt{2g|y_3 - y_2|}
\end{aligned}
$$

subject to the conditions $|y_1 - y_3| \neq 0$, $|y_3 - y_2| \neq 0$, and $|y_2 \neq 0$.

The first-order derivatives $\dot{y}_i$ in (24)–(26) are estimated by establishing second-order sliding modes with sliding manifolds $\sigma_i = z_{0i} - y_i$ and $\dot{\sigma}_i = z_{0i} - \dot{y}_i$, such that $\ddot{y}_i$ is bounded by the respective Lipschitz constant C_i in the sense of [15], i.e., $|\ddot{y}_i \leq C_i$, using the first-order differentiator/observer (16), where $i = 1, 2, 3$

$$
\begin{aligned}
\dot{z}_{0i} &= v_{0i} \\
v_{0i} &= -2C_i^{1/2}|z_{0i} - y_i|^{3/4}\operatorname{sign}(z_{0i} - y_i) + z_{1i} \\
\dot{z}_{1i} &= v_{0i} \\
v_{0i} &= -1.5C_i^{1/2}|z_{1i} - v_{0i}|^{2/3}\operatorname{sign}(z_{1i} - v_{0i}) + z_{2i} \\
\dot{z}_{2i} &= -1.1C_i^{1/2}\operatorname{sign}(z_{2i} - v_{1i}). \qquad (27)
\end{aligned}
$$

The derivative estimates z_{1i} converge to $\dot{y}_i$ in a finite time in the absence of measurement noise if the sliding variables σ and u are measured without noise. For measurements with noise bounded by $\epsilon^{2/3}$, where $\epsilon \geq 0$, the derivative estimation error is $|z_{1i} - \dot{y}_i(t)| \leq \ell_i C_i^{1/2}\epsilon^{2/3}$, $\ell_i > 0$, and the corresponding viscosity coefficient estimation error is given as $|\hat{\mu}_i - \mu_i| \leq \kappa_i \ell_i C_i^{1/2}\epsilon^{2/3}$, $\kappa_i > 1$.

The parameter-estimation method developed in the previous sections has been used for the estimation of the viscosity coefficients μ_1, μ_2, and μ_3 of the uncertain three-tank system. The parameters used for system simulation and estimation are given in Table I. The system is working in feedback configuration with a standard SMC to establish the desired steady state by tracking the desired water levels of 0.6 and 0.4 m in tanks 1 and 2, respectively. The initial conditions for all estimator inputs and outputs and parameters to be estimated are set to "zero." Simulations are carried out for the case $x_1 > x_2 > x_3$ to satisfy the full-rank conditions for parameter identifiability (23). Using the measured inputs u_1 and u_2, flat outputs y_1, y_2, and y_3, and their derivative estimates (27), the estimated parameters are shown in Fig. 2. After a very short transient process of the reachability phase, the sliding motion is established. The parameter estimates converge to their nominal values quickly.

TABLE I
THREE-TANK-SYSTEM PARAMETERS AND THEIR NOMINAL VALUES

Parameter	Description	Units	Nominal Values
S	Surface area of tanks	m^2	0.0154
S_p	Surface area of pipes	m^2	5x10^{-5}
u_{1max}, u_{2max}	Input Flow rates	ml/sec	100
y_{i_max}, ***i:1,2,3***	Maximum level in tanks	m	0.62
μ_1, μ_2, μ_3	Viscosity or flow coefficients	--	0.5, 0.675, 0.5
$\mu_{1max}, \mu_{2max}, \mu_{3max}$	Maximum value of flow coefficients	--	1.0, 1.0, 1.0
$\mu_{1min}, \mu_{2min}, \mu_{3min}$	Minimum value of flow coefficients	--	$\eta_1, \eta_2, \eta_3 > 0$

