

# Equations integrated in PYTELTOOLS

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August 2017

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# 1 Variables notations

Notation of 2D and 3D variables are described on this wiki page : <https://github.com/CNR-Engineering/PyTelTools/wiki/Notations-of-variables>. Variable notations used in this document are based on the `varID` column of the tables presented on latter webpage.

# 2 Constants

Equations may contain following constants:

Notation	Description	Value and unit
$g$	gravitational acceleration on earth	$9.80665 \text{ m.s}^{-2}$
$\rho$	water density	$1000 \text{ kg.m}^{-3}$
$\kappa$	Von Kármán constant	0.4

These constants are set in `slf/variable/variables_utils.py`.

# 3 Equations in 2D

## 3.1 Hydraulic variables

$$\begin{aligned}H &= S - B \\S &= H + B \\B &= S - H \\M &= \sqrt{U^2 + V^2} \\I &= H \times U \\J &= H \times V \\Q &= \sqrt{I^2 + J^2} \\C &= \sqrt{gH} \\F &= \frac{M}{C}\end{aligned}$$

## 3.2 Sediment transport variables

$$\begin{aligned}HD &= B - RB \\B &= HD - RB \\RB &= B - HD \\QS &= \sqrt{QSX^2 + QSY^2} \\QS &= EF + DF \\QSBL &= \sqrt{QSBLX^2 + QSBLY^2} \\QSSUSP &= \sqrt{QSSUSPX^2 + QSSUSPY^2} \\QSX &= QS \frac{U}{M} \text{ if } M > 0 \text{ else } 0 \\QSY &= QS \frac{V}{M} \text{ if } M > 0 \text{ else } 0 \\QSBLX &= QSBL \frac{U}{M} \text{ if } M > 0 \text{ else } 0 \\QSBLY &= QSBL \frac{V}{M} \text{ if } M > 0 \text{ else } 0 \\QSSUSPX &= QSSUSP \frac{U}{M} \text{ if } M > 0 \text{ else } 0 \\QSSUSPY &= QSSUSP \frac{V}{M} \text{ if } M > 0 \text{ else } 0\end{aligned}$$

### 3.3 Friction velocity

$$US = \sqrt{\frac{1}{2} C_f M^2}$$

Law	Coefficient
Chézy	$C$
Strickler	$K$
Manning	$m$
Nikuradse	$k_S$

#### 3.3.1 Chézy

#### 3.3.3 Manning

$$C_f = \frac{2g}{C^2}$$

$$C_f = \frac{2gm^2}{H^{1/3}}$$

#### 3.3.2 Strickler

#### 3.3.4 Nikuradse

$$C_f = \frac{2g}{K^2 H^{1/3}}$$

$$C_f = \frac{2\kappa^2}{\left[ \log \left( \frac{30}{e^1} \frac{H}{k_S} \right) \right]^2}$$

### 3.4 Bed shear stress, Rouse number and diameter

$$TAU = \tau = \rho US^2$$

$$R_0 = \frac{w_s}{\kappa US}$$

$$FROTP = MU DMAX$$

$$DMAX = \begin{cases} 1.4593 \times \tau^{0.979} & \text{if } \tau > 3.4 \\ 1.2912 \times \tau^2 + 1.3572\tau - 0.1154 & \text{if } 0.1 < \tau \leq 0.34 \\ 0.9055 \times \tau^{1.3178} & \text{if } \tau \leq 0.1 \end{cases}$$

## 4 Equations in 3D

$$M = \sqrt{U^2 + V^2 + W^2}$$

$$NU = \sqrt{NUX^2 + NUY^2 + NUZ^2}$$