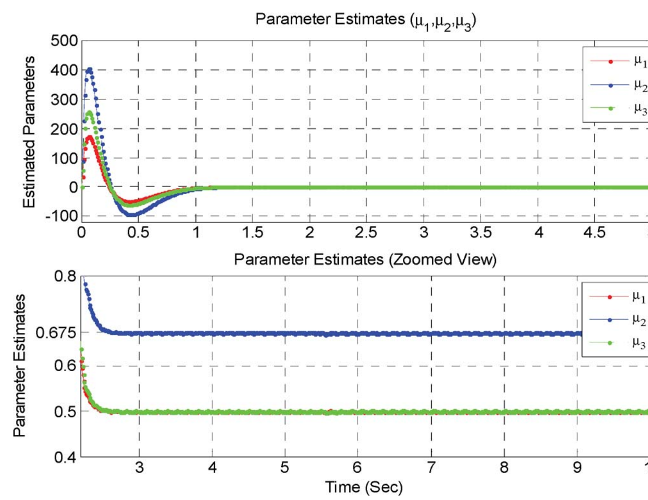

Fig. 2. Estimates of viscosity coefficients during the initial phase. Initial peaks are due to different initial conditions and the transient process of the reachability phase of sliding manifold to the sliding phase. Once the 2-sliding motion is established, the algorithm tracks the parametric variations precisely. The lower graph shows the convergence of parametric values to their nominal values of 0.5, 0.675, and 0.5, respectively.

A small chattering is observed in the estimates of parameters. This is due to the chattering effect of the control input of the first-order sliding-mode control, which acts as high-frequency noise. In order to minimize the chattering effect and high-frequency measurement noise, parameter estimates are filtered using a low-pass filter with transfer function $1.9/(s + 1.9)$, as recommended by Levant [11]. The convergence time is less than 2 s, which is reasonably fast as compared with the

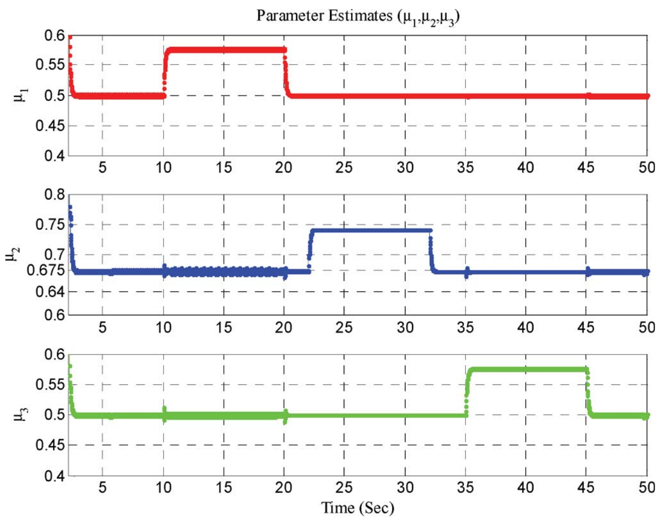

Fig. 3. Estimates of piecewise changing viscosity coefficients. The estimates converge to their nominal values after a short transient process (< 1.5 s). Increases of 15%, 10%, and 15% in μ_1, μ_2, and μ_3, respectively, are detected by the estimator instantaneously.

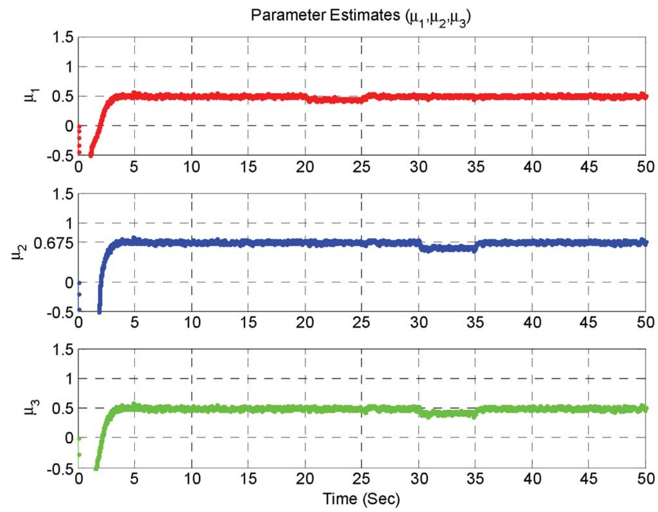

Fig. 4. Estimates of viscosity coefficients in steady state in the presence of noise with variance of 0.001. Convergence is a little slower due to noisy measurements. Also, changes in μ_1, μ_2, and μ_3 at 20–25 and 30–35 s induced by system leakage in tanks 1 and 2 show the sensitivity of system parameters to external disturbances and faults.

convergence time of existing similar techniques [4], [10], [25]. The convergence time is much shorter than the convergence times obtained using HOSM-based parameter estimation in [10] and is faster by orders of magnitude compared with sensitivity-model-based adaptive filters [25].

A piecewise increase in the parameters' nominal values is introduced to investigate the response of the parameter estimator to time-varying parameters given as follows:

$$\mu_1(t) = \begin{cases} 0.5, & t \le 10 \text{ s} \\ 0.575, & 10 < t < 20 \text{ s} \\ 0.5, & t \ge 20 \text{ s} \end{cases} \tag{28}$$

$$\mu_2(t) = \begin{cases} 0.675, & t \le 22 \text{ s} \\ 0.7425, & 22 < t < 32 \text{ s} \\ 0.675, & t \ge 32 \text{ s} \end{cases} \tag{29}$$

$$\mu_3(t) = \begin{cases} 0.5, & t \le 35 \text{ s} \\ 0.575, & 35 < t < 45 \text{ s} \\ 0.5, & t \ge 45 \text{ s.} \end{cases} \tag{30}$$

The observer has perfectly detected an instantaneous increase of 15%, 10%, and 15% in μ_1, μ_2, and μ_3 at times 10, 22, and 35 s, respectively, which lasted for 10 s, as shown in Fig. 3. Fig. 4 shows the estimates of viscosity coefficients in the presence of noise with zero mean and variance of 0.001. The results in Figs. 4 and 5 show that the estimates are convincingly good in the presence of measurement noise, indicating the better convergence and accuracy of estimates.

A diminutive change in μ_1, μ_2, and μ_3 at 20–25 and 30–35 s, respectively, is eminent in Fig. 4. This change is induced by system leakage faults in tanks 1 and 2, respectively. This sensitivity aspect of response to minor changes in system behavior can be utilized for fault diagnosis of uncertain nonlinear systems [8]. The estimation errors of viscosity coefficient with reference to their nominal values in the presence of measurement noise are shown in Fig. 5. The estimation error has zero mean and variance of 2.4×10^{-4}. The average estimation error remains below 2% of the nominal values of the viscosity coefficients in

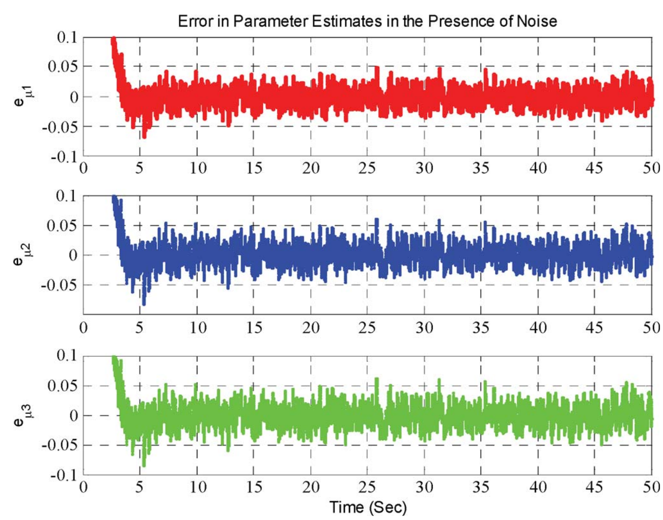

Fig. 5. Estimation error of viscosity coefficients relative to their nominal values of 0.5, 0.675, and 0.5, respectively. The average estimation error is about 2%–3% of the nominal values, showing the robustness and accuracy of the proposed algorithm. The estimation error has zero mean and variance of 2.4×10^{-4}. The estimation errors shown are for the simulation without system leakage faults.

the presence of the discussed noise magnitude, exhibiting the robustness and accuracy of the proposed method. The variance of the estimation error is smaller than the introduced noise, and this effect may be attributed to the response of filter to high-frequency noise.

B. Experimental Results

Case II—Automotive Engine: This section discusses the description of the automotive-engine parameters to be estimated with reference to nonlinear MEVM. The two important engine parameters, namely, throttle discharge coefficient (C_d) and load torque (τ_L), necessary for efficient control and diagnosis of an automotive engine are considered. In the automotive industry, air-to-fuel ratio (AFR) is a critical parameter in controlling an engine for the reduction of pollutant exhaust emissions. This ratio shall be maintained close to the stoichiometric ratio for normal and efficient functioning of an automotive engine and for economic fuel consumption. The correct prediction of air mass flow rate through the throttle body of air intake path is very critical for the control of AFR closer to the desired value which, in turn, depends upon the throttle discharge coefficient (C_d) of the throttle valve. The throttle discharge coefficient is incorporated in the modeling of air-intake-path dynamics to account for modeling inaccuracies, geometric deviations, nonlinearities of air flow dynamics, and other assumptions. This parameter is normally taken as constant equivalent to 0.9 by many researchers [10], [20], [30], but this is a mere approximation; in fact, this parameter varies in a nonlinear fashion with the throttle angle as well as intake manifold pressure and engine speed [10], [22], [24]. Most of these schemes are used to estimate these parameters offline or have longer convergence times, steady-state error, and loss of tracking during transients. In our proposed scheme, all these issues are addressed amicably.

Load torque τ_L is another important parameter that can affect engine performance seriously if it is not accounted for properly in the control loop. Normally, there is no special sensor available in the engine for the direct measurement of load torque. Automotive vehicles have various accessories such as air conditioning compressor, power steering pump, or alternator intermittently coupled to the engine drive. If any of these accessory loads is engaged to engine drive in addition to engine normal drive-train loads, it consumes a certain amount of engine load and the driver can feel the drive degrading. For better engine control and improved drive feel, load torque τ_L needs to be estimated and engine control needs to be adjusted accordingly; hence, degradation in overall vehicle performance is avoided. The load torque is estimated in real time using the engine model and measured engine speed following different parameter-estimation schemes [10], [22], [24]. In this paper, discharge coefficient C_d and load torque τ_L are estimated using the proposed scheme in Section II, which eliminates the steady-state estimation error with better tracking and transient response.

A generic two-state nonlinear MVEM of a four-stroke four-cylinder gasoline engine for parameter estimation is adapted from [10] and [20]. In this model, each cylinder process is repeated after an angular displacement of 4π radians. The fluctuations due to pressure variations within the cylinder because of burnt gas expansion are neglected and averaged by mean effective pressure. In this model, the volumetric efficiency is also taken to be 80% for the estimation of discharge coefficient (C_d) and load torque (τ_L). Further details about the derivation of MVEM can be found in [10] and [20]. The two-state nonlinear generic model is represented by a set of dynamical equations given as follows:

$$\dot{p}_m = -C_s\eta_v p_m\omega + A_k C_d f(p_m)u(\theta) \tag{31}$$

$$\dot{\omega} = a_1 p_m - a_2\omega - a_3\omega^2 - \tau_L. \tag{32}$$

The variables and constants and their units used for the 1.3-l vehicle engine adapted from [29] are given in Table II.

TABLE II
NOMENCLATURE OF PARAMETERS, VARIABLES, CONSTANTS, AND THEIR VALUES

Parameter	Description	Units	Value
θ	throttle opening angle	degree	
A_E	effective throttle area	m^2	0.0020
C_s	speed-density constant $(V_d/V_m * \pi * 240)$		0.0015
C_d	throttle discharge coef.		0.8
ω	engine speed	rad/s	
γ	ratio of specific heat capacities		1.4
m_{ai}	air flow rate across throttle	kg/s	
p_a	ambient air pressure	kPa	101.3250 1
p_m	intake manifold pressure	kPa	
R	universal gas constant	kJ/kgK	0.2871
T_m	intake manifold temperature	k	
T_a	ambient air temperature	k	
η_v	volumetric efficiency		0.8
V_d	engine displacement	m^3	0.001294
V_m	intake manifold volume	m^3	0.005127
C_r	compression ratio		9.6
C_v	specific heat capacity at constant volume	kJ/kg.K	0.717
a_1	Indicated torque parameter		2.6
AFR	Air-fuel ratio		14.7
H_k	%age of heat converted into brake work		(≈ 25%)

Here, P_m, ω, and throttle angle are assumed to be measurable system outputs. Assumptions 1 and 2 are analyzed and validated for system (31), (32). For the identifiability condition (Assumption 3), the aforementioned system takes the form

$$f(p_m,\omega,t) = \begin{bmatrix} -C_s\eta_v p_m\omega + A_k C_d f(p_m) \\ a_1 p_m - a_2\omega - a_3\omega^2 - \tau_L \end{bmatrix} \tag{33}$$

$$g(p_m,\omega,t) = \begin{bmatrix} A_k C_d f(p_m) \\ 0 \end{bmatrix} \tag{34}$$

$$h(p_m,\omega,t) = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} p_m \\ \omega \end{bmatrix}. \tag{35}$$

The function $f(p_m,\omega,t)$ can be easily represented in terms of $(\alpha_1,\alpha_2) \to (C_d,\tau_L)$ and $\xi_1(p_m,\omega)$ and $\xi_2(p_m,\omega)$, as described in Assumption 1. The identifiability Jacobian comes out as

$$J = \left[\frac{\partial}{\partial_{p_j}} L_f h_i\right], \qquad i = j = 1, 2. \tag{36}$$

For system (33), (34), the uncertain parameters are $(p_1,p_2) \to (C_d,\tau_L)$ and $(\partial/\partial\tau_L)L_f h_1 = (\partial/\partial C_d)L_f h_2 = 0$, and the determinant of the identifiability Jacobean comes out as

$$\begin{aligned} \det(J) &= \frac{\partial}{\partial_{\tau_L}} L_f h_2 \frac{\partial}{\partial_{C_d}} L_f h_1 \\ &= -A_k f(p_m) \\ &\neq 0, \qquad \text{for } p_m > 0. \end{aligned}$$

The identifiability Jacobian has full rank for $p_m > 0$, ensuring the identifiability of C_d and τ_L. Utilizing the measured data for p_m, ω, and throttle angle θ, the derivatives $\dot{p}_m$ and $\dot{\omega}$ are estimated by using differentiator (16) as $[\dot{p}_m]_e$ and $[\dot{\omega}]_e$, respectively.

In view of (17), the parameter of system (31), (32) becomes

$$[C_d]_e = \left([\dot{p}_m]_e + C_s \eta_v p_m \omega\right) / A_k f(p_m) u(\theta) \tag{37}$$

$$[\tau_L]_e = a_1 p_m - a_2 \omega - a_3 \omega^2 - [\dot{\omega}]_e. \tag{38}$$

The denominator takes on practical values, thus barring it to become zero. Let us define the following two sliding manifolds for the estimation of the time derivatives of $[\dot{p}_m]_e$ and $[\dot{\omega}]_e$

$$\sigma_1 = z_{0p_m} - p_m(t) \tag{39}$$

$$\sigma_2 = z_{0\omega} - \omega(t). \tag{40}$$

The objective here is to make σ_1, σ_2 and $\dot{\sigma}_1$, $\dot{\sigma}_2$ vanish in a finite time such that their second derivatives $\ddot{p}_m$ and $\ddot{\omega}$ are bounded by their respective Lipschitz constants C_{p_m} and C_ω, i.e., $|\ddot{p}_m| \le C_{p_m}$ and $|\ddot{\omega}| \le C_\omega$. Using differentiator observer (13), the first-order derivative of $\dot{p}_m$ and $\dot{\omega}$ is computed by the differentiator observer

$$\begin{aligned} \dot{z}_{0p_m} &= v_{0p_m}, v_{0p_m} = -2C_{P_m}^{1/2} |\sigma_1|^{3/4} \mathrm{sign}(\sigma_1) + z_{1P_m} \\ \dot{z}_{1P_m} &= v_{1P_m}, v_{1P_m} = -1.5C_{P_m}^{1/2} |\sigma_1|^{2/3} \mathrm{sign}(\sigma_1) + z_{2P_m} \\ \dot{z}_{2P_m} &= -1.1C_{P_m}^{1/2} \mathrm{sign}(\sigma_1) \end{aligned} \tag{41}$$

$$\begin{aligned} \dot{z}_{0\omega} &= v_{0\omega}, v_{0\omega} = -2C_{\omega}^{1/2} |\sigma_2|^{3/4} \mathrm{sign}(\sigma_2) + z_{1\omega} \\ \dot{z}_{1\omega} &= v_{1\omega}, v_{1\omega} = -1.5C_{\omega}^{1/2} |\sigma_2|^{2/3} \mathrm{sign}(\sigma_2) + z_{2\omega} \\ \dot{z}_{2\omega} &= -1.1C_{\omega}^{1/2} \mathrm{sign}(\sigma_1). \end{aligned} \tag{42}$$

Estimates $z_{1p_m} \to \dot{p}_m$ and $z_{1\omega} \to \dot{\omega}$ in a finite time in the absence of the measurement noise of p_m and ω.

Lipschitz constants C_{p_m} and C_ω are calculated using the maximum frequency and magnitude of both measurements [15]. Using throttle angle, p_m, ω, and the estimated derivatives $[\dot{p}_m]_e$ and $[\omega]_e$ in (37) and (38), the engine parameters C_d and τ_L can be determined. The parameter-estimation scheme developed so far has been tested using experimental data for the estimation of the throttle discharge coefficient C_d and load torque τ_L of an automotive vehicle engine. The vehicle used for experimentation is a 1.3-l OBD-II-compliant production vehicle. The experimental setup used for the estimation of engine parameters is shown in Fig. 6. The engine data consisting of manifold pressure, engine speed, and throttle angle are obtained while the vehicle is idling, and engine input (throttle angle) is varied with the accelerator pedal. The air conditioning system is activated at 250 s. Other loads are windows' motorized glasses, wiper motors, and other miscellaneous minor loads which may occur during a vehicle operation.

The acquired data pertains to two significant periods. In the first interval, three different throttle angles are applied without the application of additional accessory loads (see Fig. 7). In the second period, air conditioning and other loads are applied, and

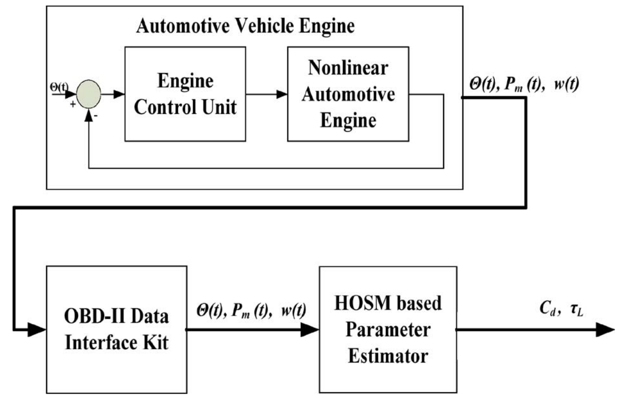

Fig. 6. Experimental setup for the parameter estimation of automotive-engine parameters. Experimental data from an automotive-engine throttle angle, manifold pressure, and engine rpm are obtained using an OBD-II data interface kit and is utilized for the estimation of engine parameters C_d and τ_L.

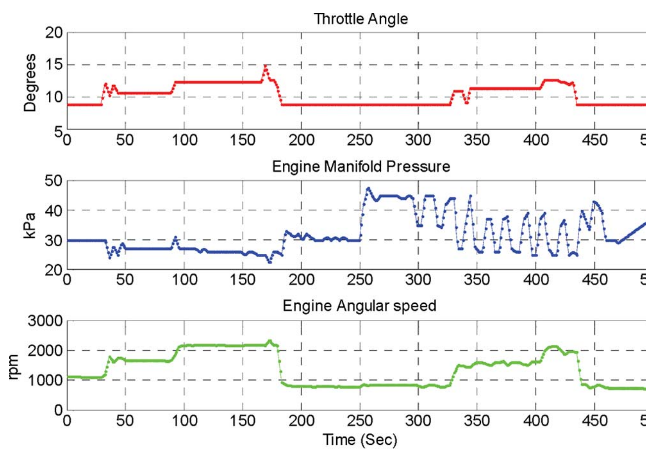

Fig. 7. Measured engine parameters, namely, engine control input and output signals, throttle angle, manifold pressure, and angular speed, from a real 1.3-l OBD-II-compliant automotive vehicle engine.

the throttle angle is varied through three different positions. The corresponding dips in engine speed signify the load variations. The engine data used as input to the proposed parameter estimator are throttle angle variations, manifold pressure, and angular speed. The estimated parameters C_d and τ_L are shown in Fig. 8. The estimate of throttle discharge coefficient C_d comes out to be ~0.4 in steady state. The estimate of throttle discharge coefficient C_d converges to its steady-state value in less than 8 s. The detailed analysis shows that the response of the proposed robust estimator to transients is also much better, as shown in Fig. 8. The steady-state estimate of $C_d \sim 0.4$ is in agreement with the experimentally estimated values quoted as 0.35, 0.5, and < 0.6 in [10], [19], and [30], respectively. The slight difference could be due to different operating conditions and the particular engine test setup. The estimated load torque shown in Fig. 8 also converges to its steady-state range of ~70 $\mathrm{N \cdot m}$. To further validate the accuracy of parameter estimates, the estimated values of engine parameters are used to tune theoretical nonlinear model (31), (32), following the similar procedure detailed in [10]. The untuned and tuned nonlinear MVEM models based on estimated parameters, along with the actual measured manifold pressure and the error between tuned and measured manifold pressures, are shown in Fig. 9. The error remains

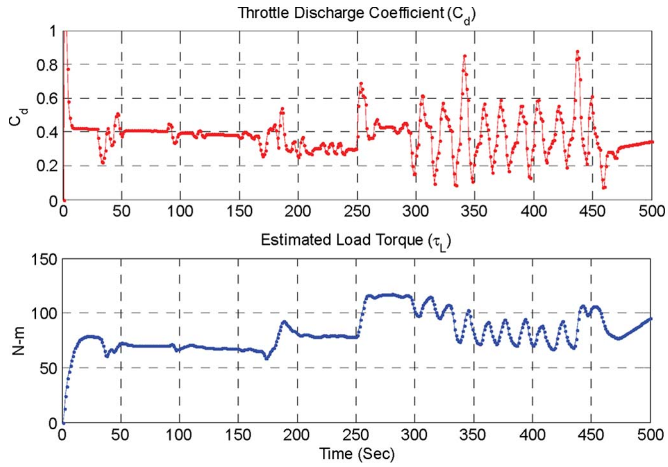

Fig. 8. Estimated engine parameters: Throttle discharge coefficient and load torque.

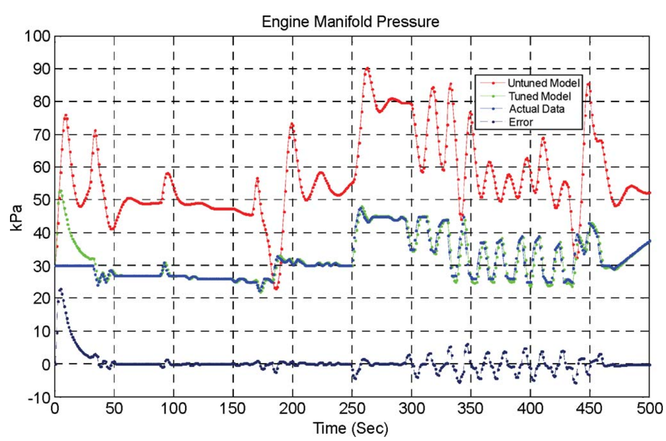

Fig. 9. Engine manifold pressure, untuned model, tuned model, and actual data along with tuning error. The tuning error is almost zero as compared with the 25–35 kPa of untuned model error.

close to zero under steady-state conditions. Analysis of Fig. 9 reveals an improvement in the estimate of the engine manifold pressure with the use of the estimated parameters; on average, the modeling error of 25–35 kPa is reduced to almost $\sim$0 kPa in steady state, a considerable improvement showing the accuracy of parameter estimates. Also, the transient response of the estimator is very good by looking at the tracking performance of the tuned and actual manifold pressures. The small nonzero error during transients can be attributed toward the response time of the estimator algorithm.

V. Conclusion

A scheme for the parameter estimation of uncertain nonlinear systems using a robust differentiator observer has been presented. The convergence and accuracy results are demonstrated using simulations and experimental data. The concept of an arbitrary-order differentiator can be utilized to estimate multiple parameters using a single nonlinear dynamical equation. The proposed scheme is computationally simple and can be employed for system identification, control, and fault diagnosis of uncertain nonlinear systems.

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